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ELEMENTS OF FINANCE

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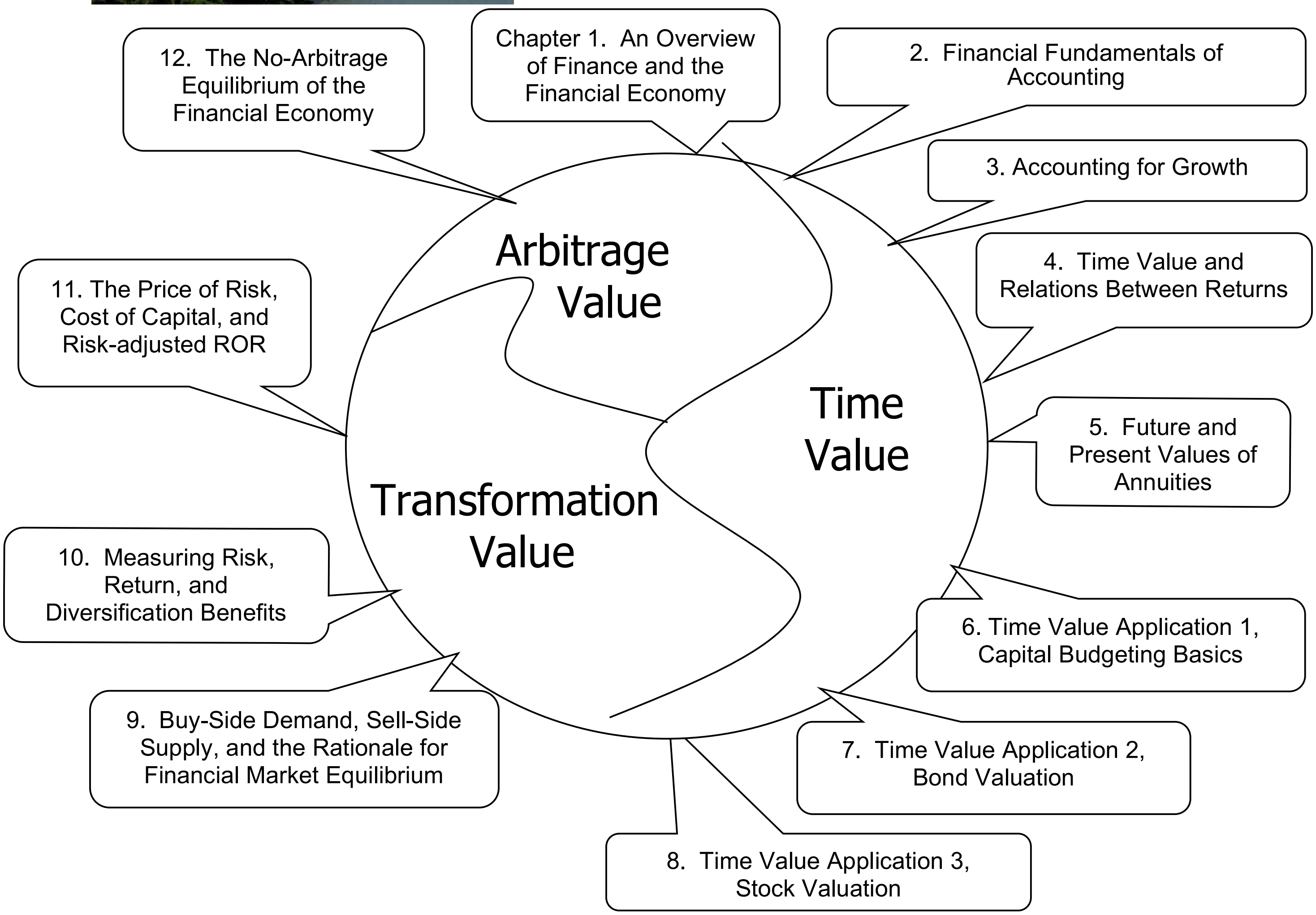

Figure 1 *Elements of Finance* chapters relevant to the three sources of economic profit.

 View this book at http://elementsOfFinance.net/elements.pdf . Order a print version from: http://www.lulu.com/content/paperback-book/lessons-about-the-structure-of-finance/13542597

Jump here (p. 510?) to read how the Golden Gate bridge (see coverphoto) was made possible by exploiting elements of finance.
Jump here (p. 524?) to read Endnotes including the Author Note, Acknowledgement, and Referenced Readings.

Table of Contents

List of Tables

List of Figures

List of Streetbites and Snippets

List of Calculator Clues

List of Formulas

List of Rules and Definitions

Clue 2.1 Setting the calculator...49?

How the Golden Gate Bridge (see the coverphoto) relates to finance

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San Francisco's *Golden Gate Bridge* opened in 1937 and for decades was the world's longest suspension bridge. Two structural traits of this modern marvel connect the beauty of the bridge by the bay to financial science and practice.

The San Francisco – Oakland *Bay Bridge* opened in 1936 (see http://baybridgeinfo.org/history). The two adjoining bridges required raising from public markets huge record setting sums of financing. One bridge pier contains more concrete than New York's Empire State Building and is bigger than Egypt's largest pyramid (see *San Francisco, the Story of a City,* by John Bernard McGloin, Presidio Press, http://www.sfmuseum.net/hist9/mcgloin.html 1978). Putting these structures in-place was as great a financing as an engineering feat.

The second connection comes from the natural number *e*, 2.7182818284…. In May 1690 a challenge was formalized by Jacob Bernoulli, one of the elders from a famous family of 8 mathematicians that for more than a century contributed scientific discoveries used today in nearly every industry and science. Jacob, a professor of mathematics in Basel, Switzerland, was well informed about the properties of circles and complex shapes such as the catenary, the shape a gold chain takes when held loosely between your index fingers. Scientific challenges during this era were thrown like gauntlets across time and distance to be answered by pioneers like Napier, Newton, Kepler, and the most innovative of all, Leonhard Euler (see http://en.wikipedia.org/wiki/Leonhard_Euler and *Elements of Algebra* 1771).

Jacob's challenge to the scientific world was publish this solution: "And now let this problem be proposed: To find the curve assumed by a loose string hung freely from two fixed points." The shape of the catenary had been debated for a while (see http://mathworld.wolfram.com/Catenary.html). Galileo had previously offered an answer, the parabola, but was proven wrong in 1669. Finally, in 1691 three men submitted the one correct solution: Gottfried Leibnitz (he and Isaac Newton independently wrote the initial descriptions on the calculus of numbers), scientist Christian Huygens, and Jacob's brother Johann Bernoulli. The answer: $y = (e^{x}+e^{-x})/2$ where y measures vertical deflection of the string and x is the relative distance between the two fixed points and e is the number 2.72 . Accomplishments like the *Golden Gate Bridge* and much more depended on that fundamental formula.

Leonhard Euler's natural number, $e = 2.72$, is the most important man-made number. He discovered, described, and published *circa* 1750 that $2.72^{3.14\,i} = +1/-1$. That formula, the famous *Euler Identity,* is the bedrock on which engineering, astrophysics, finance and all measurement arts and sciences draw strength. Among the five numbers, the transcendental 3.14 is ratio of a circumference to diameter for a circle, a number beyond the making of humans. The ratio $+1$ divided by -1 is i^{0}/i^{2}, the flip with its flip-side that always is whole. The number i^{1} is like a child that plays boomerang tag between its parents i^{0} and i^{2}. The number i^{1} being neither positive nor negative is, like a soul, a prime number unique unto itself. This leaves 2.72, a number on which financial science arguably has a natural claim since $2.72 measures the wealth accumulating from $1 continuously compounded for one-year at an annual percentage rate of 100%.

CHAPTER 1: AN OVERVIEW OF FINANCE AND THE FINANCIAL ECONOMY

1. So Just What Is "Finance" Anyway?
   1.A. Finance in the corporate pyramid
   1.B. Finance sub-disciplines
   1.C. Finance certifications
2. The Company Cash Flow Cycle
   2.A. Stakeholders
   2.B. Financial markets
   2.C. Agency problems
      C1. Shareholder versus management
      C2. Creditor versus management/shareholder
      C3. Employee versus management/shareholder
      C4. Household versus government
   2.D. Wealth creation and the company goal
3. Clones of the Company Cash Flow Cycle
   3.A. Corporate business
   3.B. Noncorporate business
   3.C. Households as companies

1. So Just What Is "Finance" Anyway?

Finance is an unusually rich subject. Notice it is one of the few college courses for which the course name has two definitions, one as action verb and the other as noun.

DEFINITION 1.1 Usage of "finance" in common vocabulary

(verb): To *finance* means the act of borrowing money.
Example: How did you finance your car?

(noun): *Finance* is the study of wealth management.
Example: This book explains basic principles of finance.

(noun): *Financial science* is the specification of processes determining cash flow rates and wealth balances.
Example: Financial science studies equilibrating forces that when incentivized right might help sustain the wealth of households, companies, maybe even the planet.

Although many types of wealth exist, society and history seem to suggest that money is a very important type. *Finance*, accordingly, has a lot to do with money. Because everyone somewhat relies on money, everyone benefits from understanding basic principles of finance.

Finance is a relatively new science. The prestigious *Journal of Finance* began in 1942. In 1990 the Nobel Prize in Economic Science was awarded for the first time to discoveries taught primarily in finance classes. The prize was shared by Harry Markowitz from the City University of New York, William Sharpe of Stanford University, and Merton Miller at the University of Chicago. Again in 1996 a finance discovery by William Vickrey of Columbia University received the Nobel Prize in Economic Science. The discovery receiving the 1997 Nobel Prize by Myron Scholes (Stanford) and Robert Merton (Harvard) uses insights from heat transfer physics to model the valuation of financial stock options! The 2013 prizes recognized advancements in financial economics by Robert Shiller (Yale University), Eugene Fama (University of Chicago), and Lars Peter Hansen (University of Chicago).

The rigor of financial science assists a large financial industry with the theory for applications. The practical implications of most financial principles is diverse. Keeping track often requires strong familiarity with accounting fundamentals. In short, *finance* is a wonderful science because it uses the pragmatism of accounting to apply the rigor of economics to the study of wealth management. A rich subject indeed!

People in households, business, and government use financial knowledge everyday to make wealth management decisions. But exactly how, and who, uses finance? Glean insight from several perspectives.

1.A. Finance in the corporate pyramid

Figure 1.1 takes a perspective focusing on the "finance group" within the corporate pyramid. This particular hierarchy is for IBM at a particular moment decades ago. Though the names on the Board today are different the historical snapshot nonetheless is still relevant.

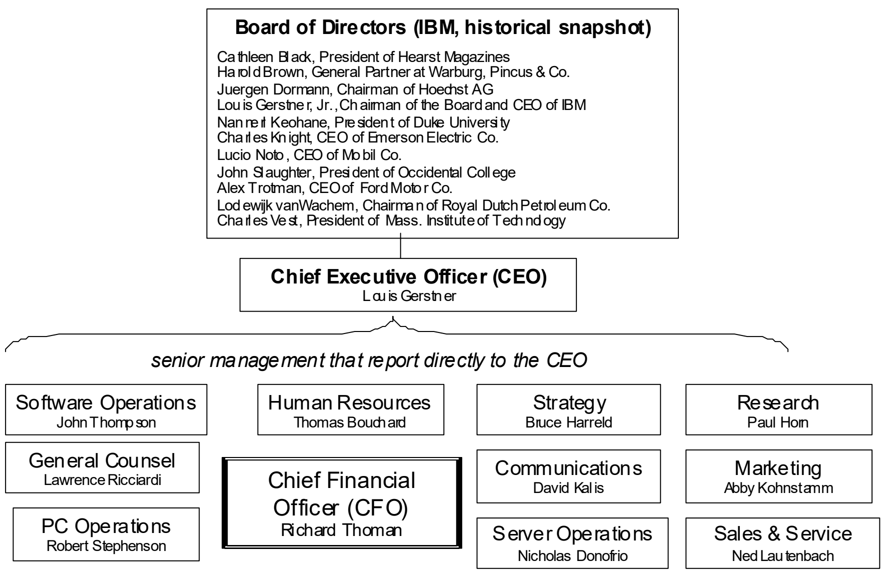


FIGURE 1.1 Finance in the corporate hierarchy

The finance group generally reports to the Chief Financial Officer ("CFO"). The CFO reports directly to the Chief Executive Officer ("CEO"). The figure shows eleven different groups at IBM report to the CEO. The finance group headed by the CFO is very important, but equally important is the Strategy group that ponders question of corporate mission, the Marketing group that creates a positive corporate image, the Sales & Service group that generates revenues, the General Counsel group that keeps the company compliant, etc. There is a lot more to business than simply finance, but finance is essential.

The CEO is the most senior employee. The CEO reports to the Board of Directors. The Board of Directors hires and fires the CEO. Persons on the Board of Directors typically are not employees of the company. Directors are individuals with careers unrelated to IBM's mission. The figure shows that on IBM's board is a president of a publishing empire, several university presidents, oil company presidents, etc.

Presumably, external Directors see the big picture and assess the sensibility of the company's efforts. The IBM Board of Directors in 2019 is 16 persons and shows more gender and ethnic diversity today than the figure 1.1 snapshot from decades ago.

Who hires and fires the Board of Directors? Shareholders, that's who! Shareholders of a corporation elect Directors from a slate of nominees. Typically one share of common stock casts one vote. The chain-of-command is thus: shareholders elect or oust the Board of Directors; the Board of Directors hires and fires the CEO, the CEO hires and fires senior management, senior management hire and fire middle management, and middle management hire (seldom fire, hopefully) college graduates that are just starting-out. When you buy common stock, you get more than hoped-for profits; you get control over management (don't get too excited, though, IBM has over half-billion shares outstanding and it's one vote per share!)

Figure 1.2 shows typical units within the finance group.

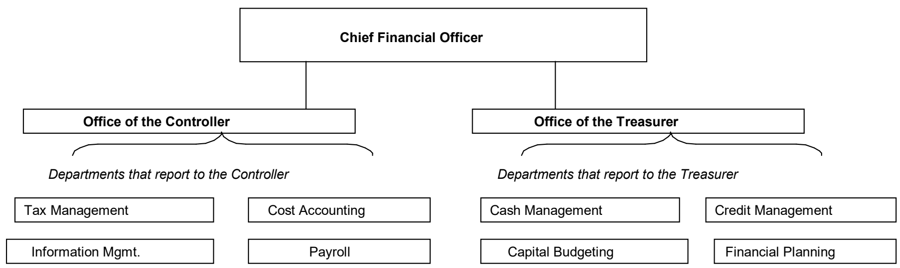


FIGURE 1.2 Typical finance group in the company

Generally speaking, the Controller administers accounting functions whereas the Treasurer administers finance functions. The Cash management department assures that checks don't bounce and that a prudent amount of cash is on-hand. Credit management pertains to customer loan policies, collections, and payments. Capital budgeting pertains to long-term decisions about investing or financing of plant, property, and equipment. Financial planning assesses current and future financial health given likely trends. Basic finance functions in the Office of the Treasurer generally monitor how the company's wealth is or should be allocated, where the wealth is coming from, and where the wealth is going. These essential business functions rely on employees that use financial science.

1.B. Finance sub-disciplines

Glean another perspective of finance by examining traditional sub-disciplines:

Corporate Finance — topics include working capital management, capital budgeting, obtaining financing, capital structure decisions, and dividend payout policies;

Investments — topics include company and security analysis, portfolio theory and management, futures and options;

Markets and Institutions — topics include banking, analysis of interest rates, and financial market microstructure;

Specialty Areas — topics include real estate, insurance, law and financial economics, personal financial planning, enterprise finance, risk management, etc.

Many universities offer one or more courses organized around these sub-disciplines. Until the mid-1990's, in fact, the accrediting agency "American Assembly of College Schools of Business" (AACSB) required that undergraduate business programs include two finance courses. Typically one course was "Corporate Finance" and the other was "Markets and Institutions." The AACSB rescinded the two-finance-course rule and as a result most undergraduate business students today study only one finance course. The curriculum for today's singular introductory course overviews all traditional sub-disciplines of finance. The maturation of financial science blurs lines separating traditional sub-disciplines and a continuous discipline of finance is emerging.

1.C. Finance certifications in industry and practice

One final perspective about finance comes from inspecting professional certifications. Perhaps most widespread is the Chartered Financial Analyst ("CFA"). The CFA designation for finance is analogous to the CPA designation that accountants earn (the "Certified Public Accountant"). Earning the CFA title requires taking a series of exams, as well as satisfying on-the-job experience requirements. Other designations often are seen following individual names on office doors or in professional advertisements. Each certification suggests that the individual satisfies the stringent criteria promulgated by the respective professional organization. Many of the more visible certifications appear in Table 1.1.

Certificate Title	Description	Contact
Chartered Financial Analyst (CFA)	The CFA is a recognized standard of competency for financial analysts in more than 70 nations worldwide	http://www.cfainstitute.org/programs/cfaprogram/Pages/index.aspx Association for Investment Management & Research
NASD Stock Broker's License	NASD brokers represent more than 5,500 securities firms with more than 82,000 branch offices across the U.S. Acquiring the license requires multiple applications and qualifications	http://www.finra.org/ National Association of Securities Dealers
Certified Financial Consultant (CFC)	Recognized by institutions and individuals as a sign of integrity and professional excellence. Many employers require the CFC designation when hiring or promoting	http://www.ifconsultants.org/ Institute of Financial Consultants
Certified in Financial Management (CFM)	The CFM is for students, practitioners, and academicians, that understand techniques defining the field of finance	www.fma.org Financial Management Association
Certified Financial Planner (CFP)	Holders of the CFP meet rigorous requirements in banking, estate, insurance, investment, and tax planning	http://www.cfp.net/ Certified Financial Planner Board of Standards

Chartered Financial Consultant (ChFC)	Granted to individuals completing a comprehensive 10-course practical program that includes economics, taxes, insurance, and investing	http://national.societyoffsp.org/ Society of Financial Services Professionals
Chartered Life Underwriter (CLU)	This is the undisputed professional credential for persons involved in the protection and preservation of financial wealth through life insurance	http://www.theamericancollege.edu/ads/clu The American College
Commodity Trading Advisor (CTA)	A registered adviser regarding the value of securities or of the advisability of investing in securities.	www.securitiesexam.com Securities Exam Preparation, Inc.
Financial Risk Management (FRM)	Earned by individuals who have been qualified to legally give clients financial planning service and advice	www.rims.org Risk and Insurance Management Society

TABLE 1.1 Professional Certifications in Finance

2. The Company Cash Flow Cycle

There are many types of wealth, just as there are many types of businesses, markets, or human wants. All wealth, however, possesses a common characteristic:

DEFINITION 1.2 Wealth (noun)
Wealth is any capital or asset that provides returns over time.

Typical examples include: (a) This IBM common stock represents a lot of her *wealth* ; or (b) This building represents a lot of the company's *wealth*. Clearly a building is a useful asset that provides services for many years, so a building is a type of wealth. Likewise, the IBM stock promises to return money in the future, so it too is a type of wealth. Of course, a building made from bricks and mortar is quite different than a security made from paper. Both assets, nonetheless, represent wealth because they promise future returns. Likewise, a pocketful of money is wealth.

Markets transfer ownership of wealth. There are as many different markets as there are asset types. Yet at the broadest level a market is either (a) a financial market, or (b) a real asset market. Financial markets trade exclusively paper claims and obligations. Paper claims and obligations represent contracts that promise either implicitly or explicitly to deliver returns through time. These pieces of paper are called "securities," and examples include stocks, bonds, and loan obligations. The paper itself does not possess intrinsic traits that endear it; you can't eat the paper or use it for anything useful. Instead, the paper represents wealth because it promises future returns (usually more paper!). Real asset markets trade goods and services. The market that trades buildings is a real asset market. Other examples of real asset markets include the markets for books, automobiles, steel, groceries, and haircuts or massages. Real goods provide economic utility. Real goods are real things!

Financial securities and real assets represent wealth because both provide returns over time. Some economists, notably Don Patinkin (1957) and Joe Stiglitz (1982), argue that financial markets lie like a veil on top of the economic community of real assets. They question the extent to which financial wealth leads to creation of real assets. This is a complex issue. Inspection of economies around the globe reveals that nations possessing the most real assets also have the most highly developed financial markets. Whether financial markets lead to sophistication of real asset markets or vice versa is like asking whether the chicken or the egg came first. It is a largely irrelevant question, too. Long-run financial market forces, just like the weather, tend towards reducing gradients. Examine the cash flow cycle in Figure 1.3 that simply illustrates how financial and real asset markets relate.

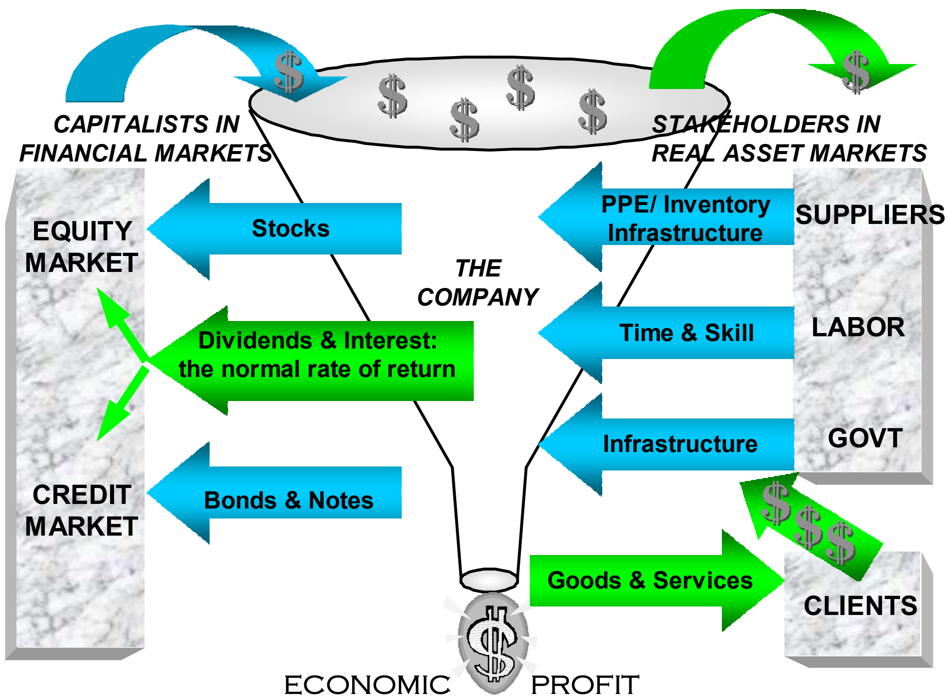


FIGURE 1.3 The Cash Flow Cycle

The “company” in figure 1.3 is any economic agent with a balance sheet and every household, enterprise, institution, nation, planet, person or collection has one. The company regardless of definition brings together resources from diverse sectors of the financial economy in order to produce goods and services. The company deals with capitalists in the financial markets and with stakeholders in the real asset markets. Positive economic profit represents worldly gain. When economic costs surpass economic income the net loss might eventuate with devastating real effects.

2.A. Stakeholders

A stakeholder is an economic entity in the real asset markets that exchanges goods and services with the company. There are four major classes of stakeholders: suppliers, labor, government, and clients. Suppliers provide the company with inventory, raw materials, and plant, property, and equipment. Those factors of production are real goods and services that the company needs to make its product. The company sends money to the suppliers. Labor includes management and employees. These individuals provide the labor services that the company needs to function. The company sends labor wages, salaries, and bonuses. The government is another major stakeholder with the company. The government provides highways and airports, they protect property and maintain civil obedience with police, firefighters, and military. The government also educates the workforce, and they maintain a regulatory environment that is politically responsive to the company. The company in return pays taxes to local, state, and federal governments. Clients, too, are stakeholders with the company. Notice in the cash flow diagram that the direction of money flow is different for the client than the other stakeholders. The company delivers goods and services to the client. The client sends money to the company.

Why does the company exist? Some people claim that the client is the sole reason for the company to exist. The argument has merit because the company stays financially healthy only as long as clients continue to buy the goods and services that the company produces.

Employees often claim that they sacrifice a lot for their company and, therefore, the company owes them a decent wage and reasonable job security. The claim that companies exist to provide people a way to make a living has merit, too.

At times in history, government has claimed that some particular company is so vital to the national interest that the government nationalizes the company. That is, the government forces the company to work toward a specific public objective. The claim that companies exist to promote national welfare has merit, too.

Between the company and each stakeholder there is a relationship. For example, a company may hire and train labor in expectation that labor will grow to become loyal and productive workers. Or a supplier may expect to make future sales to the company as long as the cost and quality of its supplies is best. Or a government may invest in infrastructure and expect the company to employ the populace. Stakeholders and the company often make decisions with the expectation of continuing a relationship. When a stakeholder relationship unexpectedly ends, then sometimes a stakeholder, the company, or both, seem to lose something valuable. So to whom does the company owe its highest allegiance? That is, what is the goal of the company? Before answering this question, let's examine the relation between the company and financial markets.

2.B. Financial markets

Companies receive money by selling securities to capitalists in financial markets. Financial securities do not provide real goods and services. Securities, just like money, are paper. More than likely, however, securities represent an ownership claim on assets or goods and services.

There are many types of financial markets, and there are many schemes for categorizing them. Table 1.2 summarizes three common schemes. One scheme is *primary* versus *secondary* financial market. In the "primary financial market" companies receive money by selling or issuing securities to capitalists. Sometimes capitalists re-sell securities to other investors in "secondary financial markets" and, in turn, the security may pass from hand-to-hand many times. IBM stock exists because once, and only once, the company issued that particular stock in a primary market transaction. Every subsequent trade between two different financial investors is a secondary market transaction. Most exchanges of marketable securities occur in the secondary market because one investor simply sells to another without direct involvement of the company.

A second scheme for categorizing financial markets is by the length of the financial contract's time horizon: *money market* versus *capital market*. The "money market" includes financing that is repayable within one year. Examples of money market transactions include short-term bank loans, overnight repurchase agreements, installment loans, and trade credit from suppliers. When a company obtains a short-term loan (or any other credit agreement) they sign a legal contract stipulating terms of repayment. The loan contract is a financial security. Sometimes the loans can be re-sold to other investors in a secondary market transaction, sometimes they can't. As the arrows in the cash flow cycle show, the company receives money from the financial market - the financial market receives a security from the company. Money market securities have a relatively short life because the financing is totally repaid within one year. "Capital markets" are at the other extreme of the time horizon because they represent long-term financing arrangements. The prominent types of capital market securities include stocks, bonds, and mortgages. Perhaps some bonds may stipulate repayment, say, in five or ten years. Stocks, in principle, possess uncertain maybe long life.

distinguishing criterion	**category 1 & description**	**category 2 & description**
new vs. seasoned security	*primary market,* stocks & bonds issued by company to investor	*secondary market,* stocks & bonds sold by one investor to another investor
length of financial contract	*money market,* financing repayable within one year	*capital market,* financing repayable in more than one year
type of repayment promise	*credit market,* trade credit, notes, and bonds that stipulate specific payments and/or interest	*equity market,* stocks that do not specify repayment but instead represent a claim on residual cash flows

TABLE 1.2 Common schemes for categorizing financial markets

A third scheme for categorizing financial markets is by type of repayment promise: *credit markets* versus *equity markets*. Credit markets include all short-term financing arrangements available in the money market. Credit markets also include all long-term debt arrangements such as bonds and mortgages. A common characteristic of credit market financing is a promise by the company to repay to the creditor all principal plus interest. Typically credit market contracts specify exact repayment terms and conditions. If the company encounters financial difficulties and is unable to satisfy the promised repayment schedule then bankruptcy may occur. Conversely, if the company strikes it rich there is no upward adjustment to the scheduled repayment. Credit markets do not receive the windfall gains that a company may earn – the repayment schedule is fixed.

Windfall gains and losses earned by the company flow to equity. Equity markets are sometimes called *stock markets*. Companies issue stocks to capitalists in exchange for money. The company does not make a legally binding promise about repaying stockholders. Instead, the motivation for stock investing is to have a controlling interest in the corporation (usually 1 vote per share of common stock), or because the investor anticipates financial gains such as dividends or share price appreciation. Stockholders may own the stocks but eventually they may sell the stocks to other investors. The company has no legal obligation to pay dividends to shareholders. Because shareholders control the Board of Directors and therefore can fire top management, however, the company likes to treat shareholders as fairly as possible.

The income statement, as the next chapter explains, shows how a company's revenues pay for the various factor costs of production. The profit, that is net income, either is paid-out as dividends to shareholders or else is plowed-back into the company to support growth. Company growth helps push-up the stock price. That benefits shareholders. Shareholders are *residual claimants* on company cash flows because they get that which is leftover after all costs have been paid. Shareholders sometimes are beneficiaries of unexpected windfall gains when the company strikes it rich, but they bear the burden of windfall losses when things unexpectedly go sour. Windfalls accrue to equity!

2.C. Agency problems

An agency problem potentially exists when a source of financing delegates decision-making authority for using the funds. The source that owns the funds is called the *principal*, and the decision-maker controlling the funds is called the *agent*. A principal-agent relationship exists when the owner of wealth is different from the controller of wealth.

An agency problem exists when objectives of the principal and agent are different. The resulting misalignment of interests may result in sub-optimal company performance. The direct and indirect decline in wealth resulting from a principal-agent problem is called an *agency cost.* The company expends substantial resources to minimize agency costs. There are four important principal-agent relationships in the financial economy.

C1. Shareholder relative to management

Shareholders supply, as the next chapter explains, a source of financing called *Stockholders equity*. Stockholders lend money to the company and management decides how to use the money. Managerial decisions generally advance the interests of shareholders because, after all, shareholders can pressure the Board of Directors to fire management. Yet managers naturally pursue their own self-interests. Sometimes, for example, management may buy a company jet or spruce up the office unnecessarily. Determining the proper amount of perquisite consumption by managers is a difficult but largely managerial decision. Perhaps managers over-invest in assets, product lines, or empire building, simply because managerial salaries tend to correlate directly with company size. Sometimes managers in pursuit of self-interest may make decisions that reduce the amount of residual wealth available to shareholders. That is an agency problem!

Several *control mechanisms* help align management and shareholder interests.

1. Shareholders can lobby to fire management.
2. Managers can be given clever incentive compensation contracts.
3. Shareholders can closely monitor and/or restrict managerial decisions.
4. Languishing stock prices can increase the threat of take-over.
5. Managers wish to maintain pristine reputations for the next job.

Operation of the first control mechanism is clear. Management cannot ignore shareholder interests too much lest they be terminated.

The second control mechanism links compensation with stock price performance. In today's world a significant fraction of management compensation depends on movement in the company stock price. These clever contracts ring similar to professional athlete contracts wherein, for example, there may be bonuses for making the all-star game or winning the conference title, etc. Managers earn big bonuses when during their tenure the company stock price rises. So managers have an incentive to make decisions maximizing the stock price. Clever compensation contracts align shareholder and managerial interests because both parties benefit from stock price increases.

Compensation contracts unexpectedly introduce agency costs when the company stock price rises for reasons unrelated to managerial actions. Strong bull markets tend to raise all share prices and, like a boat with the tide, a particular manager may be the beneficiary of an unanticipated windfall gain. The windfall would flow to equity if not for the compensation package. Instead, however, management gets more than its fair share of the windfall. Certainly hiring good managers is expensive. But when top management earns, for example, $60 million from executive stock options, then surely shareholders are paying more than management's reservation wage. Management would have performed the same services for less. The excess compensation, that is the amount above the manager's reservation wage, represents a drain from shareholder wealth and is an indirect agency cost.

The third control mechanism consists of financial audits and constraints on managerial decision-making. Audits ascertain that shareholders have full information about the company's financial actions. When management knows shareholders are watching, management is more responsive to shareholder interests. Constraints on managerial decision-making take many forms. There may be a requirement, for example, that major strategic decisions require presentation and approval at the annual shareholder meeting. This control mechanism introduces direct and indirect agency costs.

The fourth control mechanism is not written in any contract but instead is a naturally occurring market mechanism. When management does a bad job that results in a low share price the company assets become relatively cheap to acquire. Outside entrepreneurs may sense an opportunity to gain control of the company assets by purchasing the relatively cheap stock. Attainment of majority stock ownership allows the takeover group to oust extant management. The new shareholders install new management and set a new direction for the company. This control mechanism does not introduce obvious agency costs. Instead, managers have a natural incentive to maximize stock price in order to avoid being a takeover target.

The fifth control mechanism recognizes that managers have careers that evolve over time. Maintenance of reputation and dignity are important incentives. Being a good manager with one company naturally opens doors for advancement with other companies. Shareholders surely will not hire an executive who screwed shareholders at his or her previous job.

C2. Creditor relative to the management/shareholder collection

Creditors are a financing source for the company. Management decides how to use the money. The creditor is a principal, management is an agent, and there is a problem due to misalignment of interests. Because management is directly responsive to shareholders, however, we also may view this as an agency problem between creditors and shareholders. Several control mechanisms help align creditor and management/shareholder interests.

1. threat of bankruptcy induces fiduciary responsibility
2. restrictive covenants stipulate precise uses for creditor financing
3. managers wish to maintain a good reputation to maybe borrow again.

Operation of the first control mechanism is clear. If management misuses credit then the creditor can force bankruptcy and everybody loses.

Operation of the second control mechanism is less obvious. Credit agreements vary by the degree to which they restrict the use of borrowed funds. At one extreme is the line-of-credit agreement that basically allows the company to borrow money for any purpose whatsoever. At the other extreme is the mortgage bond that lends money for purchase of a specific property, perhaps ties the repayment schedule to revenues the company realizes from the property, and further disallows additional borrowing against the property. The essence of this principal-agent problem is that the creditor lends money to managers at a fixed interest rate. The manager, however, is first-most responsive to shareholders. By pursuing high-risk high-return investments the manager potentially obtains windfalls that flow through to shareholders (and perhaps his/her own bonus, too). Creditors do not want managers to invest low-cost financing in high-risk investments. Sometimes creditors may impose covenants restricting the uses of the funds.

The third control mechanism recognizes the existence of a long-term relationship between company and creditor that transcends a single credit arrangement. Irresponsible company behavior by the manager may signal to the creditor irreconcilable differences. Companies, just like households, need financing and burning a bridge with a creditor diminishes future financing opportunities.

C3. Employee relative to the management/shareholder collection

Balance sheets show, as the next chapter explains, that employees are a significant financing source for many companies. Employees usually do not give direct loans to the company. Instead, employees provide financing by deferring compensation. Under guidelines for traditional pension plans, for example, an employee may provide labor services to the company that are worth $60,000 but accept immediate compensation for only $50,000. The company retains the $10,000 of deferred compensation and promises to pay the employee a pension in the remote future. Employees for many companies are an important financing source and own the wealth, but manager/shareholders control the wealth. This is an agency problem!

Several control mechanisms help align employee and management/shareholder interests.

1. threat of litigation induces fiduciary responsibility
2. assignment of pension contributions to third parties

Dozens of court cases about pension assets dot the legal landscape. When the manager/shareholder reneges on promises it made to employees about pension benefits, employees often sue. The threat of litigation helps align the interests of employees and manager/shareholders.

Many companies are disbanding traditional pension plans and instead offer a "defined contribution plan." With this type of plan the employee is not a financing source for the company because all deferred compensation is typically transferred to a third party. Ownership of the funds clearly belongs to the employee, and fund management is not by company managers. Instead, the company usually hires an independent management company such as Scudder Management Co., or Merrill Lynch, etc. The pension management company communicates with employees to make sure employee interests are pursued.

C4. Household relative to government

Households own a lot of wealth. They own private goods as well as financial assets that represent claims on business wealth. Much of this wealth is managed by the government. The wealth that the government owns is a public good for which households are a primary financing source. The government manages the wealth of the nation in order to satisfy the wants and needs of households in the financial economy. There exists an agency problem because the households are principals that ultimately have residual claims on the wealth that government agents manage.

Government decisions generally advance the interests of the households because, after all, households either elect or tolerate the government or they revolt when the agency problem becomes intolerable. Yet governments often have difficulty managing global wealth. Difficulties arise for well-known reasons and stories of severe agency costs induced by government mismanagement dot the historical landscape. One difficulty is that households have different objectives and endowments. Another difficulty is that we lack complete scientific knowledge about how the real economy interacts with the financial economy. Governments manage the global wealth claimed by the populations that share the planet. That is an agency problem that we've been working to solve for millennia!

2.D. Wealth creation and the company goal

The cash flow cycle in figure 1.3 shows the many wealth transfers that exist between the company, stakeholders in real asset markets, and capitalists in financial markets. The next chapter explains that wealth transfers represent a flow per time period, and later chapters explain the crucial relation between time and the valuation process. The occurrence in the real world every period of all the flows in the diagram creates a very complicated mosaic of wealth transfers between capitalists, stakeholders and company. Grab hold of the financial economy by focusing in figure 1.3 on economic profit (or loss), the droplet whether golden honey gains, charred earth losses, or money.

DEFINITION 1.3 Economic profit
Periodic economic profit equals company economic income minus economic cost.

Participants in real asset and financial markets compete for compensation from the company. Suppliers want to charge the highest possible price for plant, property, and equipment, but the company wants to pay the least possible purchase price (all else equal). Labor wants the highest possible wages, but the company pays only what the market will bear. Governments collect taxes to build infrastructure in response to political pressures, and companies hire lobbyists to inform politicians about constraints on business. Capitalists look for investments providing the highest risk-adjusted real rate of return, yet companies search for the best available financing terms. And of course, clients and customers always are on the lookout for the best deal and, in competitive markets, pay a price exactly equal to their perceived value of the good or service. Economic theory suggests *in the long-run you get what you pay for and equilibrium economic profits equal zero!*

The company adds value by transforming real factors of production into a good or service that clients demand. The sales transaction between company and client consummates wealth creation. The real value of the sales revenue exceeds the real value of the inputs from stakeholders and capitalists by the amount of real wealth created. The wealth created equals the transformation value from production. The company subsequently distributes the new wealth to stakeholders and capitalists. This simplistic scenario in which economic profit equals zero is not as bad as it sounds – stakeholders and capitalists receive fair prices, wages, and rates of return and wealth accumulates through time.

Economic theory establishes that in realistic and dynamic situations, economic profits (and losses) exist because of population and demand growth, technological innovation, market maturation, product development, and myriad other phenomena such as fads, changing preferences, or plain old luck. Company management, too, potentially creates economic profit through superior foresight and decision-making.

Consider the distribution of economic profit in the cash flow cycle. One fact is clear: residual wealth flows to equity. If no stakeholders or creditors lobby for the economic profit then surely it accrues to equity. But when stakeholders and creditors see the wealth droplet forming, mouths move toward the funnel. Competition and market structure determines who sips from the font of economic profit.

For example, perhaps suppliers make technological breakthroughs that reduce economic costs of production. Surely in the long run, and in the absence of barriers, the company pays a lower price for supplies and clients pay a lower price for goods and services. In the short-run, however, suppliers and company management negotiate the distribution of economic profit. Suppliers want to pocket the profit by charging the same price for supplies as before. Management, under the shadow of an agency problem, tries to pay lower costs but charge the same price for the final product and pass the economic profit on to capitalist shareholders. So who gets it? Well, it depends on who has the strongest negotiating position. Capitalists and stakeholders most likely share economic profit in our modern financial economy.

Other examples abound. Consider a company venturing into a new cheaper production process for which there is a scarcity of labor with exactly the right skill package. Who gets the economic profit? Skilled and highly sought labor tries to get it, or possibly even managers lobby for bonuses claiming superior foresight for product development. Clearly in the absence of action the residual flows to equity. But don't count on inaction by stakeholders. They are just as smart and greedy as capitalists!

The creation of wealth in a competitive economy helps the world, irrespective of who gets it (as long as none is worse off in which case the situation has tradeoffs). This leads to a statement of a proper and ethical objective for wealth managers:

RULE 1.1 The objective for company management
Management should maximize wealth creation by the company irrespective of how economic profit distributes between capitalists, stakeholders, and factors of production subject to laws of a landscape shaped by principal-agent and contractual relations.

The management job is tough. A good manager faces the difficult challenge of ascertaining fair and competitive prices and wages for stakeholders. Sometimes those payments include economic profit, but not always. Sometimes equity receives all economic profit, sometimes economic profit is zero. Yet as long as managers transfer wealth to creditors, labor, and suppliers at competitive market prices, and likewise sell final goods and services to clients at competitive market prices, the company maximizes wealth creation for capitalists and stakeholders alike.

The change through time of the equity stock price is a useful indicator of managerial effectiveness. A later chapter explains that stock prices depend on market assessments of long-run company profitability. A rising stock price indicates (all else equal) a rising forecast of residual cash flows to equity. A rising stock price (all else equal) generally confirms that management pursues wealth-creating projects. In cases when stakeholders with inelastic competitive positions capture a share of economic profits, the stock price still rises because the fair rate of return to capitalists includes a real increase in wealth. The stock price would rise even faster, of course, if the manager could negotiate lesser payments for stakeholders and divert all economic profit toward equity. That cannot always be accomplished because stakeholders sometimes control scarce factors of production. Still, employing scarce factors of production and getting shareholders half of something is better than getting them all of nothing. In the rare case where a stakeholder holds all the cards and receives all economic profit, management should pursue a policy that creates incremental wealth for the stakeholder, even though none flows to the shareholder. The fair rate of return to shareholders excludes economic profit yet still provides a wealth-increasing risk-adjusted rate of return. The world becomes a better place when the company pursues policies that maximize wealth creation, irrespective of the distribution of economic profit, as long as all capitalist and stakeholder exchanges with the company occur at competitive prices and rates.

A falling stock price (all else equal) is an indication that management either is overpaying stakeholders or is selling the final good or service for less than the economic cost of production. Once again, running a company is a tough balancing act and the stock price is an indicator variable for managerial effectiveness.

3. Clones of the Company Cash Flow Cycle

The cash flow cycle in figure 1.3 depicts economic activity applicable for many different scenarios. Minor interpretive changes allow the cash flow cycle to depict scenarios in which the company is (1) a corporate business, (2) a noncorporate business, and (3) a household or individual. Principles of financial science pertain to all these economic entities with surprising uniformity. It behooves us to discuss the listed alternative scenarios.

3.A. Corporate business

The preceding sections identify the company as a corporation. Understanding basic characteristics of the corporate sector is important and generally eye opening. The corporate sector in the U.S.A. contains business names that almost every citizen recognizes: General Electric, AT&T, Microsoft, Disney, etc. Table 1.3 presents information about several of these American corporate icons.

Corporation Name	Ticker symbol	Number of full-time employees	Total assets	Annual sales	Annual net income	Market cap
Apple	AAPL	84,400	207,000	170,910	37,037	428,700
Walt Disney Company	DIS	175,000	81,241	45,041	6,136	116,082
Exxon Mobil Corporation	XOM	75,000	346,808	390,247	32,580	438,702
Ford Motor Company	F	181,000	202,026	146,917	7,155	60,853
General Electric Company	GE	307,000	656,560	142,937	13,057	282,006
General Motors Corp.	GM	219,000	166,344	155,427	5,346	61,305
IBM	IBM	431,212	126,223	99,751	16,483	197,772
Pfizer Inc.	PFE	77,700	172,101	51,452	22,003	196,001
AT&T Corporation	T	243,360	277,787	128,752	18,249	183,757
Wal-Mart Stores, Inc.	WMT	2,200,000	204,751	474,259	16,022	241,440
Microsoft Corporation	MSFT	99,000	142,431	77,849	21,863	287,691

TABLE 1.3 American corporate icons.
All dollars in millions, 2013, compiled by author.

Every corporation has common stock. The common stocks that trade on a stock exchange have a company identifier called the “ticker symbol”. The table shows, for example, that the ticker for Exxon Mobil is XOM, and for Ford it simply is F. Armed with a ticker symbol one can easily find on the internet a recent stock price and other news or statistics for exchange-traded companies.

The range in number of full-time employees is huge. Over 2.2 million walk the floors of Wal-Mart. At Exxon Mobil, which has roughly three-quarters as much annual sales, the number of employees is about one-thirtieth as large. Exxon Mobil has significantly more total assets than Wal-Mart, too. Clearly, sales per employee and sales per dollar of asset is much higher at Exxon Mobil than Wal-Mart. But the energy and retail industries differ so significantly that this comparison is simpy amusing trivia.

The rightmost column of table 1.3 lists company market capitalization. Market “cap,” as the next chapter explains, measures the company’s value in the stock market. Exxon Mobil in 2013 had the largest stock market value of any company in the U.S.A., $438 billion. Next highest was Apple at $428 billion. In 2014 (not in table) the market cap of Apple grew bigger than Exxon Mobil . Notice that net incomes in year 2013 for IBM and Wal-Mart were very similar ($16.5 billion versus $17.0 billion; the next chapter explains that net income is the company profit from the income statement), but the number of employees and annual sales are far apart. Compare Pfizer with IBM. The two have roughly the same stock market value but comparison of employees, assets, and sales shows differences of incredible magnitude.

The corporate icons in table 1.3 are not typical corporations. Public stock exchanges in the U.S.A. trade stocks for more than 10,000 different corporations. Table 1.4 shows quartile breakpoints for a large sample of these listed companies circa year-end 2013.

% of exchange-traded nonfinancial corporations with less than number at right	Number of full-time employees	Annual sales	Total assets	Market cap
100%	2,200,000	$ 474,259	$ 346,808	$ 438,702
75%	5,016	1,802	2,337	2,419
50%	875	338	430	484
25%	120	38	76	72

TABLE 1.4 Quartile breakpoints on key variables for 3,412 public companies.
All dollars in millions, 2013, excludes financial companies (SIC 60) and companies with incomplete data, full sample size is over 9,000 companies, compiled by author.

While true that Wal-Mart has over 2 million employees, 75% of exchange-traded nonfinancial companies have less than 5,016 employees. One-fourth have fewer than 120 full-time employees. The American corporate icons are huge compared to average.

Similar tendencies apply to the other variables in table 1.4. The maximum annual sales of any corporation in the U.S.A. is $474,259 million (WMT) yet over 50% of all exchange-traded nonfinancial companies have annual sales less than $338 million. Total assets and market cap are skewed, too. Comparing the biggest company to the median is like comparing Jane and John Doe's wealth to Microsoft's Bill and Melinda Gates. Yet the same financial principles apply to all.

The biggest companies may be atypical, and there may be few of them, but they command huge influence in the financial and real asset markets. Table 1.5 lists the sum for all companies in the sample within each quartile.

Table entry is sum for all nonfinancial corporations in sample quartile	Number of full-time employees	Annual sales	Total assets	Market cap
biggest quartile	28,906,971 (92.4%)	$ 11,322,787 (92.8%)	$ 14,250,088 (92.5%)	$ 14,934,398 (92.5%)
upper middle	1,992,035 (6.4%)	740,916 (6.1%)	994,972 (6.1%)	995,430 (6.2%)
lower middle	334,989 (1.1%)	129,092 (1.1%)	181,359 (1.2%)	196,268 (1.2%)
smallest quartile	34,940 (0.1%)	10,124 (0.1%)	21,703 (0.1%)	22,444 (0.1%)
TOTAL	31,268,935	$ 12,202,919	$ 15,398,122	$ 16,149,540

TABLE 1.5 Aggregate distribution of resources for 3,412 public companies
All dollars in millions, 2013, parentheses list percent of sample total, quartiles rebalanced by sorting on each header variable. See table 1.4 for a description of the data.

There are 31.3 million full-time employees for this sample of exchange-traded nonfinancial corporations. Rank all companies by number of employees and count the total employees in the biggest 25 percent. The table shows that the biggest 25% employ 92.4 percent of all employees in the company sample. Combine the smallest two quartiles to see a more startling fact: 50 percent of companies in the sample employ only 1.2 percent of all employees working for exchange-traded nonfinancial corporations.

Glean insight on the size of the sample relative to the entire US economy.

According to the Bureau of Census there are 143.9 million full-time employees in the USA in 2013 working in business forms of organization. The sample of 31.3 million represents about 22% of total US employment. The upshot is that our sample of exchange-traded nonfinancial corporations primarily includes bigger than average companies. Within our sample, already tilted toward large NFC, employment tends to tilt toward the largest among the large.

The tendency is true for all variables in the table. Consider this: almost 92½ percent of NFC stock market wealth in the U.S.A. is due to only 25 percent of listed companies; 75 percent of corporations represent 7½ percent of stock market wealth. (The clustering is true even with inclusion of financial corporations.)

Perhaps small corporations represent a relatively small proportion of corporate employment, sales, total assets, and stock market value, but do not trivialize the small company. Any individual is a big-time success who can establish a corporation that grows to employ 120 people, generates $38 million in annual sales, has $76 million of total assets, and has a value in the stock market of $72 million. You can bet, too, that this successful entrepreneur owns a big share of that $72 million and, to 120 different families and countless stakeholders, he or she is a very important person.

About ten thousand different corporate stocks trade on stock exchanges in the U.S.A. Each active corporation in the nation files a tax return. The Internal Revenue Service received about 6.7 million corporate tax returns in 2010. This suggests that exchange-traded corporations represent less than one percent of all corporations. The apparent motivation for organizing a company as a corporation is <u>not</u> so that its stocks can trade on an exchange. Rather, the corporate form of business organization offers other advantages. These include:

1. limited liability for the owner(s)
2. easy transferability and sharing of ownership
3. potentially infinite life (at least it might surpass the owner's lifespan)
4. easier access to the financial markets

Several significant disadvantages include:

5. legal obligations for corporations are sometimes quite complex
6. corporate income is subject to "double taxation"

Shareholders are the corporation owners. Limited liability for the owners implies that the maximum shareholder wealth at risk equals the value of the shares. The shareholder of a corporation protects personal wealth (house, car, savings, etc.) from litigation against the company. Someone suing a corporation may receive, in an extreme case, a judgment bankrupting the company, and perhaps sometimes an unscrupulous manager may be thrown in jail because of actions on-the-job. But the personal wealth of shareholders is untouchable, as though shielded behind a firewall.

Advantages 2 and 3 occur because every corporation has common stock and whoever owns the common stocks possesses control over management and has claims on residual cash flows. The owner/manager of a small company cannot sell company shares on stock exchanges without satisfying many government and exchange rules. But the owner/manager may privately sell (or give) shares to key employees, family members, or capitalists. Selling or sharing common stock implies sharing of corporate control and profits. The transferability of shares gives the company a lifespan that potentially is infinite.

Advantage 4, access to financial markets, largely occurs because advantages 1 – 3 encourage purchase of shares by outside investors. Furthermore, government regulations place stringent requirements on corporate financial reporting. These information disclosures, even though they impose a burden on corporations because preparing reports takes time and money, increase the company's attraction to capitalists.

Tax policies for corporate income are interesting and controversial. Imagine that a man-on-the street generates sales and profits from goods in his shopping cart. The personal tax on the profit is the same as if the income for the man were from wages instead of business profit. Suppose instead that the man-on-the street is organized as a corporation. The man pays corporate tax on the profit, distributes the profit to himself since he is the owner, and then as a capitalist he pays personal tax on the distribution. This is “double-taxation.” Both the corporation and capitalist pay taxes on the same income stream.

Economists persuasively argue that double-taxation biases the allocation of capital and makes the economy less wealthy. Politicians, conversely, argue for popular votes by pointing fingers at mighty corporations that pay few taxes. Politicians generally ignore the claim by economists that if double-taxation were abolished, corporations would either (a) pass along the tax savings to capitalists where it would incur personal taxes, or (b) pass along the tax savings to stakeholders in the form of higher wages or purchases. The U.S.A. is the only major economy on the globe that has not abolished double-taxation of corporate income.

3.B. Noncorporate business

Companies often operate in an organization that is not a corporation. There are two primary forms of noncorporate business: sole proprietorship and partnership. Table 1.6 provides insight by listing number of tax returns received by the Internal Revenue Service in 2010.

	Approximate number of tax returns received by the IRS from the economic entity at left
nonfarm sole proprietorship	29.7 million
partnership	3.4 million
corporation (form 1120)	3.3 million
S-corporation (form 1120S)	4.4 million
individuals	141.5 million

TABLE 1.6 Number of tax returns by organizational form in the U.S.A.
2010, compiled by author.

The sole proprietorship is, by number, the most prevalent type of business organization. Any individual may operate a business as a sole proprietorship so long as the company is compliant with local and state regulations. The federal government does not require that sole proprietors obtain I.R.S. permission to operate. Sole proprietors file individual income tax returns (form 1040) and attach a Schedule C summarizing business income and expenses.

Almost half of all business receipts for nonfarm sole proprietorships lie within three industry groups. Retail trade is the largest industry for sole proprietors, garnering 19.1% of the total. Construction is second (15.9%), and Professional Services third (11.0%). Individuals in these industries pursue an entrepreneurial dream by setting-up shop as sole proprietor. Advantages of the sole proprietorship form of business organization include:

1. relatively easy start-up and record-keeping requirements
2. there is no double-taxation

Disadvantages generally are that the sole proprietor does not have the corporate advantages:

3. the sole proprietor has unlimited liability and his/her personal wealth is at risk
4. ownership is not easily transferred so company lifetime is somewhat limited
5. access to financial markets is linked to collateral provided by the proprietor

A hybrid form of sole proprietorship is the “S-corporation.” The Internal Revenue

Service allows companies meeting certain conditions, such as fewer than 75 shareholders, to organize as S-corporations. An S-corporation files form 1120S but does not pay any corporation taxes. Instead, all profits pass through to shareholders. The shareholders file a Schedule E with their individual tax form 1040. Profits earned by S-corporations incur personal taxes but avoid double-taxation.

Partnerships are the least common form of business organization. There are two types: general partnerships and limited partnerships. The general partnership has the same advantages and disadvantages as the sole proprietorship. The income and profits of the partnership are distributed among the partners, and each partner declares his/her share on an individual tax return. The partnership also files a tax return (form 1065), but double-taxation does not exist.

The limited partnership allows each partner to insulate personal wealth from litigation against the partnership. This attribute is similar to the limited liability enjoyed by corporate shareholders.

The cash flow cycle in figure 1.3 is a fair depiction for sole proprietorships and partnerships. The only difference occurs in definition of the equity market. These two forms of business organizations do not have shareholders as a financing source. The source of equity financing for these business forms is, respectively, the sole proprietor or the partners.

3.C. Households as companies

Many people realize that managing a household may be as complex as managing a company or institution. Households, like corporate and noncorporate companies, bring together resources from financial markets and stakeholders in order to create real goods and services. Households are the largest economic entity in the U.S.A. Financial forecasts of economic recession or expansion often begin by stating the importance of household financial behavior on aggregate spending, employment and production. Two-thirds of U.S. gross national product is the result of household spending for real goods and services. Table 1.7 compares corporate and household balances.

Total assets are nearly three times larger in the household sector ($92.7 trillion) than in the nonfinancial corporate sector ($34.7 trillion). The two largest types of assets for households include real estate and pension assets. Pension assets are financial assets, and households own a lot of financial assets. A lot of the financial assets that households own include equities issued by the nonfinancial corporate sector. People living in households are, after all, the shareholders that own corporations.

Perhaps you are surprised how much wealth households own compared to businesses. But drive around town and notice all the real goods and services that you see. Businesses undeniably have a lot of stuff: stores and service stations seemingly are everywhere. Keep driving, though, and begin to notice that people own so much more: houses, cars, plus tens of trillions of dollars in financial assets causing a feeding frenzy by institutions competing for management of household financial assets!

The wealthiest households, just like the biggest companies, are atypical and there are few of them. Yet they command huge influence in politics and financial and real asset markets. Recent statistics from the Internal Revenue Service report that the top 1 percent of taxpayers now furnishes more than one-third of income tax receipts. The top 50 percent pay 96 percent of revenues that the government receives in personal income taxes.

The company cash flow cycle in figure 1.3 depicts households, too. The funnel represents the household acting as a company. Managing households requires many of the same functional skills as managing companies. Households depend on credit markets as a financing source for buying homes, cars, and college education. Households, like noncorporate companies, do not have shareholders. Still, the household inhabitants are a source of equity financing, and households accumulate net worth. Net worth equals total assets minus total liabilities. Household net worth of $78.9

trillion is more than four times larger than corporate net worth of $19.1 trillion.

Nonfarm nonfinancial corporations		**Household and nonprofit economic entities**	
total assets	**$34,708**	***total assets***	**$92,660**
real estate	10,217	real estate	22,300
trade receivables	2,432	durable goods	4,942
equipment & inventories	6,409	currency & deposits	9,699
Financial & other assets	15,650	corporate equities	12,457
		equity in noncorporate business	8,971
		pension fund reserves	19,890
		other financial assets	6,675
total liabilities	**$15,591**	***total liabilities***	**$13,794**
credit market securities	7,121	credit market instruments	13,171
trade payables	1,951	other liabilities	623
other liabilities	2,527		
net worth	**$19,117**	***net worth***	**$78,866**
market value of equity	**$20,800**	***market value of equity***	**not traded**

TABLE 1.7 Balance sheets for nonfinancial corporations and for households.
Dollars in billions, year-end 2013, all entries at market value or replacement cost, household accounts include nonprofits. Source: "Flow of Funds Accounts of the United States", Board of Governors of the Federal Reserve System, tables B100 and B102.

Households also develop stakeholder relationships in real asset markets. Households go to suppliers such as grocery stores, clothiers, car dealers, etc. Households hire labor such as carpenters, carpet cleaners, baby sitters, etc. Certainly households pay taxes; table 1.6 shows that households are filing over 141 million tax returns in year 2010.

Clients of households are of two types. First, employers are clients because the household sells its labor services in exchange for wages (revenue). Second, household inhabitants are clients to themselves. Individuals and/or family units realize benefits of the goods and services that households create. And the objective of the household is to maximize the wealth that it creates. That is, the household maximizes the utility of the goods and services created from transformation of capitalist and stakeholder inputs.

Several important components of national wealth are not in table 1.7. Most significant is government wealth. The government owns schools, fire stations, parks, aircraft carriers, and on and on. It almost is incomprehensible to consider how much wealth the local, state, and federal governments own. These institutions carry-forward balance sheets that capitalize the wealth from countless millinea of human industry, discipline, and cumulative learning. In almost every way, the citizenry are sources of equity, shareholders and owners of government wealth subject to incredible agency cost.

Financial science is about the study of wealth management. Irrespective of where wealth exists, and regardless of who owns it or how it is distributed, there is benefit for all of us to understand basic lessons about the elements of finance.

PART 1: ECONOMIC PROFIT#1; NPV AND TIME VALUE

Measurement sciences specify formulas that rigorously link flows and balances. Hydrology, for example, predicts floods by specifying relations between reservoir standing water capacities and rainfall minus runoff and absorption rates. The quantity of reservoir standing water is a balance. Rainfall and runoff and absorption rates are flows. Finance, too, specifies measureable relationships balances and flows. The quantity of capitalized economic value *V* is a balance, a snapshot existing solely for a moment in time. The value *V* changes as time flows and a wealth transfer occurs. Part 1 looks closely at how a stream of flows relates to the balance. Read the game plan below to get the big picture for figuring out finance.

Fundamental principles in chapters 2 and 3 examine how financial statements measure financial balances. Examples merge balance sheets instantly in time revealing intrinsic wealth transfers between raider and target shareholders. The spread between $\%\Delta V^{Raider}$ and $\%\Delta V^{Target}$ is a premium that simply relates to the ratio of company market capitalizations. Subsequent examples examine effects of revenue and cost flows on balance sheet snapshots. Learn the golden rule that balances at two points in time differ by the sum of flows during the interval. The rule is true for real streams, nominal streams, source streams as well as use streams. Solve word problems around the two differential processes that connect the balance sheet halves. The right-side process specifies the change in net worth (*ΔStockholder's equity*). The left-hand side specifies the change in net long term assets (*ΔPlant, property, and equipment*). Find many examples and problems that relate $ROR^{stockholder}$ to price-to-book ratio, sales, net profit margin, breakeven points, and the balance sheet sustainable growth rate.

Chapters 4 and 5 focus on the time value structural specification of flows and balances within simple straightforward formulas. *Time value* is the simple worth of an asset equal to the present value of future inflows. For some scenarios the inflow stream is an explicit contract specifying delivery time and commodity standards, like a loan payment schedule or steady stream of Benjamin Franklin hundred dollar bills. For other scenarios the inflow stream is implicit with periodic services of use and real value like a house providing habitation. Chapter 6 on capital budgeting teaches that the first source of economic profit equals *Net present value*, the difference between discounted inflows and outflows. Chapter 7 applies time value principles to bond and credit market scenarios for fixed income financial securities. Chapter 8 offers lessons on how speculative and fundamental factors sustain equity market share prices and company balance sheet measurements. And that is just part 1 of the lesson plan! Parts 2 and 3 cover the other two sources of economic profit: *diversification benefits* and *arbitrage profit!*

CHAPTER 2: FINANCIAL FUNDAMENTALS OF ACCOUNTING

"Accounting is the language of business." This phrase is well known to people in business. Accounting and accountants play an important role in our economy. Large corporations have their own accounting staff, and most businesses and many households often seek the advice of accountants. Professional accountants sometimes are ridiculed as "bean-counters" yet, at the same time, they are admired and envied for their skill and mastery of the business language.

Accounting is important to finance because it forces exactness – accountants insist on documenting expenditures and revenues. Records help a company, or a potential investor in the company, analyze the financial situation and anticipate or solve money problems. Knowing and using accounting is important for companies and investors, large and small.

A good financial analysis requires a lot more than accounting skills, however. Simply keeping good records is no assurance that a business or individual makes sound financial decisions. A thorough understanding of financial principals helps assure financial success. For better or for worse, finance and accounting form the foundations of business sciences. Learning finance requires familiarity with accounting fundamentals.

This chapter presents fundamental accounting information necessary for understanding finance. There are four sections. The first discusses conceptual foundations about flows and balances. Section 2 elaborates on basic financial statements and section 3 looks at financial ratios. The last section links the income statement to balance changes through time.

1. The Relation Between Flows and Balances

Accounting and finance recognize differences between cash flows and balances. Cash flows represent transfers of wealth. Balances represent accumulations of wealth. Cash flows and balances obviously are related because they both pertain to wealth. There are differences, however, in the way cash flows and balances relate to time.

1.A. The role of time

A cash flow is a transfer of wealth per unit of time. Consider, for example, a $2,000 loan payment that is due on the 15th of every month. This $2,000 expense is a cash flow and, even though it occurs at a specific moment, like 2pm on the 15th, it is a payment "per month". Even if one were to talk collectively about payments for the last 12 months, the payments represent a cash flow because they measure a transfer of wealth per unit of time: the payments equal $24,000 per year.

A company's profit is akin to a cash flow. Consider a company that earns $1,000,000. Maybe the time that it took the company to earn a million dollars was a day, a month, a year, or a decade. The number $1 million tells us nothing about the pertinent number of "units" of time. The cash flow number is incomplete without the number of time periods number. Nonetheless, the profit accrues throughout some time interval and therefore this sum is a cash flow. A cash flow <u>always</u> embodies a time period, albeit the period might be very short (e.g., an overnight interest rate or hourly wage) or very long (e.g., lifetime career or company earnings).

Businesses generally realize a cash inflow when they make a sale and receive income, or perhaps when they take out a loan and receive money from the bank. Conversely, a cash outflow for the business may be the wages it pays workers, or perhaps a payment the business makes to a bank, or to a supplier from whom it's buying a new machine. Notice that with cash flows there is a type of "duality": one concern's outflow is at the same time another concern's inflow. A loan payment, for example, is an outflow for the company but an inflow for the bank. Measuring and monitoring cash flows is important because cash flows signal <u>changes</u> in wealth.

Balances embody time differently than cash flows. A balance is *not* a *transfer* of wealth per unit of time – a balance is the accumulation of wealth at a point in time. In this regard, a balance is like a snapshot at a single point in time; it pictures the momentary status of an account. Companies, and households too, typically have many different types of balances. There is a balance in a checking account, another for a savings account and also there is a balance representing the value of investments in marketable securities. All the preceding balances are financial balances, but likewise there are many types of nonfinancial balances such as Inventories, or Plant, Property, and Equipment.

The balances in the preceding paragraph represent wealth that the company owns — they are asset balances. Also, however, the company owes wealth to other market participants. The wealth that a company or individual owes to other market participants are liability balances. For example, a loan balance is an amount the company owes the bank, and so the loan is a liability. Other common liabilities include Accruals that represent wages owed to labor, Payables that represent sums owed to suppliers, and Stockholders' equity that represents the claim to company wealth by shareholders.

As time passes, of course, balances probably change. The snapshot of a company's balances nonetheless provides substantial information about the company or individual. For example, every morning the company conceivably can call the bank and ask for the outstanding balance on a loan. The balance from day-to-day equals the previous day's balance, plus any new accrued interest expense, minus any credits for loan payments. The loan balance represents the wealth that the company owes. The loan balance also is the wealth that the bank owns. This exemplifies the duality property for balances: one concern's asset is another concern's liability. For any economic entity, whether it represents a single household, a business, or an entire economy, the following financial identity always is true: *the sum of all asset balances equals the sum of all liability balances.*

1.B. Cash flows accumulate to form balances

Balances and cash flows are interrelated because they both pertain to wealth. As an illustration of the interrelation, consider a savings account balance. Suppose that at the beginning of the week the balance is $12,000. During the next few days say that two thousand dollars is deposited, twenty dollars of interest is credited, and one thousand dollars is withdrawn. The balance at the end of the week obviously is $13,020. Each deposit, addition of interest, and withdrawal represents a cash flow; the cash flows for the savings account are $+2,000, $+20, and $-1000. The sum of the cash flows is $1,020. Also, the balance changes by exactly $1,020. This illustrates an important rule of finance.

RULE 2.1 The identity linking flows and balances
The difference between balances at different times equals the sum of the cash flows during the time interval.

1.C. Accrued versus realized flows and balances

When a business sends its loan payment to the bank there is an obvious transfer of wealth from the business to the bank. This exemplifies a *realized* cash flow: a realized cash flow is a transfer of wealth accompanied by a flow of funds. There is an alternative way, however, by which wealth transfers. On an outstanding loan, for example, the bank may assess a daily interest charge against the company. The accrual of this interest charge in the loan account represents a transfer of wealth per unit of time; according to the definition, therefore, the accrual of interest is a cash flow. Yet because funds do not transfer, this is not a realized cash flow. Instead, this is an *accrued* cash flow. An accrued cash flow is a transfer of wealth per unit of time that is not accompanied by a flow of funds. All wealth transfers per unit of time are cash flows, and all cash flows are either realized or accrued.

Some events transfer wealth while other events create or destroy wealth. Consider, for example, a change in the value of a company's land. As the land becomes more valuable then obviously the company owners become wealthier. There has been a transfer of wealth to the company, but from where? Unlike other cash flows for which the duality principal requires that the increase in one participant's wealth equals the decrease in another participant's wealth, the creation (or destruction) of economic wealth is a net gain (or loss) to the financial economy. It is as though a wealth transfer occurs between an active market participant and the ever present "invisible hand."

When the company owns land before its value goes up and they own it afterwards, too, wealth *accrues.* If the company wishes to *realize* the increased wealth, they must take action. To realize the new wealth they must realize a cash inflow, which they might do by selling the land, or using the land as loan collateral. Each action has offsetting tradeoffs. Regardless, all wealth changes contribute toward economic income whether realized or accrued, positive or negative, and fundamental rule 2.1 still holds true.

An implication of rule 2.1 for analysis of finance statements is: (1) The difference between realized balances at different times equals the sum of the realized cash flows occurring during the time interval; and (2) the difference between accrued balances at different times equals the sum of the accrued cash flows occurring during the time interval. Traditional financial statements are hybrids that contain both accrued and realized line items. This mix obviates the importance of analysis, agency issues, audit and disclosure policies, and market transparency.

Insert Exercises 2.1 here. Click to view.

2. Representing Flows and Balances on Financial Statements

The purpose of financial statements is two-fold. First, the statements provide the company with information for monitoring or analyzing its financial situation. Second, the statements provide external analysts with audit information for gleaning insight about the company's financial health. Most descriptions in this chapter apply directly to corporate financial statements. The previous chapter explains, however, that many companies are not corporations. The principals herein also apply to situations in which the company is a noncorporate company or a household. Indeed, many households construct personal financial statements.

Several organizations in the USA promulgate procedures for preparing financial statements. The most influential organization is the *Financial Accounting Standards Board* (FASB). FASB issues rules followed by almost all preparers of financial statements. These "generally accepted accounting principles" (GAAP) result primarily in two types of statements showing flows (the income statement and the statement of cash flows), and one type of statement showing balances (the balance sheet). The following discussion elaborates about balance sheets and income statements. Discussion on cash flows appears in the next chapter.

2.A. The Balance sheet

Balance sheets summarize the many different types of wealth that the company owns (assets) and owes (liabilities). Large companies prepare a quarterly balance sheet every 3 months. Each one shows the company's financial position at that moment. Companies almost always also report a fiscal year-end balance sheet. The year-end balance sheet does not equal the sum of line items for the four separate quarterly balance sheets. Instead, the year-end balance sheet is identical to the last quarterly balance sheet since they are a snapshot of the same moment in time.

The diversity in the collection of balance sheets is phenomenal. Analysis of data in Chapter 1 for a large sample of nonfinancial U.S. corporations showed Walmart with the most employees and General Electric with the most *Total Assets*. The NFC includes many industries: manufacturing companies, transportation companies, utilities, many more. Still, the NFC excludes a lot of U.S. and global financial economic activity. Examine the 10,000 company common stocks, sort on company *Total Assets*, and find the 40 largest ones in Table 2.1. The NFC companies are the red rows.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
FNMA	$3,270,108	7	$83,963	$122,606	$ 17,332
JPM	2,415,689	251	17,923	105,790	219,657
DB	2,220,348	98	918	58,812	49,172
BAC	2,102,273	242	11,431	101,697	164,914
FMCC	1,966,061	5	48,668	75,311	9,425
C	1,880,382	251	13,673	92,543	157,854
SAN	1,537,238	183	6,021	91,876	102,794
WFC	1,527,015	265	21,878	88,069	238,675
UBS	1,134,164	60	3,562	39,549	72,538
MET	885,296	65	3,368	68,180	60,500
TD	862,532	79	6,557	30,744	87,750

RY	860,819	79	8,331	38,760	100,949
MS	832,702	56	2,932	36,848	60,991
BNS	743,788	84	6,422	28,417	76,612
GE	656,560	307	13,057	142,937	282,006
AIG	541,329	64	9,085	67,375	74,740
BMO	537,299	46	4,183	20,851	46,777
BRK.B	484,931	331	19,476	182,042	292,405
CM	398,389	43	3,403	16,984	35,413
BK	374,310	51	2,111	15,326	39,910
USB	364,021	66	5,836	21,059	73,720
XOM	346,808	75	32,580	390,247	438,702
PWCDF	345,005	33	1,029	29,642	14,705
POFNF	341,711	32	2,027	28,830	25,602
GWLIF	325,905	21	2,408	26,421	32,730
PNC	320,296	54	4,220	16,872	41,350
COF	297,048	42	4,159	24,176	43,873
HIG	277,884	18	176	25,949	16,423
T	277,787	243	18,249	128,752	183,757
VZ	274,098	177	11,497	120,550	140,639
CVX	253,753	65	21,423	211,664	239,028
STT	243,291	29	2,136	10,295	31,861
LNC	236,945	9	1,244	11,883	13,571
DDAIF	232,201	275	9,428	162,567	92,985
VOYA	221,023	7	601	8,672	9,198
BLK	219,873	11	2,932	10,180	53,396
PFG	208,191	15	913	9,156	14,556
AAPL	207,000	84	37,037	170,910	428,700
WMT	204,751	2200	16,022	474,259	241,440
F	202,026	181	7,155	146,917	60,853
SLF	199,531	16	1,060	13,489	22,864

TABLE 2.1 The 40 largest U.S. listed corporations based on *Total assets*, 2014
Red rows are nonfinancial corporations, all others are financial corporations. All list common stock on a U.S. stock exchange.

Everyone knows that Exxon-Mobil (XOM) and Apple Computer (AAPL) and GE are huge but you would have to look far down the top-40's table for a company that "small." Inclusion of financial corporations (Standard Industrial Classification code 60) shows that JPM, as in JP Morgan Chase (John Pierpoint Morgan), has put total assets on its balance sheet exceeding by a factor of 10 those owned by Apple Computer, $2,400 billion versus $207 billion. Financial institutions, especially ones on Table 1.4, have a lot of financial securities on both sides of the balance sheet. Apple has some factories and other factors of production on its balance sheet, plus relatively few financial assets. FNMA with $3.2 *trillion* of financial securities on the asset side reports 7,000 employees, quite a few less than WMT. The market cap of many financial corporations is often small relative to its total assets. Company BAC is Bank of America, for example, and the liability side of BAC lists a lot of loans from customers who make deposits at the bank. When a client deposits a paycheck at the bank then the client is a source of financing for the bank. The client lends money to the bank, the bank lists that deposit as a liability. Besides a lot of deposits on the BAC right side of the balance sheet, there also is *Stockholders equity* from issuing common stock to raise cash. The table shows the market cap of the BAC stock is $164.9 billion. The FNMA market cap is relatively small, only $17 billion, smaller than any on Table 1.3 of American icons. A lot of diversity for sure!

Table 2.2 presents a historical snapshot of an IBM balance sheet followed by a discussion on main line items. These are typical items for many NFC companies.

ASSETS	$millions	LIABILITIES AND STOCKHOLDERS' EQUITY	
Cash	$ 7,687	$ 4,767	Payables
Inventories	5,870	2,950	Accruals
Receivables	16,515	12,957	Short Term Notes
Other Current Assets	10,623	13,326	Other Current Liabilities
Total Current Assets	*$40,695*	*$34,000*	*Total Current Liabilities*
Net PP&E	17,407	9,872	Long Term Debt
Other Long Term Assets	23,030	21,628	Total Stockholders' Equity (509,070,542 shares)
	.	15,632	Other Long Term Liabilities
Total Assets	*$81,132*	*$81,132*	*Total Liabilities & SE*

TABLE 2.2 International Business Machines Corporation and Subsidiary Companies, Consolidated balance sheet, historical snapshot.

A1. Liabilities are financing sources

The balance sheet's right-hand side shows the liabilities from which the company already has obtained its financing. In that regard, the right-hand side lists wealth that the company owes to its historical sources of financing. These sources have financial claims on the firm's wealth and are called "claimants." It is conventional to separate claimants into two groups. *Current liabilities* include claims that tend to be repaid in the near term, such as one or two years. *Long term liabilities* include claims generally outstanding over longer periods of time.

Current Liabilities

The sum of all short term claims is *Current liabilities*. *Current liabilities* represent financing already made available to the company. Analogously, though, *Current liabilities* portend the amount of future payments that the company must repay in the near future. The discussion below elaborates on the more significant *Current liabilities*.

Payables are financing, or trade credit, made available by suppliers for purchasing *Inventory* and other assets. *Payables* rise, for example, whenever a delivery truck backs up to the company's receiving dock, unloads supplies, and the warehouseman signs a docket attesting to delivery. The receiving company owes the deliverer money for the shipment — the increase in *Payables* represents a loan made to the company by the supplier. *Payables*, just like any other liability, represent wealth that the company owes to another concern.

The *Payables* entry on the balance sheet is not a cash flow. The value of all deliveries within a particular week, for example, is a cash flow. But the balance sheet entry shows the sum of all deliveries ever made, minus all payments ever sent by the company to the suppliers — the balance sheet entry is the sum of all previous cash flows for the *Payables* account.

Another *Current liability* on the balance sheet is *Accruals*. This is money the company owes its workers for jobs they have completed but not received pay. *Short term notes* typically represents short-term bank loans; longer term notes usually appear under *Long term liabilities*.

Long term Liabilities

Long term liabilities represent wealth the company repays in the remote future. *Long term debt* typically includes financing from banks or other capitalists that are repayable over three years, five years, ten years or even longer. The specific details for each loan stipulate the company's obligations regarding the amount of interest, and timing of principal payments. Quite often banks are the sources of long term loans.

Companies, just like individual households, sometimes take out a 30-year mortgage from a bank for purchasing a building. Such a loan is a component of *Long term debt*.

Another common component of *Long term debt* is *Bonds*. The bond is simply a financial security that the company sells to capitalists in exchange for money. The bond is like an "I-O-U" in which the company promises to repay money in the future. Between time of issuance and time of maturity, the company agrees to pay interest. Each bond owner has a claim on part of the company, and the category *Long term debt* documents the claim.

The IBM balance sheet lists significant *Other long term liabilities*. This line item represents claims on IBM's wealth largely by retired or nearly-retired employees. According to generally accepted accounting principals (GAAP), the company computes future retirement benefits that already it has promised employees. Some of these employees currently are in retirement whereas others may be a decade or more from retirement. The employees presumably exchanged with the company labor services that exceeded the value of wages received. IBM promised future retirement benefits.

The pension obligation is, according to GAAP, a *Long term liability.* Even though employees did not give a cash loan to IBM, they contributed wealth to the company in the guise of labor services. IBM in exchange transfers wealth to its employees. Some of the wealth transfer is a realized cash flow that employees receive as wages. The rest of the wealth transfer is an accrued cash flow. It accrues when IBM signs contracts promising employees future retirement benefits. The balance sheet clearly shows, for all to see, that IBM owes its retirees a substantial chunk of change.

Perhaps the most important of all long term liabilities is *Stockholders' equity*. Stockholders are the owners of the corporation. By voting and appointing management, shareholders determine the company mission and the actions by which the company pursues its mission. In a simplistic setting, *Stockholders' equity* measures all the cash lent to the firm by the stockholders. There are several ways that cash flows occur between stockholders and the corporate treasury.

Stockholders may provide the firm with cash inflow by directly purchasing shares from the company. As a result of this primary market transaction the investor receives a share of stock and the corporation realizes a cash inflow. In effect, the company borrows money from the shareholder and the share of stock represents an I-O-U that the investor owns and safeguards. The liability on the balance sheet labeled *Stockholders' equity* rises when the company issues new shares.

STREET-BITE 2.1 A thrilling financing source: initial public offerings (IPOs)

The Internal Revenue Service receives more than 5 million corporate tax returns each year (see table 1.6). Yet fewer than one percent of these corporations raise money by selling shares to the public. When finally, however, a corporation grows to a certain level of maturity one natural next step is "going public."

An *initial public offering* ("IPO") occurs when a corporation sells its shares for the very first time to the public markets. The main reason for going public is to expand possible financing sources by tapping into the huge public financial markets. IPOs represent a relatively small fraction of trading volume but they nonetheless receive fantastic media coverage. Table 2.3 hints why.

Annual sales of issuing firm ($ millions)	Number of IPOs	Average initial return
< $10 million	267	9.0%
$10M – 20M	67	12.4%
$20M – 50M	191	14.5%
$50M – 100M	251	20.3%
$100M – 200M	215	17.1%
> $200 million	555	11.3%
All	1,546	13.6%

TABLE 2.3 Data on initial public offerings, 2001-2013.
Source: http://bear.warrington.ufl.edu/ritter/IPOs2014Statistics.pdf

The far right column is a fantastic daily return. The initial return for an IPO is the percentage change from the offer price early in the day until trading ends later the same day. The average one-day return for the 1,546 IPOs in the sample is 13.6%. That huge sum exceeds the return that most stocks earn in an entire year – yet this accrues in one day. No wonder there is so much media hype about IPOs!

Do issuing companies view huge initial returns as a sign that their going-public process was a success? The answer is: not totally. Consider the case of MarketWatch.com offered in the primary market at $17 a share on January 15, 1999. The company issued 2.7 million shares at $17 each and raised $46.8 million of financing (=$17 x 2.7 million). On the liability side of the balance sheet *Stockholders' equity* rises to reflect the $46.8 million financing source. The asset side also rises by $46.8 million; probably *Cash* is the account which first receives all monies.

Two hours after MarketWatch.com's IPO the stock was trading in the secondary market at $130 a share. That represents an incredible increase of 665%. And what happened to the company during that time? Nothing. The balance sheet did not change at all during those two hours. To the contrary, managers at MarketWatch.com watched as some investors who bought shares for $17 were able to sell them hours later for $130. Investors made money from the price run-up, but not the corporation (maybe managers personally owned some stock that became worth millions). MarketWatch.com managers were wondering why they offered the stock for only $17. The company arguably sold underpriced stock and "left money on the table."

The going-public process is complex with many tasks. Generally, however, tasks fall into one or more of three functional categories.

1. *Regulatory* Transition from a privately-held to a publicly-traded corporation involves huge changes in regulatory responsibilities. Once a company goes public they are required to disclose a lot of information that perhaps previously manager/owners considered confidential. The "Securities Act of 1933" and "Securities Exchange Act of 1934" impose strong reporting requirements. The "Corporate Accountability Act of 2002" imposes strong penalties for fraudulent statements (that Act, also known as the Sarbanes-Oxley law, created the "Public Company Accounting Oversight Board" that inspects, registers, and disciplines companies that prepare accounting statements). Companies going public submit to the SEC (Securities and Exchange Commission) a detailed registration statement as well as a preliminary prospectus. The company hopes to eventually distribute the prospectus to potential investors. The prospectus informs investors about the company, its mission and management, and other financial information useful for analyzing the company. SEC approval is required before a company goes public. After going public, the company must forevermore

frequently file forms with the SEC (www.sec.gov is an interesting government website allowing online viewing of corporate filings and forms).

2. *Underwriting* Companies going public most likely are already successfully delivering a product that customers want. The managers, however, probably don't know much about successfully delivering stocks to the public financial markets. So they hire an investment banker to advise the company on procedural and financial tasks. Especially interesting is the underwriting process. Underwriting by an investment banker(s) is the analysis of new security issuances in order to properly assign prices and collect fees. Several investment bankers may join together for relatively large deals and form an "underwriter syndicate." Two extreme fee structures exist. (a) With a "firm commitment" contract the investment banker purchases the security from the issuing company for a mutually agreeable price. The issuing company receives a specified sum of money and the underwriter (that is, the investment banker or syndicate) receives the security. The underwriter subsequently sells the security in the primary market. The underwriter's income depends on the spread between the price paid to the issuing company and the price received upon re-sell. The underwriter has relatively high risk exposure with a firm commitment contract because, for example, the IPO may not sell-out. Perhaps the underwriter gets stuck with a lot of stock that nobody values. The issuing company has relatively low risk exposure because they receive a firm commitment of capital. (b) With a "best efforts" contract the investment banker receives a mutually agreeable fee from the issuing company and sells the security for them in the primary market at the best possible price. The underwriter has relatively low risk exposure with a best efforts contract because they never take ownership of the stock. The issuing company, however, has relatively high risk exposure. Perhaps they do not raise as much money as desired due to lackluster investor interest.

3. *Distribution* Perhaps the most difficult task is setting the offer price for issuing stocks in the primary market. The underwriter compares existing public companies to the IPO company and sets a preliminary price range for the new issue. They then gauge potential investor interest by conducting pre-market activities: roadshows tell institutional investors about the company; order books fill with tentative purchase agreements; allocation agreements stipulate which underwriters, brokers, and customers get the IPO stocks. Underwriters set the final offer price the day before shares begin trading. Then, like horses out of the gate, the shares flow out of the primary market and begin trading in the secondary market. The stock price moves up or down in response to supply and demand forces.

Not all IPOs are small companies. Shares for well-known Kraft Foods, Inc., began trading through an IPO on June 13, 2001. Prior to the IPO the company was a wholly owned subsidiary of Philip Morris. Kraft Foods booked almost $35 billion in sales for the year preceding the IPO. Phillip Morris spun off Kraft and gave it an identity of its own. They offered and sold 280 million shares at $31 per share, thereby raising $8.7 billion of cash for Phillip Morris. By the way, Phillip Morris retained ownership of 1.4 billion shares in Kraft Foods that represent a potential financing source for the future. Other examples of large IPOs abound.

History reveals that IPOs are especially popular during strong bull markets when prices are rising and investors seem to like anything resembling stock. IPO volume drops off during downturns and bear markets.

Contrary to the impression given by the fantastic initial returns in table 2.3 most investments in IPO stocks underperform market averages in the long-run. This excerpt from the *Wall Street Journal* about one of the more famous IPO booms says a lot:

"When the history of the 1990s bull market is written, one of the more intriguing postscripts will be that it has given birth to so many new companies that created so

> little wealth. From May 1988 through the market's record high on July 17 of this year [1998], stocks of America's blue-chip companies rose more than fourfold. In that period, about 4,900 companies had initial public offerings. Their fate? Just 71% are still trading regularly, and of those, on July 17 fully 44% were below their offering prices … the median annual return as of the July market peak was a minuscule 2.4%, not even beating Treasury bills." Greg Ip, "Bull market has sired a lot of new stocks, but few become stars", *Wall Street Journal*, September 15, 1998.

The bottom-line is that on average IPOs are great investments for investors making the purchase in the primary market and selling the stock within a month in the secondary market. But late-comers watch out! And, by the way, the SEC found that many investment bankers and brokers pursued illegal and unethical allocation agreements during most bull markets, millions of dollars in fines punctuate the history. The 2004 Google IPO allowed investors to bid directly for IPO shares on the internet. That and changes yet to come will make IPOs even more thrilling.

Another way that a cash flow passes between the firm and stockholder is through share repurchases. A share repurchase occurs when the company buys shares of its own stock from its shareholders. The company either inventories or shreds the stock they repurchase. Share repurchases occur for a variety of reasons and by a variety of methods. Companies in the S&P 500 index bought $500 billion of their own shares in 2013, considerably greater than the $300 billion they paid in dividends. Many share repurchases occur when a company's managers believe that their shareprice is too "cheap." The increased demand for shares puts upward pressue on the share price. Other times repurchases occur when companies accumulate large cash balances and shareholders pressure companies to return the cash. Regardless of reason, a stock repurchase cancels a liability owed to the shareholder and is reflected on the balance sheet as a decline in *Stockholders' equity*.

The sale and repurchase of stock by the company affects two components of *Stockholders' equity*: *Paid-in-equity capital* and *Equity capital surplus*. This and many others often lump these subcategories together into the broader entry called *Stockholders' equity*.

Stockholders' equity sometimes is called *Net worth*. Recall the discussion from the previous chapter about the company cash flow cycle. *Net worth* for nonfinancial corporations ($19.1 trillion in 2013) was compared to household *Net worth* ($78.9 trillion). In both cases the *Net worth* equals *Total assets* minus *Total liabilities*. A section below discusses *Total liabilities* in more detail. Equity is the residual claimant on company wealth, and *Net worth* measures the value of the claim. *Net worth* belongs to stockholders for a corporate company, to sole proprietors and partners for noncorporate companies, and to households for family balance sheets.

Corporations issue many shares of stock to put money on the balance sheet. Dividing total *Stockholders' equity* by the number of shares outstanding gives the equity book value per share:

FORMULA 2.1 Equity book value per share

The equity book value per share is the worth of each common stock and equals the ratio of two numbers from the balance sheet:

$$\begin{pmatrix} equity \\ book\ \ value \\ per\, share \end{pmatrix} = Stockholders'equity \Big/ number\ of\ shares\ outstanding\ .$$

Equity book value per share is an important ratio. If accounting statements properly reflect true values, and if the stock market properly prices shares, then market shareprices should equal equity book value per share. Indeed, market analysts often compare market shareprice to equity book value per share in order to assess whether a stock seems fairly priced. The ratio of stock market shareprice to equity book value is the “equity price-to-book” ratio:

FORMULA 2.2 Equity price-to-book ratio

The equity price-to-book ratio is the common stock’s shareprice relative to book value:

$$\begin{pmatrix} equity \\ price-to-book \\ ratio \end{pmatrix} \equiv \left(P \big/ B \right)$$

$$= market\, shareprice \Big/ equity\ \ book\ \ value\ \ per\ \ share$$

This important financial ratio is referred to as the “*P/B* ratio.”

A *P/B* ratio equal to 1.0 has a special meaning. For a company with *P/B* at unity a dollar of company assets has a price in the sharemarket of exactly one dollar. When *P/B* differs from unity, a dollar of assets according to the books has a stock market price different from $1. There is not, of course, any “law” that requires equality between market shareprice and balance sheet book value. Shareprices are forward-looking assessments of value that are supported by expectations about future profits. Book values are backward-looking allocations of historical measurements.

For a very small *P/B*, for example say 0.50, the stock market trades a dollar of company assets for only 50 cents. If an individual believes that book value correctly measures the “true value” of the company’s assets, then the implication is that the sharemarket undervalues the assets. An investor finding an undervalued company probably thinks the shares are a good investment. They think that when the market corrects its mistake, the share price should rise and *P/B* should go towards unity. Conversely, if an individual believes that the sharemarket reflects true value, the implication is that book value overstates true value. Perhaps the very small *P/B* is not a result of market undervaluation as much as inaccurate accounting of book value. There always is uncertainty whether a small P/B implies market undervaluation or accounting overstatement.

A relatively large *P/B* means that a dollar of company assets trades in the sharemarket for more than its accounting value. Once again, two interpretations of an anomalous *P/B* are plausible. Maybe it is the stock price, or maybe it is the book value, that is a more accurate measure of true value. Unfortunately, one never knows for sure whether a relatively large *P/B* signifies that the stock market is overvaluing a share, in which case the share represents a bad investment that should be avoided. Alternatively, perhaps the large *P/B* occurs because the stock market properly prices the share whereas book value understates assets.

Many investors examine company *P/B* ratios before buying stocks. Generally speaking, the rumor on the street is that a company with a relatively low *P/B* signifies an undervalued stock, a good prospect for investment. Conversely, according to the rumor, relatively high *P/B* stocks might be overvalued shares that are ripe for a major market correction. Careful studies on this issue find some support for this conjecture. The situation is cloudier than the discussion implies, however, because *P/B* ratios distribute around a number that generally is not 1.0. Inferring that a target *P/B* is relatively small (or large) requires identifying the proper benchmark for comparison. That is tricky.

Before concluding that *P/B* ratios contain a secret for successful stock-picking, be forewarned. Stock prices are very “noisy” – they jump all over the place. No single ratio or statistic does a good job predicting whether a particular stock price will rise or fall. Among all the many poor measures that one can examine, however, the *P/B* ratio is among the best. There is statistical evidence that, in the long-run and on-average, *P/B* ratios vibrate around a persistent number.

Substitute formula 2.1 for equity book value per share into formula 2.2 for the *P/B* ratio. Rearrange and obtain a formula

$$\begin{pmatrix} \textit{equity} \\ \textit{price-to-book} \\ \textit{ratio} \end{pmatrix} = \textit{market shareprice} \div \left(\frac{\textit{Stockholders'equity}}{\textit{number of shares outstanding}} \right)$$

$$= \frac{(\textit{market shareprice x number of shares outstanding})}{\textit{Stockholders'equity}}$$

The product in the numerator of the lower line equals the company’s current stock price times the number of shares outstanding. This number represents the company’s market capitalization.

FORMULA 2.3 Market capitalization

Market capitalization (also known as “market cap”) is the total stock market value for all outstanding common stocks at the current shareprice:

$$\begin{pmatrix} \textit{company} \\ \textit{market} \\ \textit{capitalization} \end{pmatrix} = \textit{market share price x number of shares outstanding}.$$

A table in the previous chapter lists market capitalization for several American corporate icons. The market cap of Apple Inc., for example, is over $400 billion. The formulas above establish that *P/B* actually measures two algebraically identical ratios: (i) share price over equity book value per share, and (ii) total company market capitalization over *Stockholders’ equity.*

The financial press often reports with fanfare about corporate mergers. Mergers are one mechanism that chapter 1 discusses for controlling principal-agent problems. Besides the real-world importance of mergers, however, mergers also provide a wonderful setting to work with balance sheets. A merger is like a marriage of balance sheets – two snapshots pulled together at a moment devoid of time. Before working with the financial effects of mergers, however, read the accompanying *Street-bite* for background information.

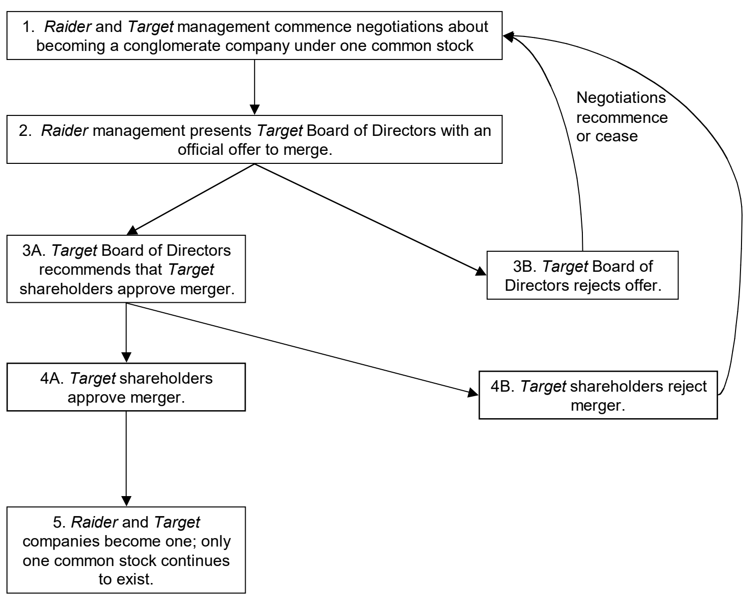
STREET-BITE Mergers, acquisitions, and contests for corporate control .

Common stockholders are the company owners. Whoever owns the common stock controls the Board of Directors and sets the company strategic mission. Inspect any *Wall Street Journal* and likely there is an article about one company taking control of another. That's what mergers and acquisitions are all about: one company, call them the *Raider*, getting a controlling amount of common stock in another company, call them the *Target*.

Before two companies merge they each possess their own unique common stock and company name. After the merger perhaps the name of the combined company is a blend of the individual company names, perhaps not. Almost surely, however, where there used to be two unique common stocks there is, after the merger, only one. In most situations a large *Raider* assimilates the smaller *Target* and consequently the *Target* loses its independent identity.

While every deal is different, figure 2.1 shows typical steps in the merger process. Initial discussions lead *Raider* management to tender an offer for acquiring *Target* company common stock. This "tender offer" may specify (1) the cash price or number of *Raider* shares that is offered in exchange for a share of *Target* stock, (2) a deadline at which the offer becomes void, and (3) other conditions such as withdrawal of the offer if the *Raider* fails to obtain a majority of *Target* shares. The *Target* Board of Directors must obtain approval of its shareholders. After all, the *Target* shares will cease to exist so this surely affects *Target* shareholders.

FIGURE 2.1 Typical managerial steps for mergers and acquisitions



Summary statistics about a window of mergers and acquisitions activity in the USA appear in table 2.4. Data over long horizon are unpredictably cyclical, the ones in this table are somewhat typical. Thousands of mergers occur each quarter of the year representing hundreds of billions of dollars of corporate wealth. There are generally two methods by which the *Raider* pays for *Target* stock. One, perhaps the *Raider* pays cash to *Target* shareholders. Two, perhaps the *Raider* offers shares of *Raider* stock as payment. These stock swaps are very common, especially when *Raider* management perceives their own stock is highly valued. Many *Raiders* pay with a combination ("combo") of cash and stocks.

Year and quarter	Total completed transactions		Mode of payment (M&A subsample)			Average stock price premium
	Number	Value ($ bil.)	Cash	Stock	Combo	
Apr-Jun 1997	1,793	$209	54%	29%	18%	32.9%
Jul-Sep 1997	2,034	214	57%	26%	17%	33.5%
Oct-Dec 1997	2,448	231	59%	26%	15%	25.4%
Jan-Mar 1998	2,419	233	60%	24%	17%	24.5%
Apr-Jun 1998	2,619	272	56%	27%	17%	25.6%
Jul-Sep 1998	2,754	376	57%	25%	18%	23.1%
Oct-Dec 1998	2,315	464	61%	28%	11%	35.2%
Jan-Mar 1999	2,410	292	60%	27%	14%	31.0%
Apr-Jun 1999	2,260	362	59%	26%	14%	37.2%
Jul-Sep 1999	2,340	271	61%	26%	13%	33.6%
Oct-Dec 1999	2,191	498	57%	29%	14%	31.3%
Jan-Mar 2000	2,206	287	53%	31%	16%	33.6%
Apr-Jun 2000	2,207	723	51%	33%	16%	34.0%
Jul-Sep 2000	1,866	258	52%	32%	16%	33.3%

TABLE 2.4 Historical snapshot of mergers and acquisitions activity in the USA
Compiled by author.

The rightmost column shows the average stock price premium that the *Raider* pays to *Target* shareholders. The average premium is about 30%. This implies, for example, that a *Target* shareholder whose stock is trading at $50 before the merger announcement receives an offer from the *Raider* in cash or stock worth $65 per share [= $50 *x* (1 + 0.30)]. The *Raider* pays more for *Target* stock than its current trading price in order to entice *Target* shareholders to approve the deal. This hefty premium motivates some investors to be on the lookout for potential target companies. Irrespective of the *Target's* profit from operations, buying stock in a company that subsequently attracts a *Raider* leads to big stock returns. Identifying potential *Targets*, however, is tricky, too.

Raider management explains to its own shareholders that overpayment to *Target* shareholders is sensible due to the existence of long-run benefits. Statistical studies show, conversely, one generally cannot reject the hypothesis that post-merger sales, net income, and market value for the conglomerate equal the sum of the pre-merger items for the *Raider* and *Target*. That is, the whole equals the sum of the parts - mergers generally do not create value, they simply transfer wealth from *Raider* to *Target* shareholders.

Typically, but not always, both companies welcome the merger. Sometimes *Target* managers believe the merger is a bad idea. They may take anti-takeover measures called shark repellants to block the merger. A short-list of common shark repellants includes:

1. *Golden parachutes* are compensation contracts in which *Target* management is guaranteed huge bonuses in event of termination due to takeover. This is analogous, in some ways, to wage contracts for coaches and athletes that specify a

huge sum payable in event of firing or trading. The existence of golden parachutes for *Target* management diminishes the attraction to the *Raider* of the merger.

2. *Poison pills* are covenants approved by *Target* shareholders that increase the difficulty for a *Raider* to obtain a controlling position in *Target* common stock. For example, perhaps a poison pill enables *Target* management to issue large amounts of new stocks at relatively low prices to existing shareholders or management in event of a takeover attempt.

3. *White knights* are third-party companies with whom the *Target* seeks a merger in order to avoid the *Raider*. Sometimes the *Target* realizes that independent corporate status is impossible because the *Raider* is attempting the takeover. Yet the *Target* does not like this *Raider* so it looks for another. The *white knight* merges with the *Target* under conditions that presumably are more favorable for *Target* management.

The financial media focus a lot on mergers, and almost everybody hears about the big ones. The news stories sometimes sound like daytime soap operas. Often, however, billions of dollars and thousands of jobs may be at stake in merger contests for corporate control.

Tax effects and accounting policies for mergers are in reality complex and controversial. Hordes of lawyers, politicians, and accountants debate proper policy. The regulatory environment is ever changing, too. Our objective is simply to focus on fundamental effects from arithmetic relations. Consequently, we adopt simplifying assumptions that the whole (conglomerate) equals the sum of parts (*Raider* and *Target*) with regards to market capitalization, sales, costs, and net income. The preceding numbers for the *Raider* plus *Target* sum to that of the *Conglomerate*. The simplistic setting shows how rates of return differ between *Raider* and *Target* shareholder due to an effect known as leverage.

Each merger in actuality is historically unique. Management of *Raider* and *Target* companies seek to satisfy the objective in Rule 1.1: Maximize wealth creation irrespective of its distribution. *Raider* management often perceives and argues that economic advantages from horizontal or vertical efficiencies create long-run wealth. The *economic value added* by the merger, often referred to as *EVA*, equals the present value of expected incremental economic profit (= *Net present value* in formula P1). The existing structural framework below easily can be made more complex but as is most lessons already can be found. The management of both companies make decisions under the umbrella of principal-agent relations between themselves and shareholders or creditors or employees or him/herself to distribute perceived *EVA*.

EXAMPLE 1 Merger mechanics

The Raider Company’s balance sheet at year-end 2525 is below.

Raider Company Balance Sheet, 12/31/2525			
	Assets	*Liabilities*	
Cash	$ 400	$ 600	Debt
PP&E	2,460	2,260	Stockholders' Equity
Total Assets	*$2,860*	*$2,860*	*Total Liabilities & Equity*

The Raider Company has 550 common shares outstanding and their equity price-to-book ratio is 0.54. The Raider Company plans to take over the Target Company whose balance sheet appears below.

Target Company Balance Sheet, 12/31/2525			
	Assets	*Liabilities*	
Cash	$ 410	$ 350	Debt
PP&E	1,400	1,460	Stockholders' Equity
Total Assets	*$1,810*	*$1,810*	*Total Liabilities & Equity*

The Target has 220 common shares outstanding and their equity price-to-book ratio is 1.23 . The Raider Company offers 9 shares of Raider stock to Target shareholders that tender 2 Target shares; that is, the exchange ratio is 4½. Suppose synergistic and tax effects and changes in debt-to-equity ratios are nil. After the Raider takes control of all Target shares, find (i) the *P/B* ratio for the new Conglomerate Company, and (ii) the effect of the merger on each shareholders' wealth?

SOLUTION

Our simplifying assumptions mean that the balance sheet for the Conglomerate Company simply equals the sum of each line item for the separate companies.

Conglomerate Company Balance Sheet, 12/31/2525			
	Assets	*Liabilities*	
Cash	$ 810	$ 950	Debt
PP&E	3,860	3,720	Stockholders' Equity
Total Assets	*$4,670*	*$4,670*	*Total Liabilities & Equity*

Recall that *P/B* equals the ratio of company market cap to *Stockholders' equity*. The problem gives each company's *P/B* and *Stockholders' equity*. Solve for the unknown market cap as follows:

P/B = market capitalization / Stockholders' equity ,

so for the Raider

0.54 = market capitalization / $2,260

or *market capitalization = $1,220.*

For the target,

market capitalization = $1,460 x 1.23

= $1,796 .

Market capitalization for the Conglomerate Company is $3,016 , the sum of the component company market values (= $1,220 + $1,796). The *P/B* ratio for the Conglomerate Company equals its market capitalization divided by total *Stockholders' equity*, which is 0.81 (= $3,016 ÷ $3,720). This is the answer for part (i) of the question.

The wealth effects for each shareholder depend on the Conglomerate shareprice after conclusion of the merger. Already we have found the total stock market value of the Conglomerate (its market cap) equals $3,016. Market cap equals price per share times number of shares, so a necessary step is finding the total number of Conglomerate shares outstanding. Each share of Raider stock outstanding before the merger remains intact even after the merger. For the Target Company, however, its shares cease to exist because the Raider absorbs the Target's shares. Each Target shareholder receives 9 shares of Raider stock for every 2 Target shares he/she tenders. The 220 Target shares outstanding before the merger exchange into 990 new Raider shares (= 220 x (9 / 2)). The Conglomerate Company therefore consists of the 550 Raider shares outstanding prior to the merger, plus the 990 Raider shares that the merger creates:

$$\begin{pmatrix} \text{number of} \\ \text{Conglomerate} \\ \text{shares} \end{pmatrix} = \begin{pmatrix} \text{number of} \\ \text{original} \\ \text{Raider shares} \end{pmatrix} + \begin{pmatrix} \text{number of} \\ \text{new Raider} \\ \text{shares issued} \\ \text{to Target shareholders} \end{pmatrix}$$

$$= \quad 550 \quad + \quad 220 \times \left(\frac{9}{2}\right)$$

$$= \quad 1{,}540$$

In total, therefore, the Conglomerate Company has 1,540 shares outstanding. Now use the definition of market capitalization to find the Conglomerate's shareprice:

$3,016 = (price per share) x 1,540 shares ,

or *price per share = $1.96*

One share in the Conglomerate Company is worth $1.96 after the merger. A Conglomerate share after the merger is physically identical to a Raider share before the merger, albeit the company name printed on each share may (or may not) be different than before. The merger affected the shareprice, however. Compute the shareprice before the merger with formulas 2.1 and 2.2:

P/B = (price per share) / (Stockholders' equity / #shares outstanding)

so for the Raider

0.54 = (price per share) / ($2,260 / 550)

(price per share) = $2.22

The Raider has a share worth $2.22 before the merger and $1.96 afterwards. The capital loss is 26 cents per share (a loss of almost 12 percent).

For the Target shareholder before the merger:

1.23 = (price per share) / ($1,460 / 220)

(price per share) = $8.16

A target stockholder tenders two Target shares worth $16.32 (= 2 x $8.16) and receives nine Conglomerate shares worth $17.64 (= 9 x $1.96). Wealth increases by $1.32 for every two shares tendered, or $0.66 per share, or about 8 percent. This is much less than the average premium shown in table 2.4.

The assumption of a zero sum merger of market caps means that the merger neither creates nor destroys wealth. Simply put, Raider companies overpay Target shareholders. Target shareholders gain wealth from Raider shareholders. The total wealth transfer from Raider to Target shareholders for Example 1 is about $143 (= 550 x (2.22 – 1.96)).

Example 1 for a zero sum merger finds that the total wealth transfer from Raider to Target equals $143. Thus 4.75% of total wealth redistributes between balance sheet claimants (143/3016 = 0.0475). The wealth transfer, however, neither creates nor destroys wealth!

Since $MC_{Conglomerate} = MC_{Targe} + MC_{Raider}$

then divide all three market caps by the same number ΔW that measures the balance sheet transfer, $143 in this case, to Target from Raider. This writes as

$$\frac{MC_{Conglomerate}}{\Delta W} = \frac{MC_{Target}}{\Delta W} + \frac{MC_{Raider}}{\Delta W}.$$

Recognize that each term measures the *reciprocal* of the percentage change. That is, $\frac{\Delta W}{MC_{Target}}$ measures the percentage change in the market capitalization of all Target shares; The percentage loss for Raider shareholders is $-\frac{\Delta W}{MC_{Raider}}$.

IDENTITY 2.1 The percentage changes of Raider and Target claims for a zero sum merger

Let $\frac{\Delta W}{MC_{Target}}$ measure the percentage change in the market capitalization of all Target shares. When $MC_{Conglomerate} = MC_{Target} + MC_{Raider}$ then the internal wealth transfer *W* distributes to Target and Raider shareholders like this:

$$\left(\frac{\Delta W}{MC_{Conglomerate}}\right)^{-1} = \left(\frac{\Delta W}{MC_{Target}}\right)^{-1} + \left(\frac{\Delta W}{MC_{Raider}}\right)^{-1}.$$

Each right-hand-side term is absolute value of the *reciprocal* of the percentage change in market cap.

For Example 1 the Raider market cap is down nearly 12% whereas the Target shareholders get an 8% increase in wealth. Identity 2.1 requires that:

$$4.75\%^{-1} = 7.96\%^{-1} + 11.72\%^{-1}.$$

Compute the reciprocal of each number (i.,e, $4.75\%^{-1}$ = 1/.0475 = 21.09) and see

$$21.09 = 12.56 + 8.53.$$

Basic math requires that when the whole is the sum of the parts and there is an internal wealth transfer then the percentage changes for each part satisfy Identity 2.1.

Insert Exercises 2.2A here. Click to view.

A2. Assets are financing uses

The balance sheet's left-hand side shows the assets that the company owns. It is conventional to separate assets into two groups. *Current assets* include ones that tend to be liquidated or sold within the near term. *Long term assets* include items generally owned by the firm for longer periods of time. Discussion below elaborates on the more important asset accounts.

Current Assets

The *Cash* line item includes funds in the checking and savings account, and possibly other short-term demand deposits or liquid interest-earning securities. Quite often, a separate line item labeled *Marketable securities* may appear on the balance sheet. *Inventories* represent items the company uses for making goods and services. Perhaps there are inventories in many different stages of progress. There may be, for example, unfinished goods as well as finished goods. Regardless, the balance sheet entry for *Inventory* lists the funds that already the company has used for purchasing and finishing the goods; the value of inventory includes product costs but does not include hopeful profits.

Receivables represent sales for which money remains uncollected. Many businesses find that issuing credit to customers increases the client base. When a credit sale is made, the customer effectively signs an IOU to the company that legally defines the customer's obligations regarding eventual payment. Until paid, however, credit sales remain on the balance sheet as *Receivables* because they represent wealth that the company owns.

Long term Assets

The most significant long term asset that a company typically owns is its buildings, factories, furniture, computers, etc. These fixed assets are measurable and their aggregate value goes on the balance sheet as "Net Plant, Property, & Equipment", referred to as *PP&E*. *PP&E* changes when old assets lose value or new assets are acquired. There are many well-known problems with assessments of *PP&E*. Consider the difficulties for the simpler problem of measuring the value of a house and all of its contents. When one gets down to the nitty-gritty, there are many different defensible ways for defining a used asset's value and, without selling it, there are very few ways for verifying such a valuation estimate. For the more complex problem of valuing an entire company, vagaries abound.

IBM lists a line item, *Other Long Term Assets*, that actually is larger than *PP&E*. These other assets predominantly include the capitalized value of long term leases that IBM makes available to its customers. This line item illustrates the basic yet important relation between leases and the balance sheet. Suppose that a university wishes to use a large, $12 million mainframe computer made and sold by IBM. The university may not be in a position to purchase outright such an expensive machine. IBM finds, like many other companies, that the customer base expands by making leases available to customers. The customer signs a long term contract in which they agree to pay IBM for usage of the machine throughout the next 3 or 5 years. Before leased, the unsold mainframe computer appears on the IBM balance sheet as *Inventory*. While leased, however, the asset really is not inventory. Yet IBM definitely owns something — they own a university's commitment to provide IBM with future cash flows. The value of the lease on the balance sheet relates directly to the future cash flows the contract promises to deliver to IBM.

Net Working Capital

Financial statements sometimes list a line item, net working capital ("NWC"), as an asset on the balance sheet. NWC actually represents the difference between current assets and current liabilities:

FORMULA 2.4 Net working capital
Net working capital equals current assets minus current liabilities and is an indicator of the company's short term financial solvency.

Net working capital = Current assets – Current liabilities.

An increase in any asset represents a use of funds, whereas an increase in a liability represents a source of funds. Typically with a growing company both current assets and current liabilities increase over time. There consequently is an increase in net working capital.

Insert Exercises 2.2A here. Click to view.

2.B. The Income Statement

An income statement summarizes many types of flows that occur throughout a specific time period. The time period may be a month, quarter, year, etc. A quarterly income statement reflects activity occurring only during the specified quarter, for example, whereas an annual income statement reflects activity for a specified year. An annual income statement could be constructed by summing line-by-line all items on 4 consecutive quarterly income statements.

The table below shows an income statement for the IBM Company. The actual statement in IBM's annual report is very complicated because IBM is a huge, complex company. The table shows a condensed version that summarizes the main line items from the income statement. Notice the income statement clearly shows that the relevant unit of time is a calendar year. This statement summarizes activity occurring between January 1st and December 31st of an historical year.

Sales	$76,654
– Cost-of-goods sold	46,815
– Selling, general, and administrative expenses	16,854
– Depreciation	3,676
= Operating income (EBIT)	$9,303
– Interest expense	716
= Taxable income	$8,587
– Taxes	3,158
= Net income	$5,429
– Dividends	706
= New retained earnings	$4,723

TABLE 2.5 International Business Machines Corporation and Subsidiary Companies, Consolidated income statement, January 1 — December 31, historical snapshot.

This income statement documents all wealth transfers occurring during the year. The balance sheet that the previous section discusses is like a snapshot taken at an exact moment, one snapshot when the income statement starts and another snapshot when it concludes. The discussion below explains several significant line items from the income statement.

Sales represents the company's only cash inflow on the above statement. Above all else, a company must make sales to survive. Financial statements often list *Net* *revenues* instead of *Sales*; *Net revenues* equals *Sales* minus returned items and discounts. All lessons in this book refer to the top-line as *Sales*. The *Cost-of-goods sold* represents the company's primary cash outflow associated with production costs, while *Selling, general, and administrative expenses* represent periodic overhead expenses such as advertising, marketing analyses, coffee and copying costs, etc. Generally, these expenses are realized cash flows representing payments to stakeholders for factors of production.

Depreciation represents a cost related to Plant, Property, and Equipment ("*PP&E*"). Notice that the income statement does not show a line item associated with purchase of *PP&E*. The company, nonetheless, often purchases *PP&E*. Instead of subtracting from *Sales* the full purchase price of *PP&E*, however, the company only subtracts *Depreciation*: an amount roughly equal to the periodic decline in the value of *PP&E*. *Depreciation* is an accrued cash flow (that is, a cash flow unaccompanied by a flow of funds), also sometimes called a non-cash charge.

As the statement shows, *Operating income* equals *Sales* minus the traditional costs of business. Financial statements sometimes list *Operating revenues* instead of *Operating income* but these two are synonyms. Financial analysts sometimes apply the acronym *EBIT* to *Operating income*; *EBIT* stands for "earnings before interest and taxes." The items beneath *Operating income* are outflows pertaining to financing costs and taxes. *Interest* is the cost paid for borrowing money from creditors. Notice that the income statement does not show as an inflow the principal a company borrows. Neither does the income statement show the principal that a company repays. Only shown is *Interest*, the amount repaid in excess of principal.

Subtract *Interest* from *Operating income* to compute *Taxable income*. This implies that *Interest*, just like *Depreciation* and *Cost of-goods sold*, is a tax deduction; every dollar of *Interest* reduces *Taxable income* by one dollar. *Interest* and *Depreciation* are two of the larger ways that businesses shield income from taxes: *Interest* and *Depreciation* are tax shields.

The *Taxes* that the business owes the federal government are based on tax tables contained in laws passed by the U.S. Congress. Congress delegates responsibility for collecting taxes to the Internal Revenue Service, a division in the Treasury Department. The tax rate often exceeds 30%, which implies that *Taxes* often are a fairly significant cash outflow for business. Sometimes, in fact, a business finds it easier to increase profits through thoughtful tax planning rather than changes in production or pricing policies.

Subtract *Taxes* from *Taxable income* in order to compute *Net income*. Sometimes financial statements also refer to *Net income* as "earnings after taxes." *Net income* represents wealth available for shareholders. Shareholders take possession of *Net income* either of two ways. First, some *Net income* might be paid directly to shareholders as a dividend. Companies record shareholder mailing addresses and typically pay out dividends quarterly. Second, some *Net income* might be kept by the business in order to finance growth and operations. The funds kept by the business, that is, *Net income* minus *Dividends*, equals *New retained earnings*. Sometimes *New retained earnings* is called "additions to retained earnings", or "internal financing", or "plowback", because it represents funds accessible to the corporation without explicitly borrowing from financial markets.

Insert Exercises 2.2B here. Click to view.

3. Financial ratios

A financial ratio compares in fraction form two different financial measures. Ratios typically convey important information about financial health. Contrast the insight gleaned about financial health in the following illustration. Suppose that two companies in the same line of business each have annual sales of $100,000. This fact conveys information about company size but little else. Suppose you learn that the "asset turnover ratio" equals five for the first company and ½ for the second company. The definition of the asset turnover ratio is *Sales* divided by *Total assets*. Knowing that the asset turnover ratio for the first company is five implies the company generates $5 of sales for every $1 of assets. The second company's asset turnover ratio of ½ implies a half-dollar of sales per dollar of assets. Even though the *Sales* figures for these two companies may be equal and suggestive of similar health, inspection of the asset turnover ratio suggests extremely different situations.

Ratio analysis is extremely important in the real world, even for you. When you apply for a personal bank loan, for example, the loan officer generally analyzes several of your ratios in order to decide whether or not to authorize the loan. The loan officer computes, for example, the ratio of monthly fixed expenses divided by monthly income. She then compares your ratio to a standard declared by the bank's policy committee. If your ratio exceeds the standard then you do not qualify for the loan because your expenses are too high relative to income. Alternatively, if your ratio is low then the bank determines that your financial health is good and you qualify for the loan.

3.A. Ratio categories

There are five broad categories of financial ratios. The table below lists well-known ratios in each category and offers a few anecdotal examples.

Ratio category	**ratio name and formula**	**company examples (historical snapshot)**
liquidity ratios	*current ratio* current assets / current liabilities	Coca Cola 0.7 IBM 1.2 GAP, Inc. 1.3
	quick ratio (cash + marketable securities + receivables) / current liabilities	Southern Co. 0.4 Gateway, Inc. 1.3 Daimler Chrysler 1.4
	times interest earned (pretax income + total interest expense) / total interest expense	Anheuser-Busch 7.9 Ann Taylor Stores 10.8 Men's Wearhouse 38.8
debt management ratios	*debt to capital* debt due more than a year later / (debt due more than a year later + stockholders' equity)	Pier One 5% Honda Motor Co. 23% Georgia Pacific 57%
	debt to equity total liabilities / stockholders' equity	Qualcomm, Inc. 0.1 3M Co. 1.2 Quaker Oats 10.1
	debt to assets total liabilities / total assets	Qualcomm, Inc. 9% 3M Co. 55% Quaker Oats 91%
	equity multiplier total assets / stockholders' equity	Qualcomm, Inc. 1.1 3M Co. 2.2 Quaker Oats 11.1
profitability ratios	*gross margin* (sales – cost of goods sold) / sales	Halliburton Co. 10% Best Buy Co. 16% Litton Industries 22%

	operating margin (sales – total operating expenses) / sales	7-Eleven, Inc. 1% Hertz Corp. 9% AG Edwards 21%
	net margin net income / sales	Walt Disney, Inc. 5% General Mills, Inc. 9% Bristol-Myers Squibb 21%
	return on assets net income / total assets	Brunswick Corp. 1% Black & Decker 7% T. Rowe Group 18%
	return on equity net income / stockholders' equity	Viacom, Inc. 3% Chevron Corp. 12% Proctor & Gamble 33%
	payout ratio dividends / net income	Circuit City 8% Wal-mart 16% Electronic Data Systems 43%
	retention ratio new retained earnings / net income	Circuit City 92% Wal-mart 84% Electronic Data Systems 57%
asset management ratios	*asset turnover* sales / total assets	Lucent Technology 0.7 Goodyear Tire & Rubber 1.0 Office Depot, Inc. 2.4
	inventory turnover annual cost-of-goods sold / inventory	Dollar General 3.9 Intel 15.0 Six Flags, Inc. 39.3
	average age of inventory 365 x inventory / annual cost-of-goods sold	Dollar General 94 days Intel 24 days Six Flags, Inc. 9 days
	average collection period 365 x receivables / annual sales	
	average payment period 365 x payables / annual cost-of-goods sold	
market-based ratios	*price-to-book* stock price / equity book value per share	Saks, Inc. 0.88 Phelps Dodge, Inc. 1.17 Texas Instruments 5.94
	price-to-earnings stock price / net income per share	Ford Motor Co. 9.8 Caterpillar, Inc. 14.7 Clorox Corp. 20.9
	price-to-cash flow stock price / {(net income + depreciation) ÷ # shares}	Sears, Roebuck, & Co. 5.2 Bear Stearns Co. 12.0 International Speedway 44.3
	price-to-free-cash flow stock price / {(net income + depreciation – dividends – capital expenditures) ÷ # shares}	General Motors, Inc. 6.6 Service Master Co. 15.6 Johnson Controls, Inc. 49.0
	price-to-sales stock price / sales	Boeing Co. 0.9 Harley-Davidson, Inc. 4.6 Applied MicroCircuits 50.0

	dividend yield annual dividend per share / stock price	Motorola Inc. 0.8% E.I. Dupont de Nemours 3.3% Host Marriott 8.1%
	stockholders' rate-of-return (ending stock price + dividend per share – beginning stock price) / beginning stock price	Walt Disney -38% Exxon Mobil Co. 6% Pfizer Inc. 139%

TABLE 2.6 Ratio categories, formulas, and company examples

For some ratios the numerator and denominator are from the same financial statement. The current ratio, for example, equals current assets divided by current liabilities. Both obviously are from the same balance sheet. The net profit margin divides net income by sales and both of these obviously are from the (same) income statement. Consider the asset turnover ratio, however: divide sales from the income statement by total assets from the balance sheet. But which balance sheet? There is one balance sheet at the beginning of the income statement's horizon and another one at its end. For any financial ratio that relates a flow with a balance three plausible definitions exist. Consider three widely used definitions for asset turnover:

$$\frac{Sales_t}{Total\ assets_{t-1}} \qquad \frac{Sales_t}{Total\ assets_t} \qquad \frac{Sales_t}{\left(Total\ assets_{t-1} + Total\ assets_t\right)/2}$$

Suppose that *Sales* is from the Income statement for year 2525. The first formula divides with *Total assets* from the year-end 2524 balance sheet. The middle formula divides with *Total assets* from the year-end 2525 balance sheet. The rightmost formula divides by average *Total assets*. All three formulations are defensible definitions. Different books quite often use different formulas for financial ratios. The point is to exercise care and awareness when comparing a flow with a balance. In order to eliminate ambiguities this book always presents time subscripts for ratios that relate a flow with a balance.

The discussion below elaborates on general importance of the five ratio categories.

Liquidity ratios enable insights about the ability to pay bills as they come due. Perhaps you have heard the phrase "land rich but cash poor." This phrase sometimes applies to businesses or households that own a lot of long term assets (in this way they are "rich"), but they don't have enough money to pay the bills (this is the "poor" part). The balance sheet for this unfortunate concern probably lists ample assets for *PP&E* or *Land*, but very little for current assets such as *Cash* or *Marketable securities*. This household or business has a "liquidity" problem.

It is relatively difficult and time-consuming to convert *PP&E* or *Land* into a form useful for paying bills; such long term assets are "illiquid." *Cash* or *Marketable securities*, on the other hand, are very liquid. An individual or business with a lot of liquid assets might only be a phone call away from obtaining cash for paying bills.

Liquidity ratios often focus on the relation between *Current assets* and *Current liabilities*. For example, the "current ratio" equals *Current assets* divided by *Current liabilities* (recall that *Net working capital* equals *Current assets* minus *Current liabilities*; the current ratio conveys a relative sense of scale that NWC does not convey). A high current ratio implies the company is likely to have sufficient cash on hand throughout the near term for paying bills. A reasonably high liquidity ratio is, generally speaking, a sign of good short term health.

Debt management ratios enable insights about the company's ability to take out new loans. Businesses sometimes face unexpected crises, or perhaps opportunities, and effective handling of the situation might require borrowing money. "Excess debt capacity" refers to the amount of additional debt that firms can safely borrow without endangering financial health. A company with zero excess debt capacity is already stretched to the limit — they have not managed their debt very well — and any unexpected crisis might push them into bankruptcy.

Debt management ratios often focus on the amount of debt relative to assets or net worth. A large debt-to-equity ratio, for example, implies the company already has a lot of debt, and perhaps not much excess debt capacity remains. When the debt-to-equity ratio gets too high, the company's financial health is in jeopardy. When the debt-to-equity ratio is too low, conversely, the company may be making itself too attractive as a take-over target to a raider looking to gobble up heaps of excess borrowing capacity.

Many lessons in corporate finance discuss debt ratios. Often it is convenient to recall that three unique ratios in the debt management category are algebraic reformulations of each other: "debt-to-assets" equals *Total liabilities* divided by *Total assets*; "debt-to-equity" equals *Total liabilities* divided by *Stockholders' equity*; and "equity multiplier" equals *Total assets* divided by *Stockholders' equity*. Given a number for any one of these three ratios then the other two must take on specific values as shown below:

FORMULA 2.5 Alternative specifications for debt ratios

$$equity\ multiplier = 1 + (debt - to - equity)$$

$$equity\ multiplier = \frac{1}{1 - (debt - to - assets)}$$

$$(debt - to - equity) = \frac{(debt - to - assets)}{1 - (debt - to - assets)}$$

Suppose, for example, the debt-to-assets ratio is 0.40. The debt-to-equity ratio must therefore equal 0.67 and the equity multiplier must equal 1.67.

Profitability ratios enable insights about the company's pricing or cost containment policies. Profitability ratios are large when the gap between sales revenues and production costs is large. For example, the "net profit margin," defined as *Net income* divided by *Sales*, measures net income per dollar of sales. Managers probably prefer the largest possible profit margin for the business. The ratio rises by increasing sales revenue while holding constant costs, or equivalently, by decreasing costs while holding constant sales revenues. Several sources exert downward pressure on profit margins. First, customers want the lowest possible prices. Second, stakeholders and creditors want the most from the company they can get. The previous chapter discusses the goal of the firm and suggests that perhaps the most essential yet difficult managerial responsibility is balancing revenues and costs.

Asset management ratios enable insights about the efficiency of operations. The "asset turnover ratio" (= *Sales* ÷ *Total assets*) is one of the more important ratios in this category. The asset turnover ratio is analogous to the amount of "blood squeezed from stone" – the blood is the sales, and the stone is the assets. For a given bundle of assets, call it a pile of bricks and mortar, the company prefers to extract as many sales as possible. It is possible, of course, to run assets so intensely in pursuit of short-run sales that long-run prospects are damaged. As almost every student knows, if you run an engine too fast for too long it is more likely to break too soon. The same goes for factories and workers!

Market-based ratios often relate the company's common stock price to an item from the balance sheet or income statement. The most common include: price-to-book (*P/B*), price-to-earnings (*P/E*), price-to-sales, and price-to-cash flow (*P/CF*). A previous section discusses *P/B*, and the next chapter looks at *P/CF*. The *P/E* ratio equals the stock price divided by earnings per share ("*eps*"). A company's *eps* simply equals net income from the income statement divided by the number of common shares outstanding. The financial press focus on *eps* because it represents the profit (that is, earnings) per share of common reported by the company.

There is, as the table below shows, substantial variation in market-based ratios.

Corporation name	P/B	P/E	P/Sales	P/CF
AOL-Time Warner, Inc.	17.0	108.6	14.2	72.4
AT&T Corporation	0.8	24.4	1.3	4.8
Exxon Mobil Corporation	4.2	17.9	1.2	13.6
Ford Motor Company	2.5	9.8	0.3	3.4
General Electric Company	9.6	36.4	3.5	23.7
General Motors Corp.	1.0	9.1	0.2	2.9
IBM	9.4	24.7	2.2	24.8
Microsoft Corporation	7.0	33.6	13.6	33.8
Pfizer Inc.	17.3	76.1	9.5	63.2
Wal-Mart Stores, Inc.	8.4	39.1	1.3	26.8
Walt Disney Company	2.7	53.9	2.5	11.3

TABLE 2.7 Market-based financial ratios for American corporate icons.
P/B is price-to-book, P/E is price-to-earnings, and P/CF is price-to-cash flow from operations. Historical snapshot.

In all cases the ratio measures the equity market valuation for one dollar of the variable in the denominator. The *P/E* is 76.1 for Pfizer pharmaceuticals, for example, and 9.1 for General Motors. A dollar of earnings may be "purchased" in the stock market for $9.10 if the company is General Motors, but the dollar of earnings costs $76.10 from Pfizer. The rumor on the street is that a high *P/E* ratio signifies stock overvaluation. A low *P/E* signifies, some argue, a bargain stock. As the chapter on stock valuation explains, however, there are a lot of legitimate reasons why *P/E* ratios vary from company to company, and most of those reasons shed no insight on whether a stock is under or overvalued.

The example below combines the balance sheet with several financial ratios in order to find the stockholders' expected rate of return over the next year. The stockholder rate of return formula appears in the bottom row of table 2.6, reprinted below:

FORMULA 2.6 Shareholders' rate of return

The shareholder rate of return ("ROR") for time period *t* represents the percentage change in wealth and equals the stock price change plus dividend relative to beginning price.

$$\text{shareholders' } ROR_t = \frac{P_t + \text{dividend per share}_t - P_{t-1}}{P_{t-1}},$$

where P_t is the stock price at end of period *t.*

EXAMPLE 2 Find the stockholders' rate of return given ratios and balance sheet

At year-end 2525 the company has Total assets of $4,900 financed by Debt of $2,400 and Stockholders' equity of $2,500. For 800 common shares outstanding, the equity price-to-book ratio at year-end 2525 is 1.8. During 2526, the company expects an asset turnover ratio (= $Sales_t \div Total\ assets_{t-1}$) of 3.5 and an operating margin of 16%. Interest charges will equal 12% of Debt. Corporate taxes equal 34% of taxable income and the payout ratio always is 40%. Your analyst tells you that at year-end 2526 the company price-to-earnings ratio will equal 7. Find the shareholders' rate of return for year 2526.

SOLUTION

We must find all information for computing the shareholder rate of return ("ROR") from formula 2.6. Find first the stock price at the beginning of the horizon, that is, at year-end 2525. Use formulas 2.1 and 2.2 for P/B and equity book value per share:

$$1.8 = (\ price\ per\ share_{2525}\ )\ /\ (\$2,500\ /\ 800)$$

$$(\ price\ per\ share_{2525}\ )\ =\ \$5.62$$

The shareholder owns a stock at end-of-year 2525 worth $5.62. During year 2526 she expects to receive a dividend plus a capital gain equal to the change in stock price.

Use the ratios from the problem to construct the income statement for year 2526:

sales	$17,150	(=$4,900 x 3.5)
- total operating costs	14,406	(=$17,150 x (1 – 0.16))
= operating income	2,744	(=$17,150 x 0.16)
- interest	288	(=$2,400 x 0.12)
= taxable income	2,456	
- taxes	835	(=$2,456 x 0.34)
= net income	1,621	
- dividends	648	(=$1,621 x 0.40)
= new retained earnings	973	

All line items are straightforward. The 800 common shares outstanding receive total company dividends of $648, implying a dividend per share of $0.81. Earnings per share ("eps") equals net income divided by number of shares, thereby implying

$$(P/E)_t = price_t\ /\ eps_t$$

$$= price_t\ /\ (net\ income\ /\ \#shares)_t\ .$$

The P/E for year-end 2526 relates to eps as:

$$7 =\ price_{2526}\ \ /\ (\$1,621\ /\ 800)$$

$$price_{2526}\ =\ \$14.18$$

The stock at year-end 2526 garners a dividend of 81 cents and has a price of $14.18. All information needed for computing the shareholder ROR is ready for substitution into formula 2.6 :

$$shareholder's\ ROR_{2526} = \frac{\$14.18 + \$0.81 - \$5.62}{\$5.62}$$
$$= 167\%$$

The scenario describes a fairly hefty rate of return for shareholders equal to 167% for year 2526!

For the special case when the number of shares is constant over the reporting period then another approach obtains the identical answer as above. Multiply the right-hand-side by (#shares ÷ #shares), which really is the same as multiplying by one and thus the answer is still 167%. $Price_t$ times #shares equals $Market\ cap_t$ and $Dividend_{2526}$ times #shares equals $Net\ Income_{2526}$. Thus,

$$shareholder's\ ROR_{2526} = \frac{Market\,cap_{2526} + Dividend_{2526} - Market\,cap_{2525}}{Market\,cap_{2525}}$$
$$= \frac{(7 \times \$1621) + \$648 - (1.8 \times \$2,500)}{1.8 \times \$2,500}$$
$$= 167\%\,.$$

When the number of shares is constant then the *ROR* for one share is the same as for all shares!

EXERCISES 2.3A

Conceptual

1. Identify the ratios in table 2.6 that relate a flow with a balance and so are subject to alternative definitions differing only because of time subscripts.

2. Your company's latest financial statements list annual *Net income* of $132,500 and *Stockholders' equity* of $980,000. On last year's balance sheet *Stockholders' equity* was $905,000. Your boss informs you that the return-on-equity ("*ROE*") for this company's peer group equals 14.0%. Find whether your company's *ROE* is larger or smaller than its peer group's *ROE*.

Numerical quickies

3. The company balance sheet lists *Total liabilities* of $85,000 and *Stockholders' equity* of $114,000. Find the company equity multiplier, debt-to-assets, and debt-to-equity ratios. ©FA11

4. Today the company announces net income equals $42 million. They have 20 million shares outstanding, and today's share price is $136.80 . Find the company's price-to-earnings ratio. ©FA8

5. The company reports that sales equal $129,000 and the net profit margin is 6.4%. How much is net income? ©FA9

6. During year 2526 company expects sales of $32,000 and a gross margin of 26%. Depreciation is expected to equal $1,030 and interest charges will equal $2,500. Corporate taxes equal 34% of taxable income. What is net income for 2526? ©FA12

7. How much was the company's *Total assets* if they just announced earnings per share

of $1.25, there are 2,500 shares outstanding, the net profit margin is 11.5%, and the asset turnover ratio (= $\textit{Sales}_t \div \textit{Total assets}_t$) is 3.6? ©FA13

Challengers
8. Shareholders had a good year, earning a 32% annual rate of return. The P/E ratio today is 26.4 and the company just announced earnings per share of $4.50. The company has a 35% payout ratio. How much did the stock price rise over the past year? ©FA14

9. The company last year had annual cost-of-goods sold equal to $50,000 and a 9.5 inventory turnover ratio (= $\textit{Annual cost-of-goods sold}_t \div \textit{Inventory}_t$). The company wants to decrease the average age of inventory by 5 days. If everything else remains exactly the same, how much is the resultant source of funds? ©EFN7

Problems 10-12 refer to this setup
At year-end 2525 the company has Total assets of $6,400 financed by Debt of $2,600 and Stockholders' equity of $3,800. For 325 common shares outstanding, the equity price-to-book ratio at year-end 2525 is 3.60. During 2526, the company expects an asset turnover ratio (= $\textit{Sales}_t \div \textit{Total assets}_{t-1}$) of 3.2 and an operating margin of 12%. Interest charges will equal 8% of Debt. Corporate taxes equal 30% of taxable income and the payout ratio always is 35%. Your analyst tells you that at year-end 2526 the company price-to-earnings ratio will equal 11½.

10. What is net income for 2526? ©FA15a

11. What is the expected stock price at year-end 2526? ©FA15b

12. What is the shareholders' rate of return for year 2526? ©FA15c

3.B. Ratio Relationships

Interpreting financial ratios is art as much as science. There are several elementary situations, though, when the importance of ratio analysis is apparent. The discussion below explains ratios within these useful contexts.

B1. Ratio norms

Financial ratios are especially useful in either a "time series" or "cross-sectional" analysis. A time series analysis examines movement over time in a ratio. The table below shows the operating margin for Home Depot, Inc. throughout a six-year window:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
7.6%	7.8%	7.8%	8.8%	9.9%	9.8%

TABLE 2.8 Time series of operating margin for Home Depot, Inc.
Each entry reflects activity for the fiscal year ending in January of the respective column.

The steady increase in operating profit margin may lead Home Depot management to conclude that price-cuts are allowable because costs seem under control. Likewise, an external analyst may conclude that Home Depot stock merits a buy recommendation because the company looks healthy.

A cross-sectional analysis compares for a specific time period one company's ratio to other companies or averages. Table 2.9 shows a cross-section of operating margins for "retail home improvement" companies at the end of year 6.

Ag Services of America	Building Materials Hldg.	Calloway's Nursery Inc.	D.I.Y. Home Warehouse	Home Depot	HomeBase, Inc.	Lowes, Inc.	Wickes Inc.
3.7%	4.5%	5.4%	-9.0%	9.8%	-1.3%	7.2%	2.8%

TABLE 2.9 Cross-section of operating margins for companies in the retail home-improvement sector at "Year 6" of Table 2.8.

The operating margin is substantially larger for Home Depot than its competitors. This implies that Home Depot's costs are small relative to revenues. Perhaps management of its competitors should sneak into the nearest Home Depot and glean insight on their operations.

Ratio averages differ by industry. Table 2.10 lists the average for selected ratios and industries. The current ratio, which equals *Current assets* divided by *Current liabilities*, ranges from 0.6 for Fast Foods to 2.1 for Department Stores. This is consistent with the observation that fast food businesses maintain little inventory compared to department stores. The high debt-to-equity ratio for Fast Food indicates heavy reliance on debt financing. Once again this contrasts starkly with Department Stores, which have the lowest debt-to-equity ratio. Grocery Stores possess a very high asset turnover ratio. Perhaps this is not surprising because some supermarkets that are little more than a warehouse generate huge amounts of sales. This contrasts with Electric Services, where multibillion dollar power plants generate relatively little revenue. Profit margins vary widely, from about 1% for grocery stores to over 10% for Electric Services.

	LOW	MEDIUM	HIGH
Liquidity category Current ratio	Fast Food (0.6) Electric Services (0.9) Mortgage Bankers (1.2) Petroleum (1.3) Groceries (1.3)	Soft Drinks (1.4) Motor Vehicles (1.4) Brokers (1.5) Steel (1.5)	Computers (1.8) Clothing (1.9) Department Stores (2.1)
Debt Management Category Debt-to-Equity	Department Stores (1.0) Brokers (1.5) Clothing (1.6)	Steel (1.7) Computers (1.7) Petroleum (1.8) Electric Services (2.1) Groceries (2.3)	Soft Drinks (2.5) Motor Vehicles (2.7) Mortgage Bankers (3.2) Fast Food (3.7)
Asset Utilization Category asset turnover	Mortgage Bankers (0.4) Electric Services (0.5) Brokers (0.7)	Steel (1.9) Computers (2.0) Soft Drinks (2.1) Department Stores (2.1) Clothing (2.4) Petroleum (2.5) Motor Vehicles (2.7)	Fast Food (3.3) Groceries (5.8)
Profitability Category Net profit margin (%)	Groceries (1.0) Clothing (1.2) Motor Vehicles (2.0) Department Stores (2.4) Computers (2.7)	Petroleum (3.0) Soft Drinks (3.3) Fast Food (3.3) Steel (4.2)	Mortgage Bankers (8.2) Brokers (10.0) Electric Services (10.5)

TABLE 2.10 Average values for important financial ratios in different industries.
Historical snapshot, compiled by author.

B2. Return on equity and the DuPont analysis

The return on equity ("ROE"), a profitability ratio, is a key financial ratio for shareholders. Obviously shareholders care about stock prices, but there is a lot about stock prices beyond anyone's control. The company's books, on the other hand, are under management's control, and ROE is the book measure of shareholder returns.

The ROE equals *Net income* divided by *Stockholders' equity*. The denominator equals the book value of stockholder claims on the company. The numerator equals the earnings available to shareholders. The ROE consequently represents the rate of return, according to the books, that shareholders earn. Notice that because ROE is the ratio of a flow to a balance several alternative definitions exist that differ only by time subscripts.

The shareholders' ROR from formula 2.6 represents the rate of return based on prices from the stock market. Glean important insights on the relation between ROE ($NI_t \div SE_{t-1}$) and shareholders' ROR for the special case when the equity P/B ratio is constant.

FORMULA 2.7 Numerical Relation between ROE and shareholders' ROR when P/B is constant

The shareholder rate of return ("ROR") for time period *t* relates to the reported return-on-equity ("ROE") as shown below only when the equity P/B ratio is constant and the company does not issue any new shares:

$$shareholder's\ ROR_t = \left\{ 1 - \frac{\begin{pmatrix} payout \\ ratio \end{pmatrix} \left(P/B - 1 \right)}{P/B} \right\} ROE_t \ .$$

The formula simplifies even further for two limiting cases. Inspect the formula and notice that if always P/B were unity then always shareholders' ROR would equal book ROE . The rate of return according to the books equals the market rate of return when P/B equals 1! Next, suppose that always the company pays out 100% of net income as dividends. Simplify the formula to see:

$$shareholder's\ ROR = ROE \left(P/B \right)^{-1}$$

The shareholders' ROR equals ROE times the reciprocal of P/B.

For a given ROE, a relatively small P/B ratio requires a relatively large ROR. Vice versa, a large P/B requires a smaller ROR. With a 14% ROE (and 100% payout ratio), for example, the shareholder ROR is doubled (28%) for a company with P/B of ½, and ROR is halved (7%) for a company with P/B of 2. The P/B ratio inversely amplifies stockholder rates of return. The inverse amplification occurs because P/B measures the relative size of the equity base that receives net income – small P/B implies big ROR because net income distributes over a relatively small equity base (and vice versa). We return to this issue later, after studying the connection between *Stockholders' equity* and the income statement.

The findings easily summarize into a rule that always is true when P/B is constant.

RULE 2.2 Qualitative relation between ROE and shareholders' ROR when P/B is constant

The shareholder rate of return ("ROR") for time period *t* relates to the reported return-on-equity ("ROE") as shown below for the limiting case when the equity P/B ratio is constant and the number of common shares is constant:

$$shareholders'\ \ ROR_t \quad \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} \ ROE_t \ .$$

$$whenever \qquad P/B_t \quad \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \ 1.0$$

The P/B ratio inversely amplifies stockholder rates of return – small P/B implies big ROR because net income distributes over a relatively small equity base (and vice versa).

In realistic settings with noisy financial markets the P/B ratio is not constant. The motion in P/B ratios is rich with information content. The measurable content qualitatively shifts the numerical relation between ROR and ROE but the qualitative inverse relation toward the P/B ratio is a structural foundation. The example below shows a simple application of formula 2.7.

EXAMPLE 3 Find the shareholders' ROR given expected ROE and constant P/B

Company X reports that next year they expect a 22% return-on-equity. Company Z expects exactly the same ROE. Both companies also have a 40% dividend payout ratio. You figure the best assumption is that the equity P/B ratios are likely to stay constant. The current P/B equals 0.75 for company X and 2.25 for company Z. Find the expected shareholder rates of return for each company.

SOLUTION

Substitute the numbers for company X into the formula:

$$shareholder's\ ROR = \left\{1 - \frac{0.40\,(0.75 - 1)}{0.75}\right\} 0.22$$

$$= 24.9\%$$

The market rate of return for shareholders in company X includes dividends plus stock price capital gains and equals 24.9%. A similar computation for company Z shows that their shareholder rate of return equals 17.1%. Even though the accountants proclaim the two companies provide equal returns on equity, differences in P/B ratios translate into almost an eight percentage-point gap in stockholder rates of return.

The ROE varies substantially across time or across companies. The "DuPont analysis" is a technique that enables insight about the source of variation in ROE. The DuPont analysis partitions the ROE into three components:

FORMULA 2.8 The DuPont decomposition

The DuPont decomposition breaks the ROE into three components:

$$Return-on-equity = \frac{Net\ income}{Stockholders'\ equity}$$

$$= \left(\frac{Net\ income}{Sales}\right)\left(\frac{Sales}{Total\ assets}\right)\left(\frac{Total\ assets}{Stockholders'\ equity}\right)$$

$$= (net\ profit\ margin)(asset\ turnover)(equity\ multiplier)$$

The ROE is the ratio of a flow to a balance so the caveat about timing raised in section 3A applies. That is, some applications may define ROE as $Net\ income_t \div Stockholders'\ equity_t$ whereas others may define as $Net\ income_t \div Stockholders'\ equity_{t-1}$, etc.

The DuPont analysis decomposes the ROE into the multiplicative product of three financial ratios. The "net profit margin" is from the profitability category, the "asset turnover ratio" is from the asset management category, and the "equity multiplier" is from the debt management category. We obtain an advantage by computing the ROE as the product of these three ratios, even though we certainly obtain the identical answer if we compute ROE directly as *Net income* divided by *Stockholders' equity*. The advantage occurs, as the example below illustrates, because the decomposition helps identify the source of variation in the ROE.

EXAMPLE 4 Apply the Dupont analysis to IBM

Use a DuPont analysis to explain why IBM's return-on-equity differs from the industry average.

SOLUTION

The IBM balance sheet (table 2.2) and income statement (table 2.4) contain the numbers to plug into the DuPont formula:

$$Return-on-equity = \left(\frac{Net\,income}{Sales}\right)\left(\frac{Sales}{Total\,assets}\right)\left(\frac{Total\,assets}{Stockholders'\,equity}\right)$$

$$= \left(\frac{\$5{,}429}{\$76{,}654}\right)\left(\frac{\$76{,}654}{\$81{,}132}\right)\left(\frac{\$81{,}132}{\$21{,}628}\right).$$

$$= (7.1\%)(0.9)\,(3.8)$$

$$= 25.1\%$$

The DuPont analysis for the industry depends on numbers for the computer industry contained in table 2.10. Notice the table does not present the ROE. It presents, however, the industry net profit margin (2.7%), the asset turnover ratio (2.0), and debt-to-equity ratio (1.7). According to formula 2.5, the equity multiplier equals one plus the debt-to-equity ratio, we write for the industry that:

$$ROE = (2.7\%) \quad (2.0) \quad (1.7 + 1)$$

$$= \quad 14.6\%.$$

The ROE is 25.1% for IBM and 14.6% for the computer industry. The relatively high ROE for IBM stems from two sources. First, IBM's profit margin is much larger than the industry average (7.1% vs. 2.7%), suggesting that IBM either controls its costs very effectively or receives a premium price for its product. Second, IBM relies on debt more than the average computer company (3.8 vs. 2.7). The reliance on debt amplifies the ROE. It is interesting to point out that even though IBM's ROE is relatively high, its asset turnover ratio is much smaller than the industry average (0.9 vs. 2.0). If IBM could increase their productive efficiency up to the norm, its ROE would be even higher.

The DuPont analysis reveals one of the more important lessons in finance: *Leverage amplifies the ROE*. The equity multiplier really should be called a debt multiplier since the bigger the debt ratio the more amplified is the ROE.

The principal of leverage amplifying returns illustrates easily. Consider the scenario in which a $100,000 asset instantly increases 50% in value to $150,000. If the debt-to-total assets ratio were zero, meaning that equity was the exclusive source of purchasing the asset, then the ROE equals the profit of $50,000 divided by the equity investment of $100,000 and is 50%. With one asset and one financing source, namely equity, then the economic income from using the asset flows to one source and the return on assets ROA is identical to the ROE. With a debt-to-asset ratio of zero the equity multiplier is 1 and an ROA of 50% yields an ROE of 50% which equals the 50% asset price change.

Now watch the bounce for another scenario in which equity finances $10,000 of the acquisition and obtains creditor financing of $90,000. The return from the asset is invariant to leverage, a 50% increase in asset value to $150,000 for this illustration. Equity immediately repays the creditor $90,000 plus interest which, given instantaneity, is zero. The residual economic income that flows to equity equals $60,000 (= $150,000 - $90,000). The ROE equals ($60,000 - $10,000)/$10,000, which is 500%. With a debt-to-asset ratio of 90% the equity multiplier is 10 and the ROE of 500% is ten times larger than the 50% return-on-assets. Generally speaking, the leverage effect amplifies the ROE by an amount equal to the reciprocal of the equity-to-asset ratio.

Leverage amplifies equity rates of return on the downside as well as the upside. Consider that equity were to supply $10,000 for a $100,000 asset that declines in value by 10% to $90,000. Equity pays off the creditors and is left with zero. The ROE is *minus* 100% even though the asset price declines by only 10%. The economic income separates into cash flow to creditors or to equity as a consequence of the natural equality between periodic sources and uses. Risk does not influence that mechanical process, but risk certainly changes because of it!

EXERCISES 2.3B

Conceptual

1. Suppose that the company P/B ratio is constant. Is the shareholders' rate of return smaller or larger than the reported return-on-equity?

Numerical quickies

2. The return on equity (= net income ÷ Stockholders equity) is one of the more important measures of company performance. Suppose a company's net profit margin (= net income ÷ sales) is 9.1%, asset turnover (= sales ÷ total assets) is 1.7, and debt-to-assets ratio is 60%. What is this company's return on equity? ©BA7

3. Company X reports that next year they expect a 26% return-on-equity. Company Z expects exactly the same ROE. Both companies also have a 30% dividend payout ratio. You assume that the equity P/B ratios are likely to stay constant. The current P/B equals 1.35 for company X and 2.75 for company Z. What is the difference between expected shareholder rates of return for each company? ©FA16

4. Find below the Company's financial statements.

Balance Sheet, 12/31/2525				*Income, 1/1 – 12/31/2526*	
$2,805	Current assets	$2,600	Debt	Sales	$7,140
<u>$2,295</u>	<u>PP&E</u>	<u>$2,500</u>	<u>Stockholders equity</u>	<u>all costs</u>	<u>$6,640</u>
$5,100	Total assets	$5,100	Liabilities & equity	net income	$500
				dividends	$250

The equity price-to-book ratio is constant at 1.36 and the company neither issues nor repays debt or equity. Find for year 2526 the return-on-equity (=*Net income*$_{2526}$ ÷ *Stockholders equity*$_{2525}$, "ROE") and stockholder's rate of return ("ROR"). ©TQ4

Challengers

5. The P/B ratio is 0.8 for company X and 2.75 for company Z. Both have 25% dividend payout ratios. Company X expects an 18% return-on-equity. What ROE for company Z provides its shareholders with exactly the same rate of return that company X shareholders receive? ©FA17

6. The DuPont formula relates return on equity to the company's net profit margin , asset

turnover (= $sales_t \div total\ assets_t$), and equity multiplier. This Company is in an industry where the average net profit margin is 5.10%, the debt-to-asset ratio is 37.8%, and return on equity is 30.90%. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525				*Income, 1/1 – 12/31/2525*	
$1,750	Current assets	$2,450	Debt	Sales	$24,130
<u>$4,600</u>	<u>PP&E</u>	<u>$3,900</u>	<u>Stockholders equity</u>	<u>all costs</u>	<u>$22,470</u>
$6,350	Total assets	$6,350	Liabilities & equity	net income	$1,660

Contrast the company and industry ROE and explain the source of differences. ©BA6

3.C. Breakeven ratios

Breakeven analysis relies on the stylized income statement below to estimate the amount of sales required for realizing specific income targets.

Sales revenue	*p Q*
- Total fixed costs	*F*
- <u>Total variable costs</u>	*<u>v Q</u>*
= Earnings before interest & taxes	*EBIT*
- Interest	*I*
- <u>Taxes</u>	*<u>T</u>*
= Net Income	*NI*
- <u>Preferred dividends</u>	*<u>PD</u>*
= Earnings available for common	*EAC*

The variable *Q* represents the quantity of product that the company sells, *p* is the unit sales price of the product (*p* times *Q* equals total *Sales revenue*), and *v* is the variable cost per unit. The preceding income statement categorizes operating costs as either "fixed" or "variable." This simplification, favored by economics textbooks and models (as you may dare remember!), is very useful even though somewhat inconsistent with financial reporting practices. Actual income statements do not list line items labeled "fixed costs" and "variable costs." Cost analysis reveals, however, that some production costs (like wages) rise proportionately with sales whereas other costs (like depreciation) are somewhat independent of short-run sales fluctuations. This stylized model allows incredible insights!

The "operating breakeven point" occurs when company *Sales revenue* equals fixed plus variable operating costs. That is, a company at the operating breakeven point has *EBIT* equal to zero. Usually, however, companies prefer a target *EBIT* that is bigger than zero. The operating breakeven ratio allows computation of any target *EBIT.*

FORMULAS 2.9a and 2.9b Operating breakeven ratio

The "operating breakeven ratio" equals the amount of sales that generates a target *Earnings before interest and taxes (EBIT^target^).*

$$\begin{pmatrix} Sales\,revenue \\ @EBIT^{target} \end{pmatrix} = \frac{Total\ fixed\ costs + EBIT^{target}}{1 - \dfrac{Total\ variable\ costs}{Sales\,revenue}}$$

$$\begin{pmatrix} Sales\,quantity \\ @EBIT^{target} \end{pmatrix} = \frac{F + EBIT^{target}}{p - v}$$

where *p* is the unit price of the product, *v* is the variable cost per unit, and *F* is *Total fixed costs*. The upper and lower formulas compute, respectively, sales in dollars and sales in number of units. This model assumes the ratio of *Total variable costs* to *Sales revenue* is constant. At the operating breakeven point $EBIT^{target}$ equals zero.

Consider this example.

EXAMPLE 5 Find the operating breakeven point

The most recent annual report lists company *Sales revenue* at $175,000. Cost analysis suggests that annual *Total fixed costs* equal $42,000 and *Total variable costs* equal $108,000. Find the company's operating breakeven *Sales revenue*.

SOLUTION

Notice that the company's annual report lists total operating costs of $150,000 (= $42,000 + $108,000) and *Sales revenue* of $175,000. Their actual *EBIT* therefore is $25,000 (= $175,000 – 150,000). The operating breakeven point is the amount of sales at which *EBIT* equals zero. To find the *Sales revenue* that generates *EBIT* of zero, plug numbers into formula 2.9a:

$$\begin{pmatrix} Sales\ revenue \\ @EBIT\ =\ \$0 \end{pmatrix} = \frac{\$42{,}000\ +\ \$0}{1 - \dfrac{\$108{,}000}{\$175{,}000}}$$

$$= \$109{,}701$$

Sales revenue at the operating breakeven point is $109,701. If the company were at the operating breakeven point then the upper half of the income statement looks like this (notice that the ratio of *Total variable costs* to *Sales revenue* equals 61.71%):

Sales revenue	*$109,701*	
- Total fixed costs	*42,000*	
- Total variable costs	*67,701*	(= 0.6171 *x* $109,701)
= Earnings before interest & taxes	*$0*	

At the operating breakeven point the *Sales revenue* pays operating costs but nothing else. Fortunately, the company at $175,000 annual sales is far beyond breakeven.

The preceding example solves for the amount of *Sales revenue* that pays all operating costs but nothing else. The same formula, however, also is useful for finding the amount of sales that provide a specific amount of operating income. Consider this example.

EXAMPLE 6 Find sales quantity that provides target operating income

The company computes that each unit of production incurs variable operating costs of $36 and sells for $48. The company's fixed costs are $74,000 per year. How many units per year must the company sell to earn $35,000 of operating income?

SOLUTION

The company must generate sufficient sales to cover the $74,000 fixed costs to suppliers plus the $35,000 operating income to them self. Use formula 2.9b and find the quantity of production at the operating breakeven point.

$$\begin{pmatrix} Sales\,quantity \\ @EBIT = \$35{,}000 \end{pmatrix} = \frac{\$74{,}000 + \$35{,}000}{\$48 - \$36}$$

$$= 9{,}083\ units$$

Reconstruct the income statement and verify that *Sales revenue* is $436,000 (= 9,083 *x* $48) and *Total fixed costs* equal $74,000 and *Total variable costs* equal $327,000 (= 9,083 *x* $36). *EBIT*, also known as operating income, equals the desired target of $35,000 (= $436,000 - $74,000 - $327,000).

Economic theory finds (as you may dare forget!) that equilibrating forces depend on relationships between marginal revenue and marginal costs. The denominator for the preceding problem, *p – v* from formula 2.9b*,* provides significant economic insights about market equilibrium. Suppose with the stylized income statement above, for example, that the company uses only labor and capital to create its product. The marginal revenue is the income from selling one extra unit of production and equals *p*. The marginal cost of labor is the variable cost of making one extra unit and equals *v*. The difference *p – v* is the pre-tax revenue, net of variable costs, from selling one extra unit of production. The measure *p – v* is the *contribution margin*. The contribution margin tends toward the *user cost of capital.* When contribution margin exceeds user cost, however producers see incentives for making additional investments thus capturing economic profit.

For purposes, herein, however, simply note the intuition underlying the breakeven ratio in formula 2.9b. The upstairs term is the sum of fixed costs and target *EBIT*. The downstairs term, *p – v*, is the revenue net of variable costs that each unit provides. Breakeven ratio 2.9b basically divides “total money required” by “money provided per unit.”

Breakeven ratio 2.9a relies on similar intuition. Divide each term of *(p – v)* by *p* and obtain *(1 – v/p),* an expression that appears in the denominator of formula 2.9a. This expression represents operating income per dollar of sales. Hence, breakeven ratio 2.9a basically divides “total money required” by “money provided per dollar of sales.” Consider this example.

EXAMPLE 7 Find sales that generate a target operating margin.

The company computes that each unit of production incurs variable operating costs of $28 and sells for $35. The company’s fixed costs are $44,000 per year. How many units per year must the company sell to attain a 15% operating margin [= (*Sales revenue – Total operating costs)* ÷ *Sales*] ?

SOLUTION

Solving this problem requires a little algebra. Realize that *Total operating costs* equals *F* + *vQ*. Write the definition for the operating margin with variables from the stylized income statement:

$$\begin{pmatrix} operating \\ margin \end{pmatrix} = \frac{pQ \ - (F + vQ)}{pQ}$$

$$= 1 - \frac{F}{pQ} - \frac{v}{p}$$

Now solve for Q:

$$Q = \frac{F}{p\left(1 - \frac{v}{p} - \begin{pmatrix} operating \\ margin \end{pmatrix}\right)}$$

Insert numbers from the problem setup to find Q, the quantity sold when the operating margin equals 15%:

$$Q = \frac{\$44{,}000}{\$35\left(1 - \frac{\$28}{\$35} - 0.15\right)}$$

$$= 25{,}143 \ units$$

Reconstruct the income statement and verify that with sales at 25,143 units the *Sales revenue* equals $880,000 (= 25,143 x $35); *Total variable costs* equal $704,000 (= 25,143 x $28); *Total fixed costs* equal $44,000; *EBIT* equals $132,000 (= $880,000 - $704,000 - $44,000); and the operating margin equals 15% [= {$880,000 – ($704,000 + $44,000)} ÷ $880,000] . Ain't algebra wonderful!

The bottom-half of the income statement documents cash flows from company to capitalists in financial markets. The company sends interest to creditors and dividends to shareholders. Analysis of the preceding stylized income statement allows further insights on how breakeven ratios allow computation of sales needed to reach specific targets.

The *Earnings available for common* (*EAC*) equals *EBIT*, minus *Interest expense (IE)*, minus *Taxes*, minus *Preferred dividends (PD)*. Some but not all companies issue preferred stock (see chapter 8). When *PD* equals zero then *EAC* is identical to *Net income.* When *Taxes* are proportional to taxable income we may write:

EAC = (EBIT – IE) (1 – tax rate) - PD .

The "total breakeven point" occurs when the *Sales revenue* pays all operating costs, interest to creditors, taxes, and preferred dividends, and *EAC* equals zero

FORMULAS 2.10a and 2.10b Total breakeven ratio

The “total breakeven ratio” equals the amount of sales that generates a target *Earnings available for common* (EAC^{target}). At the total breakeven point EAC^{target} equals zero.

$$\begin{pmatrix} \text{Sales revenue} \\ @EAC^{target} \end{pmatrix} = \frac{\text{Total fixed costs} + IE + \left(\dfrac{PD + EAC^{target}}{1 - \text{tax rate}} \right)}{1 - \dfrac{\text{Total variable costs}}{\text{Sales revenue}}}$$

$$\begin{pmatrix} \text{Sales quantity} \\ @EAC^{target} \end{pmatrix} = \frac{\text{Total fixed costs} + IE + \left(\dfrac{PD + EAC^{target}}{1 - \text{tax rate}} \right)}{p - v}$$

where p is the unit price of the product, v is the variable cost per unit, IE is periodic *Interest expense,* and PD equals periodic *Preferred dividends*. The upper and lower formulas compute, respectively, sales in dollars and sales in number of units. This model assumes the ratio of *Total variable costs* to *Sales revenue* is constant.

Consider this example.

EXAMPLE 8 Find how far sales are beyond the total breakeven point

The most recent annual report lists company *Sales revenue* at $175,000. Cost analysis suggests that annual *Total fixed costs* equal $42,000 and *Total variable costs* equal $108,000. The company has annual *Interest* expense of $8,500 and pays $2,000 in *Preferred dividends*. They pay taxes equal to 30% of taxable income. Find how far company sales must fall if the company unfortunately were to fall to its total breakeven point.

SOLUTION

At the total breakeven point EAC^{target} equals zero. To find the *Sales revenue* that generates *EAC* of zero, plug numbers into formula 2.10a:

$$\begin{pmatrix} \text{Sales revenue} \\ @EAC = \$0 \end{pmatrix} = \frac{\$42{,}000 + \$8{,}500 + \$2{,}000 / (1 - 0.30)}{1 - \dfrac{\$108{,}000}{\$175{,}000}}$$

$$= \$139{,}366$$

If *Sales revenue* were at the total breakeven point then the income statement looks like this:

Sales revenue	*$139,366*	
- *Total fixed costs*	*42,000*	
- *Total variable costs*	*86,009*	(= *0.6171* x $*139,366*)
= *Earnings before interest & taxes*	*11,357*	
- *Interest*	*8,500*	
- *Taxes (@30% tax rate)*	*857*	

= Net income	*2,000*
- Preferred dividends	*2,000*
= Earnings available for common	*$ 0*

The actual *Sales revenue* of $175,000 surpasses by a good amount the total breakeven point of $139,366. Sales would have to decline by 20.4% [= ($139,366 - $175,000) ÷ $175,000] for the company to fall to its total breakeven point.

This example shows that breakeven analysis dovetails nicely with the DuPont decomposition to provide insight about company profitability.

EXAMPLE 9 Find sales increase required to reach target ROE

The most recent annual report lists company *Sales revenue* at $56,125. Cost analysis suggests that annual *Total fixed costs* equal $21,375 and *Total variable costs* equal $25,375. The annual *Interest* expense is $2,550 and there is no preferred stock. The company pays 35% of taxable income as taxes. The annual report also shows ROE, that is return on equity (=$\textit{Net income}_t \div \textit{Stockholders' equity}_t$), equals 12.8%. The company wants to increase ROE to a target of 19.0%. They plan to hold constant *Stockholders' equity*, *Total assets, Total fixed costs*, *Interest,* and the ratio of *Sales revenue* to *Total variable costs.* Find the target *Sales revenue* that provides the target ROE.

SOLUTION

Reconstruct the stylized income statement from the annual report:

Sales revenue	*$56,125*
- Total fixed costs	*21,375*
- Total variable costs	*25,375*
= Earnings before interest & taxes	*9,375*
- Interest	*2,550*
- Taxes (@35% tax rate)	*2,389*
= Earnings available for common (=Net income)	*$4,436*

When there is no preferred stock then *Earnings available for common* is synonymous with *Net income*; it equals $4,436 in the most recent annual report. The ROE is 12.8% but the company wants to increase it to 19%.

Because *Stockholders' equity* remains constant we easily find *Net income* at the target ROE. Recall that ROE = $\textit{Net income}_t / \textit{Stockholders' equity}_t$. Thus, in the annual report

0.128 = $4,436 / Stockholders' equity so *Stockholders' equity = $34,658*.

With a target ROE of 19% find the target *Net income:*

0.19 = target Net income / $34,658 so *target Net income = $6,585.*

With *Net income* of $6,585 the ROE equals 19%. Plug numbers into formula 2.10a to find the *Sales revenue* that generates *EAC* of $6,585:

$$\begin{pmatrix} \text{Sales revenue} \\ @EAC = \$6{,}585 \end{pmatrix} = \frac{\$21{,}375 + \$2{,}550 + \$6{,}585 \big/ (1-0.35)}{1 - \dfrac{\$25{,}375}{\$56{,}125}}$$

$$= \$62{,}159$$

If the company increases sales to $62,159 then the ROE rises to its target 19%. This is a $6,034 sales increase.

What will drive the ROE higher? In the most recent annual report the net profit margin (=*Net income* ÷ *Sales revenue*) equals 7.90% (= $4,436 ÷ $56,125). At the target ROE of 19% the net profit margin is 10.59% (= $6,585 ÷ $62,159). The company obtains a higher ROE by pursuing policies that result in a larger profit margin.

The larger profit margin occurs because average costs decline as sales increase. Notice that *Total variable costs* in the annual report equal 45.21% of *Sales revenue* – this ratio presumably is constant. Every extra dollar of sales contributes 54.79 cents (= $1 – 0.4521) to pre-tax revenue. Taxes attract 35% of revenue, implying that net income rises by 35.61 cents [= 54.79 x (1 – 0.35)] for every extra dollar of sales. Analysts sometimes refer to this number as the "after-tax contribution margin". It measures the change in net income per dollar increase in sales. Compute the after-tax contribution margin as *(1 – tax rate) x { 1 – (Total variable costs ÷ Sales revenue)}*.

Total fixed costs, of course, do not increase with sales. As sales increase then fixed costs as a proportion of sales diminishes and the net profit margin increases (and ROE increases, too). For the preceding example the company finds that raising ROE to 19% requires extra net income of $2,149 (= $6,585 – $4,436). Divide the extra net income by the after-tax contribution margin of $0.3561 and compute that the company requires $6,034 of extra sales to reach its target.

Analysis of the total breakeven ratio is useful but it's a bad idea to operate precisely at the total breakeven point where *Earnings available for common* equals zero. Providing shareholders with nothing is a sure way for a manager to get fired. Shareholders expect and deserve a fair rate of return. Shareholders represent a source of capital and management should pursue policies that compensate shareholders for their required cost of capital. Formulas and lessons in later chapters specify the cost of capital and shareholder required rates of return. Here, simply recall from chapter 1 about the company cash flow cycle and that equilibrium economic profit equals zero. The "economic breakeven point" occurs when economic profit is zero and stakeholders receive competitive prices or wages and capitalists receive required costs of capital. At that point the contribution margin equals the *user cost of capital.* The *economic value added* equals the surplus of contribution margin above user cost which at the economic breakeven point equals zero!

FORMULAS 2.11a and 2.11b Economic breakeven ratio

The "economic breakeven ratio" equals the amount of sales that generates a target *rate of return for common shareholders (ROR^{target})*. At the economic breakeven point economic profit equals zero and ROR^{target} equals $ROR^{required}$ (See definition 9.2 and formula 11.2 for discussion of the required rate of return).

$$\begin{pmatrix} \text{Sales} \\ \text{revenue} \\ @ROR^{target} \end{pmatrix} = \frac{F + IE + \dfrac{PD}{1-\text{tax rate}} + \left(\dfrac{ROR^{target} \times SE \times P/B}{\left(1-\text{tax rate}\right) \times \left[P/B - \left(\text{payout ratio}\right)\left(P/B - 1\right)\right]} \right)}{1 - \dfrac{\text{Total variable costs}}{\text{Sales revenue}}}$$

$$\begin{pmatrix} \text{Sales} \\ \text{quantity} \\ @ROR^{target} \end{pmatrix} = \frac{F + IE + \dfrac{PD}{1-\text{tax rate}} + \left(\dfrac{ROR^{target} \times SE \times P/B}{\left(1-\text{tax rate}\right) \times \left[P/B - \left(\text{payout ratio}\right)\left(P/B - 1\right)\right]} \right)}{p - v}$$

where p is the unit price of the product, v is the variable cost per unit, F is *Total fixed costs*, IE is periodic *Interest expense, PD* equals periodic *Preferred dividends*, *P/B* is the equity price-to-book ratio, and *SE* is *Stockholders' equity*. The upper and lower formulas compute, respectively, sales in dollars and sales in number of units.

The "economic breakeven point" occurs when the *Sales revenue* pays all operating costs, interest to creditors, taxes, and preferred dividends, and common shareholders receive the required rate of return. The state of zero economic profit does *not* imply that zero wealth is created. The employees, the creditors, shareholders and stakeholders, all receive compensation from the company for providing a factor of production. Consider this example. The contribution margin pays for it all!

EXAMPLE 10 Find the economic breakeven point

The most recent annual report lists company *Sales revenue* at $87,675 and *Stockholders' equity* at $109,200. Cost analysis of the annual report suggests that *Total fixed costs* equal $38,475 and *Total variable costs* equal $30,450. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. The price-to-book ratio and payout ratio, also constant, equal 1.80 and 45%, respectively. The annual *Interest* expense is constant at $2,125. There is no preferred stock and the number of common shares is constant. The company tax rate is 35%. Advanced analysis convinces management that the shareholder required rate of return, that is the equity financing rate, equals 9.60%. Find the target annual *Sales revenue* at which this occurs.

SOLUTION

At the economic breakeven point ROR^{target} equals 9.60%. To find the *Sales revenue* that generates this rate of return, plug numbers into formula 2.11a:

$$\begin{pmatrix} Sales \\ revenue \\ @ROR^{target} \end{pmatrix} = \frac{\$38{,}475 \ + \ \$2{,}125 \ + \left(\dfrac{0.096 \times \$109{,}200 \ \times 1.80}{(1-0.35) \times \left[1.80 \ - (0.45)(1.80 \ - 1)\right]} \right)}{1 - \dfrac{\$30{,}450}{\$87{,}675}}$$

$$= \ \$93{,}091$$

If *Sales revenue* were at the economic breakeven point then the income statement looks like this:

Sales revenue	*$93,091*	
- *Total fixed costs*	*38,475*	
- *<u>Total variable costs</u>*	*<u>32,331</u>*	(= *0.3473* x $*93,091*)
= *EBIT*	*22,285*	
- *Interest*	*2,125*	
- *<u>Taxes (@35% tax rate)</u>*	*<u>7,056</u>*	
= *Net Income (=EAC)*	*$13,104*	
- *<u>Dividends(@45%)</u>*	*<u>5,897</u>*	
= *New Retained Earnings*	*$7,207*	

The next section explains that *New retained earnings* equals the *Net income* (i.e., the *EAC*) that the company does <u>not</u> payout as dividends. For this example the company plows $7,207 back into the company and *Stockholders' equity* rises by $7,207 to become $116,407 (= $109,200 + $7,207). Because the price-to-book ratio is identical, when number of shares is constant, to the ratio of market capitalization to *Stockholders' equity*, compute from the most recent annual report that market cap is $196,560 (= 1.8 x $109,200). Next year after the company pushes *Stockholders' equity* up to $116,407 then market cap, given constant *P/B*, becomes $209,533. Find that the shareholder rate of return equals the target:

$$ROR \ = \ \frac{\$209{,}533 + \$5{,}897 - \$196{,}560}{\$196{,}560} \quad ; \ = \ 9.60\% .$$

EXERCISES 2.3C

Conceptual

1. Inspection of the breakeven ratios shows that the role of taxes is limited. For example, when the company has no preferred stock then the total breakeven point at which *Earnings available for common* equals zero does not depend at all on the tax rate. Yet all companies (and households) know the importance of taxes. Why does the total breakeven ratio appear to assign limited importance to taxes?

Numerical quickies

2. The company computes that each unit of production incurs variable operating costs of $7 and sells for $12. The company's fixed costs are $12,000 per year. Find the number of units per year the company must sell to exactly cover operating costs. ©BE1a

3. The company computes that each unit of production incurs variable operating costs of $16 and sells for $22. The company's fixed costs are $34,000 per year. How many units per year must the company sell to earn $15,000 of operating income? ©BE1b

4. The most recent annual report lists company *Sales revenue* at $83,195. Cost analysis suggests that annual *Total fixed costs* equal $38,250 and *Total variable costs* equal $40,400. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. Find the company's operating breakeven *Sales revenue*. ©BE2a

5. The most recent annual report lists company *Sales revenue* at $95,525. Cost analysis suggests that annual *Total fixed costs* equal $42,500 and *Total variable costs* equal $50,500. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. Find the target *Sales revenue* per year at which the company earns $6,300 of operating income. ©BE2b

6. The most recent annual report lists company *Sales revenue* at $97,450. Cost analysis suggests that annual *Total fixed costs* equal $34,000 and *Total variable costs* equal $50,500. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. The annual *Interest* expense is $3,825 and the company pays *Preferred dividends* of $500 per year. They also pay 30% of taxable income as taxes. Find the *Sales revenue* and *EBIT* at the total breakeven point. ©BE5b

7. The most recent annual report lists company *Sales revenue* at $91,350. Cost analysis suggests that annual *Total fixed costs* equal $29,750 and *Total variable costs* equal $50,500. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. The annual *Interest* expense is $2,550 and the company pays *Preferred dividends* of $350 per year. They also pay 25% of taxable income as taxes. Find the target annual *Sales revenue* at which the company has $7,900 of *Earnings available for common*. ©BE6

8. The most recent annual report lists company *Sales revenue* at $745,200 and *Net income* at $125,000. Cost analysis suggests that *Total variable costs* equal $475,000. The company pays 30% of taxable income as taxes and there is no preferred stock. The ROE currently is 14%, but the company wants to increase *Net income* so that ROE rises to 18%. They plan to hold constant *Stockholders' equity*, *Total assets, Total fixed costs*, *Interest,* and the ratio of *Sales revenue* to *Total variable costs.* By how much must *Sales revenue* increase?

Challengers

9. The most recent annual report lists company *Sales revenue* at $66,900 . Cost analysis suggests that annual *Total fixed costs* equal $34,000 and *Total variable costs* equal $25,250. The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. Find the percentage decline in annual *Sales revenue* that would cause the company to fall to its operating breakeven point. ©BE3

10. The company computes that each unit of production incurs variable operating costs of $33 and sells for $45 . The company's fixed costs are $25,500 per year. Find the number of units per year the company must sell to attain a 18% operating margin [= (*Sales revenue – total operating costs) ÷ Sales*] . ©BE4a

11. The company computes that each unit of production incurs variable operating costs of \$21 and sells for \$30 . The company's fixed costs are \$34,000 per year. Find the annual *Sales revenue* at which the company attains a 20% operating margin [= (*Sales revenue – total operating costs) ÷ Sales*] . ©BE4b

12. The most recent annual report lists company *Sales revenue* at \$107,175. Cost analysis suggests that annual *Total fixed costs* equal \$42,750 and *Total variable costs* equal \$45,675. The annual *Interest* expense is \$3,825 and there is no preferred stock. The company pays 30% of taxable income as taxes. The annual report also shows ROE, that is return on equity (=$\textit{Net income}_t \div \textit{Stockholders' equity}_t$), equals 15.7%. The company wants to increase its ROE to a target of 24.0%. They plan to hold constant *Stockholders' equity*, *Total assets, Total fixed costs*, *Interest,* and the ratio of *Sales revenue* to *Total variable costs.* Find the target *Sales revenue* and net profit margin (=*Net income ÷ Sales revenue*) that provides the target ROE. ©BE7b

13. The most recent annual report lists company *Sales revenue* at \$102,900 and *Stockholders equity* at \$73,900 . Cost analysis of the annual report suggests that *Total fixed costs* equal \$38,475 and *Total variable costs* equal \$45,675 . The company believes that the ratio of *Sales revenue* to *Total variable costs* is constant. The price-to-book ratio and payout ratio, also constant, equal 2.00 and 30%, respectively. The annual *Interest* expense is constant at \$2,975 . There is no preferred stock and the number of common shares is constant. The company tax rate is 25%. Advanced analysis convinces management that the shareholder required rate of return, equals 12.5%.
a. Find the target annual *Sales revenue* at which this most likely occurs. ©BE10a
b. At the economic breakeven point find the total *Dividends* that shareholders receive. ©BE10c

4. The income statement links adjacent balance sheets

The balance sheet is a momentary snapshot of the company assets and liabilities. After the snapshot is taken, time elapses and the income statement records all cash flows that occur. Then, at the end of the period, another snapshot captures the balance sheet showing the new amounts for all assets and liabilities. As rule 2.1 states, the change in balances from one snapshot to the next equals the sum of intervening cash flows.

Two processes link the income statement with the two adjacent balance sheets. These differential processes are given below:

FORMULAS 2.12a and 2.12b Differential processes for Stockholders' Equity and PP&E

Two formulas link the income statement with its adjacent balance sheets.

$$\begin{pmatrix} \textit{Stockholders'} \\ \textit{equity} \end{pmatrix}_t = \begin{pmatrix} \textit{Stockholders'} \\ \textit{equity} \end{pmatrix}_{t-1} + \textit{Net equity issues}_t + \begin{pmatrix} \textit{New} \\ \textit{retained} \\ \textit{earnings} \end{pmatrix}_t .$$

$$\textit{PP\&E}_t = \textit{PP\&E}_{t-1} + \textit{Capital expenditure}_t - \textit{Depreciation}_t .$$

For each formula the left-hand-side represents a line item on the balance sheet from the snapshot taken at the end of the reporting period. *Stockholders' equity* is on the liability side of the balance sheet, and *PP&E* is on the asset side. The first right-hand-side term for each formula represents the line item from the balance sheet at the beginning of the reporting period (that is identical to the end of the previous period). The rightmost right-hand-side terms (*New retained earnings* and *Depreciation*) in each formula appear on the income statement. The middle terms (*Net equity issues* and *Capital expenditure*) represent realized cash flows that affect the respective balance but do not appear on the income statement.

The formulas show that the balance this period equals the balance last period plus adjustments for cash flows. The process is analogous to the water level of a lake. The water level this period equals the level last period, minus evaporation and runoff, plus rain and inflow. For the *PP&E* account the *Depreciation* is analogous to the evaporation. *Depreciation* causes the balance of *PP&E* to fall. *Capital expenditure* is like the rainfall and causes the balance of *PP&E* to rise.

The explanation for *Stockholders' equity* is similar. The term *Net equity issues* equals the value of shares sold by the company to the public, minus the value of shares that it repurchases. The term *New retained earnings* equals *Net income* minus *Dividends*. Suppose, for example, that a company makes $100,000 of *Net income*. The company certainly can payout all of the $100,000 as *Dividends*. These dividends represent a return to shareholders for investing in the company's stock. The company may choose, however, that instead of paying out 100% of *Net income* as *Dividends* they will retain some, say 60%, within the firm. This *New retained earnings* of $60,000 represents internal financing for the company. Managers plowback earnings into the company when they believe they can put the money to good use.

A company that keeps *New retained earnings* incurs a liability to shareholders. The money could have been paid-out as a dividend to shareholders but, instead, the company implicitly borrows money from shareholders. Consequently, the appearance on the income statement of *New retained earnings* causes an increase on the balance sheet of *Stockholders' equity*.

Some balance sheets partition *Stockholders' equity* into several components: Accumulated retained earnings, paid-in-equity capital, equity capital surplus, etc. Throughout this book we lump these components together into the broader category called *Stockholders' equity*.

The differential processes in formula 2.12 shape evolution of the balance sheet through time. Examples below provide practice linking the income statement with its adjacent balance sheets. First, use the relation for *Stockholders' equity*.

EXAMPLE 11 Use the differential process for *Stockholders' equity* to find next period's book value

At year-end 2525 the company has Total assets of $4,400 financed by Debt of $1,700 and Stockholders' equity of $2,700. For year 2526 the company forecasts an asset turnover ratio (= $Sales_{2526} \div Total\ assets_{2525}$) of 3.9, a net profit margin of 6.4%, and a dividend payout ratio of 45%. There are 270 shares outstanding. If no additional shares are issued, what is the equity book value per share at year-end 2526?

SOLUTION

Use formula 2.12a to find *Stockholders' equity* at year-end 2526:

$$\begin{pmatrix} \textit{Stockholders'} \\ \textit{equity} \end{pmatrix}_{2526} = \begin{pmatrix} \textit{Stockholders'} \\ \textit{equity} \end{pmatrix}_{2525} + \textit{Net equity issues}_{2526} + \begin{pmatrix} \textit{New} \\ \textit{retained} \\ \textit{earnings} \end{pmatrix}_{2526}$$

$$= \$2{,}700 + 0 + \begin{pmatrix} \textit{New} \\ \textit{retained} \\ \textit{earnings} \end{pmatrix}_{2526}$$

Necessary information is *New retained earnings* from the income statement for year 2526. Reconstruct the income statement from the information given: sales equals 3.9 *x* $4,400; net income equals 0.064 *x* sales; and *New retained earnings* equals 0.55 times net income (45% of net income is paid-out as dividends, 55% is retained). Thus, *New retained earnings* for year 2526 equals $604 (= 3.9 *x* $4,400 *x* 0.064 *x* 0.55). Substitute this amount above to find that $\textit{Stockholders' equity}_{2526}$ equals $3,304. Divide by the 270 shares outstanding to find equity book value per share of $12.24.

PP&E differential process in formula 2.12 offers one more lesson about *Capital expenditures.* There exist two motivations for company management to invest in real capital: *replacement investment* to offset effects of depreciation or obsolescence and *expansion investment* to increase the asset base. Rearrange and relabel formula 2.12 to find:

$$\textit{Capital expenditure}_t = \underbrace{PP\&E_t - PP\&E_{t-1}}_{\textit{expansion investment}_t} + \underbrace{\textit{Depreciation}_t}_{\textit{replacement investment}_t}$$

Lessons in chapter 6 examine assessment tools for determining whether capital investments create, maintain, or destroy wealth. For current purposes, however, learn to recognize the difference between replacement and expansion investment.

EXAMPLE 12 Find and partition *Capital expenditures* into replacement and expansion investment

The balance sheet shows $\textit{PP\&E}_{2525}$ equals $46,400 and $\textit{Total assets}_{2525}$ equal $64,000 . $\textit{Sales}_{2526}$ are $86,000 . The company sets its target asset turnover ratio (= $\textit{Sales}_{2526} \div \textit{Total assets}_{2526}$) at 1.20. *PP&E* is the only asset that increases during 2526. Suppose $\textit{Depreciation}_{2526}$ equals 18% of beginning of year *PP&E*. Find the *Capital expenditures* in 2526. Partition the expenditure into two components: that which replaces depreciating assets and that which expands the asset base.

SOLUTION

Because $\textit{Sales}_{2526} \div \textit{Total assets}_{2526}$ equals 1.20 and $\textit{Sales}_{2526}$ equals $86,000 then $\textit{Total assets}_{2526}$ equals $71,667 (= $86,000 ÷ 1.20). Between year-ends 2525 and 2526 the increase in *Total assets* is $7,667 (= $71,667 – $64,000). Because *PP&E* is the only asset that increases during 2526 then *PP&E* also increases by $7,667. Rearrange formula 2.12 to find $\textit{Capital expenditures}_{2526}$ and substitute that *Depreciation* equals 18% of beginning of year *PP&E*.

$$\textit{Capital expenditure}_{2526} = \Delta PP\&E + \textit{Depreciation}_{2526}$$

$$= \$7{,}667 + (0.18 \times \$46{,}400)$$

$$= \$7,667 \; + \; \$8,352$$

$$= \$16,019.$$

The company spends a total of $16,019 purchasing *PP&E*. Of that expenditure, more than half ($8,352) is replacement investment with sole purpose of replacing depreciating assets. Expansion investment, that is the net increase in the fixed asset stock, equals $7,667. This example highlights the realistic fact that a large proportion of the economy's investment in fixed capital assets does not expand the asset base, but simply replaces that which is obsolete, out-of-favor, or just plain worn out.

Linking *Stockholders' equity* from one year to the next provides an opportunity to strengthen understanding of the relation between book and market measures of the shareholders' rate of return.

EXAMPLE 13 Contrast market ROR with book ROE given financial ratios and constant P/B

For year 2526 the company forecasts sales of $70,000 , an asset turnover ratio (= *Sales*$_t$ ÷ *Total assets*$_{t-1}$) of 2.9,a net profit margin (= *Net income* ÷ *Sales*) of 4.2%, a dividend payout ratio (= *Dividends* ÷ *Net income*) of 70%, and a debt-to-equity ratio (= *Total debt* ÷ *Stockholders' equity*) of 135%. The company expects the equity price-to-book ratio to remain constant and they do not intend to issue any new shares. Contrast for year 2526 the shareholders' book and market rates-of-return given the P/B ratio is (a) 1.00, or (b) 0.60, or (c) 1.40.

SOLUTION

The book return-on-equity does not depend on the P/B ratio, so ROE is the same number for case a-b-c. Define ROE for this example as *Net income*$_{2526}$ ÷ *Stockholders' equity*$_{2525}$. Multiply *Sales* of $70,000 by the net profit margin of 4.2% to obtain that *Net income* is $2,940. Obtain *Stockholders' equity*$_{2525}$. by combining information about the asset turnover and debt-to-equity ratios:

$$\frac{Sales_{2526}}{Total\ assets_{2525}} = 2.9\ ;\quad or\quad \frac{\$70{,}000}{Total\ assets_{2525}} = 2.9\ ;\quad so\ Total\ assets_{2525} = \$24{,}318$$

$$\frac{Total\ debt_{2525}}{Stockholders'\ equity_{2525}} = 1.35\quad or\quad Total\ debt_{2525} = 1.35\ x\ Stockholders'\ equity_{2525}$$

$$(Total\ debt_{2525} + Stockholders'\ equity_{2525}) = \$24{,}318$$

$$(1.35\ x\ Stockholders'\ equity_{2525} + Stockholders'\ equity_{2525}) = \$24{,}318$$

$$Stockholders'\ equity_{2525} = \$24{,}318\ /\ 2.35\ ;\quad = \$10{,}271$$

$$ROE = \frac{\$2{,}940}{\$10{,}271}\ ;\quad = 28.6\%$$

ROE is the book measure for the shareholders' rate-of-return and equals 28.6%. Notice that the DuPont decomposition in formula 2.8 arrives at the identical answer [that is, 28.6% = 4.2% x 2.9 x (1+1.35)].

The market ROR depends on the shareprice. The ROR differs between cases a-b-c because the P/B ratio differs and B, the equity book value, is constant. Formula 2.6 shows the ROR on a per share basis. The identical number is found, however, on the accumulated basis when number of shares outstanding is constant:

$$ROR_t = \frac{market capitalization_t + total\ dividends_t - market capitalization_{t-1}}{market capitalization_{t-1}}$$

,

Already we have found that *Net income*$_{2526}$ is \$2,940 which, combined with a payout ratio of 70%, implies that *Total dividends*$_{2526}$ equals \$2,058 and *New retained earnings*$_{2526}$ equals \$882.

Now let's examine cases a-b-c.

<u>case a: P/B =1.0</u> Previous examples have taught us that the company's market capitalization equals its *Stockholders' equity* times the P/B ratio. Already we computed that *Stockholders' equity*$_{2525}$ is \$10,271. Formula 2.12 shows that *Stockholders' equity*$_{2526}$ equals *Stockholders' equity*$_{2525}$ plus *New retained earnings*$_{2526}$. Thus, let's find *ROR* with P/B = 1 as follows:

$$ROR_t = \frac{\{1.0\ x\ (\$10{,}271\ +\ \$882)\} + \$2{,}058\ - (1.0\ x\ \$10{,}271)}{1.0\ x\ \$10{,}271}$$

$$= \frac{\$2{,}058\ + (1.0\ x\ \$882)}{1.0\ x\ \$10{,}271}\ ; = \frac{\$2{,}940}{\$10{,}271}\ ;\ = 28.6\%$$

The ROR of 28.6% is identical to the ROE found previously. That's because P/B= 1.0.

<u>case b: P/B =0.60</u> Recompute the above expression:

$$ROR_t = \frac{\{0.60 \text{ x } (\$10{,}271 \; + \; \$882)\} + \$2{,}058 \; - (0.60 \text{ x } \$10{,}271)}{1.0 \text{ x } \$10{,}271}$$

$$= \frac{\$2{,}058 \; + (0.60 \text{ x } \$882)}{0.60 \text{ x } \$10{,}271} \quad ; \quad = 42.0\%$$

The market ROR of 42.0% is substantially greater than the ROE of 28.6%. This result is expected because the P/B is less than one.

case c: P/B =1.40 Recompute the above expression, but this time let's jump to the second line showing the numerator equals *Dividends* plus the product of P/B times *New retained earnings*:

$$ROR_t = \frac{\$2{,}058 \; + (1.40 \text{ x } \$882)}{1.40 \text{ x } \$10{,}271} \quad ; \quad = 22.9\%$$

The market ROR of 22.9% is smaller than the ROE of 28.6%. This result is expected because the P/B is bigger than one.

The example below further brings *Depreciation* into the analysis. This complexity typically arises in the real world. This means that usually you have to work simultaneously with the PP&E and SE relations.

EXAMPLE 14 Use the PP&E relation to find next period's *Stockholders' equity* and ROR

Find below the Company balance sheet for year-end 2525.

Balance Sheet, 12/31/2525

Current Assets	$1,700	$3,200	Debt
PP&E	$6,200	$4,700	Stockholders equity (500 shares)
	$7,900	$7,900	Total

For year 2526 the company forecasts an asset turnover ratio (= sales_{2526} ÷ total assets_{2525}) of 2.8, a net profit margin (= net income ÷ sales) of 7.7%, a dividend payout ratio (=dividends÷net income) of 55%, and depreciation that is 21% of beginning-of-year PP&E. Throughout year 2526 Debt remains unchanged, and the P/B ratio of 1.4 is expected to remain constant. The company expects to make *Capital expenditures* such that for the year-end 2526 balance sheet PP&E is $400 larger than on the 2525 balance sheet. Suppose the *Capital expenditure* is financed exclusively by issuing new shares at the stock price of year-end 2525. Find (i) Stockholders' equity at year-end 2526, and (ii) the shareholders' ROR for year 2526.

SOLUTION

Realize that $\textit{Depreciation}_{2526}$ equals $1,302 (=.21 x $6,200). Furthermore, the company wants $\textit{PPE}_{2526}$ to equal $6,600 (=$6,200 + 400). Use formula 2.12 to find the *Capital expenditure* that occurs during year 2526:

$$\textit{PP\&E}_{2526} = \textit{PP\&E}_{2525} + \textit{Capital expenditure}_{2526} - \textit{Depreciation}_{2526}$$

$$\$6{,}600 = \$6{,}200 + \text{Capital expenditure}_{2526} - \$1{,}302$$

$$\text{Capital expenditure}_{2526} = \$1{,}702$$

By investing $1,702 in plant and equipment the company more than offsets effects of depreciation and *PP&E* rises to $6,600 on the year-end 2526 balance sheet.

The company pays a supplier $1,702 for the PP&E. The setup tells us they obtain this financing by selling new stock. *Stockholders' equity* increases by $1,702 plus the amount of *New retained earnings*$_{2526}$. Use the logic from the previous example to compute that *New retained earnings* equals $766 (= 2.8 *x* $7,900 *x* 0.077 *x* 0.45). Thus,

$$\begin{pmatrix}\text{Stockholders'}\\ \text{equity}\end{pmatrix}_{2526} = \begin{pmatrix}\text{Stockholders'}\\ \text{equity}\end{pmatrix}_{2525} + \text{Net equity issues}_{2526} + \begin{pmatrix}\text{New}\\ \text{retained}\\ \text{earnings}\end{pmatrix}_{2526}$$

$$= \$4{,}700 + \$1{,}702 + \$766$$

$$= \$7{,}168$$

The answer to question (i) is that Stockholders' equity at year-end 2526 is $7,168.

Find the answer to question (ii) by using formula 2.6 for shareholders' ROR. The stock price at year-end 2525 uses the definition for the P/B ratio:

$$1.4 = (\text{ price per share })_{2525} \;/\; (\$4{,}700 / 500)$$

$$(\text{ price per share })_{2525} = \$13.16$$

The company raises $1,702 by selling new stock at $13.16 a share. The company issues 129 shares (= $1,702 ÷ $13.16), thereby bringing to 629 shares (= 500 + 129) the total outstanding at year-end 2526. Now use the P/B definition again:

$$1.4 = (\text{ price per share })_{2526} \;/\; (\$7{,}168 / 629)$$

$$(\text{ price per share })_{2526} = \$15.95$$

The shareholder has a stock worth $13.16 at the beginning and $15.95 at the end, plus they receive a dividend. The dividend per share is $1.49 (= 2.8 *x* $7,900 *x* 0.077 *x* 0.55 ÷ 629).

$$\text{shareholder's ROR}_{2526} = \frac{\$15.95 + \$1.49 - \$13.16}{\$13.16}$$

$$= 32.5\%$$

The scenario describes a fairly hefty rate of return for shareholders equal to 32.5% for year 2526! By the way, notice that formula 2.7 relating ROR and ROE is a little biased for scenarios when the number of common shares changes.

The last example in this chapter simply illustrates how an income statement connects two adjacent balance sheets.

EXAMPLE 15 Balance sheet dynamics

The Company's balance sheet for December 31, 2525, and its income statement for the year 2526 appear below.

Company Balance Sheet, 12/31/2525			
	Assets		*Liabilities*
Cash	$50,000	$100,000	Debt
PP&E	250,000	200,000	Stockholders' Equity (10,000 shares)
Total Assets	*$300,000*	*$300,000*	*Total Liabilities & Equity*

Company Income Statement, Jan. 1 — Dec. 31, 2526	
Sales	$500,000
- expenses (includes depreciation of $40,000)	450,000
= Net Income	$50,000
- Dividends	20,000
= *New Retained Earnings*	$30,000

The company does not plan to acquire new assets nor change their *Debt* or shares outstanding. Based solely on this information, what does the balance sheet look like on December 31, 2526?

SOLUTION

The balance sheet at year-end 2526 reflects an increased *Stockholders' equity* due to *New retained earnings*. Recall formula 2.12 that governs how Stockholders' equity changes over time. We are told that the company does not repurchase nor issue shares, so substitution shows:

$$SE_{2526} = \$200{,}000 + \$30{,}000$$

$$= \$230{,}000$$

On the balance sheet's right-hand side, *Debt* is unchanged and *Stockholders' equity* rises to $230,000. On the balance sheet's left-hand-side, *PP&E* falls due to the effects of *Depreciation*. We are told the company does not acquire new assets. Use formula 2.12 that governs the relation between depreciation and capital expenditures.

$$PP\&E_{2526} = \$250{,}000 - \$40{,}000$$

$$= \$210{,}000$$

Summarizing our results shows the year-end 2526 balance sheet as follows:

Company Balance Sheet, 12/31/2526			
	Assets		*Liabilities*
Cash	$?	$100,000	Debt
PP&E	210,000	230,000	Stockholders' Equity (10,000 shares)
Total Assets	*$330,000*	*$330,000*	*Total Liabilities & Equity*

Properly accounting for the information from the problem setup leads to the balance sheet above. Yet what do we know about the proper amount for the *Cash* line item. The answer: *everything*. Because Total assets must equal Total liabilities and equity, and all other items are specified, *Cash* must equilibrate the balance sheet - cash must rise to $120,000!

This example utilizes the bottom-line identity of balance sheets: the sum of the right-hand-side always equals the sum of the left-hand-side. The problem gives sufficient information in the setup for determining the balance sheet's right-hand-side: the only change is *Stockholders' equity*. On the left-hand-side, *PP&E* falls because of *Depreciation*. The question that undoubtedly pops-up, though, is how and why does *Cash* rise?

Inspection of the income statement shows that *Depreciation* was subtracted in order to compute *New retained earnings*. The company, though, never had to "pay" or write a check for *Depreciation*; this is a non-cash charge. So the company actually has cash "flowing" into the checking account equal to the $30,000 *New retained earnings* plus the $40,000 *Depreciation*. If the company does not spend this $70,000 then the *Cash* account on the balance sheet increases by $70,000 and closes the year at $120,000. Lessons in the next chapter explain that the distinction between cash flow and income is important.

EXERCISES 2.4

Numerical quickies

1. At year-end 2525 the company has total assets of $3,700 financed by Debt of $1,100 and Stockholders' equity of $2,600. For year 2526 the company forecasts an asset turnover ratio (= sales$_{2526}$ ÷ total assets$_{2525}$) of 3.8, a net profit margin of 7.80%, and a dividend payout ratio of 45%. There are 260 shares outstanding. If no additional shares are issued, what is the equity book value per share at year-end 2526? ©BA17a

2. On January 1, the company has total assets of $4,800 financed by debt of $1,670 and *Stockholders' equity* of $3,130; for 900 common shares outstanding, the equity price-to-book ratio is 0.80. During the subsequent year the company does not issue new shares. They also expect an asset turnover ratio (= *Sales$_t$* ÷ *Total assets$_{t-1}$*) of 3.5; a 9% net profit margin; and a 30% payout ratio. If the year-end equity price-to-book ratio were 0.90, what year-end shareprice is forecast? ©BA13

3. At year-end 2525, Stockholders' Equity is $4,000 and there are 170 common shares outstanding. For 2526, sales should equal $18,800, the net profit margin is 4.70%, the payout ratio is 55%, and no shares are issued or repurchased. If the equity price-to-book ratio at year-end 2525 is 0.80, and it moves to 0.90 at year-end 2526, what is the shareprice at year-end 2526? ©BA12a

Challengers

4. At year-end 2525, Stockholders' Equity is $3,800 and there are 100 common shares outstanding. For 2526, sales should equal $12,540 , the net profit margin is 6.40%, the payout ratio is 60%, and no shares are issued or repurchased. If the equity price-to-book ratio at year-end 2525 is 0.67, and it moves to 0.84 at year-end 2526, what is the shareholders' annual rate of return for 2526? ©BA12b

5. At year-end 2525 the company has total assets of $5,800 financed by Debt of $2,400 and Stockholders' equity of $3,400. For 500 common shares outstanding, the equity price-to-book ratio is 1.25. During 2526, they expect sales equal to $25,500 and a gross margin (=operating income before depreciation ÷ sales) of 22%. Depreciation is expected to equal $1,280 and interest charges will equal 12% of Debt. Corporate taxes equal 32% of taxable income, and the dividend payout ratio (=dividends ÷ net income) is 58%. Suppose the company has no intention of borrowing more money or buying more assets. What would be the percentage change in shareprice for 2526 that could be supported if the equity price to book ratio were to remain constant? ©BA2d

6. At year-end 2525 the company has total assets of $3,100 financed by Debt of $1,500

and Stockholders' equity of $1,600. For year 2526 the company forecasts an asset turnover ratio (= $sales_{2526} \div total\ assets_{2525}$) of 4.5, a net profit margin of 7.60%, and a dividend payout ratio of 40%. There are 150 shares outstanding and, at year-end 2525, the price-to-earnings ratio is 14.1. Throughout year 2526 no additional shares are issued, and the price-to-earnings ratio remains unchanged. Suppose that the net income is 7.3% larger in 2526 than in 2525. Find the shareholder annual rate of return for year 2526. ©BA14

7. At year-end 2525 the company has total assets of $4,900 financed by Debt of $1,000 and Stockholders' equity of $3,900. For year 2526 the company forecasts an asset turnover ratio (= $sales_{2526} \div total\ assets_{2525}$) of 3.5, a net profit margin (= net income ÷ sales) of 6.50%, and a dividend payout ratio (=dividends÷net income) of 55%. There are 390 shares outstanding and, at year-end 2525, the price-to-earnings ratio is 14.5. If no additional shares are issued, and the price-to-earnings ratio remains unchanged, what is the shareprice at year-end 2526? ©BA1c

8. Find below the Company balance sheet for year-end 2525.

Balance Sheet, 12/31/2525

Current Assets	$1,800	$3,600	Debt
PP&E	<u>$7,300</u>	<u>$5,500</u>	<u>Stockholders equity (700 shares</u>
	$9,100	$9,100	Total

For year 2526 the company forecasts an asset turnover ratio (= $sales_{2526} \div total\ assets_{2525}$) of 3.1, a net profit margin (= net income ÷ sales) of 8.7%, a dividend payout ratio (=dividends÷net income) of 55%, and depreciation that is 22% of beginning-of-year PP&E. Throughout year 2526 Debt remains unchanged, and the P/B ratio of 0.8 is expected to remain constant. The company expects to make capital expenditures such that for the year-end 2526 balance sheet PP&E is $400 larger than on the 2525 balance sheet. Suppose the *Capital expenditure* is financed <u>exclusively</u> by issuing new shares at the stock price of year-end 2525.

8a. Find Stockholders' equity at year-end 2526. ©BA9a

8b. Find the stockholders' rate of return for year 2526. ©BA9c

9. For year 2526 the company forecasts sales of $40,000 , an asset turnover ratio (= $sales_{2526} \div total\ assets_{2525}$) of 1.5, a net profit margin (= net income ÷ sales) of 6.1%, a dividend payout ratio (=dividends ÷ net income) of 60%, and a debt-to-equity ratio (= total debt ÷ stockholders equity) of 111%. The company expects the equity price-to-book ratio of 1.40 to remain constant. Contrast for year 2526 the shareholder's book return-on-equity (= $net\ income_{2526} \div stockholder's\ equity_{2525}$) and market rate of return. ©BA11a

ANSWERS TO CHAPTER 2 EXERCISES

EXERCISES 2.1

1. The car is an asset and only appears on one side. This is true irrespective of whether the car was bought with cash or with a loan. Suppose that a loan was taken out to completely pay for the car. The loan is on the liability side of the balance sheet. Then the liability side rose by the amount of the loan. That rise perfectly matches the increase on the asset side from adding the car. The increase in the liability was the source of financing and the increase in the asset was a use of financing. Sources equaled uses. They always do just as surely as a coin only has two sides. Suppose that the car was bought exclusively with cash. If so, then the purchase of the car caused a decline in the Cash account on the asset side; the decline in the asset account would have been a source of funds financing the investment. The investment was the use. The source and the use both happened on the asset side of the balance sheet. The liability side was unaffected. This is called “rebalancing the portfolio.”

2. Flows are events. Balances are objects. Thus, (a) the semiannual insurance payment definitely is $X dollars per 6 months. That is its flow rate. (b) An inheritance as a bequest is a list of assets with some usefulness whether immediate or deferred. That type of inheritance is a noun and is a balance. The only way that an inheritance is a flow occurs when the event of the passover occurs. During that period wealth flows from one balance sheet to one or more others. That movement of the inheritance is a flow of realized and accrued wealth. (c) The purchase of a car is a sale for somebody, and *Sales* per period is a flow. (d) The car is on the balance sheet as an asset. Assets provide returns over time and hopefully this car is not a lemon. (e) The purchase of a stock is an event and that is a flow. The certificates sitting on the shelf or on the ledger are balances. (f) Grand Central Park provides returns over time and is a public asset. Going to the park is an event and that is a flow. (g) Admirals and politicians deploy assets and a battleship is a big asset. (h) A battle definitely is an event, albeit unfortunate. The battle is bounded by the beginning and ending of a historical period regardless of length. No battle lasts forever. As an on-going historical legacy, however, the battle provides returns over time to the extent that future households and populations realize greater utility (discounted uses) flowing from the invested capital of prior households (discounted sources).

3. The change in the cash account from this year to next is a realized cash flow. If next year is bigger then the cash account was a use of financing during the year. If next year’s is smaller then the cash account was a source of financing. The change in the value of the car is an accrued cash flow. It’s becoming worth less and that diminishes this capital account on your balance sheet. A decline in an asset is a source of financing. The car was a source of returns during the year, and its return was in the form of transportation services. The value of the utility from using the car plus the change in car value equals the economic income from the car account during the year. Even though the change in the value of the car (regardless of sign) is an accrued cash flow the maintenance expenditures are realized cash flows. Every thing said about the car account is also true about the personal effects asset account. The personal effects provided utility, some wore out totally and were discarded, others should have been, and yet others were acquired. The acquisitions represent short term capital investments and are realized cash flows. If more were acquired than depreciated then the change in personal effects represented a use of funds.

The total change in assets equals the total change in total liabilities and equity. That is an unalterable fact discovered by mathematician Luca Pacioli in the sixteenth century. It is true because equity is a claim on the accrual called residual cash flow. The problem does not specify the types of liabilities owed to creditors. Interest accrues unless it is a payment in which case it's realized, principal changes are realized unless they are marked to market in which case they are accrued. Equity increases are accrued unless they are securitized, like David Bowie Bonds, in which case they are realized. Equity decreases are residual cash flows marked to market and are accrued unless bankruptcy occurs in which case it becomes an unfortunate realization.

4. ©FF22 a) "An increase in a liability account represents a source of funds" is true. If your loan account balance a month ago were \$100 and now is \$120 then it was the source for a \$20 loan taken from the account. b) "An increase in an asset account represents a use of funds" is true. If your checking account balance a month ago were \$70 and now is \$85 then it was the use for \$15 deposited into the account. c). "A decrease in an asset account represents a source of funds" is true. If your checking account balance a month ago were \$100 and now is \$60 then it was the source for \$40 taken from it.

EXERCISES 2.2A

1. Equity book value per share is (\$7,100 ÷ 250), or \$28.40. The company *P/B* therefore equals (\$24.30 ÷ \$28.40), or 0.86. *P/B* for the peer group is 0.67. The company *P/B* is a little higher than the peer group's, and to the extent that comparing their ratios is valid (the chapter on stock valuation discusses this more fully), the following inferences apply: the price the stock market assigns to a dollar of assets is larger for the company than for peer group, so either (i) the company stock is overvalued relative to the peer group, or (ii) the peer group is undervalued relative to the company.
2. Company market capitalization equals \$41 x 490 million, or \$20,090 million (just over \$20 billion). The company *P/B* equals (\$41 ÷ \$35), or 1.17.
3. The stock price today is \$23(1+.032), and today's market capitalization is \$23(1+.032)(260 million), or \$6.17 billion. Today's <u>change</u> in market cap is \$23(032)(260 million), or \$191.6 million.
4. Market cap for the Raider equals \$9,500 x 3.80, or \$36,100. Target market cap is \$3,600 x 1.50, or \$5,400. Conglomerate market cap therefore equals (\$36,100 + \$5,400), or \$41,500.
5. Use your calculator to quickly find that $\left(\frac{1.2}{47}\right)^{-1} = \left(\frac{1.2}{5}\right)^{-1} + x^{-1}$ which means x^{-1} equals 39.1667 minus 4.1667. That is, 35.0000 measures the reciprocal of percentage change in Raider shareholder wealth. Therefore, -2.86% (= 35^{-1}) is the Raider percentage change in wealth balances.
5a. Quickly find that $x^{-1} = 0.30^{-1} + 0.02^{-1}$ which means x^{-1} equals 3.3333 plus 50. That is, 53.3333 measures the reciprocal of the internal wealth transfer from Raider to Target shareholders relative to combined company equity market capitalization. The percentage change was 1.88%. For a \$1 billion company that means the wealth transfer is \$1.88 million. For a \$100 billion deal that means the wealth transfer is \$188 million.
5b. Quickly find that for the "tiny" deal $\frac{\$1\text{ billion}}{\$0.1\text{ billion}} = 0.30^{-1} + x^{-1}$ which means x^{-1} equals 3.3333 plus 10. That is, 13.3333 measures the reciprocal of the percentage change in Raider wealth balances. The percentage change was 1.88%. For a \$1 billion company that means the wealth transfer is \$1.88 million. For a \$100 billion deal that means the wealth transfer is \$188 million.
5. The Conglomerate shareprice equals its market cap divided by total number of shares outstanding. Total number of shares equals {820 + 770x(1/5)}, or 974. Conglomerate shareprice consequently equals (\$41,500 ÷ 974), or \$42.61.
6. Conglomerate *Stockholders' equity* equals (\$9,500 + \$3,600), or \$13,100. Conglomerate *P/B* therefore equals (\$41,500 ÷ \$13,100), or 3.17.
7. The value of 820 Raider shares before the merger equals its market cap, or \$36,100. The value of those 820 shares after the merger is (\$42.61 x 820), or \$34,940. The total Raider loss on all shares is (\$36,100 – \$34,940), or \$1,160. This sum also equals the total gain by Target shareholders.

8. Raider shareprice before the merger is ($9,500 x 3.80 ÷ 820), or $44.02. The loss for each Raider share due to the merger is ($44.02 – $42.61), or $1.41. This represents a 3.2 percent decline in wealth {= ($42.61 - $44.02) / $44.02; also equals $1,160 ÷ $36,100}. Target shareprice before the merger is ($3,600 x 1.50 ÷ 770), or $7.01. Target shareholders tender 5 Target shares worth $35.05 and receive one Conglomerate share worth $42.61, so their wealth increases 21.5 percent {= ($42.61 - $35.05) / $35.05; also equals $1,160 ÷ $5,400}.

EXERCISES 2.2B

1. No! Appearance on the balance sheet of a liability means that the company already has received financing from that source. For this problem, shareholders have been an historical source of $12 million. As to the future, well that is different. Maybe the company put the money in the cash account, in which case the funds are available for paying bills. If the company bought plant and equipment that over the next few decades should return profits, then right now maybe the money for paying bills is unavailable. The bottom line: insights about sources of funds provides little insight about uses of funds. Paying bills is a use of funds, *Stockholders' equity* is a source of funds, and one implies little about the other.

2. Cash is $6,800 on the balance sheet for 12/31/2525 and $7,900 for 12/31/2526. An increase in an asset is a use of funds. Maybe during year 2525 the company took out a loan for $1,100 and put the money into its cash account. The increase in the liability (loans) would have been a source of funds. Realize the importance of temporal perspective, here. The increase in *Cash* was a use of funds during year 2526. As for the future, well because more money is in the account then potentially the company can withdraw more *Cash* than otherwise. When that draw-down occurs, the decline in *Cash* (or any asset) will represent a source of funds. Financial statements document history, the increase in *Cash* represents a historical use of funds. But more *Cash* means more potential future sources – financial statements, however, do not document the future!

3. *Net working capital* equals *Current assets* minus *Current liabilities*. Components of *Current assets* include *Receivables* (decreases $4,400) and *Cash* (increases $6,100). Components of *Current liabilities* include *Payables* (increases $6,800) and *Short-term notes* (increases $5,600). Long-term accounts such as *PP&E* and *Long-term debt* do not affect *Net working capital*. The *Current assets* increase $1,700 (= $6,100 - $4,400), while *Current liabilities* increases $12,400 (= $6,800 + $5,600). Because *Current liabilities* increases more than *Current assets* there is a decline in *Net working capital*. The change in *NWC* equals $-10,700 (= $1,700 - $12,400) and is a source of funds.

EXERCISES 2.3A

1. When F and B represent a flow and balance variable, respectively, the plausible definitions for a financial ratio include F_t and either B_t or B_{t-1}. Perhaps the ratio, for example, equals F_t / B_t or F_t / B_{t-1} or $F_t \div \{(B_t + B_{t-1})/2\}$, or B_t / F_t, etc. Ratios in table 2.6 that are a flow divided by a balance include return-on-assets, return-on-equity, asset turnover, inventory turnover, and dividend yield. Ratios that are a balance divided by a flow include average age of inventory, average collection period, average payment period, price-to-earnings, price-to-cash flow, price-to-free cash flow, and price to sales.

2. Table 2.6 shows the formula for return-on-equity is *Net income* ÷ *Stockholders' equity*. Because this formula relates a flow to a balance, however, three defensible computations for your company's *ROE* exist: 13.5% (= $132,500 ÷ $980,000); or 14.6% (= $132,500 ÷ $905,000); or 14.1% {= $132,500 ÷ (($980,000 + $905,000)/2)}. While you know the peer group *ROE* is 14.0%, you do not know which formula they use. So you cannot say

with certainty whether your company's *ROE* is larger or smaller than the peer group's. They are pretty close, though, irrespective of formula.

3. *Total assets* equals the sum of *Total liabilities* ($85,000) and *Stockholders' equity* ($114,000), so *Total assets* is $199,000. The equity multiplier equals *Total assets* ÷ *Stockholders' equity*, which is 1.75. The debt-to-assets ratio is 43% (=$85,000/$199,000), and debt-to-equity ratio is 0.75 (=$85,000/$114,000).

4. *Earnings per share* equals $42 million divided by 20 million shares, or $2.10. The P/E ratio is $136.80 / $2.10, or 65.1. This is much larger than the traditional average P/E that ranges between 10 and 15.

5. *Net income* equals $129,000 *x* 0.064, or $8,256.

6. The income statement looks like this:

sales	$32,000	
- cost of goods good	not needed	
= earnings before depreciation	8,320	(= $32,000 *x* 0.26)
- depreciation	1,030	
= operating income	7,290	
- interest	2,500	
= taxable income	4,790	
- taxes	1,629	(= $4,790 *x* 0.34)
= net income	3,161	

7. *Net income* equals $3,125 (= $1.25 *x* 2,500). *Net income* divided by *Sales* is 0.115, so *Sales* equals $27,174 (= $3,125 ÷ 0.115). Therefore *Total assets* equals $7,548 (= $27,174 ÷ 3.6).

8. A P/E of 26.4 with earnings of $4.50 implies that the shareprice is $118.80 (= 26.4 *x* $4.50). The end-of-period shareprice is $118.80 and the dividend is $1.57 (= $4.50 *x* 0.35). The ROR of 32% equals ($118.80 + $1.57 – P_{t-1}) / P_{t-1} . Solve the preceding for P_{t-1} and find that the previous price was $91.19 [= ($118.80 + $1.57) / (1 + 0.32)]. The shareprice capital gain to $118.80 from $91.19 equals $27.61.

9. An inventory turnover at 9.5 with annual cost-of-goods sold of $50,000 implies that (a) *Inventory* equals $5,263 (= $50,000 ÷ 9.5), and (b) the average age of inventory is 38.4 days (= 365 ÷ 9.5). The average age of inventory decreases by 5 days to become 33.4 days. The new inventory turnover becomes 10.9 (= 365 ÷ 33.4), suggesting that the company turns over inventory more frequently than before. Compute that *Inventory* therefore equals $4,578 (= $50,000 ÷ 10.9). The decline in *Inventory* on the balance sheet to $4,578 from $5,263 suggests a source of funds equal to $685 (= $5,263 - $4,578).

10. Find *Net income* from the ratios in the setup. *Sales* equals 3.2 *x* $6,400; *Taxable income* equals *Sales* *x* 0.12 – (0.08 *x* $2,600). *Net income* equals *Taxable income* *x* (1 – 0.30). Thus, *Net income* equals $1,575 [= { (3.2 *x* $6,400 *x* 0.12) – (0.08 *x* $2,600) } *x* 0.70].

11. The P/E of 11½ given *Net income* of $1,575 and 325 shares outstanding implies a stock price of $55.72 (= 11.5 *x* $1,575 ÷ 325).

12. The P/B at year-end 2525 of 3.6 with *Stockholders' equity* of $3,800 and 325 shares outstanding implies a stock price of $42.09 (= 3.6 *x* $3,800 ÷ 325). The stock price begins at $42.09 and a year later equals $55.72. Also, the share receives a dividend of

$1.70 (= $1,575 x 0.35 ÷ 325). The stockholders' rate of return therefore equals 36.4% {= ($55.72 + $1.70 - $42.09) ÷ $42.09}.

EXERCISES 2.3B

1. When P/B is constant then P and B are changing at the same rate: the capital gains rate equals the percentage change in book value. The explanation therefore can focus on the role of the dividend yield ($Dividend_t \div P_{t-1}$ and $Dividend_t \div B_{t-1}$). When the P/B < 1 then the shareholders' ROR ≥ ROE because $Dividend_t \div P_{t-1} > Dividend_t \div B_{t-1}$. With a relatively small stock price the market dividend yield exceeds the book dividend yield (a relatively small price buys a relatively large dividend). Conversely, when the P/B > 1 then the shareholders' ROR ≤ ROE because $Dividend_t \div P_{t-1} < Dividend_t \div B_{t-1}$. The equality for both cases occurs when the payout ratio is zero.

2. The ROE equals 9.1% x 1.7 x (equity multiplier). The debt-to-asset ratio of 60% means the equity-to-asset ratio is 40%, and the reciprocal of 40% is the equity multiplier (2.5). Now multiply the three parts together and find that the ROE equals 38.7%.

3. Use formula 2.7. For company X find the unknown ROR:

ROR = 26% { 1 – (0.30)(1.35 - 1)/1.35 }; ROR = 24.0%.

Now for company Z use the formula:

ROR = 26% { 1 – (0.30)(2.75 - 1)/2.75 }; ROR = 21.0%.

The difference between ROR equals 3 percentage points.

4. The simplest way to answer this is compute that ROE is 20% (= $500 ÷ $2,500). Then use formula 2.7 to compute that ROR is 17.4% (= 20% x [1 - ($250/$500)(1.36 - 1)/1.36]. You also could have taken the longer route to find that SE_{2526} = $2,750 (= $2,500 + $250). Thus $Market\ cap_{2526}$ is $3,740 (= 1.36 x $2,750). Find ROR as (1.36x$2,750 + $250 - 1.36x$2,500) ÷ (1.36x$2,500), which of course is 17.4%. You also could have done that on a per share basis. The ROR is smaller than the ROE because price-to-book is bigger than one.

5. Use formula 2.7. For company X find the unknown ROR:

ROR = 18% { 1 – (0.25)(0.8 - 1)/0.8 }; ROR = 19.1%.

Now for company Z use the formula to find the unknown ROE (given the ROR of 19.1%).

19.1% = ROE { 1 – (0.25)(2.75 - 1)/2.75 }; ROE = 22.7%.

The ROE must be 4.7% bigger for company Z than company X in order for the two to provide the same shareholders' ROR.

6. Compute the company numbers directly from the financial statements. The table below summarizes everything:

	ROE	NI/Sales	Sales/TA	TA/SE
company	42.6%	6.9%	3.8	1.6
industry	30.9%	5.1%	??	1.6

Find the industry equity multiplier from the debt-to-assets ratio as follows using formula 2.5 [or reason that if debt-to-assets is 37.8% then equity-to-assets is 62.2% and equity multiplier equals $(62.2\%)^{-1}$.] Find the unknown industry Sales/TA as 0.309 x $(5.1\%)^{-1}$ x 62.2%; that equals 3.8. The company ROE is larger than the industry average because the company net profit margin is larger (and all else is equal).

EXERCISES 2.3C

1. The basic reason that the tax rate does not enter the total breakeven point when

Preferred dividends equal $0 is because taxes are a constant proportion of taxable income. With taxable income of zero the *Taxes due* equal zero, the *EAC* equals zero, and the company is at the total breakeven point. Taxes matter when the company is beyond the total breakeven point. Suppose, for example, that the tax rate is 25%. To earn an extra $1 of *EAC* the company must earn an extra $1.33 of taxable income [=*$1/(1 – 0.25)*]. Taxes are a big deal, only not at the breakeven point. That is true when the contribution margin is exogenous.

2. Straightforward application of formula 2.9b shows that the quantity of production at the operating breakeven point equals 2,400 units [= $12,000 ÷ ($12 - $7)].

3. The company must cover the $34,000 fixed costs to suppliers plus the $15,000 operating income to them self, so they must net $49,000 (= $34,000 + $15,000). Use formula 2.9b and find that the quantity of production at the operating breakeven point equals 8,167 units [= $49,000 ÷ ($22 - $16)]. Reconstruct the income statement and verify that *EBIT*, also known as operating income, equals $15,000 when *Sales revenue* is $179,667 (= 8,167 *x* $22) and *Total fixed costs* equal $34,000 and *Total variable costs* equal $130,667 (= 8,167 *x* $16).

4. Use formula 2.9a and find that the *Sales revenue* at the operating breakeven point equals $74,359 [= $38,250 ÷ (1 - $40,400/$83,195)]. Verify that *EBIT* equals $0 when *Sales revenue* is $74,359 and *Total variable costs* is $36,109 (= $74,359 *x* $40,400/$83,195).

5. Use formula 2.9a and find that with $EBIT^{target}$= $6,300 the *Sales revenue* equals $103,534 [= ($42,500 + $6,300) ÷ (1 - $50,500/$95,525)]. Verify that *Total variable costs* equal $54,734 (= $103,534 *x* $50,500/$95,525).

6. Use formula 2.10a and find that the *Sales revenue* at the total breakeven point equals $79,993 [= ($34,000 + $3,825 + (500/(1-0.30))) ÷ (1 - $50,500/$97,450)]. *EBIT* equals $4,539 and may be computed as $79,993 x (1 - $50,500/$97,450) - $34,000.

7. Use formula 2.10a and find that the *Sales revenue* is $96,829 when EAC^{target} = $7,900 [= ($29,750 + $2,550 + (350+$7,900)/(1-0.25))) ÷ (1 - $50,500/$91,350)]. *EBIT* equals $13,550 and may be computed as $96,829 x (1 - $50,500/$91,350) - $29,750.

8. Easily find that the ratio of variable cost to sales is 0.6374 [= $475,000/$745,200]. Variable costs rise and fall with sales but always the ratio is 0.6374. Every $100 increase in sales raises variable costs by $63.74. Every $100 increase in sales contributes $36.26 to operating income (EBIT). The contribution margin of 0.3626 is the denominator of the breakeven formulas. The company in this problem has net income of $125,000 and therefore is clearly paying all its fixed costs and interest expenses. A further increase in sales of $100 increases EBIT by $36.26 and *Net income* also would increase by $36.26 except that the company must pay 30% of the $36.26 as taxes. Incremental sales of $100 contributes $25.38 [= (1 - 0.30) x 0.3626] to *Net income.* The *after-tax* contribution margin is 0.2538. If sales were to rise from $745,200 to $745,300 then *Net income* would rise from $125,000 to $125,025.38.

The problem states that the ROE (= *Net income / Stockholders equity*) is 14% when *Net income* equals $125,000. Thus *Stockholders equity* is $892,857. To reach an ROE of 18% then *Net income* must equal $160,714 [= $892,857 x 0.18]. Thus, if *Net income* rises by $35,714 [= $160,714 - $125,000]. Because a $100 increase in sales raises *Net income* by $25.38, then to get an increase of $35,714 in *Net income* means sales must rise by $140,712 (= $35,714 ÷ 0.2538) in order to reach the 18% ROE.

Notice that this solution uses the same old breakeven formulas and introduces no new formulas. Instead, it requires a thorough understanding of what formulas mean.

9. Use formula 2.9a and find that the *Sales revenue* at the operating breakeven point equals $54,612 [= $34,000 ÷ (1 - $25,250/$66,900)]. This is a decline relative to the original *Sales revenue* of –18.4% [= ($54,612 - $66,900) ÷ $66,900].

10. . ©BE4a Example 7 shows the relation between quantity sold when the operating margin is

Q = F / [p (1 – v/p – operating margin)]

Insert the numbers to find Q, the quantity sold when the operating margin equals 18%:

Q = $25,500 / [$45 (1 – 33/45 – 0.18)]
= 6,538 units

Verify that with sales at 6,538 units the *Sales revenue* equals $294,231 (= 6,538 x $45); *Total variable costs* equal $215,769 (= 6,538 x $33); *Total fixed costs* equal $25,500; and the operating margin equals 18%, as requested [= ($294,231 - $215,769 - $25,500) ÷ $294,231].

11. ©BE4b Like the previous problem, find the quantity sold when the operating margin is 20%:

Q = $34,000 / [$30 (1 – 21/30 – 0.20)]
= 11,333 units

With sales at 11,333 units the *Sales revenue* equals $340,000 (= 11,333 x $30).

12. Reconstruct the stylized income statement from the annual report and find that *Net income* equals $10,448. The ROE of 15.7% with *Net income* of $10,448 implies that *Stockholders' equity* is $66,545. To get an ROE of 24% the *Net income* must rise to $15,971 (an increase of $5,523). Compute the requisite *Sales revenue* either of two ways. Method 1: Plug numbers into formula 2.10a and find that *Sales revenue* is $120,925 [= ($42,750 + $3,825 + ($15,971)/(1-0.30)) ÷ (1 - $45,675/$107,175)]. Method 2: Divide the after-tax contribution margin of 0.4017 [= (1 – 0.30) x (1 - $45,675/$107,175)] into the extra *Net income* to find the extra *Sales revenue* is $13,750 (= $5,523 / 0.4017). Thus, sales must rise to $120,925 (= $107,175 + $13,750).

13a. Compute the requisite *Sales revenue* from formula 2.11a and find that *Sales revenue* is $100,590 [= {$38,475 + $2,975 + (0.125 x $73,900 x 2.0)/{(1-0.25) x [2.0 - 0.30 x (2.0 - 1)]}} ÷ (1 - $45,675/$102,900)].
13b. *EAC* equals $10,868 [= (0.125 x $73,900 x 2.0)/[2.0 - 0.30 x (2.0 - 1)]. Thus, *Dividends* equal $3,260 [= $10,868 x 0.30].

EXERCISES 2.4

1. *Sales* equals 3.8 x $3,700; *Net income* equals 0.078 x *Sales*; *New retained earnings* equals (1 – 0.45) x *Net income*; SE_{2526} equals *New retained earnings* plus $2,600; equity book value per share equals SE_{2526} ÷ 260. Thus, equity book value per share = { (3.8 x $3,700 x 0.078 x 0.55) + $2,600 } ÷ 260; which is $12.32.

2. *Sales* equals 3.5 x $4,800; *Net income* equals 0.09 x *Sales*, *New retained earnings* equals (1 – 0.30) x *Net income*; SE_{2526} equals *New retained earnings* plus $3,130; $price_{2526}$ equals SE_{2526} x 0.90 ÷ 900. Thus, $price_{2526}$ = { (3.5 x $4,800 x 0.09 x 0.70) + $3,130 } x 0.90 ÷ 900; which is $4.19.

3. *Net income* equals $18,800 x 0.047; *New retained earnings* equals (1 – 0.55) x *Net income*; SE_{2526} equals *New retained earnings* plus $4,000; equity book value per share equals SE_{2526} ÷ 170; $price_{2526}$ equals 0.90 x equity book value per share. Thus, $price_{2526}$ = { ($18,800 x 0.047 x 0.45) + $4,000 } ÷ 170 x 0.90; which is $23.28.

4. *Net income* equals $12,540 x 0.064; *New retained earnings* equals (1 – 0.60) x *Net*

income; SE_{2526} equals *New retained earnings* plus $3,800; equity book value per share equals SE_{2526} ÷ 100; $price_{2526}$ equals 0.84 *x* equity book value per share. Thus, $price_{2526}$ = { ($12,540 *x* 0.064 *x* 0.40) + $3,800 } ÷ 100 *x* 0.84; which is $34.62. The beginning shareprice, $price_{2525}$, equals ($3,800 ÷ 100) *x* 0.67, which is $25.46. The dividend per share equals {($12,540 *x* 0.064 *x* 0.60) ÷ 100, which is $4.82. The shareholders' ROR equals ($34.62 + $4.82 - $25.46) ÷ $25.46, which is 55%.

5. With a constant P/B ratio the percentage changes in P and B are equal. Finding the percentage increase in shareprice therefore requires finding the percentage increase in *Stockholders' equity*. SE_{2525} is given as $3,400. SE_{2526} equals $3,400 plus *New retained* $earnings_{2526}$. Find *New retained earnings* by reconstructing the income statement. *Taxable income* equals $25,500 *x* 0.22 - $1,280 – (0.12 *x* $2,400); *Net income* equals (1 – 0.32) *x* *Taxable income*; *New retained earnings* equals (1 – 0.58) *x* *Net income*. Thus, *New retained earnings* = { $25,500 *x* 0.22 - $1,280 – (0.12 *x* $2,400) } *x* 0.68 *x* 0.42; which is $1,154. SE_{2526} equals $4,554; the percentage increase is $1,154 ÷ $3,400; which is 34%.

6. With a constant P/E ratio the percentage changes in P and E are equal. The setup states that E increases 7.3% during 2526, so therefore the stock price increases 7.3%, too. The shareholders' ROR equals ($price_{2526}$ + $dividend_{2526}$ - $price_{2525}$) ÷ $price_{2525}$, which also may be written as *%∆price* + ($dividend_{2526}$ ÷ $price_{2525}$). Find $dividend_{2526}$ and $price_{2525}$ as follows. *Sales* equals 4.5 *x* $3,100; *Net income* equals 0.076 *x* *Sales*; *Dividends per share* equals *Net income* *x* 0.40 ÷ 150. Thus, $dividend_{2526}$ = 4.5 *x* $3,100 *x* 0.076 *x* 0.40 ÷ 150, which is $2.83. Notice that $price_{2526}$ equals 14.1 *x* *earnings per* $share_{2526}$; thus $price_{2526}$ = 14.1 *x* 4.5 *x* $3,100 *x* 0.076 ÷ 150; which is $99.66. We know $price_{2526}$ is 7.3% larger than $price_{2525}$. Thus, $price_{2525}$ = $99.76 ÷ (1 + 0.073); which is $92.88. Therefore, $dividend_{2526}$ ÷ $price_{2525}$ equals $2.83 ÷ $92.88; which is 3.0%. The shareholders' ROR equals 7.3% + 3.0%, which is 10.3%.

7. *Sales* equals 3.5 *x* $4,900; *Net income* equals 0.065 *x* *Sales*; *Earnings per share* equals *Net income* ÷ 390; $price_{2526}$ equals 14.5 *x* *Earnings per share* . Thus, $price_{2526}$ = { (3.5 *x* $4,900 *x* 0.065 ÷ 390) *x* 14.5 }, which is $41.45.

8a. *Sales* equals 3.1 *x* $9,100; *Net income* equals *Sales* *x* 0.087; *New retained earnings* equals (1 – 0.55) *x* *Net income*; SE_{2526} equals $5,500 plus *New equity issues* plus *New retained earnings*; *New equity issues* equals the *Capital expenditure*, (0.22 *x* $7,300 + 400), which equals $2,006. Therefore, SE_{2526} = { (3.1 *x* $9,100 *x* 0.087 *x* 0.45) + $5,500 + $2,006}; which is $8,610.

8b. The beginning shareprice, $price_{2525}$, equals ($5,500 ÷ 700) *x* 0.80, which is $6.29. The ending shareprice, $price_{2526}$, equals ($8,610 ÷ *#shares*) *x* 0.80. The *#shares* equals the 700 original shares plus the new issues. Issuing equity to finance the *Capital expenditure* of $2,006, at $6.29 per share, means that 319 shares were issued. Therefore, $price_{2526}$ = ($8,610 ÷ 1,019) *x* 0.80; which is $6.76. The dividend per share equals {(3.1 *x* $9,100 *x* 0.087 *x* 0.55) ÷ 1,019, which is $1.32. The shareholders' ROR equals ($6.76 + $1.32 - $6.29) ÷ $6.29, which is 29%.

9. There are two ways to solve this. First, let's use the memory intensive approach. Use the DuPont decomposition to find ROE (recall that the equity multiplier equals 1 plus debt-to-equity).

$$\begin{aligned} \text{ROE} &= \text{net profit margin } \times \text{ asset turnover } \times \text{ equity multiplier} \\ &= 6.1\% \times 1.5 \times (1 + 1.11) \\ &= 19.3\% \end{aligned}$$

Now use formula 2.7 that relates ROR to ROE when the P/B is constant. Realize that because P/B is bigger than one we expect that the ROR is smaller than 19.3%.

ROR = 19.3% { 1 – (0.60)(1.4 - 1)/1.4 }
= 16.0%.

The market ROR, as expected, is 16% and is somewhat smaller than the book ROE of 19.3%.

The second approach for solving this problem is not memory intensive, it is more intuitive, but it takes more steps. Find net $income_{2526}$ is $2,440 (= $40,000 x .061); total $dividends_{2526}$ is $1,464 (= $2,440 x 0.60); new retained $earnings_{2526}$ is $976 (= $2440 - $1,464). Next combine the asset turnover and debt-to-equity ratios to find SE_{2525} is $12,638 (D/SE = 1.11; D = 1.11 x SE; then D + SE = $40,000; 1.11SE + SE = $40,000/1.5; then solve for SE_{2525}); find that SE_{2526} is $13,614 (= $12,638 + $976). Compute ROE as the ratio of net $income_{2526}$ to SE_{2525}, which is 19.3% (= $2,440 / $12,638). Compute market cap as SE x P/B for the ROR formula and find ROR is 16.0% {= ($1,464 + 1.4x$13,614 - 1.4x$12,638)/(1.4x$12,638)}.

CHAPTER 3: ACCOUNTING FOR GROWTH

It is almost impossible for a company to remain unchanged from one year to the next. Some changes occur because of overt management actions, while other changes seem to happen all by themselves. Changes that occur over time, planned or otherwise, include “growth”.

Growth is as natural to businesses as to households and individuals. Growth is a complex yet important phenomenon. Companies sometimes grow too slow — they may fail if they do not take advantage of opportunities. Alternatively, sometimes companies grow too fast — they may fail as a result of ill-advised explosive growth. Growth is just like water, oxygen, and many other phenomena: There can be too much as well as too little. The table below illustrates that company growth rates vary widely by both type of measurement and company.

Corporation Name	**%Δ Total assets**	**%Δ Annual sales**	**%Δ net income**	**%Δ dividend / share**	**%Δ stock price**
AOL-Time Warner, Inc.	93%	46%	66%	0%	102%
AT&T Corporation	67	7	-2	0	-8
Exxon Mobil Corporation	2	-4	-9	3	n.a.
Ford Motor Company	-1	3	18	8	7
General Electric Company	15	12	6	15	23
General Motors Corp.	9	3	9	8	6
IBM	1	4	10	9	35
Microsoft Corporation	53	24	40	0	33
Wal-Mart Stores, Inc.	25	16	21	22	41
Walt Disney Company	4	4	-15	7	-21

TABLE 3.1 Growth rates for selected variables of American corporate titans.
Each entry is the annual average percentage change in the respective variable for an historical 3-year window (stock price adjusted for splits).

There are many ways to measure company growth: percentage change in total assets, net income, stock price, or even number of employees. The table reveals that huge growth in total assets usually relates to huge revenue and stock price growth, but not always. At more moderate levels of growth, the relations are even more confused. Surely growth matters, as every business and household knows, but what are its effects and how can we plan for it?

Understanding the relation between growth and financial statements assists the planning process. This chapter contains lessons examining how growth affects flows and balances. Section 1 focuses on the effect of growth on financing needs. Section 2 looks for insights about growth rates that financial statements may reveal. Section 3 examines effects of growth on cash flows.

1. Financial forecasting

A successful company anticipates financing needs long before it requires funds. Once the company identifies the need, management arranges financing just in case the need eventuates. For any business, as for any household too, it's stressful to learn today that a large sum of money is needed tomorrow. Planning financial needs is critically important.

There are two general approaches for forecasting financing needs. One relies primarily on cash flow analysis and the other primarily on balance sheets. The objective of both approaches is identical: determine whether the company in the future expects sources of funds to satisfy requisite uses. Results of the analysis suggest that the company expects either a surplus or a shortfall.

DEFINITION 3.1 Surplus and shortfall
Management should forecast future financing needs long before funds are needed.

$$\textit{When forecast sources of funds} \begin{Bmatrix} > \\ < \end{Bmatrix} \textit{forecast uses of funds}$$

$$\textit{then the company expects a} \begin{Bmatrix} \textit{surplus} \\ \textit{deficit or shortfall} \end{Bmatrix}.$$

When the company forecasts a surplus then management is in the fortunate position of debating prudent uses of surplus funds. Conversely, when the company faces an expected shortfall then management must take strategic action to avoid financial misfortune.

1.A. Cash budgeting

In a *cash budgeting* analysis the company sums expected revenues (sources of funds) and subtracts expected cash costs (uses of funds). A surplus occurs when more money comes in than goes out. Conversely, a deficit occurs when more money goes out than comes in. An advantage of the cash flow approach is its directness. Most students intuitively understand cash budgeting, and most routinely conduct one: estimate tuition and costs of living, estimate likely revenue sources, and make arrangements to cover shortfalls. The company analysis is analogous, as the example below simply illustrates.

EXAMPLE 1 Find the surplus or shortfall

The third quarter just concluded. September's monthly sales were $60,000 and the company paid shareholders quarterly dividends totaling $8,000. The end-of-September quarterly balance sheet lists *Cash* at $10,500. The company never wants the cash account to drop below $10,000 because the cash buffer provides protection against forecasting errors. The company forecasts the following events during the next four months (October – January):

a. monthly sales forecasts equal $64,000; $80,000; $95,000; and $60,000; 40% of all sales revenue is collected in month of sale and the remainder is collected the subsequent month
b. cash variable costs for taxes and supplies equal 78% of the previous month's sales
c. cash fixed costs equal $12,000 per month
d. an extraordinary debt payment of $14,000 is due in November
e. quarterly dividends payable in December will be 4% larger than September's dividends

Find the company's pretax net cash flow each month and determine financing needs.

SOLUTION
Summarize numbers from the setup into tabular form.

	October	November	December	January
S_t, sales	$64,000	$80,000	$95,000	$60,000
collections from S_{t-1}	$36,000	$38,400	$48,000	$57,000
collections from S_t	$25,600	$32,000	$38,000	$24,000
variable costs	$46,800	$49,920	$62,400	$74,100
fixed costs	$12,000	$12,000	$12,000	$12,000
extraordinary costs		$14,000	$8,320	
Beginning of month cash	$10,500	$13,300	$7,780	$11,060
cash inflows	$61,600	$70,400	$86,000	$81,000
cash outflows	$58,800	$75,920	$82,720	$86,100
pretax net cash flow	$2,800	-$5,520	$3,280	-$5,100
End of month cash	$13,300	$7,780	$11,060	$5,960

All entries are straightforward. During October the company has collections equal to 60% of September's sales (S_{t-1}), plus 40% of October's sales (S_t). The other 60% of October's sales is collected in November. Variable costs equal 78% of the previous months sales, and fixed costs always equal $12,000 per month. Extraordinary costs include the debt payment in November and dividends in December ($8,320 = $8,000 x 1.04).

The bottom panel presents the beginning of month cash balance. Cash inflows (that is, the sum of collections) increase the balance, whereas cash outflows (that is, the sum of variable plus fixed and extraordinary costs) decrease the balance. Pretax net cash flow equals the sum of inflows minus outflows – net cash flow equals sources minus uses of cash and measures that month's surplus or shortfall. In October the net cash flow of $2,800 raises the end of month cash balance to $13,300 (=$10,500 + $2,800). The company expects a surplus in October, and its cash balance exceeds the target minimum of $10,000.

In November the company expects a deficit of $5,520. Furthermore, the ending cash balance of $7,780 is less than the allowable minimum. To raise the balance to $10,000 the company must borrow an additional $2,220 (=$10,000 - $7,780). In December the company expects a surplus, however, and the short-term loan could be partially repaid. Finally, in January there is a deficit of $5,100. January's ending cash balance is $4,040 less than the desirable minimum (=$10,000 - $5,960), suggesting that management has a few months to arrange for financing or to institute pricing or cost polices that lessen the shortfall.

The most difficult task with a cash budgeting analysis is getting good forecasts of cash flows. Otherwise, cash budgeting is mechanical. Few financial exercises, however, are as important as cash budgeting – figuring whether checks are going to bounce because of a cash shortfall is necessary, as we all intuitively know, for good financial health.

One temporal trait of cash budgeting merits mention. It is impossible to detect a surplus or deficit at a frequency that is shorter than the cash flow period. The example above employs a monthly frequency for tabulating cash flows. This means that daily or weekly deficits are undetectable. Consider the October scenario, for example. The monthly surplus is $2,800. It is possible, though, that the cash outflows of $58,800 may occur during the first half of the month whereas the inflows of $61,600 occur during the latter half. Because the checking account begins with $10,500 then the checks written to suppliers may start bouncing before the money from customers arrives. The key point is that monthly cash budgets detect monthly shortfalls, but are incapable of detecting weekly shortfalls. For that, construct a weekly cash budget!

With cash budgeting the details overwhelm the analysis. Forming long range plans with cash budgets is difficult because detailed forecasts of cash flows are difficult to obtain. For long-run planning, balance sheets come to the rescue!

1.B. Balance sheet forecasts

This section examines a method for forecasting financing needs that depends on the balance sheet identity that *Total assets* equals *Total liabilities & Stockholders' equity*. The balance sheet approach shines because of its stark simplicity and irrefutable logic: make a forecast of (a) expected *Total assets*, and (b) expected *Total liabilities & Stockholders' equity*. The "External Financing Needs" (*EFN*) equals the difference

FORMULA 3.1 External financing needs

EFN is a positive number when expected uses of cash exceed expected sources.

$$\begin{pmatrix} External \\ Financing \\ Needs \end{pmatrix} = \begin{pmatrix} expected \\ Total\ \ assets \end{pmatrix} - \begin{pmatrix} expected \\ Total\ \ liabilities \\ \&\ Stockholders' \\ equity \end{pmatrix}$$

When *EFN* is positive, there are insufficient funds to finance the company's expected assets and the company should arrange additional financing to cover the shortfall. A negative *EFN*, conversely, implies a surplus — the company expects to have more than enough financing to support expected assets.

B1. Fundamentals of forecasting with reliance on balance sheets

This example illustrates fundamental principles about forecasting financing needs with reliance on balance sheets.

EXAMPLE 2 Find EFN in a static setting . ©EFN1b

Suppose a company's balance sheet looks as follows:

Company Balance Sheet			
	Assets	*Liabilities*	
Cash	$ 100	$ 200	Current Liabilities
Inventory	400	350	Long term Debt
PP&E	500	450	Stockholders' Equity
Total Assets	*$1,000*	*$1,000*	*Total Liabilities & Equity*

Also suppose that *Sales* equal $2,500 and that *Cost-of-goods sold* equal $1,875. The company realizes that if they cut by 30 days the length of time that inventory stays on the shelf before sold, and all else remains the same, the company reduces the amount of inventory required. If the company proceeds with this inventory policy change, what is the effect on external financing needs?

SOLUTION

The first step in solving this problem is to compute the length of time inventory stays on the shelf before sold. The definition from table 2.6 for the "average age of inventory" is:

$$\begin{pmatrix} average \\ age \ \ of \\ inventory \end{pmatrix} = \frac{365 \ x \ (Balance \ \ Sheet \ \ Inventory)}{(Annual Cost - of - goods sold)}$$

$$= \frac{365 \, x \, \$400}{\$1,875}$$

$$= 77.8 \ \ days$$

Reducing by 30 the number of days that inventory remains on the shelf lowers the average age of inventory to 47.8 days.

In the preceding definition, set average age of inventory to 47.8 and hold cost-of-goods-sold the same as before at $1,875. Solve for the new balance sheet *Inventory* as:

$$47.8 \ \ days \ = \ \frac{365 \ x \ (Balance \ \ Sheet \ \ Inventory)}{\$1,875}$$

$$Balance \ \ Sheet \ \ Inventory \ = \ \$245$$

The balance sheet after the policy change lists *Inventory* at $245. The original balance sheet lists Inventory at $400. This policy change decreases the amount of inventory that the company keeps on hand.

What does the new balance sheet look like, and how much is *EFN*? This elementary yet important question deserves discussion. So far, we deduce the following:

Preliminary Forecast Company Balance Sheet			
Assets		*Liabilities*	
Cash	$ 100	$ 200	Current Liabilities
Inventory	245	350	Long term Debt
PP&E	500	450	Stockholders' Equity
Total Assets	*$845*	*$1,000*	*Total Liabilities & Equity*

EFN is found as

$$\begin{pmatrix} External \\ Financing \\ Needs \end{pmatrix} = \begin{pmatrix} expected \\ Total\ \ assets \end{pmatrix} - \begin{pmatrix} expected \\ Total\ \ liabilities \\ \&\ Stockholder's \\ equity \end{pmatrix}$$

$$= \quad \$845 - \$1{,}000$$

$$= \quad \$-155$$

The analysis shows the policy change results in a surplus of $155.

The preceding analysis finds that the company expects a surplus. The analysis has not yet finalized a prediction about appearance of the forecast balance sheet. The actual balance sheet that eventuates will have identical bottom line numbers that depend on the policy that management pursues. Carefully consider the options for the ways it can play out.

The final balance sheet cannot look like the preliminary one forecast above. The forecast one does not equalize the right and left-hand side bottom-lines. The reason a balance sheet must "balance" is not because it is a government or trade law, nor because it is the ethical thing to do. Instead, the balance sheet balances because it is <u>impossible</u> for it to be unbalanced. It is just as likely to have a coin with only one side as it is to have an unbalanced balance sheet. It just won't happen!

Consider the company with its surplus of $155. What can it do? While there are many possibilities, consider the following three.

Case 1: Maybe the company chooses to hold the surplus as cash, in which case the final balance sheet appears as follows:

Forecast Company Balance Sheet (Final, case 1)			
Assets		*Liabilities*	
Cash	$ 255	$ 200	Current Liabilities
Inventory	245	350	Long term Debt
PP&E	500	450	Stockholders' Equity
Total Assets	*$1,000*	*$1,000*	*Total Liabilities & Equity*

Cash increases by $155. That is, *Total as*sets rises to equilibrate with *Total liabilities & Stockholders' equity* and now everything balances.

Case 2: Perhaps the company uses the surplus to pay an extraordinary dividend to its shareholders. For this choice, the final balance sheet is:

Forecast Company Balance Sheet (Final, case 2)			
Assets			*Liabilities*
Cash	$ 100	$ 200	Current Liabilities
Inventory	245	350	Long term Debt
PP&E	500	295	Stockholders' Equity
Total Assets	*$845*	*$845*	*Total Liabilities & Equity*

Notice that *Stockholders' equity* decreases by $155. This occurs because the company income statement lists *Dividends* that are $155 larger than otherwise, and *New retained earnings* (and *Stockholders' equity*) are lower by $155. That is, *Total liabilities & Stockholders' equity* falls to equilibrate with *Total assets*. Everything balances on a balance sheet.

Case 3: Finally, suppose that the company uses the surplus to pay off some debt, in which case the final balance sheet is:

Forecast Company Balance Sheet (Final, case 3)			
Assets			*Liabilities*
Cash	$ 100	$ 200	Current Liabilities
Inventory	245	195	Long term Debt
PP&E	500	450	Stockholders' Equity
Total Assets	*$845*	*$845*	*Total Liabilities & Equity*

Debt decreases by $155 and everything balances.

Financial planning determines whether a surplus or deficit is likely. Once the outcome is determined, management pursues strategic decisions that advance the corporate mission: namely, maximize wealth creation for stakeholders and capitalists. Typically, long-range forecasts with balance sheets are the best analysis for seeing "the forest", whereas to see "the trees" employ the always-essential cash budgeting.

Growing a company often requires access to capital. For comparative purposes imagine how growing a household requires access to capital. Buying a car usually involves borrowing money. And certainly buying a house requires borrowing money. Paying for college, as you may already know, too often requires access to capital from financial markets. Before continuing lessons on forecasting external financing needs, learn a little about the premier forum for raising capital: the *New York Stock Exchange*.

STREET-BITE The New York Stock Exchange

The New York Stock Exchange traces origins to 1792 when two dozen New York City stockbrokers and merchants signed an agreement for trading securities. For years the trading forum was on Wall Street, so-named after a 12-foot high stockade fence constructed in 1653 as protection for the fledgling village of New Amsterdam (apparently the wall was ineffective because in 1664 England captured the town from the Netherlands and renamed it New York). The NYSE moved operations a few blocks south to Broad Street in 1865 into their glorious building constructed in 1903. Corporate operations still are on Broad Street but as of 2007 all trading floors now operate at #11 Wall Street.

The NYSE Group (New York) and the Euronext N.V. all-electronic stock exchange (Paris) merged in 2007. The NYSE Euronext consolidated balance sheet right-hand-side listed stockholder's equity for shares that publicly traded with ticker symbol NYX. Prior to 2005 the NYSE was a privately-held corporation owned by member firms that owned or leased a fixed number of "seats" on the NYSE. In 2005 the NYSE went public through a merger with the Pacific Stock exchange owner, the ARCA Inc.. Each NYSE seat owner was given $500,000 and 40,000 shares of NYX. Until 2013 one could have bought ownership in the NYSE simply by buying a share of NYX - about $18 per share at one point - and you could have bought the share at the New York Stock Exchange! In 2013 the NYSE-EuroNext was acquired for $8.2 billion by the

Intercontinental Exchange (ticker symbol is ICE). The combined ICE-NYSE-Euronext is the third largest exchange group globally, behind world No. 1 Hong Kong Exchanges and Clearing and the Chicago based CME Group. The ICE, headquartered in Atlanta, operates a network of regulated exchanges and clearing houses for financial and commodity markets in the United States, the United Kingdom, Continental Europe, Canada, and Asia.

Licensed NYSE members buy and sell securities on the trading floor. Members meet rigorous professional standards set by the Exchange. Wall Street companies such as J.P. Morgan Chase or Goldman-Sachs buy several licenses in order to trade securities for themselves and their clients. For an annual license fee of about $40,000 quite a few wealthy individuals also own licenses, too, simply to trade on their own account.

The NYSE earns revenues from several activities: They collect a fee for every trade that executes; they provide data processing services and sell proprietary information; they collect various fees from members; and they collects fees from companies like IBM that pay for the privilege to have their stocks traded at the prestigious New York Stock Exchange! The NYSE earns revenues, pays expenses, and earns *Net income* that is either paid as dividends to owners of ICE stock or retained, thereby increasing the *Stockholders' equity* on its balance sheet.

About 2,750 different companies list stocks at the NYSE; roughly 450 are foreign-based while the rest have headquarters in the U.S. Each company pays fees to list its stock, perhaps as much as half-million dollars per year. The exchange sets standards that companies must satisfy to qualify for listing. For example, aggregate pretax earnings over the previous three years for domestic companies must equal $10 million or market capitalization must exceed $100 million. Companies listed at the NYSE are basically big-to-huge. The table below shows that by far the NYSE has the largest market cap ($16.6 trillion at year end 2014) of any stock exchange in the world (in 1990 the Tokyo Stock Exchange briefly was bigger than the NYSE). About one-quarter to one-third of the entire world's stock market value resides at the Big Board. The table also shows the effect of the global financial crisis on 2008 stock market values.

	World Total	*NYSE*	*NYSE %world*	*NASDAQ*	*Tokyo*	*London*	*Deutsche Börse*	*Euronext*
2012	$54.7	$14.1	26%	$4.6	$3.5	$3.5	$1.5	$2.8
2011	$47.5	$11.8	25%	$3.8	$3.3	$3.3	$1.2	$2.4
2010	$54.9	$13.4	24%	$3.9	$3.8	$3.6	$1.4	$2.9
2009	$47.7	$11.8	25%	$3.2	$3.3	$3.4	$1.3	$2.9
2008	$32.6	$9.2	28%	$2.4	$3.1	$1.9	$1.1	$2.1
2007	$60.8	$15.6	26%	$4.0	$4.3	$3.9	$2.1	$4.2
2006	$50.8	$15.4	30%	$3.9	$4.6	$3.8	$1.6	$3.7
2005	$41.0	$13.3	32%	$3.6	$4.6	$3.1	$1.2	$2.7
2004	$36.9	$12.7	34%	$3.5	$3.6	$2.9	$1.2	$2.4
2003	$31.2	$11.3	36%	$2.8	$3.0	$2.5	$1.1	$2.1
2002	$22.8	$9.0	39%	$2.0	$2.1	$1.9	$0.7	$1.5
2001	$26.8	$11.0	41%	$2.9	$2.3	$2.1	$1.1	$1.8
2000	$30.9	$11.4	37%	$3.6	$3.2	$2.6	$1.3	$2.3
1999	$35.6	$11.8	33%	$5.8	$4.0	$2.8	$1.4	$2.4
1998	$26.3	$10.3	39%	$2.5	$2.4	$2.3	$1.1	$1.8
1997	$22.3	$8.9	40%	$1.7	$2.1	$2.1	$0.8	$1.3
1996	$20.1	$6.8	34%	$1.5	$3.0	$1.7	$0.7	$1.1
1995	$17.5	$5.7	33%	$1.2	$3.5	$1.3	$0.6	$0.9
1990	$9.6	$2.7	28%	$0.3	$2.8	$0.9	$0.4	$0.5
1980	$2.9	$1.2	41%	…	$0.4	$0.2	$0.1	$0.1

TABLE 3.2 Global Stock Market Capitalization ($ trillions)

All figures are year-end. The "World Total" includes the approximately 50 exchanges that are members of the World Federation of Exchanges. Source: http://www.world-exchanges.org > Statistics > Equity Domestic Market Capitalization.

The NYSE is the premier forum for on-going companies to raise capital. For example, an on-going listed company such as IBM (they originally listed with the NYSE in 1915) decided that they needed external financing for their pension obligations. At one point IBM registered with the Securities Exchange Commission the intention to sell up to $1.5 billion of new stock (about 19.3 million shares). The stocks were to be distributed at prevailing market prices on the NYSE (among others; IBM also is listed on the Chicago Stock Exchange). The NYSE is so large and liquid that the market fairly easily absorbs the new stocks allowing IBM to raise capital successfully securing its employee's pension plans.

The NYSE also is the premier forum for initial public offerings. Companies with IPOs on the NYSE are of course already large and well-established. Awhile ago, for example, Aramark Inc. decided to raise capital through an IPO on the NYSE. This previously privately held company is a leading provider of food and support services, uniform and career apparel services and childcare and early education – they have a big presence on college campuses. Aramark reported sales during the prior year of approximately $7.8 billion and net income of approximately $176 million. Aramark sold 30 million shares on the NYSE and raised almost $700 million of external financing in order to support their growth. NYSE IPOs raise tens of billion of dollars each year for companies going public.

Most stock exchanges in today's world are purely electronic trading systems ("ETS") in which traders click on computer screens to execute trades. The NYSE offers several ETS alternatives. For example, NYSE Direct+ automatically matches buy and sell orders up to 1,099 shares, enabling users anonymity and speed. Another ETS alternative, SuperDot, transmits orders up to specified sizes (depending on the stock) to the proper trading floor position. These orders execute on the floor as quickly as market interest and activity permit.

About 80 percent of all NYSE trades execute on the floor where members meet face-to-face in an open-outcry auction. Every listed security is assigned to a specific trading position. Furthermore, every listing company assigns its stock to a <u>specialist</u> that acts as auctioneer for, typically, between 5 and 10 different listed stocks. Specialists are NYSE members empowered to *maintain a fair and orderly market for the trading of securities at their assigned positions* (at one time there were about 450 specialists employed by 7 different firms working 18 trading posts; each post has about 2 dozen trading positions). The trading crowd around a post includes <u>floor-brokers</u> that are members offering either to buy stocks at the "bid-price" or to sell stocks at the "ask-price." The specialist records bid and ask prices, directing floor brokers to the best price in the crowd. About 85 percent of all floor trades occur between floor brokers. For the other 15 percent, however, the trade typically occurs between the specialist and one floor broker. The specialist, for example, takes one side of all orders coming to the floor by SuperDot. Also, the specialist steps in as middleman when imbalance between buy and sell orders spreads apart the bid and ask prices so far that trading stalls. Specialists supposedly do not affect direction of stock price movements, they simply keep the stock trading.

The NYSE must follow government regulations, most notably the Securities Exchange Act of 1934. That federal law was passed in response to public out roar about the stock market crash of 1929 and its seemingly deleterious effect on the economy. Politicians in the U.S. Congress responded by establishing the Securities Exchange Commission ("SEC"). Ever since then the NYSE has reported significant activities to the SEC. For some actions the NYSE must seek prior permission. Generally, however, the NYSE is "self-regulating" and devotes significant resources to maintain a fair and orderly stock market.

The U.S.A. is unique among nations for the nature of relations between the government and private sector organizations such as the NYSE, the Financial Accounting Standards Board, the American Medical Association, etc. Basically, the government pursues a hands-off approach unless the organization screws up and public pressure pushes Congress into action. Such an event occurred in summer 2003 when members of the NYSE voted a $138 million compensation package for their President and CEO Richard Grasso. The loud out roar from Main Street sent Wall Street into damage-control mode; after all, the NYSE got the $138 million from its captive audience – Main Street investors. NYSE members responded by forcing out Mr. Grasso, by revoking some of his compensation package, and most significantly, by reorganizing so as to satisfy the SEC and dissipate political pressure on Congress. The NYSE governance structure adopted in 2004 creates an independent Board of Directors ("BoD") and a Board of Executives comprised of representatives from the securities industry. The NYSE believes that independence of the BoD will enable the exchange to address issues objectively and intelligently so that, once again, the public may put its trust in the New York Stock Exchange, and its owner, the ICE.

B2. Forecasting external financing needs when internal financing is available

This method for estimating future financing needs assumes the availability of a forecast for expected future sales. The forecast sales revenue, combined with reasonable assumptions about profit margins, enables an estimate about availability of *New retained earnings*. *New retained earnings* represents internal financing available to the company and diminishes the amount of External Financing Needs.

The number "future Sales Revenue" is incredibly important. It is hard to get, too. Regardless, we assume that perhaps the marketing staff conduct surveys, the MBAs run numbers, make phone calls, or take clients to lunch, and that somehow the company obtains a forecast for expected sales. Given the sales forecast, the basic procedure for finding EFN is given below:

(1) Forecast the *Total assets* required for sustaining desired future sales. Most likely, use a financial ratio to link future sales with specific asset categories and then estimate *Total assets*.

(2) Forecast the future *Total liabilities & Stockholders' equity* that you expect to accumulate. There are 3 reasons these might change from current values:

a. Some liabilities such as *Payables* often increase spontaneously and proportionately with *Sales*
b. *Stockholders' equity* increases because expected *Sales* creates *New retained earnings*
c. Pre-commitments might cause *Notes* and *Long term debt* to change. Otherwise, these should remain constant for forecasting.

(3) Compute *EFN* as the difference between expected *Total assets* from step 1 and expected *Total liabilities & Stockholders' equity* from step 2. A summary formula for this general procedure is:

FORMULA 3.2 External financing needs, concise version

Let ΔA equal the change in *Total assets* (that is, $A_t - A_{t-1}$) expected throughout the next period, let ΔL equal the forecast change in spontaneous liabilities, and let R_t equal the forecast internal financing from *New retained earnings.* The company must arrange for financing during period t equal to EFN_t, where:

$$EFN_t = \Delta A - \Delta L - R_t$$

Examples for applying the above procedure are given below for two common situations.

EFN when balances change proportionately with sales

Consider the situation in which a company at end-of-year $t-1$ expects sales growth at rate g during year t, while at the same time it expects to remain unchanged the following ratios: the asset turnover ratio, net profit margin, and dividend payout ratio. For this situation, the summary formula specializes to become:

$$EFN_t = g\, A_{t-1} - \Delta L - (1 + g)\, R_{t-1}$$

EXAMPLE 3 Find EFN when all ratios except debt-to-equity stay constant

The Company's balance sheet for December 31, 2525, and its income statement for the year 2525 appear below:

Company Balance Sheet, 12/31/2525			
Assets			*Liabilities*
		$ 200	Payables
		150	Short term notes
		225	Long term Debt
		325	Stockholders' Equity
Total Assets	*$900*	*$900*	*Total Liabilities & Equity*

Company Income Statement, Jan. 1 — Dec. 31, 2525	
Sales	$2,500
- expenses	2,250
= Net Income	$250
- Dividends	200
= *New Retained Earnings*	$ 50

The company plans to increase sales by 12% during 2526. They hope, however, to hold constant at 2.78 the asset turnover ratio (=$Sales_t \div A_t$), the net profit margin at 10%, and the dividend payout ratio at 80%. If *Payables* rise proportionately with sales, how much are external financing needs?

SOLUTION

The situation is a perfect match for application of the formula:

$$EFN_t = g\,A_{t-1} - \Delta L - (1 + g)\,R_{t-1}$$

$$= (0.12 \times \$900) - (0.12 \times \$200) - (1.12 \times \$50)$$

$$= \$28$$

The company forecasts a shortfall of $28; this sum must be borrowed in order to support required *Total assets*.

Even though the answer for the above problem is $28, further inspection of the outcome is instructive. Consider first the income statement for 2526. *Sales* grow 12% to become $2,800; application of the 10% net profit margin and 80% payout ratio yield the following:

Company Income Statement, Jan. 1 — Dec. 31, 2526	
Sales	$2,800
- expenses	2,520
= Net Income	$280
- Dividends	224
= *New Retained Earnings*	$ 56

The balance sheet for year-end 2526 reflects the $56 internal financing as an increase in *Stockholders' equity*. The only other liability to increase is *Payables*, which rise spontaneously with sales (12%) to become $224. On the left-hand-side side, *Total assets* increases proportionately with sales ($900 x 1.12 = $1,008). The preliminary balance sheet appears here:

Preliminary Company Balance Sheet, 12/31/2526			
	Assets		*Liabilities*
		$ 224	Payables
		150	Short term notes
		225	Long term Debt
		381	Stockholders' Equity
Total Assets	*$1,008*	*$980*	*Total Liabilities & Equity*

Expected *Total assets* is $28 more than expected *Total Liabilities & Stockholders' equity*. This shortfall identifies a financing need of $28 during the year 2526. The company should make arrangements for obtaining this financing in case the need eventuates. If the funds are not raised, *Total assets* cannot grow as required.

EFN for flexible cases

The procedure outlined above easily adapts to variations in the business situation. Variations might occur, for example, as a result of the following circumstances: (1) perhaps only some *Total assets* increase proportionately with sales; (2) perhaps profit margins change. Regardless, the procedure remains the same as in formula 3.2. The two examples below illustrate the flexibility of this procedure for finding *EFN*.

The first example analyzes a company that currently underutilizes its *PP&E*. The company believes that if it works its *PP&E* harder, maybe by running an extra labor shift, the existing *PP&E* can support the sales growth.

EXAMPLE 4 Find EFN when PP&E is constant

The Company's balance sheet for December 31, 2525, and its income statement for the year 2525 appear below:

Company Balance Sheet, 12/31/2525			
	Assets		*Liabilities*
Cash	$ 40	$ 210	Payables
Inventory	170	120	Short term notes
Receivables	60	250	Long term Debt
PP&E	900	590	Stockholders' Equity
Total Assets	*$1,170*	*$1,170*	*Total Liabilities & Equity*

Company Income Statement, Jan. 1 — Dec. 31, 2525	
Sales	$3,600
- expenses	3,130
= Net Income	$280
- Dividends	220
= *New Retained Earnings*	$ 60

The company plans to increase sales by 6% during 2526. They hope, however, to hold constant the net profit margin and dividend payout ratio. *Payables* rise proportionately with sales. All assets also rise proportionately with sales, except for *PP&E* which currently is underutilized and can support fully the sales growth. How much external financing is needed?

SOLUTION

The situation implies that all assets except *PP&E* rise by 6%. Thus, the change in *Total assets* equals 16.2 {= 0.06 x ($1,170 - 900) }. Because the profit margin and payout ratios remain the same, however, the *New retained earnings* simply equals the previous year's *New retained* *earnings* multiplied by one plus the growth rate. *New retained earnings* for 2526 equal the 2525 value times 1.06. Substitution into formula 3.2 shows:

EFN = { 0.06 x ($1,170 - 900) } – (0.06 x $210) – (1.06 x $60)

= $–60

The company forecasts a surplus of $60. This sum will be available for repaying debt or acquiring new assets.

Verify for the previous example that the income statement for 2526 shows *Sales* of $3,816 and *New retained earnings* of $63.6. Also verify that the preliminary balance sheet for year-end 2526 appears as below:

Preliminary Company Balance Sheet, 12/31/2526			
	Assets		*Liabilities*
Cash	$ 42.4	$ 222.6	Payables
Inventory	180.2	120	Short term notes
Receivables	63.6	250	Long term Debt
PP&E	900	653.6	Stockholders' Equity
Total Assets	*$1,186.2*	*$1,246.2*	*Total Liabilities & Equity*

EFN equals the difference between the left and right-hand-side bottom-lines. The final balance sheet depends on the strategic decisions by management for equating the bottom lines.

The next example analyzes a company that increases its profit margin. The change in profit margin affects the availability of internal financing. The computation of *New retained earnings* should properly reflect the new profit margin, as the example below illustrates:

EXAMPLE 5 Find EFN when the profit margin changes EFN2b

The Company's balance sheet for December 31, 2525, and its income statement for the year 2525 appear below:

Company Balance Sheet, 12/31/2525			
	Assets		*Liabilities*
Cash	$ 60	$ 170	Payables
Inventory	200	230	Short term notes
Receivables	90	350	Long term Debt
PP&E	950	550	Stockholders' Equity
Total Assets	*$1,300*	*$1,300*	*Total Liabilities & Equity*

Company Income Statement, Jan. 1 — Dec. 31, 2525	
Sales	$4,200
- expenses	3,850
= Net Income	$350
- Dividends	275
= *New Retained Earnings*	$ 75

The company plans to increase sales by 7% during 2526. They plan to hold constant the dividend payout ratio. *Payables* rise proportionately with sales. All assets also rise proportionately with sales. The company intends, however, to institute cost cutting measures so that the net profit margin rises by one percentage point. For this scenario, how much external financing is needed?

SOLUTION
The situation implies that all assets rise by 7%. Thus, the change in *Total assets* equals { 0.07 x ($1,300) }. *New retained earnings* for 2526 is based upon the new *Sales* figure of $4,494 (= 1.07 x $4,200), the new net profit margin of 9.33% (= .01 + $350/$4,200), and the retention ratio of 21.43% (= 100% - $275/$350). Substitution into formula 3.2 shows:

$$EFN = \{ 0.07 \times (\$1{,}300) \} - (0.07 \times \$170) - (1.07 \times \$4{,}200 \times .0933 \times .2143)$$

$$= \$-10.8$$

The company forecasts a surplus of $10.8 . This sum will be available for repaying debt or acquiring new assets.

EXERCISES 3.1

Numerical quickies
1. Analysts report that the company has *Total assets* of $287,000. Over the next year the *Total assets* should increase 5.4%, spontaneous financing should equal $3,870 and *New retained earnings* should equal $9,530. Other liabilities are unchanged. What is the forecast EFN ("external financing needed")? ©EFN5a

Challengers
2. Company *Sales* equal $58,000 for the year ending December 31, the *Costs-of-goods sold* (cogs) equal 75% of *Sales*, and *Inventory* was replaced about every 65 days (inventory turnover in days = 365 ÷ inventory turnover ratio; inventory turnover ratio = annual cogs ÷ *Inventory*). The Company is considering a change in their inventory ordering policy. As a result, they believe that *Sales* would remain constant in the forthcoming year, yet the length of time that *Inventory* stays on the shelf would increase by 33 days. If the financing rate for inventories is 14% per year, by how much would they change their annual inventory financing costs? ©EFN1a,b

3. For year 2525 the company's *Sales* were $360,000 and annual *Cost-of-goods-sold* equaled 75% of *Sales*. The company followed a policy that set the average payment period (= *Payables* ÷ daily-cost-of-goods-sold) at 56 days. The company realizes that relying on *Payables* as a financing source is free, whereas relying on *Debt* costs 19% per annum. Suppose they institute a policy that causes the average payment period to increase by 30 days. Further, suppose the policy has no effect on the firm's *Total Assets* or *Sales*. Based on the numbers for year 2525, how much would the new policy affect annual financing costs due to the company's switch between high-cost *Debt* and low-cost (free) *Payables*? ©EFN3a

4. Find below the Company's financial statements for year 2525.

	Balance Sheet, 12/31/2525			*Income, 1/1 – 12/31/2525*	
$525	Cash & securities	$540	Current liabilities	Sales	$16,600
$735	Inventory	$620	Debt	total costs	$16,200
$1,700	PP&E	$1,800	Stockholders' equity	Net income	$400
$2,960	Total assets	$2,960		Dividends	$140
				New retained earnings	$260

For 2526 the company plans 17.50% sales growth. They plan to hold constant the asset turnover (=*Sales*÷*Total assets*) and payout ratio (=*Dividends*÷*Net income*). They plan to increase *Current liabilities* spontaneously with *Sales*, while holding *Debt* constant. Suppose the company holds constant their net profit margin (=*Net income ÷ Sales*). Given the above plan, how much external financing is needed for year 2526? ©EFN2a

5. Find below the Company's financial statements for year 2525.

	Balance Sheet, 12/31/2525			*Income, 1/1 – 12/31/2525*	
$635	Cash & securities	$310	Current liabilities	Sales	$19,500
$655	Inventory	$620	Debt	total costs	$18,800
$1,790	PP&E	$2,150	Stockholders' equity	Net income	$700
$3,080	Total assets	$3,080		Dividends	$340
				New retained earnings	$360

For 2526 the company plans 12% sales growth. They plan to hold constant the asset turnover (=*Sales÷Total assets*) and payout ratio (=*Dividends÷Net income*). They plan to increase *Current liabilities* spontaneously with sales, while holding *Debt* constant. Suppose the company institutes cost-cutting measures that should increase the net profit margin (=*Net income ÷ Sales*) by 2.60% above its value of year 2525. Given the above plan, how much external financing is needed for year 2526? ©EFN2b

6. Find below the Company's financial statements for year 2525.

	Balance Sheet, 12/31/2525			*Income, 1/1 – 12/31/2525*	
$420	Cash & securities	$740	Current liabilities	Sales	$18,000
$720	Inventory	$1,000	Debt	total costs	$17,700
$2,700	PP&E	$2,100	Stockholders' equity	Net income	$300
$3,840	Total assets	$3,840		Dividends	$170
				New retained earnings	$130

For 2526 the company plans 12.30% sales growth. They plan to hold constant the asset turnover (=*Sales÷Total assets*) and payout ratio (=*Dividends÷Net income*). They plan to increase *Current liabilities* spontaneously with *Sales*, while holding *Debt* constant. Suppose the company holds constant their net profit margin (=*Net income ÷ Sales*). How much is *Total assets* for year 2526 if the forecast shortfall is not financed with external borrowing? ©EFN2c

7. The Company balance sheet on 12/31/2525 contains the following:

	Balance Sheet, 12/31/2525		
		$1,700	Current liabilities
$2,100	Current assets	$1,300	Debt
$3,000	PP&E	$2,100	Stockholders' equity
$5,100	Total assets	$5,100	

From the income statement for 2525, *Sales* equal $21,930 and *Net income* is $1,645 and *Dividends* equal $1,234. The Company expects sales growth during year 2526 of 10.7%. They expect *Current assets* and *Current liabilities* will increase spontaneously and proportionately with *Sales*. They believe, however, that they can better utilize existing *PP&E*. Consequently, they expect *PP&E* to remain constant. Also expected to remain constant are the net profit margin (= *Net income ÷ Sales*) and the payout ratio (= *Dividends ÷ Net income*). The company anticipates running a surplus during year 2526. Given that *Debt* is unchanged, how large is the forecast surplus? ©EFN4

8. Find below the Company financial statements for year 2525.

	Balance Sheet, 12/31/2525			*Income, 1/1 – 12/31/2525*	
Current assets	$2,100	$1,200	Current liabilities	Sales	$16,500
		$1,700	Debt	total costs	$15,741
PP&E	$5,400	$4,600	Stockholders equity	Net income	$759
	$7,500	$7,500	Total	Dividends	$569

For year 2526 the company forecasts sales growth of 8.8% with constant net profit margin (= *Net income ÷ Sales*) and dividend payout ratio (=*Dividends÷Net income*). The company expects that due to more efficient asset management they can increase the asset turnover ratio (= $Sales_t \div Total\ assets_t$) to 2.70 . They expect *Current liabilities* will rise spontaneously with *Sales*. According to these forecasts, find the surplus external financing needed in 2526. ©EFN6

2. Natural growth rates

What amount of growth is just right? This is a complex question and the answer always depends on the specific business situation. While there are no golden rules, there are two baseline cases that provide insight about natural growth rates.

Gain understanding of growth rates by examining simplified financial statements. Suppose that the left-hand side of the balance sheet contains *Total assets*, denoted *A*. The right-hand-side lists *Debt* and *Stockholders' equity*, denoted *D* and *SE*, respectively. The right and left-hand-sides are of course equal; that is, *A = D + SE*. Further suppose that the income statement lists only *Sales*, *Total expenses*, *Net income*, and *Dividends*. The bottom-line of the income statement is *New retained earnings*, denoted *R*, which represents internal financing available for growth.

2.A. Growth exclusively with internal financing

A natural question arises: *How fast can a company grow by relying exclusively on internal financing?* Believe it or not, obtain a straightforward answer for this question by viewing the scenario when several of the company's most important financial ratios are stable. The answer appears in the formula below.

FORMULA 3.3a, 3.3b, and 3.3c The internal growth rate

A company with constant asset turnover ratio, net profit margin, and dividend payout ratio that relies exclusively on internal financing grows at the "internal growth rate":

$$g^{internal} = \frac{R_t}{A_t - R_t}$$

$$= \frac{(retention\ ratio)(ROA)}{1-(retention\ ratio)(ROA)} \quad when\ ROA = \frac{Net\ income_t}{Total\ assets_t}$$

$$= (retention\ ratio)(ROA) \quad when\ ROA = \frac{Net\ income_t}{Total\ assets_{t-1}}$$

The variables *R* and *A* denote *New retained earnings* and *Total assets*, respectively. The "ROA" is the return-on-assets.

Derivation of formula 3.3 assumes that the following ratios are constant: asset turnover ratio; net profit margin; and dividend payout ratio. As the discussion in the previous chapter suggests, these ratios often tend toward constants.

The numerical example in table 3.3 illustrates the dynamics of this growth story. During year 2525 this company generates $90 of *New retained earnings*. The end-of-year balance sheet lists total assets of $1,000. The balance sheet bottom-line, however, already includes this $90 of *New retained earnings*. Because end-of-year total assets equal $1,000 but internal financing contributes $90 toward that total, the beginning-of-year total assets would equal $910 if all else were equal. If $910 of total assets were to generate $90 of internal financing, the company is growing 9.89% with exclusive reliance on *New retained earnings*:

$$g^{internal} = \frac{\$90}{\$1000 - \$90}$$

$$= 9.89\%$$

PANEL A: Status Quo for year 2525					
Balance Sheet, 12/31/2525				*Income Statement, 1/1 to 12/31/2525*	
		400	Debt (D)	Sales	$3,000
	______.	600	Equity (SE)	total expenses	2,850
Total (A)	$1,000	$1,000	Liabilities & Equity	Net Income	150
				Dividends	60
				New Retained Earnings (R)	90
PANEL B: Effect on year 2526 of growth at rate $g^{internal}$					
Balance Sheet, 12/31/2526				*Income Statement, 1/1 to 12/31/2526*	
		400	Debt (D)	Sales	$3,297
	______.	699	Equity (SE)	total expenses	3,132
Total (A)	$1,099	$1,099	Liabilities & Equity	Net Income	165
				Dividends	66
				New Retained Earnings (R)	99
PANEL C: Effect on year 2526 of growth at rate $g^{sustainable}$					
Balance Sheet, 12/31/2526				*Income Statement, 1/1 to 12/31/2526*	
		471	Debt (D)	Sales	$3,530
	______.	706	Equity (SE)	total expenses	3,354
Total (A)	$1,177	$1,177	Liabilities & Equity	Net Income	176
				Dividends	70
				New Retained Earnings (R)	106

PANEL D: End-of-year financial ratios	12/31/2525 status quo	12/31/2526 growth @ $g^{internal}$	12/31/2526 growth @ $g^{sustainable}$
asset turnover ratio: (sales ÷ total assets)	3.0	3.0	3.0
net profit margin: (net income ÷ sales)	5.0%	5.0%	5.0%
payout ratio: (dividends ÷ net income)	40%	40%	40%
debt-to-equity ratio: (debt ÷ Stockholders' equity)	0.67	0.57	0.67

TABLE 3.3 Natural growth rate dynamics

Panel B shows the outcome in year 2526 when the company grows at the internal growth rate of 9.89%. *Sales* become $3,297 (= $3,000(1.089)). *Net income*, too, is up 9.89% to become

\$165. Likewise, *Dividends* become \$66. The *New retained earnings* of \$99 flow into *Stockholders' equity*, thereby bringing the balance sheet's bottom line for year-end 2526 to \$1,099.

Notice in Panel D that with growth at rate $g^{internal}$ the asset turnover ratio remains at 3.0 in year 2526, exactly as it was in 2525. The net profit margin remains constant, too, at 5.0%, and the dividend payout ratio remains constant at 40%. This particular sales growth rate of 9.89% is the only rate at which exclusive reliance on internal financing holds constant these three ratios.

2.B. The sustainable growth rate

Businesses typically target a particular debt ratio as desirable. A shortcoming of growth at rate $g^{internal}$ is that the debt ratio declines over time. Inspect the numerical illustration above, for example. In year 2525, the debt-to-equity ratio equals 0.67; in 2526 it equals 0.57. If the company initially were at its target debt-to-equity ratio, growth at rate $g^{internal}$ moves the company away from its target.

Slight modification of the preceding formula leads to the following result:

FORMULA 3.4a, 3.4b, and 3.4c The sustainable growth rate

A company with constant asset turnover ratio, net profit margin, dividend payout ratio, and debt-to-equity ratio grows at the "sustainable growth rate":

$$g^{sustainable} = \frac{R_t(1+D_t/SE_t)}{A_t - R_t(1+D_t/SE_t)}$$

$$= \frac{(retention\,ratio)(ROE)}{1-(retention\,ratio)(ROE)} \qquad when\ ROE = \frac{Net\ income_t}{Stockholders\,equity_t}$$

$$= (retention\,ratio)(ROE) \qquad when\ ROE = \frac{Net\ income_t}{Stockholders\,equity_{t-1}}$$

The variables *R*, *A, D,* and *SE* denote *New retained earnings, Total assets*, *Total debt,* and *Stockholders' equity,* respectively.

These three formulations are algebraically equivalent. The top line (formula 3.4a) uses from the income statement *New retained earnings* (R_t) and from the contemporaneous balance sheet *Total assets* (A_t), *Total debt* (D_t), and *Stockholders equity* (SE_t). The middle and bottom lines (3.4b and 3.4c, respectively) use the retention ratio (that is, *1 − dividends/Net income*) and return-on-equity (*ROE*). The *ROE* is the ambiguous ratio of a flow and a balance with different definitions in-use. The two *ROE* definitions above differ because *Net income* is divided by *Stockholders equity* at either the end or beginning of period.

For the illustration begun in Panel A, we use formula 3.4a to easily compute the sustainable growth rate.

$$g^{sustainable} = \frac{\$90(1+400/600)}{\$1000-\$90(1+400/600)}$$

$$= 17.65\%$$

Panel C shows the outcome in year 2526 when the company grows at 17.65%. *Sales* are up 17.65% to become $3,530 (= $3,000(1.1765)), *Net income* becomes $176, and *Stockholders' equity* becomes $706. Because the asset turnover ratio is constant, *Total assets* also are up 17.65% to $1,177. One other aspect changes, too. *Debt* on the balance sheet rises to $471, an increase of 17.65%, which implies the company takes out additional loans of $71 (that is, external financing equals $71). Confirm that all ratios, including the debt-to-equity ratio, are the same in Panels A and C when sales grow at the sustainable growth rate.

EXAMPLE 6 Contrasting growth rates for IBM

For the IBM financial statements presented below from table 2.2 find both $g^{internal}$ and $g^{sustainable}$.

SOLUTION

The actual financial statements contain a lot of detail absent in the growth rate formulas. Consolidation consequently must occur. Partition all right-hand-side balance sheet line items into *Stockholders' equity* and *Debt.* Similarly simplify the income statement (all dollars in millions) and see

IBM Corporation financial statements

Balance Sheet, 12/31 year t				*Income Statement, 1/1 to 12/31 year t*	
		59,504	Debt (D)	Sales	$76,654
	______.	21,628	Equity (SE)	total expenses	71,225
Total (A)	$81,132	$81,132	Liabilities & Equity	Net Income	5,429
				Dividends	706
				New Retained Earnings (R)	4,723

Application now of the growth rate formulas is straightforward:

$$g^{internal} = \frac{R_t}{A_t - R_t}$$

$$= \frac{\$4,723}{\$81,132 - \$4,723}$$

$$= 6.18\%$$

and

$$g^{sustainable} = \frac{R_t(1 + D_t / SE_t)}{A_t - R_t(1 + D_t / SE_t)}$$

$$= \frac{\$4,723(1 + \$59,504/\$21,628)}{\$81,132 - \$4,723(1 + \$59,504/\$21,628)}$$

$$= 27.94\%$$

Computations indicate that if IBM holds constant its asset turnover ratio, net profit margin, and payout ratio, the resultant *New retained earnings* support a growth rate of 6.18%. If they also issue more debt such that the debt-to-equity ratio is constant, growth at 27.94% is supported. We note parenthetically, however, that IBM already relies on debt much more than the average computer company (see the discussion in the previous chapter about the DuPont analysis). Consequently, IBM probably should tend toward lower growth so that their debt ratio diminishes.

The balance sheet interaction with the flow of funds from the snapshot of time (1996) for the preceding discussion define the history of IBM. Cycles of motion in financial ratios occur, some repeat, some are long-term, all cycles are unique. Examine the snippet below from table 2.1 showing IBM sorted 56th by *Total assets* in the list of 11,000 U.S. companies circa beginning of year 2014.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
SCHW	$143,642	14	$ 1,071	$ 5,539	$ 33,719
MSFT	$142,431	99	21,863	77,849	287,691
PG	$139,263	121	11,312	84,167	211,132
JNJ	$132,683	128	13,831	71,312	258,341
FITB	$130,443	19	1,836	6,864	17,987
IBM	$126,223	431	16,483	99,751	197,772

SNIPPET from table 2.1 in chapter 2: IBM

IBM *Total assets* grew to $126 billion (6th row above) from $81 billion in table 2.2 for an average annual growth rate of about 2.5%. Likewise, *Sales* grew relatively slowly at 1.6%. For sure, IBM tended toward lower growth. The *Stockholder's equity* (not shown) for the recent date equals $22.7 billion for IBM. This implies the IBM debt ratio rose higher rather than decline, not good. Still, the IBM equity price to book ratio rose during the time horizon to 13.6 from 9.4 and the common stock returned a 350% cumulative *ROR*. The company known as *Big Blue* successfully maintained a relatively large *equity multiplier* to amplify adequate return on assets into fine shareholder *ROR*.

EXERCISES 3.2

Numerical quickies

1. Find below items from the company's income statement.

	Income, 1/1 - 12/31/2525
Sales	$10,000
all costs	$9,400
Net income	$600
Dividends	$390
New retained earnings	$210

Total assets at 12/31/2525 equal $2,635. If the company is growing at their internal growth rate, what are *Total assets* at 12/31/2526? ©GR4

2. Find below items from the company's income statement.

	Income, 1/1 - 12/31/2525
Sales	$12,000
all costs	$10,800

Net income	$1,200
Dividends	$920
New retained earnings	$280

Total assets at 12/31/2525 equal $3,650 and the debt-to-assets ratio is 45%. If the company is growing at their sustainable growth rate, what are *Total assets* at 12/31/2526? ©GR1

3. Analysts report that the company successfully grows at their sustainable growth rate of 7.4%. Today the company's total assets equal $105,600 and this past year their sales were $48,000 . What is the likely increase over the next year in the company's sales? ©GR5

4. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$450	Current assets	$1,650	Debt
$3,200	PP&E	$2,000	Stockholders' equity
$3,650	Total assets	$3,650	

Income, 1/1 – 12/31/2525

Sales	$21,500
total costs	$21,100
Net income	$400
Dividends	$250
New retained earnings	$150

For 2526 the asset turnover (*Sales÷Total assets*), net profit margin (=*Net income ÷ Sales*), and payout ratio (=*Dividends÷Net income*) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. What is the sales growth rate? ©GR2a

Challengers

5. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$300	Current assets	$1,600	Debt
$3,400	PP&E	$2,100	Stockholders' equity
$3,700	Total assets	$3,700	

Income, 1/1 – 12/31/2525

Sales	$14,100
total costs	$13,700
Net income	$400
Dividends	$275
New retained earnings	$125

For 2526 the asset turnover (=*Sales÷Total assets*), net profit margin (=*Net income ÷ Sales*), payout ratio (=*Dividends÷Net income*) and price-to-earnings ratio (now 14.6) will be constant. The number of shares outstanding is 110. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. What is the equity book value per share at year-end 2526? ©GR2c

6. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$375	Current assets	$875	Debt
$2,400	PP&E	$1,900	Stockholders' equity
$2,775	Total assets	$2,775	

Income, 1/1 – 12/31/2525

Sales	$8,900
total costs	$8,200
Net income	$700
Dividends	$380
New retained earnings	$320

For 2526 the asset turnover (=*Sales÷Total assets*), net profit margin (=*Net income ÷ Sales*), payout ratio (=*Dividends÷Net income*) and price-to-earnings ratio (now 20.4) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. What is the equity price-to-book ratio at year-end 2526? ©GR2d

7. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$450	Current assets	$1,650	Debt
$3,200	PP&E	$2,000	Stockholders' equity
$3,650	Total assets	$3,650	

Income, 1/1 – 12/31/2525

Sales	$21,500
total costs	$21,100
Net income	$400
Dividends	$250
New retained earnings	$150

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), payout ratio (=*Dividends*÷*Net income*) and price-to-earnings ratio (now 24.6) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. What is the debt-to-equity ratio at year-end 2526? ©GR2e

8. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$360	Current assets	$960	Debt
$2,500	PP&E	$1,900	Stockholders' equity
$2,860	Total assets	$2,860	

Income, 1/1 – 12/31/2525

Sales	$13,200
total costs	$12,600
Net income	$600
Dividends	$220
New retained earnings	$380

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), payout ratio (=*Dividends*÷*Net income*) and price-to-earnings ratio (now 14.3) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. For the shareholder that buys a share at year-end 2525 and holds the stock through year-end 2526, what is the rate of return? ©GR2b

9. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$4,350	Current assets	$8,350	Debt
$21,000	PP&E	$17,000	Stockholders' equity
$25,350	Total assets	$25,350	

Income, 1/1 – 12/31/2525

Sales	$76,100
total costs	$72,000
Net income	$4,100
Dividends	$2,870
New retained earnings	$1,230

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), and payout ratio (=*Dividends*÷*Net income*) will be constant. The price-to-earnings ratio, 29.2 at year-end 2525, is expected to equal 22.5 at year-end 2526. The number of shares outstanding is 8500. The firm seeks maximum growth by relying exclusively on retained earnings; external financing will be zero. For the shareholder that buys a share at year-end 2525 and holds the stock through year-end 2526, what is the rate of return? ©GR3a

10. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$450	Current assets	$1,650	Debt
$3,200	PP&E	$2,000	Stockholders' equity
$3,650	Total assets	$3,650	

Income, 1/1 – 12/31/2525

Sales	$21,500
total costs	$21,100
Net income	$400
Dividends	$250
New retained earnings	$150

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), and payout ratio (=*Dividends*÷*Net income*) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying on internal and external financing such that the debt-to-equity ratio remains constant. What is the equity book value per share at year-end 2526? ©GR2g

11. Find below the Company's financial statements for year 2025.

Balance Sheet, 12/31/2525

$375	Current assets	$1,275	Debt
$2,600	PP&E	$1,700	Stockholders' equity
$2,975	Total assets	$2,975	

Income, 1/1 – 12/31/2525

Sales	$8,900
total costs	$8,000
Net income	$900
Dividends	$560
New retained earnings	$340

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), payout ratio (=*Dividends*÷*Net income*) and price-to-earnings ratio (now 29.5) will be constant. The number of shares outstanding is 90. The firm seeks maximum growth by relying on internal and external financing such that the debt-to-equity ratio remains constant. What is the equity price-to-book ratio at year-end 2526? ©GR2h

12. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$510	Current assets	$910	Debt
$2,400	PP&E	$2,000	Stockholders' equity
$2,910	Total assets	$2,910	

Income, 1/1 – 12/31/2525

Sales	$16,000
total costs	$15,600
Net income	$400
Dividends	$180
New retained earnings	$220

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), payout ratio (=*Dividends*÷*Net income*) and price-to-earnings ratio (now 22.0) will be constant. The number of shares outstanding is 100. The firm seeks maximum growth by relying on internal and external financing such that the debt-to-equity ratio remains constant. For the shareholder that buys a share at year-end 2525 and holds the stock through year-end 2526, what is the rate of return? ©GR2j

13. Find below the Company's financial statements for year 2525.

Balance Sheet, 12/31/2525

$4,050	Current assets	$9,450	Debt
$39,000	PP&E	$33,600	Stockholders' equity
$43,050	Total assets	$43,050	

Income, 1/1 – 12/31/2525

Sales	$245,400
total costs	$233,100
Net income	$12,300
Dividends	$8,490
New retained earnings	$3,810

For 2526 the asset turnover (=*Sales*÷*Total assets*), net profit margin (=*Net income* ÷ *Sales*), and payout ratio (=*Dividends*÷*Net income*) will be constant. The price-to-earnings ratio, 19.4 at year-end 2525, is expected to equal 21.8 at year-end 2526. The number of shares outstanding is 16,800. The firm seeks maximum growth by relying on internal and external financing such that the debt-to-equity ratio remains constant. For the shareholder that buys a share at year-end 2525 and holds the stock through year-end 2526, what is the rate of return? ©GR3B

3. Focus on cash flow

Accounting earnings on the income statement equals *Net income*. Quite often, however, analysts want other flow measures. The rate of return to capitalists, for example, relates directly to cash flow the company provides equity investors and other creditors. Perhaps higher cash flow correlates with higher *Net income*, but not always. The fact is, cash flows signal wealth changes and many different stakeholders or capitalists want information about distribution of the company wealth. This section offers lessons for measuring cash flow.

3.A. Cash Flow from the corporation to the financial markets

The Cash Flow Cycle from chapter 1 diagrams flows between the corporation and the financial markets. Straightforward definitions stem from this diagram. Focusing first on shareholders:

FORMULA 3.5 Cash flow to shareholders

$$CF^{\text{to shareholders}} = \text{Dividends} - \text{Net equity issues}$$

$CF^{\text{to shareholders}}$ measures the amount of wealth the company transfers to shareholders as a realized flow of funds. It increases as *Dividends* rise. Conversely, $CF^{\text{to shareholders}}$ falls when the company sells shares, and rises when the company repurchases shares.

Focus now on all other financial market participants that lend money to the corporation. Group these lenders together under the label *Creditors*, and define $CF^{\text{to creditors}}$ as follows:

FORMULA 3.6 Cash flow to creditors

$$CF^{\text{to creditors}} = \text{Interest} - \text{Net debt issues}$$

Net debt issues equals the change in principal of all the company's outstanding credit obligations. $CF^{\text{to creditors}}$ diminishes when the company takes out new loans because the company is the recipient of a wealth transfer from the financial or trade creditors. Conversely, the repayment of loan principal causes an increase in $CF^{\text{to creditors}}$. Finally, $CF^{\text{to creditors}}$ rises when the company pays interest. In short, $CF^{\text{to creditors}}$ measures the net wealth transfer from the company to its financial or trade market creditors.

The sum of cash flows to shareholders and creditors is the total wealth transfer from the corporation to the financial markets. This summation equals the cash flow to capitalists:

FORMULA 3.7 Cash flow to capitalists

$$CF^{\text{to capitalists}} = CF^{\text{to shareholders}} + CF^{\text{to creditors}}.$$

Company value in the financial markets depends on the cash flow that the company is forecast to deliver to capitalists throughout the foreseeable future.

Financial markets supply financing to companies because of expectations about future cash flows. If cash flows promise to be "big enough", then capitalists open the spigot and let flow the source funds for the company: The funds enable the employment and purchase of factors of production required for producing goods and services. Capitalists in pursuit of profit determine which companies to finance, which industries to nurture, and which goods and services to produce — successful capitalists predict the types of goods and services the client markets want.

EXAMPLE 7 Compute cash flow measures for IBM

Find below simplified historical financial statements for IBM. Definitions of *PP&E* and *Total debt* for this illustration have been expanded so that these line items contain the "Other Long Term" balance sheet entries (all dollars in millions). From the statements, find these three cash flow measures for year *t*: $CF^{to\ creditors}$, $CF^{to\ shareholders}$, and $CF^{to\ capitalists}$.

IBM Consolidated Balance Sheet, 12/31 year t-1			
Assets		*Liabilities*	
Cash	$ 7,259	$ 31,648	Current liabilities
Other current assets	33,432	26,221	Total Debt
PP&E	39,601	22,423	Stockholders' Equity
Total Assets	*$80,292*	*$80,292*	*Total Liabilities & Equity*

IBM Income Statement, Jan. 1 — Dec. 31, year t	
Sales revenue	$76,654
- Cost-of-goods-sold	46,815
- Selling and general expenses	16,854
- Depreciation	3,676
= Operating income	$9,303
- Interest expense	716
= Taxable income	$8,587
- Taxes	3,158
= Net income	$5,429
- Dividends	706
= *New Retained Earnings*	$4,723

IBM Consolidated Balance Sheet, 12/31/ year t			
Assets		*Liabilities*	
Cash	$ 7,687	$ 34,000	Current liabilities
Other current assets	33,008	25,504	Total Debt
PP&E	40,437	21,628	Stockholders' Equity
Total Assets	*$81,132*	*$81,132*	*Total Liabilities & Equity*

SOLUTION

$CF^{to\ creditors}$ equals the net wealth transfer to long term debt. Notice from the income statement that IBM pays interest to creditors of $716 million. Also notice from the two balance sheets that during year *t* the outstanding balance of long term debt (expanded definition) changes by $–717 million (= $25,504 – $26,221); that is, debt declines. Both of these items represent a transfer of wealth from IBM to creditors. Substitution of the above into the cash flow definition shows that $CF^{to\ creditors}$ is $1,433 million (= $716 – ($–717)).

$CF^{to\ shareholders}$ equals the net wealth transfer to shareholders. Notice from the income statement that IBM pays *Dividends* to shareholders of $706 million. Also notice from the definition of *Stockholders' equity* that *Net equity issues* must equal $–5,518 million (= $21,628 – $22,423 – $4,723). Once again, both of these items represent a transfer of wealth from IBM to shareholders. Substitution of the above into the cash flow definition shows that $CF^{to\ shareholders}$ equals $6,224 million (= $706 – ($–5,518)).

The total transfer of wealth from IBM to capitalists in year *t*, $CF^{to\ capitalists}$, equals the sum of $CF^{to\ shareholders}$ and $CF^{to\ creditors}$, which is $7,657 million (= $1,433 + $6,224). IBM transfers over $7.6 billion to the financial markets in this one year alone. This huge sum represents about 9.5% of total assets. In addition to this realized cash flow to capitalists, financial market investors in IBM also accrued wealth increases because stock and bond prices rose substantially in this year.

3.B. Other Cash Flow Measures

With perfectly fluid and competitive economic markets the principal and profit from company assets return to the financial markets. This gives rise to the following definition.

FORMULA 3.8 The link between assets and capitalists

$CF^{from\ assets} = CF^{to\ capitalists}$

The right-hand-side of the above equation contains items (*Interest* and *Dividends*) that appear on the income statement. Substitution and rearrangement of several formulas results in a very useful alternative expression for $CF^{from\ assets}$.

FORMULA 3.9 Cash flow from assets, expanded version

$CF^{from\ assets} = EBIT + Depreciation - Taxes - \Delta NWC - Capital\ expenditures$

where *EBIT* represents "earnings before interest and taxes". Sometimes *EBIT* also is called "operating income" or "operating revenue."

$EBIT = Sales - COGS - SGA - Depreciation,$

COGS represents *Cost-of-goods-sold* and *SGA* represents *Selling, general, and administrative expenses. NWC* represents net working capital (*NWC* is *Current assets* minus *Current liabilities*; ΔNWC is its change), and *Capital expenditures* represent spending on *PP&E*.

Publicly traded corporations generally report *Net income* every 3 months. Often, however, analysts want a measure of cash flow resulting directly from the company's operations:

FORMULA 3.10 Cash flow from operations

$$CF^{from\ operations} = EBIT + Depreciation - Taxes$$

$$= EBITDA - Taxes$$

EBITDA, "earnings before interest, taxes, depreciation and amortization," is increasingly popular in the financial press:

$EBITDA = EBIT + Depreciation$

The difference between *EBITDA* and $CF^{from\ operations}$ is *Taxes*. When the analyst believes that a company's particular tax bill is unusual, s/he may choose to focus on *EBITDA* since it is unaffected by taxes. Regardless, $CF^{from\ operations}$ often portrays a better picture than *Net income* of financial health and wealth creation.

Comparison of formulas 3.9 and 3.10 reveals that the *Cash flow from assets* equals *Cash flow from operations* when the company spends absolutely nothing on *Net working capital* and *PP&E*. Usually, though, some funds get used on *Net working capital* or *PP&E* so not all *Cash flow from operations* immediately returns to capitalists. Combining several of the preceding formulas yields a key expression.

FORMULA 3.11 Cash flow from assets, summary version

$CF^{from\ assets} = CF^{from\ operations} - \Delta NWC - capital\ expenditures$

Besides cash flow, many analysts watch the company's *Cash surplus*. The *Cash surplus* is simply the change in *Cash* from one balance sheet to the next. Rearrangement of the above definitions shows:

FORMULA 3.12 Cash surplus

$$\begin{aligned} \textit{cash surplus} &= \Delta \textit{ Cash} \\ &= CF^{\textit{from operations}} - \Delta NWC(\textit{excluding Cash}) - \textit{Capital expenditures} - CF^{\textit{from assets}} \end{aligned}$$

The example below applies cash flow definitions to IBM.

EXAMPLE 8 Get detailed cash flow measures for IBM

From the financial statements for IBM presented for example 7, find the following for year *t*: (i) $CF^{\textit{from operations}}$; (ii) ΔNWC ; (iii) *Capital expenditures*; (iv) $CF^{\textit{from assets}}$; and (v) the *Cash surplus*.

SOLUTION

(i) Compute $CF^{\textit{from operations}}$ as *EBIT* ($9,303), plus *Depreciation* ($3,676), minus *Taxes* ($3,158). *Cash flow from operations* is $9,821 million.

(ii) NWC_t equals total current assets minus total current liabilities, and is $9,043 million (= $7,259 + $33,432 – $31,648); NWC_t equals $6,695 million (= $7,687 + $33,008 – $34,000). From year-end *t-1* to *t*, the ΔNWC is $–2,348 million. The decline in net working capital represents a source of funds for IBM in year *t* of over $2.3 billion.

(iii) Apply formula 2.12 and find that *Capital expenditures* equals $4,512 million (= $40,437 – $39,601 + $3,676). This sum represents a use of $4.5 billion for increasing the balance of long term assets (recall that the simplified balance sheet for this example consolidates *PP&E* with *Other Long Term Assets*).

(iv) $CF^{\textit{from assets}}$ equals $CF^{\textit{from operations}}$ ($9,821), minus ΔNWC ($–2,348), minus *Capital expenditures* ($4,512). The *Cash flow from assets* is $7,657 million. Notice that this answer is identical to $CF^{\textit{to capitalists}}$ from example 7.

(v) The *Cash surplus* equals the change in *Cash* on the balance sheet. From year-end *t-1* to *t*, the *Cash surplus* is $428 million (= $7,287 – $7,259). The increase in *Cash* represents a use of funds by IBM during year *t* to pad their checking account. Reliance on formula 3.12 obtains the identical answer:

$$\begin{aligned} \textit{Cash surplus} &= CF^{\textit{from operations}} - \Delta NWC(\textit{excluding Cash}) \\ &\quad - \textit{Capital expenditures} - CF^{\textit{from assets}} \\ &= \$9{,}821 - \{(\$33{,}008 - \$34{,}000) - (\$33{,}432 - \$31{,}648)\} \\ &\quad - \$4{,}512 - \$7{,}657 \\ &= \$428 \textit{ million} \end{aligned}$$

IBM has a cash surplus in year *t* and the amount in their checking account increases even though they return a huge sum to capitalists.

The next example combines many different formulas from chapters 2 and 3 and hones basic accounting skills to the degree required for doing well at finance.

EXAMPLE 9 Combine the balance sheet with income statement items to find cash flows

Find below the Company's balance sheet at year-end 2525.

Balance Sheet, 12/31/2525

Cash	$ 90	$200	Current liabilities
Other current assets	300	180	Debt
PP&E	680	690	Stockholders equity (80 shares)
	$1,070	$1,070	Total

For year 2526 the following are forecast: the stock price should end the year at $7.50 per share; *Sales* equal $2,100; operating margin (= *EBIT* ÷ *Sales*) is 14%; *Depreciation* is 15% of *PP&E*; *Interest* is 10% of debt; *Taxes* are 35% of taxable income; *Dividends* are 60% of *Net income*. Suppose the company makes sufficient *Capital expenditures* during 2526 so that it holds *PP&E* constant. It finances these *Capital expenditures* by issuing 10 shares during 2526 for $7 per share. The remainder of the *Capital expenditures* is financed internally. All else remains the same. Find all the different cash flow measures as well as the price-to-earnings, price-to-book, and price-to-cash flow.

SOLUTION

For convenience forecast the income statement from the preceding facts.

Company Income Statement, Jan. 1 — Dec. 31, 2526

Sales revenue	$2,100
- Other costs (see discussion)	. . . .
- Depreciation (@ 15% of PPE)	102
= EBIT (@ 14% of sales)	$294
- Interest (@ 10% of debt)	18
= Taxable income	$276
- Taxes (@ 35% tax rate)	97
= Net income	$179
- Dividends (@ 60% payout)	108
= *New Retained Earnings*	$ 71

The operating margin allows computation of *EBIT* from *Sales*. It is unnecessary to compute *Other costs*. Notice that the income statement does not show in any way the *Capital expenditures* nor equity issues. These latter items, however, affect the balance sheet for year-end 2526. Apply formula 2.12 to compute that SE_{2526} equals $832 (= 690 + 72 + 70). For convenience show all that is known about the balance sheet for year-end 2526:

Balance Sheet, 12/31/2526

Cash	$?	$200	Current liabilities
Other current assets	300	180	Debt
PP&E	680	832	Stockholders equity (80 shares)
	$1,212	$1,212	Total

Cash is found such that it equalizes the bottom line. That is, $Cash_{2526}$ equals $232 (= $1,212 – 300 – 680). Now apply the cash flow formulas and find that $CF^{\text{to creditors}}$ equals interest paid on debt and equals $18 (no new loans are taken out); $CF^{\text{to shareholders}}$ equals *Dividends* minus new issues and equals $38 (= $108 – $70); $CF^{\text{from assets}}$ equals the sum of $CF^{\text{to creditors}}$ plus $CF^{\text{to shareholders}}$ and is $56. The *Cash surplus* is the change in *Cash* on the balance sheet and is $142 (= $232 – $90). As an aside, confirm that $CF^{\text{from assets}}$ and *Cash surplus* are consistent with formulas 3.9 and 3.12, respectively.

One final comment about this example pertains to the source of the $102 *Capital expenditure* that the Company pays to the capital goods supplier. The Company sells new shares and raises \$70; the remaining \$32 is paid in cash. Confirm that if *Capital expenditures* were zero and no shares were issued, *Cash* is \$32 higher at \$264, and *Total assets* is \$1,142.

The next example uses accounting skills to answer a very relevant financial question.

EXAMPLE 10 Construct and interpret the price-to-operating cash flow multiple ©CF3a

Find below the Company's balance sheet for year-end 2525.

Balance Sheet, 12/31/2525

Cash	\$455	\$490	Current liabilities
Inventory	\$725	\$870	Debt
PP&E	\$3,800	\$3,620	Stockholders equity
	\$4,980	\$4,980	Total

For 2525 the Company's asset turnover ratio ($Sales_{2525} \div Total\ assets_{2525}$) is 3.2. *Depreciation* equals 15% of *PP&E*, and the operating profit margin (= earnings before interest and taxes ÷ *Sales*) is 12.40%. *Interest* expense equals 8.20% of *Debt*. *Taxes* equal 35% of taxable income, and the payout ratio (=*Dividends*÷*Net income*) is 40%. There are 180 shares outstanding and the price-to-earnings ratio at year-end 2525 equals 21.4. There are no other items on the income statement for 2525.

As a prospective investor in the Company's shares, you are especially interested in their financial ratios. You know that the ratio of price-to-operating cash flow for this company's peer group is 6.2. What is the company's price-to-operating cash flow ratio, and does it make the company's stock look cheap or expensive?

SOLUTION

First, work down the income statement in order to compute components necessary for applying formula 3.10 to find $CF^{from\ operations}$. *Sales* equals \$15,936 (= 3.2 x \$4,980). Next use the operating profit margin of 12.40% to find that *EBIT* equals \$1,976 (= 0.1240 x \$15,936). Notice that *EBIT* subtracts from *Sales* expenses that include *Cost-of-goods sold, Depreciation,* and *Selling, general, and administrative expenses.* Formula 3.10 requires knowledge that *Depreciation* equals \$570 (= \$3,800 x 0.15). Subtract *Interest* from *EBIT* to compute that *Taxable income* equals \$1,904 (= \$1,976 – (.0820 x \$870)). With a 35% tax rate the *Taxes* equal \$667 (= \$1,904 x 0.35)). Now use formula 3.10 to find $CF^{from\ operations}$

$$CF^{from\ operations} = EBIT + Depreciation - Taxes$$

$$= \$1,976 + 570 - \$667$$

$$= \$1,879$$

Second, recognize that the ratio of price to operating cash flow for one share is identical to the ratio of the entire company's market capitalization to total operating cash flow. Find market capitalization by using the price-to-earnings ratio of 21.4. Earnings, that is *Net income*, equals *Taxable income* minus *Taxes* and is \$1,237 (= \$1,904 – \$667). Thus,

$$\frac{market\ capitalization}{Net\ income} = 21.4$$

and market cap equals \$26,472 (= 21.4 x \$1,237). The numerical answer is:

$$\frac{price\ per\ share}{operating\ cash\ flow\ per\ share} = \frac{market\ capitalization}{total\ operating\ cash\ flow}$$

$$= \frac{\$26,472}{\$1,879}$$

$$= 14.1$$

The qualitative interpretation is this. For the peer group the price-to-cash flow averages 6.2. This means that the stock market pays $6.20 for a dollar of cash flow from this type of company. For this specific company, however, the stock market pays $14.10 for a dollar of cash flow. A possible inference is that the company's stock price is expensive relative to the other companies in the peer group. Perhaps the stock is *too* expensive for the cash flows that the company creates. Chapter 8 explores in more detail the use of price multiples for making valuation inferences.

The final example for this chapter shows a simplified setting for a company that obtains financing from a venture capitalist. The venture capital industry is an alternative financing source for many companies.

STREET-BITE An American invention: venture capitalists

Companies need money to grow and quite often venture capitalists are a tremendous financing alternative. Table 3.3 summarizes U.S. venture capital financing during a past snapshot.

Year	Number of companies	Average per company ($ millions)	Sum of venture capital financing ($ millions)
1990	1,471	1.95	2,862
1991	1,279	1.79	2,285
1992	1,415	2.54	3,593
1993	1,209	3.20	3,868
1994	1,239	3.39	4,200
1995	1,901	4.04	7,683
1996	2,656	4.36	11,582
1997	3,250	4.66	15,160
1998	4,203	5.11	21,473
1999	5,684	9.68	54,995
2000	8,208	12.96	106,391
2001	4,691	8.76	41,082
2002	3,028	6.99	21,155
….			
2013	4,041	7.3	29,500

TABLE 3.4 Venture capital financing
Source: Compiled by author.

Companies receiving venture capital financing number in the low thousands. In 2013, $29.5 billion was invested in 3,382 companies through 4,041 deals. This number is small relative to the millions of corporations that operate and/or receive business loans from commercial banks or other sources. In the record year of 2000 the average venture capital deal of almost $13 million was spread over 8,200 companies. All in all, however, venture capital is a relatively small but extremely important financing source for economic innovation and growth. This financing source is absent from nearly every other country on planet earth, although in Europe venture capital firms have started to appear.

Venture capital firms, like banks, lend money. When banks lend, however, they loan to well-established companies, they give the company a payment book, and they collect interest over the life of the loan. Here are several ways that venture capitalists differ.

1. Venture capital firms take an equity stake in the company. That is, when the venture capitalist lends money they actually purchase shares directly from the company. The venture capitalist is not pursuing a fixed interest rate of return. Instead, they expect the stock eventually to rise in value.

2. The company may have reached its debt capacity as far as bank loans go, yet venture capital still may be available. Banks tend to rely on historical records for lending decisions, whereas venture capitalists look toward the future. “Seed investing” occurs for companies that are at very early stages before there is a real product. Venture capitalists also invest in rapidly growing companies in their “expansion stage.” And sometimes venture capitalists invest in

"later stage" companies that are on the verge of going public. In rare occasions, too, venture capitalists may invest in companies that already are publicly traded.

3. Venture capitalists are activists. They use their experience to help managers of growing companies make sound decisions about strategic marketing, planning, and development. Venture capitalists are like farmers who love to grow successful companies. They are entrepreneurs first and financiers second.

4. Venture capitalists grow the company but eventually intend to liquidate their equity stake. The average venture capital investment lasts between 4 and 7 years. The different ways that the venture capitalist liquidates the investment include: (a) the venture capitalist sells the stock back to the company at a negotiated repurchase price; (b) the company is taken-over or merges with an established company and the venture capitalist swaps their equity for cash or acquiring-company stock; (c) the company goes public and the venture capitalist sells their equity through an IPO; (d) the company goes bankrupt or reorganizes and the venture capitalist's stock becomes worthless.

The success story for a venture capital deal occurs when the company grows to a level of maturity that an IPO occurs (see the *Street-bite* on initial public offerings in chapter 2). Table 3.4 provides information about venture-backed IPOs during a historical snapshot.

Year	Number of U.S. IPOs - 1 -	Number of U.S. venture-backed IPOs - 2 -	Average venture-backed offer size ($ millions) - 3 -	Average venture-backed post-offer value ($ millions) - 4 -
1997	537	131	35.9	159.1
1998	329	75	48.3	224.5
1999	480	233	76.4	493.0
2000	354	226	93.3	470.5
2001	88	35	82.6	383.4
2002	94	22	86.8	373.6
…				
2013	169	81	137.0	784.0

TABLE 3.5 Venture-backed initial public offerings
Compiled by author.

About 38% of all companies going public during the early 6-year window of the table received venture capital financing before the IPO. That increased to almost half in 2013. Column 3, "offer size", is the amount of financing that the company raises by selling stock during the IPO. Column 4, "post offer value", is the amount the stock is worth one-day later in secondary market trading. The huge gap between offer size and post offer value suggests that venture capitalists grow companies in which public markets have a lot of interest. The gap suggests, too, that these companies underprice their stock during the IPO and leave a lot of money on the table! These tendencies still are evident today. In 2014 led by biotechnology companies the venture backed exit activity posted the best full year for new IPO listings since the year 2000!

Other important non-bank sources of financing for young companies include the Small

Business Administration (read about “small business investment companies” at www.sba.gov/INV/overview.html) and angel investors (see www.angeldeals.com, for example).

Consider this example for finding the cash flows for a venture capitalist.

EXAMPLE 11 Find cash flows for venture capitalist ©BA4a

A venture capitalist lends a start-up company $100,000 at year-end 2525 and acquires 500 shares. The company also obtains $200,000 by selling its owner/manager 1000 shares, and by borrowing for 2-years a balloon loan of $40,000 on which it pays 10% annual interest. The company’s net profit margin (= *Net income* ÷ *Sales*) is forecast at 12%. Sales should equal $400,000 during year 2526, and they should grow 18% for year 2527. The Company sets a 20% dividend payout ratio (= *Dividends* ÷ *Net income*). They promise to repurchase the shares from the venture capitalist at year-end 2527 for 140% of their book value. For this scenario, sketch the cash flows to equity and to creditors.

SOLUTION

The first stage for solving this problem is to sketch the initial balance sheet. For clarity, partition *Stockholders’ equity* into several components.

Company Balance Sheet, 12/31/2525

Assets	*Liabilities*		
	$ 40,000	Debt	
	300,000	Total Stockholders’ Equity	
		200,000	manager/owner (1,000 shares)
		100,000	venture capitalist (500 shares)
		0	accumulated retained earnings
$340,000	$340,000		

Notice the original equity book value per share is $200 (= $300,000 / 1,500 shares). *Stockholders’ equity* changes over subsequent years as a result of *New retained earnings*. *New retained earnings* simply equals *Net income* (12% of *Sales*) minus *Dividends* (20% of *Net income*). For year 2526 when *Sales* equal $400,000 *Net income* is $48,000 and *Dividends* are $9,600 and *New retained earnings* are $38,400. For year 2527 when *Sales* are 18% higher at $472,000 *Net income* is $56,640 and *Dividends* are $11,328 and *New retained earnings* are $45,312. The balance sheet at year-end 2527, before the loan and venture capitalist are repaid, appears as follows:

Company Balance Sheet, 12/31/2527 (before restructure)

Assets	*Liabilities*		
	$ 40,000	Debt	
	383,712	Total Stockholders’ Equity	
		200,000	manager/owner (1,000 shares)
		100,000	venture capitalist (500 shares)
		83,712	accumulated retained earnings
$423,712	$423,712		

At this point in time, the balloon loan of $40,000 is repaid. Presumably the firm liquidates an asset, say money in its cash account, to pay off the loan. Both assets and liabilities decline by $40,000. Furthermore, the equity book value per share equals $255.81 (= $383,712 / 1,500 shares). Equity book value increases over the two-year horizon primarily because the company earns profits and plows back for growth. This is the ideal situation for start-ups. They are a lot like seeds that store their energy so that when eventually they sprout, they survive and thrive.

The company repurchases shares from the venture capitalist at 140% of book value. The repurchase price is therefore $358.13 per share (= $255.81 x 1.4). For 500 shares, the total payment to the venture capitalist is $179,066 (= $358.13 x 500). The company repurchases the shares from the venture capitalist, further drawing down their cash account. Repayment of loan and venture capitalist reduces company cash by a total of $219,066 (=$40,000 + $179,066). *Total assets* declines by exactly this amount, and it falls to $204,646 (=$423,712 - $219,066). The balance sheet after restructuring appears as follows:

Company Balance Sheet, 12/31/2527 (after restructure)

Assets	*Liabilities*		
	$ 0	Debt	
	204,646	Total Stockholders' Equity	
		200,000	manager/owner (1,500 shares)
		0	venture capitalist (0 shares)
		4,646	accumulated retained earnings
$204,646	$204,646		

The *Stockholders' equity* was derived so that it equaled the known *Total assets* of $204,646. Notice that *Accumulated retained earnings* is less after the restructuring than before. In effect, the company took a charge against retained earnings because they used equity to pay off the venture capitalist.

What did the bank and venture capitalist obtain from the company? The answer, of course, is <u>cash flow</u>. For the bank, the cash flows are given by

12/31/2525	12/31/2526	12/31/2527
$–40,000	+$4,000	+$44,000

The rate of return for this cash flow stream, as the next chapter explains, is 10%. The bank's rate of return equals the interest rate that it charges.

Cash flows for the venture capitalist equal their share of *Dividends* (500/1500, or 1/3rd of total dividends, which equals $3,200 in 2526 and $3,776 in 2527) plus the repurchase sum:

12/31/2525	12/31/2526	12/31/2527
$–100,000	$3,200	+$182,842

The rate of return for this cash flow stream, as the next chapter explains, is about 37% per year. While this certainly is higher than the rate earned on savings accounts, the venture capitalist finances a lot of projects, and some of them fail. The high earnings on winners compensate the venture capitalists for the losses suffered on losers.

EXERCISES 3.3

Numerical quickies

1. The Company balance sheet for year 2525 shows that *Total assets* of $4,300 are financed by *Debt* of $400 and *Stockholders' equity* of $3,900. There are 430 shares outstanding at year-end 2525. The company plans to obtain venture capital by selling 140 additional shares at their current book value to a venture capitalist. The company agrees to repurchase the shares at year-

end 2526 at a price equal to 146% of that year's book value. For year 2526 the company forecasts *Sales* of $29,240 and a net profit margin (= *Net income* ÷ *Sales*) of 9.20% and a dividend payout ratio (= *Dividends* ÷ *Net income*) of 30%. Assume *Debt* remains unchanged. How much total cash flow (*Dividends* plus repurchase price) does the venture capitalist receive at year-end 2526? ©BA3a

2. The Company balance sheet for year 2525 shows that *Total assets* of $3,800 are financed by *Debt* of $600 and *Stockholders' equity* of $3,200. There are 270 shares outstanding at year-end 2525. The company plans to obtain venture capital by selling 120 additional shares at their current book value to a venture capitalist. The company agrees to repurchase the shares at year-end 2526 at a price equal to 125% of that year's book value. For year 2526 the company forecasts *Sales* of $25,080 and a net profit margin (= *Net income* ÷ *Sales*) of 10.50%, and a dividend payout ratio (= *Dividends* ÷ *Net income*) of 35%. Assume *Debt* remains unchanged. How much is the equity repurchase shareprice at year-end 2526? ©BA3b

Challengers

3. The Company balance sheet for year 2525 shows that *Total assets* of $4,900 are financed by *Debt* of $400 and *Stockholders' equity* of $4,500. There are 640 shares outstanding at year-end 2525. The company plans to obtain venture capital by selling 240 additional shares at their current book value to a venture capitalist. The company agrees to repurchase the shares at year-end 2526 at a price equal to 130% of that year's book value. For year 2526 the company forecasts *Sales* of $25,400 and a net profit margin (= *Net income* ÷ *Sales*) of 7.50%, and a dividend payout ratio (= *Dividends* ÷ *Net income*) of 25%. Assume *Debt* remains unchanged. What is the rate of return for the venture capitalist on this investment? ©BA3c

4. The Company balance sheet for year 2525 shows *Total assets* of $3,500 financed by *Debt* of $400 and *Stockholders' equity* of $3,100. There are 340 shares outstanding at year-end 2525. The company plans to obtain venture capital by selling 70 additional shares at their current book value to a venture capitalist. The company agrees to repurchase the shares at year-end 2527 at a price equal to 120% of that year's book value. For year 2526 the company forecasts *Sales* of $21,700 , a net profit margin (= *Net income* ÷ *Sales*) of 13.50%, and a dividend payout ratio (= *Dividends* ÷ *Net income*) of 35%. For year 2527, *Sales* should be higher by 19% but the net profit margin and payout ratio should remain constant. Also, assume that *Debt* always remains unchanged. How much total cash flow (*Dividends* plus repurchase price) does the venture capitalist receive at year-end 2527? ©BA4a

5. The Company balance sheet for year 2525 shows *Total assets* of $4,900 financed by *Debt* of $300 and *Stockholders' equity* of $4,600. There are 660 shares outstanding at year-end 2525. The company plans to obtain venture capital by selling 260 additional shares at their current book value to a venture capitalist. The company agrees to repurchase the shares at year-end 2527 at a price equal to 124% of that year's book value. For year 2526 the company forecasts *Sales* of $38,710 and a net profit margin (= *Net income* ÷ *Sales*) of 7.80%, and a dividend payout ratio (= *Dividends* ÷ *Net income*) of 20%. For year 2527, *Sales* should be higher by 22% but the net profit margin and payout ratio should remain constant. Also, assume that Debt always remains unchanged. How much is the equity repurchase shareprice at year-end 2527? ©BA4b

6. Find below the Company's balance sheet at year-end 2525.

Balance Sheet, 12/31/2525

Cash	$555	$1,255	Debt
PP&E	$3,100	$2,400	Stockholders equity
	$3,655	$3,655	Total

For the year 2526, the following items are forecast: *Depreciation* is $430; *Capital expenditures* equal $360; *Interest* expense is $110; *Net income* is $480; *Dividends* equal $154; *Cash flow from assets* is –$114; *Net debt issues* is $138 (that is, debt increases). There is no preferred stock or extraordinary items, and there are no other non-cash expenses. The balance sheet for year-end 2526 contains only the same line items as appear above. For year 2526, how much is *Net equity issues*? ©CF1e

7. Find below the Company's balance sheet at year-end 2525.

Balance Sheet, 12/31/2525

Cash	$555	$1,055	Debt
PP&E	$1,600	$1,100	Stockholders equity
	$2,155	$2,155	Total

For the year 2526, the following items are forecast: *Depreciation* is $130; *Capital expenditures* equal $110; *Interest* expense is $90; *Net income* is $320; *Dividends* equal $157; *Cash flow from assets* is $185; *Net debt issues* is $95 (that is, *Debt* increases). There is no preferred stock or extraordinary items, and there are no other non-cash expenses. The balance sheet for year-end 2526 contains only the same line items as appear above. For year 2526, how much is the *Cash flow to shareholders*? ©CF2

8. Find below the Company's balance sheet at year-end 2525.

Balance Sheet, 12/31/2525

Cash	$495	$1,095	Debt
PP&E	$2,000	$1,400	Stockholders equity
	$2,495	$2,495	Total

For the year 2526, the following items are forecast: *Depreciation* is $200; *Capital expenditures* equal $170; *Interest* expense is $100; *Net income* is $400; *Dividends* equal $132; *Cash flow from assets* is $264; *Net debt issues* is $66 (that is, *Debt* increases). There is no preferred stock or extraordinary items, and there are no other non-cash expenses. The balance sheet for year-end 2526 contains only the same line items as appear above. For year-end 2526, how much is *Stockholders' equity*? ©CF1c

9. Find below the Company's balance sheet at year-end 2525.

Balance Sheet, 12/31/2525

Cash	$420	$1,120	Debt
PP&E	$3,100	$2,400	Stockholders equity
	$3,520	$3,520	Total

For the year 2526, the following items are forecast: *Depreciation* is $430; *Capital expenditures* equal $360; *Interest* expense is $80; *Net income* is $490; *Dividends* equal $162; *Cash flow from assets* is $280; *Net debt issues* is $34 (that is, debt increases). There is no preferred stock or extraordinary items, and there are no other non-cash expenses. The balance sheet for year-end 2526 contains only the same line items as appear above. For year 2526, how much is the *Cash surplus*? ©CF1a

10. Find below the Company's balance sheet for year-end 2525.

Balance Sheet, 12/31/2525

Cash	$435	$435	Current liabilities
Inventory	$780	$780	Debt
PP&E	$3,000	$3,000	Stockholders' equity
	$4,215	$4,215	Total

For 2525 the Company's asset turnover ratio ($Sales_{2525} \div Total\ assets_{2525}$) is 3.8. *Depreciation* equals 17% of *PP&E*, and the operating profit margin (= earnings before interest and taxes ÷ *Sales*) is 10.70%. *Interest* expense equals 9.40% of *Debt*. *Taxes* equal 35% of taxable income, and the payout ratio (=*Dividends*÷*Net income*) is 35%. There are no other items on the income statement for 2525. There are 150 shares outstanding.

As a prospective investor in the Company's shares, you are especially interested in their ability to generate cash flow. How much is *Cash flow from operations* for year 2525? ©CF3b

11. Find below the Company's balance sheet for year-end 2525.

Balance Sheet, 12/31/2525

Cash	$345	$750	Current liabilities
Inventory	$795	$790	Debt
PP&E	$4,000	$3,600	Stockholders equity
	$5,140	$5,140	Total

For 2525 the Company's asset turnover ratio ($Sales_{2525} \div Total\ assets_{2525}$) is 3.6. *Depreciation* equals 18% of *PP&E*, and the operating profit margin (= earnings before interest and taxes ÷ *Sales*) is 9.70%. *Interest* expense equals 9.60% of *Debt*. *Taxes* equal 35% of taxable income, and the payout ratio (=*Dividends*÷*Net income*) is 45%. There are no other items on the income statement for 2525. There are 180 shares outstanding.

As a prospective investor in the Company's shares, you are especially interested in their financial ratios. You know the price-to-earnings ratio at year-end 2525 equals 28.3. More significant to you, however, is the price-to-cash-flow ratio (= shareprice ÷ operating cash flow per share). What is the company's price-to-cash-flow ratio? ©CF3a

ANSWERS TO CHAPTER 3 EXERCISES

EXERCISES 3.1

1. The increase in *Total assets* equals $15,498 (= .054 x $287,000). The change in *Total liabilities and Stockholders' equity* equals $13,400. Applying formula 3.2 shows that EFN equals $2,098 (= $15,498 – $13,400). The company must borrow $2,098 or else the assets cannot grow as required.

2. . ©EFN1b The first step is finding original *Inventory*. The inventory turnover ratio is 5.6154 (= 365 / 65) and original *Inventory* therefore is $7,747 (= 0.75 x $58,000 / 5.6154). After the policy change the inventory turnover ratio becomes 3.7245 (= 365 / (65+33)). The new balance of *Inventory* therefore is $11,679 (= 0.75 x $58,000 / 3.7245). The rise in *Inventory* of $3,933 (= $11,679 – $7,747) incurs additional annual financing costs of $551 (= $3,933 x 0.14).

3. Find that original *Payables* equals $41,425 (= 56 x $360,000 x 0.75 / 365). Then find that new *Payables* equals $63,616 (= (56+30) x $360,000 x 0.75 / 365). The annual financing costs that the company avoids from increasing the average payment period equals $4,216 (= 0.19 x ($63,616 – $41,425)).

4. *Total assets* increases by $518 (= 0.175 x $2,960). *Current liabilities* rises by $94 (= 0.175 x $540). *New retained earnings* for 2526 grows to $306 (= (1+0.175) x $260). Use formula 3.2 to find that EFN is $118 (= $518 – $94 – $306).

5. ©EFN2b *Total assets* increases by $370 (= 0.12 x $3,080). *Current liabilities* rises by $37 (= 0.12 x $310). *New retained earnings* for 2526 grows to $695 (= [($700 /$19,500)+0.026)] x (1+0.12) x $19,500 x ($360/$700)). Use formula 3.2 to find that EFN is –$363 (= $370 – $37 – $695). The negative sign means the company will run a surplus.

6. This question asks that if EFN equals zero how much is the allowable increase in *Total assets. Current liabilities* rises by $91 (= 0.123 x $740). *New retained earnings* for 2526 grows to $146 (= (1+0.123) x $130). Internal and spontaneous financing sum to $237 (= $91 + $146). In the absence of external financing, *Total assets* can rise to become $4,077 (= $3,840 + $237).

7. *Total assets* increases by $225 (= 0.107 x ($5,100 – $3,000). *Current liabilities* rises by $182 (= 0.107 x $1,700). *New retained earnings* for 2526 grows to $455 (= (1+0.107) x ($1,645 – $1,234). Use formula 3.2 to find that EFN is –$412 (= $225 – $182 – $455); that's the surplus.

8. The company increases *Sales* to $17,952 (= (1+.088) x $16,500). *Total assets* for year 2526 with the new asset turnover ratio becomes $6,649 (= $17,952 / 2.70). The change in *Total assets* is –$851 (= $6,649 – $7,500). The negative sign means *Total assets* decreases. Remember that a decline in an asset is a source of financing. *Current liabilities* rises by $106 (= 0.088 x $1,200). *New retained earnings* for 2526 grows to $207 (= (1+0.088) x ($759 – $569). Use formula 3.2 to find that EFN is –$1,163 (= –$851 – $106 – $207); that's a surplus.

EXERCISES 3.2

1. ©GR4 Apply formula 3.3a and find that $g^{internal}$ equals 8.66% (= $210 / ($2,635 – $210)). *Total assets* at 12/31/2526 therefore equals $2,863 (= $2,635 x (1 + .0866)).

2. Use formula 2.5 to find that the debt-to-equity ratio equals 0.8182 (= 0.45 / (1 – 0.45)). Apply formula 3.4a and find that $g^{sustainable}$ equals 16.21% (= $280 x (1 + 0.8182) / [$3,650 – $280 x (1 + 0.8182)]). *Total assets* at 12/31/2526 therefore equals $4,241 (= $3,650 x (1 + .1621)).

3. The increase in sales equals $3,552 (= $48,000 x 0.074).

4. Because the company relies exclusively on retained earnings and the key ratios (except for the debt ratio are constant) the company grows at $g^{internal}$. Apply formula 3.3a and find that $g^{internal}$ equals 4.29% (= $150 / ($3,650 – $150)). That's the sales growth rate!

5. Apply formula 3.3a and find that $g^{internal}$ equals 3.50% (= $125 / ($3,700 – $125)). Because of constant ratios find that *New retained earnings*$_{2526}$ equals $129 (= $125 x (1 + .035)). *Stockholders' equity*$_{2526}$ therefore becomes $2,229 (= $2,100 + $129), and equity book value per share is $20.27 (= $2,229 / 110).

6. Apply formula 3.3a and find that $g^{internal}$ equals 13.03% (= $320 / ($2,775 – $320)). Because of constant ratios find that *Net income*$_{2526}$ equals $791 (= $700 x (1 + .1303)). Market capitalization at year-end 2526 becomes $16,141 (= 20.4 x $791). *New retained earnings*$_{2526}$ equals $362 (= $320 x (1 + .1303)) and *Stockholders' equity*$_{2526}$ therefore becomes $2,262 (= $1,900 + $362). The ratio of *Market cap*$_{2526}$ to *Stockholders' equity*$_{2526}$ therefore equals 7.14.

7. Apply formula 3.3a and find that $g^{internal}$ equals 4.29% (= $150 / ($2,775 – $150)). Because of constant ratios find that *New retained earnings*$_{2526}$ equals $156 (= $150 x (1 + .1303)) and *Stockholders' equity*$_{2526}$ therefore becomes $2,156 (= $2,000 + $156). *Debt* is constant, so the ratio of *Debt*$_{2526}$ to *Stockholders' equity*$_{2526}$ therefore equals 0.77.

8. We find the rate of return to equity as the increase in market capitalization plus total dividends (the rate of return to one share is identical to the rate of return to all equity; this approach works when no shares are issued). Market capitalization at year-end 2525 equals $8,580 (= 14.3 x $600). Apply formula 3.3a and find that $g^{internal}$ equals 15.32% (= $380 / ($2,860 – $380)). Because of constant ratios find that *Net income*$_{2526}$ equals $692 (= $600 x (1 + .1532)). Market capitalization at year-end 2526 becomes $9,895 (= 14.3 x $692). *Dividends*$_{2526}$ equals $254 (= $220 x (1 + .1532)). The stockholder rate of return therefore equals 18.28% (= ($9,895 + $254 – $8,580) / $8,580)).

9. This problem is the same as the previous one except that there is a change in the price-to-earnings ratio. Market capitalization at year-end 2525 equals $119,720 (= 29.2 x $4,100). Apply formula 3.3a and find that $g^{internal}$ equals 5.10% (= $1,230 / ($25,350 – $1,230)). Because of constant ratios find that *Net income*$_{2526}$ equals $4,309 (= $4,100 x (1 + .0510)). Market capitalization at year-end 2526 becomes $96,954 (= 22.5 x $4,309). *Dividends*$_{2526}$ equals $3,016 (= $2,870 x (1 + .0510)). The stockholder rate of return therefore equals –16.50% (= ($96,954 + $3,016 – $119,720) / $119,720)).

10. This problem applies formula 3.4a to find $g^{sustainable}$; it equals 8.11% (= {$150 x (1 + $1,650 / $2,000)} / ($3,650 – {$150 x (1 + $1,650 / $2,000)})). Because of constant ratios find that *New retained earnings*$_{2526}$ equals $162 (= $150 x (1 + .0811)). *Stockholders' equity*$_{2526}$ therefore becomes $2,162 (= $2,000 + $162), and equity book value per share is $21.62 (= $2,162 / 100).

11. Apply formula 3.4a and find that $g^{sustainable}$ equals 25.00% (= {$340 x (1 + $1,275 / $1,700)} / ($2,975 – {$340 x (1 + $1,275 / $1,700)})). Because of constant ratios find that *Net income*$_{2526}$ equals $1,125 (= $900 x (1 + .25)). Market capitalization at year-end 2526 becomes $33,187 (= 29.5 x $1,125). *New retained earnings*$_{2526}$ equals $425 (= $340 x (1 + .25)) and *Stockholders' equity*$_{2526}$ therefore becomes $2,125 (= $1,700 + $425). The ratio of *Market cap*$_{2526}$ to *Stockholders' equity*$_{2526}$ therefore equals 15.6.

12. Find the rate of return to equity as the increase in market capitalization plus total dividends. Market capitalization at year-end 2525 equals $8,800 (= 22.0 x $400). Apply formula 3.4a and find that $g^{sustainable}$ equals 12.36% (= {$220 x (1 + $910 / $2,000)} / ($2,910 – {$220 x (1 + $910 / $2,000)})). Because of constant ratios find that *Net income*$_{2526}$ equals $449 (= $400 x (1 + .1236)). Market capitalization at year-end 2526 becomes $9,888 (= 22.0 x $449). *Dividends*$_{2526}$

equals $202 (= $180 x (1 + .1236)). The stockholder rate of return therefore equals 14.66% (= ($9,888 + $202 – $8,800) / $8,800)).

13. This problem is the same as the previous one except that there is a change in the price-to-earnings ratio. Market capitalization at year-end 2525 equals $238,620 (= 19.4 x $12,300). Apply formula 3.4a and find that $g^{sustainable}$ equals 12.79% (= {$3,810 x (1 + $9,450 / $33,600)} / ($43,050 – {$3,810 x (1 + $9,450 / $33,600)})). Because of constant ratios find that *Net income*$_{2526}$ equals $13,873 (= $12,300 x (1 + .1279)). Market capitalization at year-end 2526 becomes $302,434 (= 21.8 x $13,873). *Dividends*$_{2526}$ equals $9,576 (= $8,490 x (1 + .1279)). The stockholder rate of return therefore equals 30.76% (= ($302,434 + $9,576 – $238,620) / $238,620)).

EXERCISES 3.3

1. The venture capitalist purchases 140 shares for a total of $1,270 (= 140 x $3,900 / 430). *Stockholders' equity* therefore rises immediately to $5,170 (= $3,900 + $1,270) and there are 570 shares outstanding (= 430 + 140). Then throughout year 2526 there is *Net income* of $2,690 (= .0920 x $29,240); *Dividends* of $807 (= $2,690 x 0.30); and *New retained earnings* of $1,883 (= $2,690 x 0.70). *Stockholders' equity*$_{2526}$ consequently equals $7,053 (= $5,170 + $1,883). Equity book value per share at year-end 2526 is $12.37 (= $7,053 / 570). The company repurchases shares from the venture capitalist for $18.07 each (= $12.37 x 1.46). The total repurchase amount is $2,529 (= $18.07 x 140). The venture capitalist's total *Dividends* equal $198 (= $807 x 140 / 570). Total cash flow to the venture capitalist at year-end 2526 equals $2,727. ©BA3a

2. Collapse the logic for the previous problem into fewer steps. The venture capitalist purchases 120 shares and *Stockholders' equity* therefore rises immediately to $4,622 (= $3,200 + 120 x $3,200 / 270). Then throughout year 2526 there is *New retained earnings* of $1,712 (= .1050 x $25,080 x (1 – 0.35)). *Stockholders' equity*$_{2526}$ consequently equals $6,334 (= $4,622 + $1,712). The company repurchases shares from the venture capitalist for $20.30 each (= $6,334 x 1.25 / (120 + 270)).

3. Collapse the logic for the previous problems into fewer steps. The venture capitalist purchases 240 shares for $1,687 and *Stockholders' equity* therefore rises immediately to $6,187 (= $4,500 + 240 x $4,500 / 640). Then throughout year 2526 there is *New retained earnings* of $1,429 (= .0750 x $25,400 x (1 – 0.25)) and *Dividends* of $476 (= .0750 x $25,400 x 0.25). *Stockholders' equity*$_{2526}$ consequently equals $7,616 (= $6,187 + $1,429). The company repurchases 240 shares from the venture capitalist for $2,700 (= $7,616 x 1.30 x 240 / (240 + 640)). The venture capitalist's rate of return is 68% (= {$2,700 + [$476 x 240 / (240 + 640)] – $1,687 } / $1,687).

4. *Stockholders' equity* rises immediately to $3,738 (= $3,100 + 70 x $3,100 / 340). *Stockholders' equity*$_{2526}$ rises to $5,642 (= $3,738 + .1350 x $21,700 x (1 – 0.35)). *Stockholders' equity*$_{2527}$ rises to $7,908 (= $5,642 + .1350 x $21,700 x (1 + .19) x (1 – 0.35)). The venture capitalist at year-end 2527 receives *Dividends* of $208 (= .1350 x $21,700 x (1 + .19) x 0.35 x 70 / (70 + 340)) and a repurchase amount of $1,620 (= $7,908 x 1.20 x 70 / (70 + 340)), for total cash flow of $1,828 (= $208 + $1,620). ©BA4a

5. *Stockholders' equity*$_{2527}$ rises to $11,775 (= $4,600 + {$4,600 x 260 / 660} + {.0780 x $38,710 x (1 – 0.20)} + {.0780 x $38,710 x (1 + .22) x (1 – 0.20)}). The venture capitalist at year-end 2527 receives a repurchase price per share of $15.87 (= $11,775 x 1.24 / (260 + 660)).

6. Use the formula $CF^{from\ assets}$ = *Interest – Net debt issues + Dividends – Net equity issues*. Substitute values from the set-up: –$114 = $110 – $138 + $154 – *Net equity issues*. Compute

that *Net equity issues*$_{2526}$ equals 240 (the positive sign means the sells stock in the primary market).

7. ©CF2 Use the formula $CF^{from\ assets}$ = *Interest – Net debt issues + Dividends – Net equity issues*. Substitute values from the set-up: $185 = $90 – $95 + $157 – *Net equity issues*. Compute that *Net equity issues*$_{2526}$ equals –33 (the negative sign means the company repurchases equity). $CF^{to\ shareholders}$ is $190 (= $157 – (–33)).

8. Use the formula $CF^{from\ assets}$ = *Interest – Net debt issues + Dividends – Net equity issues*. Substitute values from the set-up: $264 = $100 – $66 + $132 – *Net equity issues*. Compute that *Net equity issues*$_{2526}$ equals –98. Now find that *New retained earnings*$_{2526}$ is $268 (= $400 – $132). *SE*$_{2526}$ consequently equals $1,570 (= $1,400 + $268 – $98).

9. Use the formula $CF^{from\ assets}$ = *Interest – Net debt issues + Dividends – Net equity issues*. Substitute values from the set-up: $280 = $80 – $34 + $162 – *Net equity issues*. Compute that *Net equity issues*$_{2526}$ equals –$72. Now find that *New retained earnings*$_{2526}$ is $328 (= $490 – $162). *SE*$_{2526}$ consequently equals $2,656 (= $2,400 + $328 – $72). Also, *PP&E*$_{2526}$ equals $3,030 (= $3,100 – $430 + $360). Construct the balance sheet for year-end 2526 by entering the known values for *Debt*$_{2526}$ and *SE*$_{2526}$ and *PP&E*$_{2526}$.

Balance Sheet, 12/31/2526

Cash	?	$1,154	Debt
PP&E	$3,030	$2,656	Stockholders equity
	$3,810	$3,810	Total

Compute that *Cash* equals $780 (= $3,810 – $3,030). The *Cash surplus* equals the change in *Cash,* which equals $360 (= $780 – $420).

10. Work down the income statement and apply formula 3.10. *Sales* equals $16,017 (=3.8 x $4,215); *EBIT* is $1,714 (= 0.1070 x $16,017); *Taxable income* is $1,640 (= $1,714 – (.0940 x $780)); *Taxes* equal $574 (= $1,640 x 0.35)); and $CF^{from\ operations}$ equals $1,650 (= $1,714 + (.17 x $3,000) – $574).

11. ©CF3a Collapse the logic for previous problems into fewer steps and find $CF^{from\ operations}$. *EBIT* is $1,795 (= 3.6 x $5,140 x 0.0970); *Taxes* equal $601 (= {$1,795 – (.0970 x $790)} x 0.35); and $CF^{from\ operations}$ equals $1,913 (= $1,795 + (.18 x $4,000) – $601). Now find market capitalization: *Net income* is $1,117 (= {$1,795 – (.0970 x $790)} x {1 – 0.35}); market cap equals $31,607 (= 28.3 x $1,117). The ratio of market cap to $CF^{from\ operations}$ is identical to the ratio of shareprice to operating cash flow per share and equals 16.5 (= $31,607 / $1,913).

CHAPTER 4: TIME VALUE AND RELATIONS BETWEEN RETURNS

Financial decisions generally require relating cash flows occurring at different times. This section explains that a dollar today is not the same as a dollar yesterday or tomorrow. Cash flows occurring at different times, even though apparently the same quantity, possess different value depending on the time of occurrence. Combining cash flows from different times requires adjusting for time value differences. This chapter explains time value relations.

1. Framework for properly measuring average rates of return

Financial decisions often require recognition about rates of return that different cash flow streams offer. Measuring rates of return can be tricky, and it's important to carefully consider what really is meant by "rate of return". ROR is the abbreviation for rate of return.

When you invest $100 and get back $120, it seems intuitively obvious that the cumulative rate of return is 20 percent. This gives rise to the following definition for the cumulative rate of return:

FORMULA 4.1 The cumulative rate of return

The cumulative rate of return measures the percentage change in wealth during the total investment horizon.

$$ROR^{cumulative} = \frac{W^{end} - W^{beginning}}{W^{beginning}}$$

The variables $W^{beginning}$ and W^{end} denote beginning and ending wealth. With the numbers above, for example, you begin with \$100 and end with \$120, so $ROR^{cumulative}$ is easily found as

$$ROR^{cumulative} = \frac{\$120 - \$100}{\$100}$$
$$= 20\%.$$

The cumulative rate of return represents the percentage change in wealth between the beginning and ending times, regardless of how much time elapses. With the numbers above, for example, the cumulative *ROR* is 20 percent regardless of whether the \$20 increase in wealth accrues overnight, over a month, or over two years. The lump-sum time value relation throughout this chapter sets to zero all dividends and distributions between beginning and end in order to focus on the basic relation between the rate of return and a balance change.

Formula 4.1 is easily rewritten to show several equivalent and often used cumulative rate of return versions:

$$ROR^{cumulative} = \frac{W^{end} - W^{beginning}}{W^{beginning}}$$

$$= \frac{\Delta W}{W^{beginning}}$$
$$= \frac{profit}{W^{beginning}}$$
$$= \frac{W^{end}}{W^{beginning}} - 1 .$$

The first alternative version computes *ROR* as change in wealth relative to beginning wealth. The middle formulation recognizes that the change in wealth equals profit, so the *ROR* equals profit over beginning wealth. Finally, the last version divides ending wealth by beginning wealth, and subtracts 1. Convince yourself that the different versions are equivalent, and that they always give the same answer. Use the one that best matches your problem's information.

Often institutions and individuals express a rate of return per period of time. Banks quote interest rates, for example, as annual rates — the relevant period of time for these is a year. Many institutions present quarterly rates of return that they earn on investments or offer on accounts. Some mutual funds report monthly or even daily returns. Irrespective of the period's length, the formula for the periodic *ROR* during period *t* is:

FORMULA 4.2 The periodic rate of return

$$ROR_t = \frac{W_t - W_{t-1}}{W_{t-1}} .$$

W_t denotes the price, value, or wealth for an investment at time *t*. ROR_t is the periodic rate of return for period *t*, and simply represents the percentage change in asset value during the time period. If one uses year-end security prices, for example, formula 4.2 computes an annual *ROR*; if one uses end-of-month prices, then periodic *ROR* are monthly rates of return; etc.

Suppose you invest $100 in a security today; in one year the security price is $60; and in two years you sell the security for $100, the original price. For the first year the periodic *ROR* is:

$$ROR_1 = \frac{\$60 - \$100}{\$100}$$
$$= -40\%$$

Wealth diminishes 40 percent the first year. The second year, however, the security price climbs back to $100. The second year's periodic *ROR* is:

$$ROR_2 = \frac{\$100 - \$60}{\$60}$$
$$= +67\%$$

Wealth increases two-thirds the second year.

The annual *ROR* is –40 percent for the first year and 67 percent for the second year. What is the average annual *ROR*? This seemingly simple yet important question has two plausible answers. The first answer uses the "arithmetic average periodic *ROR*" definition below:

FORMULA 4.3 The arithmetic average rate of return

$$ROR^{arithmetic average} = \sum_{t=1}^{N} \frac{ROR_t}{N} \ .$$

ROR_t is the periodic rate of return for period *t*, and *N* is the number of terms added together. $ROR^{arithmetic\ average}$ simply adds together all periodic *ROR* and divides by the number of periods; it is a simple average.

The average annual *ROR* for our example is found by adding together the two annual *ROR* of –40 percent and 67 percent, and dividing by two:

$$ROR^{arithmetic\ average} = \frac{-40\% + 67\%}{2}$$
$$= 13.5\%$$

The average annual ROR of 13.5 percent suggests to a casual reader that the investor earns 13.5 percent per year. The more astute reader realizes, though, that $W^{beginning}$ is $100, W^{end} two years later is $100, and $ROR^{cumulative}$ is 0 percent. In other words, the investor ends and begins with the same wealth, yet the arithmetic average annual *ROR* implies earnings of over 13 percent per year.

This curious situation motivates introduction of a second definition for the average *ROR*. We use this definition to get the second answer for our simple question "what is the average annual *ROR*?"

FORMULA 4.4 The geometric average rate of return, expanded version

$$ROR^{geometric\ average} = \sqrt[N]{(1+ROR_1)(1+ROR_2)...(1+ROR_N)} - 1$$

The geometric average periodic rate of return adds 1 to each periodic *ROR*, multiplies together all *N* of the "1 plus *ROR*'s", takes the N th root of the product, and subtracts one. For our example, the periodic *ROR* equal –40 percent and +67 percent for the first and second years, respectively. Thus,

$$\begin{aligned} ROR^{geometric\ average} &= \sqrt[2]{(1+(-.40))(1+0.67)} - 1 \\ &= \sqrt[2]{(0.60)(1.67)} - 1 \\ &= \sqrt[2]{1.00} - 1 \\ &= 0.0\% \end{aligned}$$

Notice the formula for geometric average *ROR* requires entry of the periodic *ROR* as their decimal equivalents. That is, enter sixty-seven percent as 0.67.

The geometric average *ROR* accurately relates beginning and ending wealth. A little bit of algebra and rearrangement of equation 4.4 provides a simpler definition for the lump-sum scenario:

FORMULA 4.5 The geometric average rate of return, concise version

$$ROR^{geometric\ average} = \sqrt[N]{\frac{W^{end}}{W^{beginning}}} - 1$$

To compute the geometric average rate of return divide ending wealth by beginning wealth, take the N th root, and subtract 1!

STREET–BITE Mistaken Measures: Case of the Beardstown Ladies

It seems like it should be pretty simple to correctly measure the rate of return. The issue actually can be pretty complicated. It becomes even more confusing when cash infusions, withdrawals, taxes, and other factors come into play.

One relatively well-known debacle involving mismeasurement of the rate of return is the Case of the Beardstown Ladies. In 1999 the Beardstown ladies were reaping profits from their New York Times Bestseller entitled How to Win Big in the Stock Market. These dozen senior ladies had formed an investment club a decade and a half earlier. Each contributed money, researched and analyzed possible purchases, and made buy and sell recommendations at club meetings. The fortunate women began amassing quite a fortune from their stock picks. After a while people started talking. The Beardstown ladies loved to talk, too, and became popular guests on daytime talk shows. The business press began a full-court press, tracking down the ladies, interviewing them, and critically analyzing claims. Time magazine describes what happened next.

> "The lovable ladies were unmasked as frauds--unintentional, mind you--but frauds nonetheless. Five books, hundreds of speeches and dozens of national-TV appearances later, Chicago Magazine challenged their claim of earning compound annual average returns of 23.4% in the 10 years ending in 1993. Undeterred but under pressure, the ladies went to Price Waterhouse for an audit and discovered that their actual return was a sickly 9.1%--far less, according to Lipper Analytical Services, than the Standard & Poor's 500 average annual return of 14.9% or even the average general-stock-fund return of 12.6% during that same period." [*Time*, March 30, 1998. Vol. 151, No. 12, found @ www.pathfinder.com/time]

The lovable ladies unfortunately made an honest mistake measuring the rate of return. As you can see, measuring it correctly can be very important!

Two commonly used procedures for computing an average rate of return include the arithmetic and the geometric procedures (formulas 4.3 and 4.4). Table 4.1 confirms that the alternative procedures lead to extremely different estimates. The table shows the monthly average rate of return for the respective company's common stock throughout the stated sample period. All companies trade on either the New York Stock Exchange or American Stock Exchange, and consequently represent rather large, liquid securities with considerable investor interest.

Description	ROR arithmetic average	ROR geometric average	Difference
January – December 1997 Saf T Lok, Inc.	41.12%	–1.41%	42.53%
January 1990 – December 1997 O C G Technology, Inc.	26.94	1.01	25.93
January 1980 – December 1997 Venus Exploration, Inc.	2.44	–1.00	3.44
January 1960 – December 1997 United Park City Mines, Co.	1.39	0.15	1.24
January 1940 – December 1997 Sunshine Mining & Refining, Co.	1.14	0.24	0.90
December 1927 – December 1997 Texas Instruments, Inc.	2.26	0.72	1.54

TABLE 4.1 Differences between geometric and average rates of return for selected NYSE and AMEX stocks

Each entry is the maximum among all NYSE+AMEX firms of the difference between arithmetic and geometric average monthly rates of return for the respective sample period. Compiled by author.

The first row shows that in 1997 the average of the twelve monthly rates of return for Saf T Lok common stock is 41.12 percent by the arithmetic procedure. This implies a hugely profitable investment, especially since this represents a monthly rate of return. The implication is false, however. The geometric average rate of return is –1.41 percent per month. Shareholders in this company actually lose wealth during the year. The arithmetic average overstates the geometric average rate of return by 42.53 percent per month.

The table illustrates extreme differences between these rates of return measures for long time horizons, too. For Texas Instruments common stock over the 70 years concluding with year-end 1997, the monthly average rate of return is 2.26 percent by the arithmetic procedure, and 0.72 percent by the geometric procedure. The difference exceeds 1 3/4 percent per month. Applying an incorrect rate in the time value relation grossly misstates wealth accumulation – ask any one of the Beardstown Ladies.

The table also highlights an important rule regarding the relation between arithmetic and geometric average returns:

RULE 4.1 The ranking of average ROR

The arithmetic average rate of return always exceeds or equals the geometric average rate of return, that is

$$ROR^{\text{arithmetic average}} \geq ROR^{\text{geometric average}} .$$

EXAMPLE 1 Find the average ROR

You bought a stock three years ago for \$48. Today the stock is worth \$120. What has been your average annual rate of return?

SOLUTION

The question doesn't specify whether to find the arithmetic average, or the geometric average. Consider both answers.

Use formula 4.5 to find the geometric average annual ROR:

$$\begin{aligned} ROR^{\text{geometric average}} &= \sqrt[3]{(\$120/\$48)} - 1 \\ &= \sqrt[3]{2.50} - 1 \\ &= 1.3572 - 1 \\ &= 35.72\% \end{aligned}$$

That's not bad, 35.72 percent a year for three years!

CALCULATOR CLUE 4.1 exponents and roots: Many calculators use the y^x key in order to compute the x^{th} root of a term. Recall that taking the 3^{rd} root is the same as raising to the 1/3 power. Solve the above problem on the BAII Plus© with the following keystrokes:

2.5 [y^x] 3 [1/x] [=] [–] 1 [=]

The answer of 0.3572 appears on the display.

Now consider the arithmetic annual average rate of return. Inspect equation 4.3 and notice that the computation requires each year's performance. $ROR^{\text{arithmetic average}}$ depends on the pattern between beginning and ending prices. The problem set-up only tells us that the beginning price is \$48 and, three years later, the ending price is \$120. There is not enough information to compute the arithmetic average ROR. Furthermore, there are many plausible answers. For example, the two scenarios below show plausible price histories:

	W_0	W_1	W_2	W_3	$ROR^{\text{arithmetic average}}$
Scenario 1	$48	$72	$96	$120	36.1%
Scenario 2	$48	$29	$35	$120	75.0%

In each case the price begins at $48 and ends at $120. In between, however, the price histories differ and consequently the arithmetic averages differ. There exist infinite plausible arithmetic annual averages for this problem because there are an infinite number of plausible price histories. The geometric average depends only on the beginning and ending wealth, not the price history, and at 35.72 percent is smaller than every one of those infinite arithmetic averages. The time value framework requires that uniqueness.

EXERCISES 4.1

Concept quiz

1. Suppose an asset price rises during the first period. The second period the price falls to its original value. Is the second period's absolute percentage change larger or smaller than the first period's? What does this suggest about the relation between arithmetic and geometric average rates of return?

2. True or False: A column of three prices can make one and only one column of rates of return, but a column of three rates or return can make an infinite number of columns of prices.

Numerical quickies

3. Two years ago you purchased a stock for $40 . One year ago the price had moved to $19. Today it is at $60. What are the arithmetic and geometric annual average rates of return? ©ROR3c

4. Two years ago the stock price was $20. The periodic rates of return during the subsequent two years are –15% and 65%. What is today's stock price? ©ROR5

5. A venture capitalist provides a company equity financing of $10.0 million. After 3 years the company repurchases the equity for $12.4 million. There are no other cash flows between the two. Find the average annual geometric rate of return, and also find the cumulative rate of return. ©ROR1

Numerical challenger

6. Your broker correctly tells you that your portfolio's average annual rate of return for the past two years is 20%. You know the portfolio value today of $9,150 is $850 less than when you started the account two years ago. What was the portfolio's value one year ago? ©ROR6 .

2. The lump-sum time value formula: one inflow, one outflow

The definition for the geometric average periodic rate of return yields a measure with intuitively pleasing properties. The definition easily rearranges to express the beginning wealth as a function of the ending wealth, the rate of return, and the number of periods:

$$W^{beginning} = \frac{W^{end}}{(1 + ROR^{geometric\ average})^N} .$$

The variables above are given new names for the lump-sum time value formula, but they represent identical ideas:

FORMULA 4.6 The lump-sum time value relation

$$PV = \frac{FV}{(1+r)^N}$$

PV is the beginning wealth, abbreviated as *PV* to denote "present value". *FV* is the ending wealth *N* periods later, abbreviated as *FV* to denote "future value". The rate of return *r* is, of course, the investment's geometric average periodic rate of return.

Equation 4.6 is the lump-sum time value formula. The relation involves one cash flow at the beginning, one cash flow at the end, and none in the middle. "Compounding" is the process by which beginning wealth, *PV*, grows to become ending wealth, *FV*. Perhaps you have heard about the magic of compounding. Sometimes a beginning amount accumulates such a large sum that magic seems involved. It's not, of course. The explanations below elaborate on this issue.

The lump-sum time value formula implicitly contains several other important members of the relationship: Periodic interest, total accumulated interest, and interest-on-interest.

FORMULAS 4.7, 4.8, and 4.9 Periodic components in time value

$$periodic\ interest = \begin{pmatrix} periodic\ interest \\ rate \end{pmatrix} \begin{pmatrix} beginning\ of \\ period\ balance \end{pmatrix}$$

$$\begin{pmatrix} total\ accumulated \\ market\ interest \end{pmatrix} = \begin{pmatrix} account \\ balance \end{pmatrix} - \begin{pmatrix} total\ contributed \\ principal \end{pmatrix} ,$$

$$\begin{pmatrix} total\ accumulated \\ market\ interest \end{pmatrix} = \begin{pmatrix} total\ interest- \\ on-interest \end{pmatrix} + \begin{pmatrix} total\ interest- \\ on-principal \end{pmatrix}$$

Formula 4.7 computes the periodic interest that an asset earns, or likewise that a liability owes, by multiplying the periodic interest rate times the beginning of period balance. The total market interest that accumulates in an account, specified in equation 4.8, is the difference between the account balance and the principal contributed by the depositor or investor. Formula 4.9 shows that total accumulated market interest equals the sum of total interest-on-interest plus total interest-on-principal.

Suppose, for example, a bank receives at time 0 a deposit of $1,000 into an account that earns interest at an annual rate of 4.75 percent. The lump-sum time value formula allows us to determine the account balance one year later immediately after the bank credits the account with the first year's interest. Set *PV* equal to $1,000 and *r* to 0.0475. *N* equals 1. Substitute into formula 4.6 and solve for *FV*, the account's ending balance.

$$\$1,000 = \frac{FV}{(1+0.0475)^1}$$

$$\$1,000(1+0.0475)^1 = FV$$

or $FV = \$1,047.50$

CALCULATOR CLUE 4.2 Problems that use the lump-sum time value formula sometimes are easier to solve by using the basic arithmetic keys. The time value functions, however, also may be used. These pre-programmed functions are especially useful for solving complex time value calculations. Before using time value functions prepare the BAII Plus© calculator as follows:
(1) Clear existing values from the time value memories by typing 2nd FV .
(2) Verify the compounding frequency is set to the appropriate number. For this problem, compounding frequency is once per year (a later section discusses intraperiod compounding) so type 2nd I/Y 1 ENTER 2nd CPT .

Time value functions now are ready to use. Type the following keys to solve the preceding problem.
1000 PV 4.75 I/Y 1 N CPT FV

The answer of $–1,047.50 appears on the display. The negative sign in front of the number simply means that FV is available to flow out of the account. PV and FV must have opposite signs on the BAII Plus© in all lump-sum problems.

The account grows because the bank contributes interest to the account. The amount of periodic interest obviously equals $47.50, a sum consistent with equation 4.7:

$$\begin{aligned} periodic\ interest &= (0.0475)(\$1,000) \\ &= \$47.50 \end{aligned}$$

The account balance increases each year by the same proportion, 4.75 percent, but every passing year's periodic interest gets larger and larger. For example, the account ending balance after two years is $1,097.26:

$$\$1,000 = \frac{FV}{(1+0.0475)^2}$$

or $FV = \$1,097.26$. The second year's periodic interest is

$$periodic\ interest = (0.0475)(\$1{,}047.50)$$
$$= \$49.76$$

The second year's periodic interest of $49.76 exceeds the first year's $47.50. It's important to realize why. Recall that the bank adds $47.50 to the account at time 1, and so during the second year the interest earns its own interest. The second year's periodic interest, in other words, results from two sources: interest-on-principal plus interest-on-interest. During the second year the interest-on-principal is $47.50 and the interest-on-interest is $2.26.

At the close of the third year the account's future value, given by equation 4.6, is $1,149.38:

$$\$1{,}000 = \frac{FV}{(1+0.0475)^3}$$

or $FV = \$1{,}149.38$.

CALCULATOR CLUE 4.3 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Find the ending wealth after 3 years by typing:
1000 PV 4.75 I/Y 3 N CPT FV

The answer of $–1,149.38 appears on the display.

Total accumulated market interest, given by equation 4.8, is $149.38:

$$\begin{pmatrix} total\,accumulated \\ market\,interest \end{pmatrix} = \begin{pmatrix} account \\ balance \end{pmatrix} - \begin{pmatrix} total\ contributed \\ principal \end{pmatrix},$$

$$= \$1{,}149.38 - \$1{,}000$$

$$= \$149.38$$

Because the principal earns interest each year of $47.50, the total interest-on-principal equals $142.50 (that is, $142.50 = $47.50x3). The total interest-on-interest, given by rearrangement of equation 4.9, therefore equals $6.88:

$$\begin{pmatrix} total\,interest\text{-} \\ on\text{-}interest \end{pmatrix} = \begin{pmatrix} total\ accumulated \\ market\,interest \end{pmatrix} - \begin{pmatrix} total\ interest- \\ on-principal \end{pmatrix},$$

$$= \$149.38 - \$142.50$$

$$= \$6.88\ .$$

The preceding story allows the easy computation of all the important concepts implicit in the lump-sum time value relation. The story is so simplistic, though, that interest-on-interest seems trivial. The example below illustrates the magical power of seemingly trivial interest-on-interest.

EXAMPLE 2 Powerful magic of interest-on-interest.
Two twins each receive an inheritance of $20,000 which they dutifully invest in an exceptional account that earns 12.25 percent per year. Andraya leaves each year's interest in the account so that she earns interest-on-interest. Zarcog is content to earn only interest-on-principal, so every year he goes to the bank and withdrawals all interest-on-interest. After 20 years, how much wealth has each sibling accumulated?

SOLUTION
Principal of $20,000 that earns 12.25 percent per year accumulates periodic interest during the first year equal to $2,450:

$$\begin{aligned} 1^{st} \text{ year periodic interest } &= (0.1225)(\$20{,}000) \\ &= \$2{,}450 \end{aligned}$$

At the end of the first year neither sibling takes money out of the account. At the end of the second year, however, Zarcog notices that the periodic interest is $300.12 larger than a year earlier:

$$\begin{aligned} 2^{nd} \text{ year periodic interest } &= (0.1225)(\$22{,}450) \\ &= \$2{,}750.12 \end{aligned}$$

Instead of leaving this extra interest-on-interest of $300.12 in the bank, Zarcog withdraws it. He figures, hey, $300.12 out of $2,750.12 isn't much and he might as well enjoy some of it! (By the way, notice that the interest-on-interest of $300.12 equals $2,450x0.1225). So every year Zarcog withdraws from the account any annual interest exceeding $2,450 per year. Throughout twenty years the account earns total interest-on-principal of $2,450x20, or $49,000. Zarcog began with $20,000 and the account rose by $49,000 to end at $69,000. Along the way, of course, Zarcog withdrew all interest-on-interest.

Andraya resists the temptation to spend and leaves in the account all interest-on-interest. The account's ending wealth consequently is easily deduced from the lump-sum time value equation. Set *N* equal to 20, *r* equal to 0.1225, and *PV* to $20,000. Solve for *FV*:

$$\$20{,}000 = \frac{FV}{(1 + 0.1225)^{20}}$$

or FV = $201,724 . The lump-sum time value formula gives an answer that automatically includes interest-on-interest plus interest-on-principal.

CALCULATOR CLUE 4.4 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Find the ending wealth after 20 years by typing:
20000 PV 12.25 I/Y 20 N CPT FV

The answer of $–201,724 appears on the display.

Total accumulated market interest for Andraya is found with equation 4.8:

$$\begin{pmatrix} \text{total accumulated} \\ \text{market interest} \end{pmatrix} = \begin{pmatrix} \text{account} \\ \text{balance} \end{pmatrix} - \begin{pmatrix} \text{total contributed} \\ \text{principal} \end{pmatrix},$$

$$= \$201{,}724 - \$20{,}000$$

$$= \quad \$181{,}724 \ .$$

Total accumulated market interest equals the sum of interest-on-principal plus interest-on-interest. Andraya's account earns interest-on-principal throughout 20 years totaling \$49,000. The interest-on-interest for Andraya therefore equals \$132,724 (that is, \$132,724 = \$181,724 – \$49,000).

Andraya ends up with \$201,724 and Zarcog ends up with \$69,000; Andraya's account balance is larger by \$132,724. For Andraya the interest earned interest, then the interest-on-interest earned interest, too, and the whole process compounded immensely. Zarcog spent his interest-on-interest, so the interest itself never earned more. Sure Zarcog benefited by spending money along the way (don't worry about finding the sum of withdrawals, but it equals \$57,022 (=\$300.12 x (1+2+…+19)). He sure paid for it, though, because he ended with \$132,724 less than Andraya. Allowing seemingly trivial interest-on-interest to accumulate increases wealth accumulation almost magically.

The lump-sum time value formula contains four variables: *PV*, *FV*, *r*, and *N*. The basic formula contains *PV* on the left-hand side. Arguably, *PV* is the most important variable because good financial decisions typically increase present value. Yet the formula contains four variables, and given any three, the fourth one comprises the unknown variable and takes on a value that satisfies the equality. The discussions below consider different scenarios in which each variable is the unknown answer.

2.A. Ending wealth, FV, as the unknown

Many situations require knowledge about ending wealth. For example, suppose exactly eight years ago relatives placed \$10,000 in an account that earns 7.25 percent annually. How much is the account balance today? Equation 4.6 easily solves this equation, but first decisions must be made about the different variable settings. Clearly *N* equals 8 and *r* equals 0.0725. What about *PV* and *FV*, though? The beginning wealth is \$10,000 and *PV* represents beginning wealth. *PV* equals \$10,000 and *FV* is the unknown variable. Getting the answer requires finding *FV*.

Rearrangement of formula 4.6 shows:

$$FV = PV\,(1 + r)^N \ .$$

For this particular problem,

$$FV = \$10{,}000\ (1 + .0725)^8$$

$$= \$17{,}506$$

If \$10,000 earns seven and a quarter percent for eight years, it grows to \$17,506.

The problem below combines a few of the preceding concepts.

EXAMPLE 3 Find total interest.

Today you invest \$4,500 in a security on which you expect to earn a 12.5 percent average annual rate of return (and of course, this is the geometric average, not the arithmetic average). How much total market interest will have accumulated exactly 5 years from today?

SOLUTION
The first step once again is to set the variables that equation 4.6 requires. Set *N* equal to 5, *r* equal to 0.125, and set *PV* equal to the beginning wealth of $4,500. Solve for the ending wealth, *FV*:

$$FV = \$4{,}500\ (1 + .125)^5$$

$$= \$8{,}109\ .$$

CALCULATOR CLUE 4.5 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Solve the preceding problem by typing:
4500 PV 12.5 I/Y 5 N CPT FV

The answer of $–8,109 appears on the display.

Because the ending wealth is $8,109 and the investor's contribution is $4,500 then the remainder is market interest.

Total account balance = Total principal + Total market interest ,

$8,109 = $4,500 + Total market interest ,

or *Total market interest = $3,609 .*

The market interest accruing to this investment represents a transfer of wealth to you from the market's invisible hand.

EXERCISES 4.2A

Concept quiz

1. What typically has the biggest impact on ending wealth: doubling the initial wealth, doubling the investment horizon, or doubling the rate of return?

Numerical quickies

2. Say that 375 years ago immigrants purchased the island of Manhattan from native Americans for $24. If that sum were invested at 6.25% compounded annually, and the account left-alone, what would be the accumulation today? ©LS3 .

3. A deposit of $1,000 five years ago earns an annual average return of 5.5%. Otherwise, the account has been left alone. As of today, how much total interest has accumulated on the deposit? ©LS4b .

4. Ten years ago a $5,000 stock fund was established. The account will be closed eighteen years from now. The account always earns an average annual geometric rate of return of 12%. Otherwise, the account will have been left alone. At the time the account is closed, how much is the total accumulated interest-on-principal and interest-on-interest? ©LS4c and ©LS4d .

Numerical challengers

5. A $3,500 savings account was established 4 years ago. The account earns 8.75% compounded annually. Otherwise, the account has been left alone. When the annual interest is credited to the account today, how much is credited? ©LS6a .

6. An account established four years ago is today credited with annual interest of $2,500. The interest rate is 6.75% compounded annually. Otherwise, the account has been left alone. How much is the end-of-day balance? ©LS7a .

7. Saf T Lok, Inc. had a volatile year in 2525. Their shareprice on December 31, 2524, was $12.50. During January, 2525, their shareprice was up 43.75 percent. The monthly rates of return throughout 2525 are shown below:

Jan.	Feb.	Mar.	Apr.	May	June
43.75	-13.04	70.00	-45.59	22.97	-28.57
July	Aug.	Sept.	Oct.	Nov.	Dec.
-49.23	-21.21	-50.00	607.69	-5.43	-37.93

Compute the arithmetic and geometric average monthly rates of return for Saf T Lok in 2525. For 1,000 shares of the stock, use the lump-sum time value relation to find the value at year-end 2525. Find FV by using as r both the arithmetic and geometric averages. How much are the differences in computed ending wealth, and which is correct?

2.B. *Beginning wealth, PV, as the unknown*

Financial decisions often require computing how much beginning wealth ventures require. For example, suppose a venture capitalist wants to make a deposit today that, given the interest rate, finances a series of planned future withdrawals. The unknown variable is beginning wealth, PV. For many asset management decisions, the future returns and target rate of return may be fairly known. The unknown variable is PV, the proper allocation to finance the investment.

Consider a simple illustration in which 4 years from today you wish to withdraw $80,000 from an account. The account earns 6.25 percent compounded annually, and you withdraw the money immediately after the 4th year's interest is credited to the account. You establish the account today by making a deposit. How much is today's deposit?

The answer for this question requires setting N equal to 4, r equal to 0.0625, and FV equal to $80,000. Substitution into equation 4.6 shows

$$PV = \frac{FV}{(1+r)^N}$$

$$= \frac{\$80,000}{(1+.0625)^4}$$

$$= \$62,773.$$

The answer of $62,773 is the amount which, if deposited today, grows at an annual rate of six and one-quarter percent to become equal to $80,000 in four years. The investment earns total market interest equal to $17,227 (Total market interest = $80,000 – $62,773).

CALCULATOR CLUE 4.6 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Solve the preceding problem by typing:
80000 FV 6.25 I/Y 4 N CPT PV

The answer of $–62,773 appears on the display.

Finding present value requires dividing the future sum by one plus the rate raised to an exponent. Dividing $80,000 by a number greater than one gives an answer smaller than $80,000. The answer is smaller by exactly the amount of interest accruing over the investment horizon. This process of dividing a future sum by $(1+r)^t$ is called "discounting". Finding a present value requires discounting future cash flows. The rate r often is called the "discount rate". Common conversation relies on similar word usage, as in the phrase "you have to discount what you sometimes hear them say." The accompanying Street-Bite illustrates importance of discounting.

STREET–BITE The Fed, the Discount Rate and the Stock Market

The present value of a future cash flow equals the cash flow divided by one-plus-the-discount rate raised to the t^{th} power. This discount rate is important.

DEFINITION 4.1 Discount rate
The discount rate is the periodic percentage return subtracted from the future cash flow for computing present value.

The discount rate is important because it constitutes a link between present values and future returns.

There are many different ways to measure the economy's discount rate. One important measure is the official U.S. government discount rate managed by the Federal Reserve System. The Fed is historically the most important bank in the USA. It is called the nation's "central bank" and most nations have one. Often activities of central banks are extremely politicized. Even in the USA appointment of the Fed's top management requires approval of the United States Congress. Because of influences by men such as Alexander Hamilton, appointed Treasurer by George Washington in 1789-1795 and killed in duel by Aaron Burr in 1804, the Fed today represents the interests of a broad band of participants in the US political economy.

The Federal Reserve Board of Directors led by the powerful Chairman of the Fed uses several tools to influence market activity. These include:

- the Federal Reserve Board votes whether to raise or lower the official government discount rate; this is the interest rate charged by federal district banks to member public and private banks.

- the Fed sets the reserve requirement on member bank accounts; this regulates the amount of loans that banks may lend to business and individual borrowers.

- the Fed buys and sells marketable currencies and government securities in the global financial marketplace; this affects supply and demand conditions.

The Fed meets fairly regularly. The markets have grown accustomed to paparazzi hoopla about Fed actions and inactions. So called "Fed watching" is a popular pastime to many regular viewers of CNBC and CNN. The table below shows how the stock market has responded when the Fed changed the official discount rate.

Announcement date and rate change	Stock index returns during the days after announcement of a discount rate change by the Federal Reserve Board's Open Market Committee			
	next 2 days	next 7 days	next 30 days	next 180 days
January 31, 1996 lowered to 5%	0.95%	2.00%	2.44%	14.76%
February 1, 1995 raised to 5.25%	0.45	2.32	2.98	23.37
Nov. 15, 1995 raised to 4.75%	0.03	-2.39	-1.99	20.79
August 16, 1994 raised to 4%	0.45	1.43	1.62	9.41
May 17, 1994 raised to 3.5%	0.96	1.79	-0.45	4.49
July 2, 1992 lowered to 3%	0.09	0.79	2.10	26.82
Dec. 20, 1991 lowered to 3.5%	2.16	8.57	21.52	26.74
Nov. 6, 1991 lowered to 4.5%	1.21	2.14	-3.35	19.80
Sept. 13, 1991 lowered to 5%	-0.10	0.51	2.31	25.81
April 30, 1991 lowered to 5.5%	0.61	1.43	3.68	23.30
February 1, 1991 lowered to 6%	3.30	9.53	19.44	36.37
Dec. 19, 1990 lowered to 6.5%	0.41	-0.14	12.32	48.81
average return for entire sample period	0.20	0.71	3.07	19.91

TABLE 4.2 Stock returns after changes to the Fed's discount rate

Dates are from http://www.stls.frb.org/fred/data/irates/discount. The stock returns index is the "CRSP equal-weighted, nyse+amex, with dividends, 1990.1219 to 1996.0131+180-1". Each entry is the compound index return between the event and day t-1, where t is the length of the investment horizon. Compiled by author.

Each row's date is the actual day on which the Fed executes an order to change the discount rate. Each column's entry represents the cumulative rate of return (see formula 4.1) for a big basket of stocks. For each date the cumulative rate of return is collected for different investment holding periods. The upper left number, 0.95 percent, indicates that cumulative stock returns were a little less than one percent for the two days following the Fed discount rate change on January 31, 1996. On this date the rate was lowered to 5 percent from 5.25%. Over the 7 trading days subsequent to the announcement, cumulative returns equaled 2.00 percent. Over the subsequent 180 trading days (until late summer of 1996) aggregate share values rose 14.76 percent.

That nearly 15 percent return is rather healthy. Money doubles in about five years at 15 percent per year. Still, this return is less than the overall sample average of 19.91 percent per 180-day horizon. The bottom row lists average returns in the entire sample period for different investment horizons. This sample period contains one of the most remarkable sustained bull markets in financial history.

Nine of the twelve Fed announcements are followed over the next seven days by above average stock returns. Seemingly this gives rise to conjecture that perhaps stock returns after Fed action are bigger than average. Contrary evidence exists for the 30 day horizon returns, however. In this column, eight of twelve announcement period returns lag the average. Such flip-flops in comparisons make generalization difficult. Indeed, predicting share price responses to anything, even the powerful visible hand of the Fed, seems for many, a mystery.

The following example illustrates that measurements for periodic and total market interest from equations 4.7 and 4.8 also pertain to PV computations.

EXAMPLE 4 Find the initial deposit given this year's periodic interest.

Your uncle marvels at the power of compound interest. Twelve years ago he deposited money into an account that earns 6 5/8 percent per year. Otherwise he never has deposited nor withdrawn money from the account. Just today the most recent year's interest of $4,000 was credited to the account. How much was your uncle's initial deposit?

SOLUTION

The first step is finding the account balance at the beginning of the most recent year by manipulating (1) this year's periodic interest, and (2) the periodic interest rate. Use equation 4.7:

$$periodic\ interest = \begin{pmatrix} periodic\ interest \\ rate \end{pmatrix} \begin{pmatrix} beginning\ of \\ period\ balance \end{pmatrix}$$

$$\$4,000 = (0.06625) \begin{pmatrix} beginning\ of \\ period\ balance \end{pmatrix}$$

$$\begin{pmatrix} beginning\ of \\ period\ balance \end{pmatrix} = \$60,377$$

If an account earning 6 5/8 percent per annum has a balance one year ago equal to $60,377 then how much was deposited in the account twelve years ago? Determine the answer to this question with the lump-sum time value formula. Set r to 0.06625, N to 11 (from twelve years ago until 1 year ago is 11 years), and FV to $60,377:

$$PV = \frac{\$60,377}{(1+.06625)^{11}}$$

$$= \$29,814.$$

Your uncle's initial deposit 12 years ago was $29,814.

CALCULATOR CLUE 4.7 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Solve the preceding problem by typing:
60377 FV 6.625 I/Y 11 N CPT PV

The answer of $–29,814 appears on the display.

EXERCISES 4.2B

Concept quiz

1. In the lump-sum time value relation, is *PV* always less than *FV*? Explain.

Numerical quickies

2. Today you sell your stock fund for $19,500. You bought it 4 years ago and otherwise the account has been left alone. The stocks have earned a 16% average annual rate of return. How much did you buy the stocks for? ©LS8 .

3. Eight months from now a bill of $3,500 is due. Today you deposit money such that if the account earns 1.25% per month, the bill is perfectly financed. How much do you deposit? ©LS9 .

Numerical challengers

4. In exactly 14 months a bill of $6,200 is due. Today you deposit money such that if the account earns 1.25% per month, the bill is perfectly financed. Unfortunately, your account earns only 1.05% per month. When the bill is due, how much money do you lack? ©LS10a .

5. Exactly 18 years ago your uncle deposited money into an account that earns 6.25% per year. Otherwise, he has left the account alone. Just today the most recent year's interest of $2,200 was credited to the account. How much was the initial deposit? ©LS11 .

2.C. The investment horizon, N, as the unknown

When known variables include the rate and the beginning and ending wealth, the unknown variable N adjusts to satisfy the lump-sum time value equation. Recall the formula:

$$PV = \frac{FV}{(1+r)^N} .$$

The variable N appears on the right-hand-side only as the exponent in $(1+r)^N$. Getting *N* out of the exponent requires taking the logarithm of both sides of the equation. Rearranging to isolate *N* by itself on the left-hand side yields:

$$N = \frac{\log\left(FV/PV\right)}{\log(1+r)} .$$

EXAMPLE 5 Find the investment horizon.

If you invest $1,000 today in a security that you expect to earn 12 1/8 percent per year, how many years does it take to accumulate $2,500.

SOLUTION

Use the preceding formula, set *r* to 0.12125 , *PV* to $1,000 and *FV* to $2,500:

$$N = \frac{\log\left(\$2,500/\$1,000\right)}{\log(1+0.12125)} .$$

or $N = 8.01$. In about 8 years the investment's value increases to $2,500.

CALCULATOR CLUE 4.8 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Solve the preceding problem by typing:
2500 FV 1000 +/– PV 12.125 I/Y CPT N .

The answer of 8.01 appears on the display.

EXERCISES 4.2C

Numerical quickies

1. Today you purchase some international mutual funds for $2,400. You read that they should earn a 14% average annual rate of return throughout the foreseeable future. If you leave the account alone, how long should it take to accumulate $6,000? ©LS12 .

2. A newspaper reports that a particular mid-level manager today has stocks worth $100,000. The person bought the stocks with $2,660 from a summer job while in college. The stocks have earned an average annual return of 12%. How along ago did she buy the stocks? ©LS13 .

Numerical challengers

3. Today your account was credited with its annual interest of $1,100. The account was established some time ago with a $7,000 initial deposit. Otherwise, the account has been left alone. The account earns 7.25% annual interest. How long ago was the account established? ©LS14 .

4. Some time ago a $7,000 initial deposit opened an account. Today the annual interest was credited to the account. Total lifetime interest now equals $7,095. The account earns 7.25% annual interest. How long ago was the account established? ©LS15 .

5. You are entering a creative financing arrangement that involves two different transactions. For the first transaction you will borrow $15,300 at an annual interest rate of 8.70%. For the second transaction you will invest the borrowed money today in a security that promises a future pay-off of $29,800 . Upon receiving the pay-off from the

second transaction, you will repay in-full the loan from the first transaction. It is certain that these cash flows actually will happen, but the timing of the pay-off isn't clear. What determines whether or not this is a good deal? ©LS2 .

2.D. The rate of return, r, as the unknown

Recall that r is the geometric average periodic rate of return that links the beginning and ending wealth across N periods. When r is the only unknown variable, simple rearrangement of the lump-sum time value formula shows an easy solution:

$$r = \sqrt[N]{FV/PV} - 1 .$$

This is identical to the definition of $ROR^{geometric\ average}$ from equation 4.6.

EXAMPLE 6 Find initial deposit and subsequent rate of return.

Today Ben needs $15,000 to make a deposit on a house. He invested enough money 8 years ago so that if he earned his target rate of return of 10 percent, his accumulation today would exactly equal $15,000. Instead, however, the investment did better than expected so today Ben has $19,000. By how much does the actual average rate of return exceed the target rate of return?

SOLUTION

The problem set-up states that *N* equals 8, and the ending wealth, *FV*, equals $19,000. Also we need the beginning wealth, yet *PV* is not explicitly stated. To find the beginning wealth, though, take away eight years of ten percent interest from $15,000:

$$PV = \frac{\$15,000}{(1 + .10)^8}$$
$$= \$6,998 .$$

Ben invested $6,998 eight years ago with the expectation that it would grow to $15,000. Given that eight years later it actually grows to $19,000 then *r* is found as:

$$r = \sqrt[8]{\$19,000/\$6,998} - 1 ,$$
$$= 13.30\% .$$

CALCULATOR CLUE 4.9 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Find the amount invested by typing:
15000 FV 10 I/Y 8 N CPT PV

The answer $–6,998 appears on the display. While the display still shows $–6,998, reset FV and compute the new rate of return by typing
19000 FV CPT I/Y .

The answer of 13.30 appears on the display.

Ben's actual rate of return, 13.30 percent, exceeds the 10 percent target by 3.30 percent. The investment earns 330 basis points more than expected, so Ben has $4,000 more than expected.

The preceding paragraph introduces the phrase "basis point."

DEFINITION 4.2 Basis point
A basis point is one-hundredth of a percentage point.

For example: (1) the difference between 6 percent and 7 percent is 100 basis points; (2) the difference between 6.25 percent and 6.50 percent is 25 basis points; (3) the difference between 6 3/4 percent and 6 7/8 percent is 12.5 basis points. In the real world, basis points count. Some investors win or lose thousands of dollars when interest rates move even three basis points!

EXERCISES 4.2D

Numerical quickies

1. Exactly 5 years ago you put $1,350 in an investment account. Today the account was credited with its annual interest so that its balance now is $2,750. What is the annual average rate of return for the account? ©LS16 .

2. Today you are buying some stocks for $900. In 8 years you would like the account to have accumulated $2,700. What is the desired annual average rate of return for the account? ©LS17 .

3. A sum of money doubles in 24 years. What is the annual average rate of return? ©LS19 .

4. Today your account was credited with its annual interest of $800, thereby bringing the balance to $9,000. What is the account's annual interest rate? ©LS20 .

Numerical challengers

5. In exactly 18 months a bill of $5,300 is due. Today you deposit money such that if the account earns 1.25% per month, the bill is perfectly financed. Unfortunately, your actual monthly rate of return was less than the target, so your account accumulates $5,080. What was the actual average monthly rate of return? ©LS21a

6. Investment A returns $8,000 in 11 years. Investment B costs $6,800 today and $10,000 in 7 years. If your sole objective is to buy the one investment with the largest annual average rate of return, what is the decision rule? ©LS1 .

2.E. Approximations with the rule of 72

Doing time value calculations in your head is sometimes useful. Perhaps it's useful because you want to check whether an answer on your calculator or spreadsheet seems reasonable. Perhaps it's useful because you don't walk around with a calculator and, on a rare occasion, need to know a time value answer. Fortunately, there is a very useful rule-of-thumb that often helps: The rule of 72.

RULE 4.2 The rule of 72
The approximate number of periods in which a sum of money doubles equals 72 divided by the periodic rate of return.

Consider the following application for the rule of 72. Suppose a savings account earns interest at 6 percent per year. The doubling period, according to the rule of 72, equals 72 divided by 6, which is 12. That is, at a six percent rate money doubles in about twelve years. Notice that the interest rate in the Rule of 72 formula is not a decimal.

To find the exact doubling-period, use the lump-sum time value formula. In equation 4.6 set the ratio *FV/PV* to 2.0 since wealth doubles and *FV* is twice as large as *PV*, set *r* to 0.06, and solve for *N*:

$$N = \frac{\log(2)}{\log(1 + 0.06)},$$

or $N = 11.9$ years. The rule of 72 is wrong by only one-tenth a year. Not bad for a rule of thumb!

EXAMPLE 8 Find approximate doubling period
You hear from your friend that for the past four years the stock market has averaged about 20 percent per year. You wonder approximately how much has your friend's stock fund increased?

SOLUTION
Divide 72 by 20 and get about three and-a-half. According to the rule of 72 money doubles in 3 1/2 years at 20 percent. Thus, if the funds earned twenty percent for four years, the balance more than doubled. The exact answer is found with the lump-sum formula with *FV* as the unknown: $FV = PV(1 + .20)^4$; $FV = PV \times 2.07$; this means the ending value equals 2.07 times beginning value.

EXERCISES 4.2E

Concept quiz

1. Long-run averages suggest corporate bonds return 8% and the stock market returns 12% per year. Sketch a table with two columns pertaining to the two preceding rates. Insert 3 rows corresponding to the investor's age at time of making a $10,000 deposit: 25 years of age, 40 years of age, and 55 years of age. Use the rule of 72 to approximate in each cell the ending wealth at age 70 given the respective investor age and rate of return. Comment on the relative importance of investment horizon and rate of return. {HINT: The bottom row of the first column, 8% and 55 years when saving begins, is about $35,000.}

3 Intraperiod compounding of interest

The horizon for accruing periodic interest almost always is less than a year. Corporations pay interest every 6 months to investors in corporate bonds. Credit unions pay interest every quarter to savings account deposits. Credit cards and consumer loans charge interest every month on outstanding balances. There is even an overnight market between banks, and interest accrues at an overnight interest rate. The lump-sum time value formula from the previous section properly compounds intraperiod interest regardless of period length.

3.A. The relationship between periodic components

The lump-sum time value formula with intraperiod compounding is

$$PV = \frac{FV}{(1+r)^N}$$

Still *PV* is the beginning wealth, *FV* is the ending wealth *N* periods later, and *r* is the geometric average periodic rate of return. Intraperiod compounding enters through the definition for *r*,

$$r = APR / m .$$

APR is the annual percentage interest rate and *m* is the number of compounding periods per year. For semiannual situations, for example, *m* equals 2. For monthly situations, such as credit cards or car loans, *m* equals 12. For daily compounding, *m* equals 365.

Compounding frequency affects the time value relation. Consider, for example, a depositor that puts $1,000 in an account earning interest at an annual percentage rate of 8.4 percent. Table 4.3 shows the account balance after one year for alternative compounding frequencies.

compounding frequency m per *t*	period description	future value of $1,000 after one year at 8.4%	effective annual rate see section 3.B below
1	annual	$1,084.00	8.400%
2	semiannual	$1,085.76	8.576%
4	quarterly	$1,086.68	8.668%
12	monthly	$1,087.31	8.731%
365	daily	$1,087.62	8.762%
∞	continuous	$1,087.63	8.763%

TABLE 4.3 Ending wealth of a $1,000 deposit at an 8.4 percent APR for different compounding frequencies

When interest accrues once a year then *m* equals 1 and the formula is

$$FV = \$1000\left(1 + \frac{.084}{1}\right)^1$$

$$= \$1{,}000\ (1.084)$$

$$= \$1{,}084.00$$

The bank credits the account at the end-of-year with interest-on-principal equal to $84. There is no interest-on-interest.

Suppose, however, the bank credits interest every half-year. For this case, *m* equals 2. The periodic interest rate now is a semiannual rate of 0.084÷2, or 4.2 percent per semiannum. Substitution into the formula shows that the ending wealth after 2 semiannual periods is:

$$FV = \$1000\left(1 + \frac{.084}{2}\right)^{2}$$

= $1,000 (1.042)^2

= $1,085.76

The ending wealth is higher with intraperiod compounding because interest earns interest. After six months the bank credits the savings account with a half-year of interest. The periodic interest after six months is:

$$periodic\, interest = \left(\begin{array}{c} periodic\ interest \\ rate \end{array}\right)\left(\begin{array}{c} beginning\, of \\ period\, balance \end{array}\right),$$

= (0.084 / 2) ($1,000)

= $42 .

The first half-year's periodic interest equals exclusively the $42 interest-on-principal. The bank credits periodic interest to the account and brings the account balance at the beginning of the second half-year to $1,042. During the second six months periodic interest equals:

= (0.084 / 2) ($1,042)

= $43.76 .

During this second half-year the principal once again earns interest of $42. The interest from the first six months earns its own interest, too. Interest-on-interest is $1.76 (notice that $42x0.084/2 is $1.76).

Ending wealth increases as compounding frequency increases. With daily compounding, m equals 365. The lump-sum time value formula shows the ending wealth after 365 days is:

$$FV = \$1000\left(1 + \frac{.084}{365}\right)^{365}$$

= $1,000 (1.0002301)^{365}

= $1,087.62 .

A $1,000 deposit with an 8.4 percent annual percentage rate earns interest-on-principal in one year of $84. Because the account balance indicates total market interest of $87.62, the interest-on-interest with daily compounding is $3.62 (that is, $3.62 = $87.62 – $84.00).

CALCULATOR CLUE 4.10 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories. Now access the intraperiod compounding setting by typing 2nd I/Y . To set for daily compounding, for example, type 365 ENTER . Once the compounding frequency is properly set, exit this loop by typing 2nd CPT . The N key refers to the number of periods, not number of years.

Suppose you wish to find the future value after 1 year of a $1,000 investment given an 8.4 percent APR and daily compounding. Type:

2nd I/Y 365 ENTER 2nd CPT 1000 PV 8.4 I/Y 365 N CPT FV

The answer $–1,087.62 appears on the display.

Simply knowing the annual percentage rate is not enough to know how much interest actually accrues per dollar per year. Compounding frequency matters, too. Suppose the deposit is $10 million, instead of $1,000. The difference in annual interest at an 8.4 percent annual percentage rate ranges from $840,000 with annual compounding, to $876,184 with daily compounding. As you can imagine, that extra $36,184 is pretty significant to someone. Compounding frequency is important. Its importance increases, too, as the investment horizon lengthens or size of the pot grows.

EXAMPLE 9 Find the benefit of switching compounding frequency

Ms. Williams just made a $10,000 investment that promises to pay 8.25 percent compounded semiannually for 8 years. She just learned that on 8-year investments an alternative bank offers 8.15 percent compounded monthly. She can switch out of her original deal and reinvest in the alternative. She might have to pay a penalty to the original bank, however. To switch or not to switch, that is the question!

SOLUTION

The question really asks which alternative accumulates the most wealth. Compute first the ending wealth for the original investment if Ms. Williams sticks with it for 8 years. There are 16 half-years in 8 years, so *N* is 16. *PV* is $10,000. The semiannual interest rate *r* is 0.0825 ÷ 2. Use the lump-sum time value formula:

$$\$10,000 = \frac{FV}{\left(1+\frac{.0825}{2}\right)^{16}}$$

$$FV = \$10,000\,(1.04125)^{16}$$

$$= \$19,093\,.$$

Ms. Williams receives $19,093 if she sticks with the original investment.

Now compute for the alternative investment how much she needs to invest today in order to receive $19,093 in eight years. That is, use the lump-sum time value equation with *FV* at $19,093 and *r* at 0.0815 ÷ 12 (monthly compounding), and *N* at 96 (eight years is 96 months):

$$PV = \frac{\$19,093}{\left(1+\frac{.0815}{12}\right)^{96}}$$

$$= \$9,970$$

Ms. Williams gets the same ending wealth as the original investment if, in the alternative, she invests $9,970. As long as she gets back $9,970 or more from her original $10,000 investment, she should cancel and reinvest in the alternative. In other words, $30 is the maximum penalty at which switching is profitable.

CALCULATOR CLUE 4.11 Prepare the BAII Plus© calculator by typing 2nd FV to clear the time value memories. To find the ending wealth for the original investment type:
2nd I/Y 2 ENTER 2nd CPT 10000 PV 8.25 I/Y 16 N CPT FV

The display shows $–19,093 . Now make this the ending wealth for the alternative investment and find the requisite beginning wealth. With $–19,093 still on the display, type:
FV 2nd I/Y 12 ENTER 2nd CPT 8.15 I/Y 96 N CPT PV

The answer of $9,970 appears on the display. The difference between $9,970 and $10,000 is the $30 maximum penalty at which switching is profitable.

Many asset management decisions involve finding an alternative with the best present value. For an outflow the best alternative has the smallest possible present value. For an inflow, however, the best alternative has the largest present value. This is true regardless of compounding frequency. The following example illustrates that different capital costs must be discounted before choosing the cheapest alternative.

EXAMPLE 10 Which cost has the cheapest present value.
Quickie.Dot purchases inventory from one of three suppliers. The suppliers offer different prices and payment plans for exactly the same item. Quickie.Dot must choose the least expensive alternative. The following table summarizes the choices.

Supplier	Base price	Payment plan	Grace period for full price sales	Percent discount for prompt payment
Alpha Inc.	$17,500	"1.5 in 5 or net 60"	60 days	1.5% if paid within 5 days
Kappa Inc.	$17,400	"0 net 90"	90 days	Never a discount
Zeta Inc.	$17,350	"2 in 10 or net 30"	30 days	2% if paid within 10 days

TABLE 4.4 Payment & discount plans for purchasing inventory

Suppose that Quickie.Dot finances its operations at a 12 percent annual financing rate (compounded daily). Which supplier and payment plan is cheapest?

SOLUTION
First understand data in the table. The base price of the item from Alpha Inc. is $17,500. When Quickie.Dot buys from Alpha, they can pay the full base price any time within 60 days without falling delinquent. Quickie.Dot wisely plans to stay away from delinquency. Yet if they buy from Alpha at full price the prudent strategy delays payment as long as practical. If Quickie.Dot pays for the item within 5 days, however, they receive a 1.5 percent discount. The discount price is $17,237 {= $17,500 x (1 - 0.015)}. Consequently, the alternative payment plans for Quickie.Dot if they purchase from Alpha include (1) pay $17,500 after 60 days, or (2) pay $17,237 after 5 days. The present values of these two alternatives at a 12 percent annual rate (compounded daily over 365 days) is:

$$PV(\text{full price from Alpha}) = \frac{\$17,500}{\left(1+\frac{.12}{365}\right)^{60}}$$

$$= \$17,158.$$

$$PV(\text{discount price from Alpha}) = \frac{\$17,237}{\left(1+\frac{.12}{365}\right)^{5}}$$

$$= \$17,209.$$

The least expensive purchase is one with the lowest present value of cost. The prudent payment plan if Quickie.Dot purchases inventory from Alpha Inc. is to pay the full price after 60 days.

Perhaps it seems strange that paying $17,500 at day 60 is cheaper than paying $17,237 at day 5. Those prices, however, occur at different times. Consequently, comparing them is like comparing apples and oranges. Time value relations require discounting all cash flows before making comparisons. Paying more to Alpha later has a smaller discounted cost than paying less sooner. It actually is cheaper for Quickie.Dot to forgo the discount and pay the full base price later.

The cost of forgoing the discount equals the rate of return connecting the later higher price (FV= $17,500) and sooner lower price (PV= $17,209) . That is,

$$\$17,209 = \frac{\$17,500}{\left(1+\frac{i}{365}\right)^{60-5}}$$

$$i = 11.13\% .$$

The percentage cost of forgoing the discount, 11.13 percent, is less than the 12 percent financing rate that Quickie.Dot pays on its operations. Suppose the sooner purchase were financed with a $17,209 loan at 12 percent interest for 55 days. Repayment of the loan incurs an ending principal and accumulated interest balance of

$$\$17,209 = \frac{FV}{\left(1+\frac{.12}{365}\right)^{60-5}}$$

$$FV = \$17,523 .$$

The lower purchase price on day 5 has a time value of $17,523 at day 60. This exceeds the full base price of $17,500. The cost of forgoing the discount from Alpha is less expensive than the cost of paying the full price. When the company financing rate exceeds the cost of foregoing the discount, like here 11.13% is less than 12%, then the company should forego the discount and pay full price at the end of the net period. Conversely, when the company financing rate is less than the cost of foregoing the discount then take the discount.

Quickie.Dot also might choose to purchase from Kappa and pay $17,400 after 90 days. The present value of this cost is

$$PV(full\ price\ from\ Kappa) = \frac{\$17,400}{\left(1+\frac{.12}{365}\right)^{90}}$$

$$= \$16,893.$$

Purchase from Kappa is less expensive than any payment plan that Alpha offers.

Alternative purchase plans from Zeta Inc. include (1) pay $17,350 after 30 days or (2) pay $17,003 {= $17,350 x (1 - 0.02)} after 10 days. The present values of these costs equal

$$PV(full\ price\ from\ Zeta) = \frac{\$17,350}{\left(1+\frac{.12}{365}\right)^{30}}$$

$$= \$17,179.$$

$$PV(discount\ price\ from\ Zeta) = \frac{\$17,003}{\left(1+\frac{.12}{365}\right)^{10}}$$

$$= \$16,947.$$

The payment plan offering Zeta's customers the best deal is payment at 10 days of the discount price.

Five alternative payment plans exist for Quickie.Dot. The best of the lot has the smallest present value; it might also have the smallest price, but not necessarily. Sometimes paying less now is better than paying more later, sometimes it's not. Periodic cash flows from different times must be discounted before comparisons with other cash flows are meaningful. Quickie.Dot looks at its choices and quickly finds that the best deal is purchase of inventory at full base price from Kappa with deferral of payment for 90 days.

EXERCISES 4.3A

Numerical quickies

1. A deposit five years ago of $1,200 earns 5.5% annual interest compounded monthly. Otherwise, the account has been left alone. As of today, how much total interest has accumulated on the deposit? ©CY6b .

2. Today you plan to cash in savings bonds for $15,000. You bought them exactly 10 years ago. The savings bonds have earned a 6.25% annual rate of return compounded

semiannually. How much did you pay for the savings bonds? ©CY7 .

3. In 4 years you must transfer $4,200 to associates. Today you invest money such that if it earns 12.5% per annum, compounded monthly, you'll accumulate the required funds. How much do you invest? ©CY8 .

4. Today you invest $3,100 that earns 9% per annum compounded quarterly. If you leave the account alone, how long should it take to earn $2,550 of total interest? ©CY9 .

Numerical challengers

5. In exactly 14 months you expect to receive $6,200 from an investment. Today you borrow money from an associate such that if interest accrues at an annual rate of 8.25% compounded monthly, the investment exactly repays the loan. Unfortunately, the associate charges you 250 basis points more than expected. In 14 months when the investment returns $6,200 and you repay the loan, how much money do you lack? ©CY12 .

6. Exactly 14 years ago your uncle deposited money into an account with a 5.50% annual percentage rate that compounds quarterly. Otherwise, he has left the account alone. Just today the most recent quarter's interest of $350 was credited to the account. How much was the initial deposit? ©CY13 .

7. In exactly 18 months a bill of $5,300 is due. Today you deposit money such that if the account earns the target APR of 7.25%, compounded monthly, the bill is perfectly financed. Otherwise, there are no other deposits or withdrawals. When the bill is due, the account actually has $5,080. What was the actual average APR? ©CY10a

8. A $1,200 savings account was established 42 months ago. Otherwise, there are no other deposits or withdrawals. The account earns a 7.75% annual percentage rate, compounded monthly. When the monthly interest is credited to the account today, how much is credited? ©CY14a .

9. An account established four years ago is today credited with quarterly interest of $620. The interest rate is a 5.25% APR, compounded quarterly. Except for the initial deposit, there are no other deposits or withdrawals. How much is the end-of-day balance? ©CY15a .

10. Today your account was credited with semiannual interest of $608. The account was established some time ago with a $9,000 initial deposit. Otherwise, there are no other deposits or withdrawals. The account earns a 7.5% APR, compounded semiannually. How long ago was the account established? ©CY16 .

11. Some time ago a $3,200 initial deposit opened an account. Today the monthly interest was credited to the account. Total lifetime interest now equals $1,321. The account earns a 6.5% APR, compounded monthly. How long ago was the account established? ©CY17 .

12. Suppliers X and Z are competing to sell your company supplies. The full price of supplies from supplier X is $2,200 and they offer these payment plans: 4.4% discount if you pay within 30 days, otherwise pay full price within 140 days. The full price with supplier Z is $2,060 and they offer these payment plans: 3.6% discount if you pay within 30 days, otherwise pay full price within 115 days. Your company financing rate is 12.8% compounded daily. Find the supplier and payment plan that represent the lowest present value of cost. ©CY21

13. You just signed a paper committing $40,000 to a 14-year investment that pays 7.70% annual interest compounded semiannually. Now you learn that an alternative investment has no minimum investment amount, and pays interest at 7.62% compounded monthly for 14-years. Maybe you would like to get back your money on the original investment, and switch to the alternative. You can cash in your original investment right now, but you would have to pay a penalty.
a. Is it worth switching if the penalty for terminating is $250? ©CY5a .
b. Find the penalty at which you are indifferent between the two alternatives. ©CY5b .

3.B. Annual percentage rate (APR) vs. effective annual rate (EAR)

The periodic interest rate for computing actual interest income or interest expense equals the annual percentage rate divided by the number of periods in a year. The annual percentage rate, abbreviated APR for convenience, gives rise to differing amounts of annual interest. This motivates introduction of the effective annual rate (EAR), known by many other names such as effective annual yield, etc.

DEFINITION 4.3 Effective Annual Rate
The effective annual rate is the amount of interest that accrues on one dollar in one year.

Compute the *EAR* from the *APR* with the following formula.

FORMULA 4.10 Relation between Effective Annual Rate and Annual Percentage Rate

$$\begin{pmatrix} effective \\ annual \\ rate \end{pmatrix} = \left(1 + \frac{APR}{m}\right)^m - 1 \ .$$

where *m* is the number of subperiods per year.

Table 4.5 shows the effective annual rate for different combinations of APR and compounding frequencies. The EAR for a 4 percent APR, for example, is 4.04 percent with semiannual compounding, and 4.08 percent with daily compounding. Thus, a deposit of $100 that earns a 4 percent APR accrues interest of $4.00 with annual compounding, $4.04 with semiannual compounding, and $4.08 with daily compounding.

Annual Percentage Rate	compounding frequency, m			
	2	4	12	365
4%	4.04%	4.06%	4.07%	4.08%
6%	6.09%	6.14%	6.17%	6.18%
8%	8.16%	8.24%	8.30%	8.33%
10%	10.25%	10.38%	10.47%	10.52%
12%	12.36%	12.55%	12.68%	12.75%
14%	14.49%	14.75%	14.93%	15.02%
16%	16.64%	16.99%	17.23%	17.35%
18%	18.81%	19.25%	19.56%	19.72%

TABLE 4.5 Effective annual rates for different APR and compounding frequencies

The table reveals a few general tendencies.

- The *EAR* always exceeds the *APR*. For this reason, institutions advertising loans tend to state the *APR*. The *APR* is smaller than the *EAR* and the institution wants to make the rate appear small. On the other hand, institutions advertising savings rates tend to state the bigger *EAR*.

- The EAR always increases with compounding frequency. This naturally occurs because interest-on-interest increases with compounding frequency.

- The gap between the *EAR* and *APR* is bigger for high interest rates. When a credit card charges an *APR* of 18 percent, for example, the effective annual rate is higher by 156 basis points!

Because the same *APR* on different offerings may provide varying amounts of interest, *APRs* generally cannot be compared to find the best one. Instead, valid comparisons involve the *EAR*.

EXAMPLE 11 Choose the lowest effective annual rate.

Which of the following loans is least expensive: an 8 3/4 percent loan compounded monthly, or an 8 5/8 percent loan compounded quarterly.

SOLUTION

Find the *EAR* for each deal. The smallest *EAR* is the cheapest rate. For the *APR* of 8 3/4 percent, *m* equals 12:

$$\begin{pmatrix} \textit{effective} \\ \textit{annual} \\ \textit{rate} \end{pmatrix} = \left(1 + \frac{.0875}{12}\right)^{12} - 1$$

$$= \quad 9.11\% \,.$$

For the *APR* of 8 5/8 percent, *m* equals 4:

$$\begin{pmatrix} \textit{effective} \\ \textit{annual} \\ \textit{rate} \end{pmatrix} = \left(1 + \frac{.08625}{4}\right)^{4} - 1$$

$$= \quad 8.91\% \,.$$

The quarterly loan has the lowest *EAR* and is the cheapest choice.

EXERCISES 4.3B

Concept quiz

1. For a given APR, do savers prefer increased or diminished compounding frequency? How about borrowers?

Numerical quickies

2. What is the effective annual rate (EAR) for a credit card whose annual percentage rate (APR) is 15.70% compounded monthly? ©CY1 .

3. Is it better to invest in a Certificate of Deposit that pays 9.34% compounded monthly, or 9.24% compounded daily? ©CY18 .

Numerical challenger

4. Exactly 32 months ago you put $3,250 in a cash management account. Otherwise, you have left the account alone. Today the account was credited with its monthly interest so that its balance now is $3,860. What is the account's APR? How about its EAR? ©CY19 .

4. Inflation and time value

Prices of goods change over time. The U.S. government collects information about prices and publishes the *consumer price index* ("CPI"). Think of the CPI as the price for an imaginary shopping basket full of food, clothing, transportation and housing services, and myriad other things that consumers purchase. The annual *inflation rate* measures the percentage price changes that occur during a year. The government estimates that consumer prices during 2015 increased less about one percent. This number is relatively small compared to other years during the past century. In 1990, for example, consumer prices increased over 6 percent, in 1979 they were up more than 13 percent! Annual inflation averages a little less than 4 percent per year.

A primary reason that companies and households save is to accumulate funds for making future purchases. Setting a savings target requires recognition that prices change over time. The rule of 72 suggests that with 4 percent inflation prices double in about 18 years. Thus, even with average inflation a long-term savings plan improperly accounting for inflation introduces serious errors.

Figure 4.1 illustrates that inflation and interest rates generally but not always move together. Chapter 11 explains that this positive correlation occurs because savers expecting high inflation tend to demand high interest. The actual interest rate observable in the marketplace is known as the *nominal interest rate*. The financial media often subtract the inflation rate from the nominal interest rate and report the result as the approximate *real interest rate*. The approximate real rate in figure 4.1 is shown as the dashed line. Throughout the previous 75 years the average corporate interest rate, the inflation rate, and the real rate average about 7.12 percent, 3.79 percent, and 3.33 percent, respectively. Several times during the 1970s the real rate was negative. Probably high inflation caught savers by surprise and they erred by not demanding high interest (on the chart it look like savers made up for the error in the early 1980s when inflation fell more than interest rates).

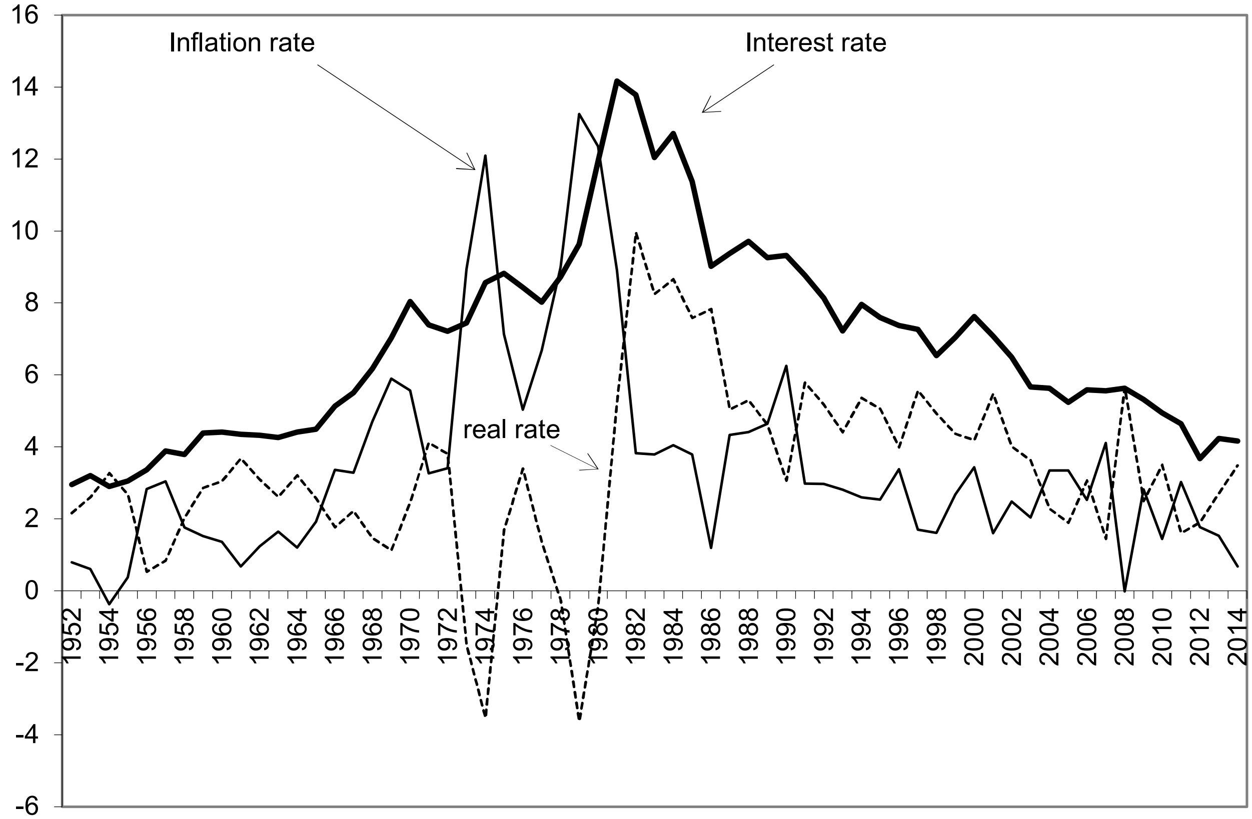


FIGURE 4.1 Interest and inflation rates

The Interest rate is the high grade corporate bond yield and the Inflation rate is the annual percentage change in the Consumer Price Index. The dashed line is the approximate real rate computed as Interest rate minus Inflation rate. Source data are from Federal Reserve Bank of St. Louis, http://research.stlouisfed.org/fred2/. Interest rate series Aaa is copyright by Moody's Investors Service; inflation series CPIAUCNS compiled by U.S. Department of Labor: Bureau of Labor Statistics.

Actual prices or cash flows at different times for the same good or service are called *current dollar prices* or *cash flows*. Adjust current dollar cash flows for inflation and obtain *constant dollar prices* or *cash flows*. Suppose, for example, that in 2014 an individual is comfortable earning income of $50,000. Say that over the subsequent 14 years the inflation rate is 3 percent per year. The current dollar income in 2029 providing the same standard of living equals $75,629 (= $50,000 × 1.03^{14}). The incomes of $50,000 in 2014 and $75,629 are current dollar cash flows – the numbers look different but after adjusting for inflation they represent equivalent purchasing power. Suppose that the person's actual current dollar income in 2029 (their paycheck!) is $70,000. The 2029 constant dollar income (measured in 2014 dollars) equals $46,278 (= $50,000 ÷ 1.03^{14}). Constant dollar income declines to $46,278 in 2029 from $50,000 in 2014, implying decline in purchasing power. This decline occurs even though current dollar income increases to $70,000 from $50,000.

Applying time value formulas in accordance with this rule properly accounts for effects of inflation.

RULE 4.3 How to integrate inflation into time value computations

Two procedures properly account for effects of inflation in the time value relation. They both lead to the same inferences. Use the one that is easiest to implement.

Rule 4.3A: Specify all present and future cash flows in current dollars and apply the nominal discount rate.

Rule 4.3B: Specify all present and future cash flows in constant dollars and apply the precise real discount rate, where

$$(1 + real\ discount\ rate) = \frac{1 + nominal\ discount\ rate}{1 + inflation\ rate}\ .$$

As an aside notice that the preceding formula precisely computes the *real rate* whereas a quick approximation formula is *real rate = nominal rate – inflation rate.*

First notice that the information requirements for applying rules 4.3 A and B differ. When one knows the discount rate and all future cash flows then apply 4.3A. When on the other hand one is more certain about constant dollar cash flows and the real rate then apply 4.3B. Notice second that computing the real rate as the nominal rate minus the inflation rate is not precisely correct for application in the time value framework; the former is a very close and easy-to-obtain approximation that's good for most but not all purposes.

The example below applies the rule to lump-sum time value formula 4.6.

EXAMPLE 12 Find the deposit for purchasing a house in an inflationary setting

There is a house that today costs $100,000 and, for peculiar reasons, in exactly 8 years you want to buy the house. You expect that because of inflation the house price will increase 7.9% per year. How much must you deposit today into an account that earns 11.4% per year such that the future purchase is perfectly financed?

SOLUTION

Applying rule 4.3A requires finding that in eight years the current dollar price of the house will be $183,726 (= $100,000 × 1.079^8). Discount the future current dollar cash flow of $183,726 with the nominal rate of 11.4 percent to find that the present value equals $77,462 (= $183,726 ÷ 1.114^8). A deposit today of $77,462 earning 11.4% for 8 years grows to become $183,726 and perfectly finances the house purchase.

Find the identical answer by applying rule 4.3B. Set the constant dollar future cash flow equal to an amount in today's dollars that purchases the target house. Constant dollar *FV* equals $100,000. Discount the constant dollar cash flow with the real rate. Given that the nominal rate is 11.4% and inflation is 7.9% then find the precise real rate equals 3.24% [1 + *real rate* = (1.114 / 1.079)]. As an aside note that the approximate real rate of 3.5% (= 11.4% – 7.9%) is 26 basis points greater than the precise real rate. Use formula 4.6 and discount the constant dollar future value of $100,000 with the real rate of 3.24% to find that the present value equals $77,462 (= $100,000 ÷ 1.0324^8). A deposit today of $77,462 earning a real rate of 3.24% for 8 years grows to become $100,000 of today's constant dollars and perfectly finances the house purchase.

The preceding analysis explains two procedures for arriving at the same correct answer. There is one often-used incorrect procedure – ignore inflation! A person observes that today's interest rate is 11.4% and that today's target house costs $100,000. It is seductive to set $100,000 as the target accumulation, to discount with 11.4% for 8 years, and to infer that today's deposit is $42,161 (= $100,000 ÷ 1.114^8). While today's deposit of $42,161 earning 11.4% for 8 years certainly grows to become $100,000 there will be insufficient funds for purchasing the house 8 years from now. The shortfall of $83,726 means missing the real target.

EXERCISES 4.4

Concept quiz

1. When inflation is high the interest rate tends to be high. Explain whether this tendency accentuates or dampens valuation errors that occur by ignoring inflation in the time value relations.

Numerical quickies

2. There is a house that today costs $168,000 and, for peculiar reasons, in exactly three years you want to buy the house. You expect that because of inflation the house price will increase 5.5% per year. The interest rate is 9.4% per year (compounded annually).
2a. Find the precise real rate of interest that the account earns. ©LS25a
2b. Find the price of the house at the time of purchase. ©LS25b
2c. How much must you deposit today into an account such that the future purchase is perfectly financed? ©LS25c

Numerical challenger

3. There is a house that today costs $148,000 and, for peculiar reasons, in exactly three years you want to buy the house. You expect that because of inflation the house price will increase 1.7% per year. The amount that you intend to deposit today is $113,000 (compounded annually). This deposit should grow so that it perfectly finances the purchase.
3a. Find the annual nominal interest rate that the account earns. ©LS26a
3b. Find the precise real annual interest rate that the account earns. ©LS26b

5. The general formula for time value

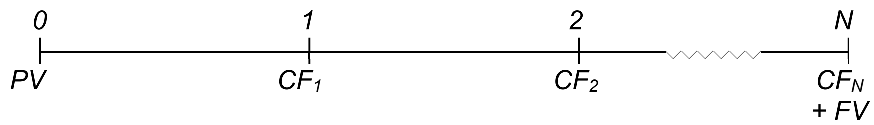
Financial situations often involve a series of cash flows. There may be one cash flow at the beginning, one at the end, and one or more in the middle. Perhaps, for example, you withdraw or deposit money several times into a savings account. These situations fit within the general time value formula:

FORMULA 4.11 The general time value relation connecting mixed cash flow streams

$$PV = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} + \frac{FV}{(1+r)^N} \ .$$

Variable definitions are the same as before, with the addition of CF_t as the cash flow at time *t*. The beginning wealth is *PV* and the ending wealth *N* periods later is *FV*. For some scenarios some variables might equal zero. That's okay. The periodic interest rate is r, where r equals the annual percentage rate i divided by m, the number of compounding periods per year. The first expression represents the summation of N different cash flows. Each cash flow, however, is divided by one plus the periodic interest rate raised to the t^{th} power. In the general time value formula each particular cash flow may have its own unique value; they don't all have to be the same number.

Properly accounting for the timing of cash flows is crucial. A time line easily illustrates the cash flow timing.



The first cash flow, CF_1, occurs exactly one period after *PV*. When finding a present value, therefore, the formula implicitly takes away from CF_1 exactly one period of interest. The second cash flow, CF_2, arrives at time 2. Dividing CF_2 by one plus the periodic rate squared finds its present value. The last cash flow, CF_N, occurs exactly N periods after *PV*. When finding the present value of CF_N and *FV*, too, since they happen at the same time, exactly N periods of compound interest are removed.

Cash flows occurring at the same time, like CF_N and *FV,* are directly comparable and additive. Cash flows occurring at different times are not directly comparable and adding them introduces error of varying severity. The discounting process properly accounts for time value differences so that cash flows from different times are comparable. For the most accurate analysis involving big bucks, especially, it is a real necessity to discount cash flows from different time periods before comparing them.

Quite a few different variables appear in formula 4.11. The compounding frequency m, however, almost always is specified. Otherwise, variables that might represent the unknown answer include *FV*, *PV*, *N*, or any one of the CF_t . To find any single unknown variable requires assigning numerical values to all other variables.

5.B. *Ending wealth, FV, as the unknown*

Suppose you make a couple of deposits and want to know the ending balance. For this scenario, FV is the unknown variable. Rearrange the general time value formula to isolate FV on the left-hand-side:

FORMULA 4.12 The general time value formula, solve for *FV*

$$FV = PV(1+r)^N - \sum_{t=1}^{N} CF_t (1+r)^{N-t} \quad .$$

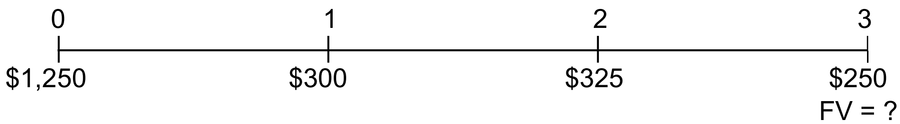
Substitute values for all right-hand-side variables and find the answer for *FV*.

EXAMPLE 13 Find the ending wealth for a mixed deposit stream

Your account earns interest at a 6.25 percent APR compounded quarterly. Today the bank credits the account with its quarterly interest and the end-of-day account balance becomes $1,250. You plan to deposit $300 one quarter from today, $325 two quarters from today, and $250 three quarters from today. What is the account's balance immediately after the last deposit?

SOLUTION

Beginning wealth, *PV*, is $1,250. The unknown variable is the account balance exactly three quarters from now. The three cash flows include CF_1 at $300, CF_2 at $325, and CF_3 at $250. The timing of cash flows is as follows:



The compounding frequency, *m*, is 4 and the annual percentage rate is 6.25 percent. The periodic interest rate *r* therefore equals 0.0625 ÷ 4. Substitution into formula 4.12 shows:

$$FV = \$1,250\left(1+\frac{.0625}{4}\right)^3 + \$300\left(1+\frac{.0625}{4}\right)^2 + \$325\left(1+\frac{.0625}{4}\right)^1 + \$250$$

$$= \$1,250\,(1.015625)^3 + \$300\,(1.015625)^2 + \$325\,(1.015625)^1 + \$250$$

$$= \$2,199.04$$

The account balance rises to $2,199.04 from $1,250 today, an increase of $949.04.

CALCULATOR CLUE 4.12 Knowing both the algebraic and time value approaches to solving this problem on the calculator are a good idea. Both approaches are discussed.

Time value computations are very sensitive to interest rate rounding. A rate of 6.25 percent generates very different answers than 6.30 percent. Typical credit card statements, for example, list the periodic interest rate to five or eight places to the right of the decimal point. It's real money so the bank pays attention to rounding errors.

Reduce rounding errors by storing values in the calculator's memories. The BAII Plus© memory stores eight significant digits, regardless of how many the display shows. That is, perhaps the display shows 0.0625 ÷ 4 equal 0.0156, but the actual value in memory is 0.01562500. The answer you get for your time value problems differs if you type in 0.0156 instead of 0.015625. The rounding error gets worse as N gets big.

Effective and efficient calculator usage stores values in memory instead of re-typing. The BAII Plus© has 10 memories, and their addresses equal the numbers 1, 2,…, 9, 0. Store the value on the display (with all eight of its significant digits) in memory by typing STO followed by the address of the desired memory. Recall the value in the memory at any time simply by typing RCL followed by the memory address.

For the algebraic solution to the preceding problem, compute and store in memory the value of 1+r, that is, one plus the periodic rate. Type

.0625 ÷ 4 + 1 = STO 1

Compute the ending wealth by typing

1250 X RCL 1 y^x 3 + 300 X RCL 1 x^2 + 325 X RCL 1 + 250 = .

The display shows $2,199.04 .

To use time value functions for finding the ending wealth of mixed cash flow streams on the BAII Plus© involves two stages. The first stage enters the cash flow stream and finds its present value. To enter the cash flow stream, type CF . The calculator allows you to enter a column of numbers. To clear any unwanted numbers that already may be stored in this column, type 2nd CE/C . Enter this problem's cash flow stream as follows:

1250 ENTER ↓ 300 ENTER ↓ ↓ 325 ENTER ↓ ↓ 250 ENTER

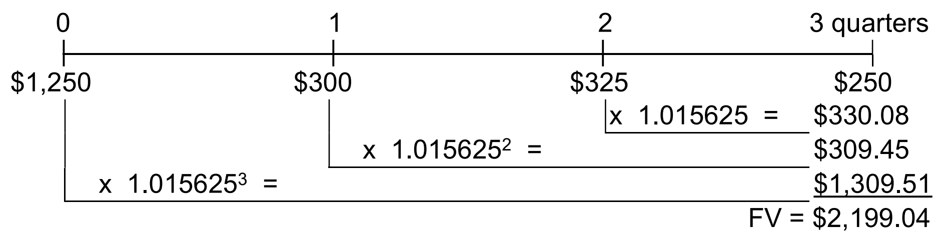
Now find the present value of the stream given the rate is 6.25 percent compounded

quarterly. Type:
NPV 6.25 ÷ 4 = ENTER ↓ CPT

The display shows $2,099. A deposit of $2,099 at time 0 compounds into exactly the same ending wealth as the problem's actual cash flow stream. That is, the ending wealth at time 3 is the same regardless of whether one deposits (1) $2,099 at time zero, or (2) $1,250 at time zero, plus $300 at time one, plus $325 at time two, plus $250 at time three. With $2,099 on the display, conclude the problem by typing:
PV 2nd I/Y 4 ENTER 2nd CPT 6.25 I/Y 3 N CPT FV

The answer of $2,199.04 appears on the display.

This graphic shows the source of the accumulation.



Total contributed principal is the sum of deposits, and equals $875. According to equation 4.8, therefore, the account earns total market interest during the next three quarters of $74.04. The graphic clearly shows the source of interest. The beginning balance of $1,250 grows to $1,309.51, implying that this term earns interest of $59.51. The time 1 deposit, $300, grows to $309.45, implying interest earnings of $9.45. Finally, the $325 deposit grows to $330.08 implying interest earnings of $5.08. The last deposit earns no interest because the account balance is checked immediately after making the deposit.

EXERCISES 4.5A

Concept quiz

1. Generally speaking, should you make a small deposit today if it means that next period's deposit will be smaller? Or is it better to make no deposit today so that next period's deposit can be larger?

Numerical quickies

2. You invest $3,200 today. One year from today you invest $4,500. Finally, two years from today you invest $5,000. Your account earns 12.5% annual interest (compounded annually). How much is in the account immediately after the last deposit? How much is in the account three years from today? ©MC1a . ©MC1b .

3. You invest $900 today. One month from today you invest $500. Finally, five months from today you invest $750. Your account earns a 1.5% monthly periodic rate of return. How much is in the account immediately after the last deposit? ©MC2a .

Numerical challenger

4. You deposit $1,000 today and exactly 9 months from today you deposit $1,200. Your account earns annual interest of 7.5% compounded monthly. How much is in the

account one year from today? ©MC2b .

5.B. *Beginning wealth, PV, as the unknown*

To solve for PV as the unknown variable, supply numerical values for all right-hand-side variables in the general time value formula 4.11:

$$PV = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} + \frac{FV}{(1+r)^N} \ .$$

EXAMPLE 14 Find the present value of a mixed cash flow stream

An investment promises to return cash flow of \$1,200 exactly six months from now, \$3,000 exactly 18 months from now, and \$4,500 exactly 24 months from today. An investor decides that the investment represents a fair deal if it can be bought at a price that provides a 15 percent annual rate of return, compounded semiannually. What is the fair price for the investment?

SOLUTION

The cash flows occur semiannually, and the time line shows:

0	1	2	3	4 semiannum
PV = ?	\$1,200	0	\$3,000	\$4,500

The unknown variable is *PV*. The right-hand side *r* equals 0.15 ÷ 2, or 0.075. The CF_t take values as shown on the time line (*N* is 4). *FV* equals zero for this scenario. *FV* takes on a non-zero value if there were some additional ending wealth, for example if revenue in addition to the \$4,500 cash flow were generated. Substituting numbers into the present value formula shows:

$$PV = \frac{\$1,200}{\left(1+\frac{0.15}{2}\right)^1} + \frac{\$0}{\left(1+\frac{0.15}{2}\right)^2} + \frac{\$3,000}{\left(1+\frac{0.15}{2}\right)^3} + \frac{\$4,500}{\left(1+\frac{0.15}{2}\right)^4}$$

$$= \frac{\$1,200}{1.075^1} + \frac{\$0}{1.075^2} + \frac{\$3,000}{1.075^3} + \frac{\$4,500}{1.075^4}$$

$$= \$6,901$$

CALCULATOR CLUE 4.13 For the algebraic solution to the preceding problem, compute and store in memory the value of one plus the periodic rate. Type
.15 ÷ 2 + 1 = STO 1

Now compute the present value by typing
1200 ÷ RCL 1 + 3000 ÷ RCL 1 y^x 3 + 4500 ÷ RCL 1 y^x 4 = .
The display shows \$6,901.

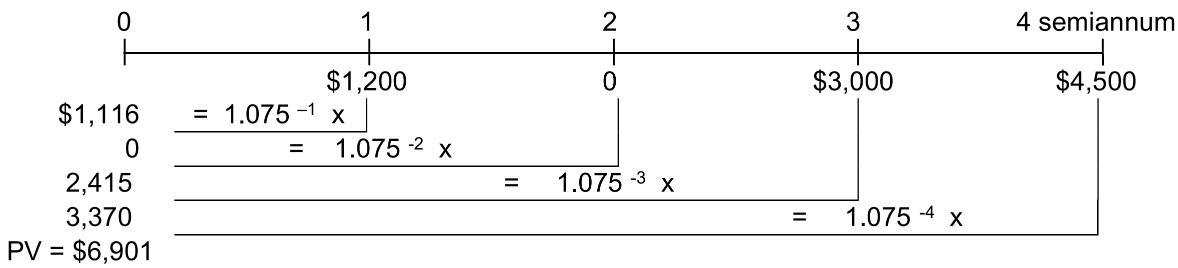
To use time value functions on the BAII Plus© type CF and clear unwanted

numbers by typing 2nd CE/C . Now enter this problem's cash flow stream as follows:
↓ 1200 ENTER ↓ ↓ 0 ENTER ↓ ↓ 3000 ENTER ↓ ↓ 4500 ENTER
Now find the present value of the stream given the rate is 15 percent compounded semiannually. Type:
NPV 15 ÷ 2 = ENTER ↓ CPT

The display shows $6,901.

Whenever possible one should check whether the calculator solution seems reasonable. In this problem, the answer suggests that an investment today of $6,901 earns a fifteen percent rate of return if followed by returns of $1,200 , $0 , $3,000 , and $4,500. The sum of returns equals $8,700. The cost of $6,901 seems "reasonably" less than $8,700 to support a fifteen percent rate of return.

The solution is correct, and the graphic below allows insight about discounting future wealth.



The above procedure divides each cash flow by one plus the periodic rate raised to some exponent. The division by a number larger than one yields an answer smaller than the cash flow. Effectively, the division removes market interest. The present value of a cash flow is the amount deposited today that grows and perfectly finances the cash flow.

For this example the investment returns $1,200 in six months, that is, in one semiannual period. The graphic shows that an $1,116 deposit today earning 7.5 percent semiannual interest grows to $1,200 after six months. The investor with a fifteen percent target annual rate of return, therefore, should pay $1,116 for this first return. After 18 months an additional cash flow of $3,000 occurs. The present value of this cash flow is $2,415. A deposit of $2,415 in a savings account that earns a 15 percent APR, compounded semiannually, accumulates market interest after 18 months equal to $585. The deposit, therefore, perfectly finances the withdrawal of $3,000. Finally, financing the final cash flow of $4,500 four semiannual periods from now requires a deposit today of $3,370. The $6,901 sum of deposits perfectly finances the planned withdrawals.

EXERCISES 4.5B

Numerical quickies

1. An investment promises to return $7,400 in one year and $9,000 in two years. What is the investment's cost today if it promises a 15 percent annual rate of return? ©MC4 .

2. You forecast bills of $900 in one month, $500 in six months, and $750 in twelve months. You wish to make a deposit today that perfectly finances the bills. Your account earns a 7.5% annual return, compounded monthly. How much is today's deposit? ©MC3 .

Numerical challengers

3. Here are two future expenses that you want to save for today: $5,300 payable in 4 years, and $7,000 payable in 9 years. You make an investment today that perfectly finances the future expenses if the investment earns a target 10.8% average annual rate of return (compounded annually).
3a. How much is your investment? ©MC5a .
3b. When it is time to pay the first expense, you make the expected withdrawal from the account. After the withdrawal what is the account balance? ©MC5b .
3c. The investment indeed grows sufficiently to finance your first expense. Unfortunately, for the entire investment horizon your actual annual rate of return falls short of the target by 90 basis points per year. When it is time to pay the second expense, how much money do you lack? ©MC5c .

4. An investment promises two cash flows: $1,000 in exactly 9 months and $2,000 in 18 months. You purchase the investment at a price that promises a target annual rate of return of 14% compounded monthly. After the investment horizon concludes, however, the actual rate of return differs from the target because the second cash flow is smaller than promised (the first cash flow is on-target). If the actual annual rate of return is 12.5% compounded monthly, how much was the second cash flow? ©MC6a .

5. Here are two future expenses that you want to save for today: $4,900 payable in 6 years, and $7,300 payable in 9 years. You make an investment today that perfectly finances the future expenses if the investment earns a target 8.0% average annual rate of return (compounded quarterly). The investment indeed grows sufficiently to finance your first expense. Unfortunately, for the entire investment horizon your actual annual rate of return falls short of the target by 240 basis points per year. When it is time to pay the second expense, how much money do you lack? ©MC13 .

5.D. Finding the rate of return, r, as the unknown variable for improper cash flow streams

The rate of return is the discount rate that equates the present value of cash flows to one other. The variable *r* in the general time value formula cannot be isolated alone on the left-hand-side because it appears in too many places with too many exponents except for case of perpetuities within the general formula. Financial calculators, however, use smart chips that easily find numerical values for *r*.

EXAMPLE 15 Find the target rate of return

Today you deposit $4,000 into an account with your broker. Your intention is to withdraw $2,000 in one year, $1,500 in two years, $1,000 in three years, and $500 in four years. Annual interest is credited to your account immediately before each withdrawal, and after the last withdrawal the account balance is zero. What annual percentage rate must the account earn?

SOLUTION

Sketch the cash flows onto the time line:

0	1	2	3	4 years
$4,000	$–2,000	$–1,500	$–1,000	$–500

The sign of the time 0 cash flow is opposite the sign of cash flows 1 through 4 because the initial cash flow goes in the opposite direction than the others. For example, at time 0 money flows into the account. At all other times, money flows out of the account.

Substitute the numbers above into general time value formula 4.11:

$$\$4{,}000 = \frac{\$2{,}000}{(1+r)^1} + \frac{\$1{,}500}{(1+r)^2} + \frac{\$1{,}000}{(1+r)^3} + \frac{\$500}{(1+r)^4}$$

Without a financial calculator the only way to solve the above equation is by trial-and-error with different values for *r*. The financial calculator, however, easily finds that *r* equals 12.16 percent.

CALCULATOR CLUE 4.14 You must use the advanced calculator functions to solve this problem because it does not have an algebraic solution. On the BAII Plus© type CF and clear unwanted numbers by typing 2nd CE/C . Now enter this problem's cash flow stream as follows:

4000 ENTER ↓ 2000 +/– ENTER ↓ ↓ 1500 +/– ENTER ↓ ↓ 1000 +/– ENTER ↓ ↓ 500 +/– ENTER

Now find the periodic rate of return that satisfies this time value formula. Type:

IRR CPT

The display shows 12.16 percent.

Table 4.6 shows more detail about the scenario. The account begins at the end of period 0 with a deposit of $4,000. During the subsequent year the account accrues interest at a rate of 12.16 percent, the solution found above. Multiplying the rate times the beginning of period balance shows the account earns interest during the first year of $486.54. The account balance rises to reflect the interest, and immediately before the first withdrawal the balance is $4,486.54. The withdrawal of $2,000 occurs at the end of the first year, thereby bringing the balance down to $2,486.54.

	t = 0	t = 1	t = 2	t = 3	t = 4
beginning of period balance	$0.00	$4,000.00	$2,486.54	$1,288.99	$445.78
periodic interest earned	$0.00	$486.54	$302.45	$156.79	$54.22
new balance before cash flow	$0.00	$4,486.54	$2,788.89	$1,445.78	$500.00
end of period cash flow	$4,000.00	$–2,000.00	$–1,500.00	$–1,000.00	$–500.00
end of period balance	$4,000.00	$2,486.54	$1,288.99	$445.78	$0.00

Table 4.6 Common components in the general time value relationship

The process repeats each year. Interest earnings equals 12.16 percent times the beginning of year balance. The interest is credited to the account and immediately the subsequent withdrawal occurs. Notice during the fourth year the beginning of year balance equals $445.78. The annual interest of $54.22 brings the account balance up to $500, exactly the amount of the last withdrawal. After the withdrawal, the account balance is zero.

The rate of return found by the general time value equation is the only meaningful rate at which the beginning wealth earns interest that perfectly finances the planned withdrawals. If the rate exceeds 12.16 percent then after the final withdrawal money still would remain in the account. Conversely, the last deposit would be underfunded with a rate smaller than 12.16 percent.

EXAMPLE 16 Find present value and the subsequent rate of return

An adventurous capitalist finds an investment that offers returns of \$2,400 in three months, \$3,500 in six months, and \$8,200 in one year. The adventurist makes an offer to purchase the investment. At that price the annual rate of return equals the target 14 percent, compounded quarterly. The seller rejects the offer price. Instead, the seller counteroffers a price that is \$400 greater than the offer. If the adventurous capitalist purchases the investment at the counteroffer price, what is the rate of return?

SOLUTION

The time line describes the scenario:

0	1	2	3	4 months
PV = ?	\$2,400	\$3,500	0	\$8,200

The immediate computation finds the offer price. Use the general time value formula to remove a 14 percent rate of return (compounded quarterly) from the cash flows:

$$PV = \frac{\$2,400}{\left(1+\frac{0.14}{4}\right)^1} + \frac{\$3,500}{\left(1+\frac{0.14}{4}\right)^2} + \frac{\$0}{\left(1+\frac{0.14}{4}\right)^3} + \frac{\$8,200}{\left(1+\frac{0.14}{4}\right)^4}$$

$$= \frac{\$2,400}{1.035^1} + \frac{\$3,500}{1.035^2} + \frac{\$0}{1.035^3} + \frac{\$8,200}{1.035^4}$$

$$= \$12,732$$

The offer to purchase the investment is \$12,732. The seller wants \$400 more; that is, the counteroffer price is \$13,132. In the above equation, set *PV* equal to \$13,132 and let *r* adjust to satisfy the time value equation:

$$\$13,132 = \frac{\$2,400}{(1+r)^1} + \frac{\$3,500}{(1+r)^2} + \frac{\$0}{(1+r)^3} + \frac{\$8,200}{(1+r)^4}$$

Use the financial calculator to find that the periodic rate, r, equals 2.42 percent per quarter. The annual percentage rate therefore equals 9.68 percent (9.68 = 4 x 2.42).

CALCULATOR CLUE 4.15 You must use the advanced calculator functions to solve this problem because it does not have an algebraic solution. On the BAII Plus© type CF and clear unwanted numbers by typing 2nd CE/C . Now enter this problem's cash flow stream as follows:

↓ 2400 ENTER ↓ ↓ 3500 ENTER ↓ ↓ 0 ENTER ↓ ↓ 8200 ENTER

Now find the present value of the stream given the rate is 14 percent compounded quarterly. Type:

NPV 14 ÷ 4 = ENTER ↓ CPT

The display shows \$12,732. You can't buy at that price, however, and must pay \$400 more. With \$12,732 still on the display, type:

+ 400 = +/– STO 1

Now plug that higher price into the cash flow stream and compute the periodic rate of

return, which must be multiplied by 4 to obtain the annual return:
CF RCL 1 ENTER IRR CPT X 4 = .

The display shows 9.68 percent.

At a price of $12,732 the cash flow stream provides a rate of return equal to the 14 percent target. At the higher a price of $13,132, however, the rate of return is 9.68 percent. The higher price reduces the rate of return to the adventurous capitalist by 432 basis points per year (432 basis points = 0.1400 – 0.0968).

EXERCISES 4.5C

Numerical quickies

1. An investment that costs $12,000 promises to return $7,400 in one year and $9,000 in two years. What is the average annual rate of return? ©MC7 .

2. A potential investment promises returns of $900 in one month, $1,000 in two months, and $3,200 in three months. You make an offer to purchase the investment for $4,000..
2a. What is the annual percentage rate of return, compounded monthly, if you buy at the offer price and receive the promised returns? ©MC8a
2b. The seller rejects your offer price, and counteroffers at $4,500. If you buy at the higher counteroffer price and receive the promised returns, by how many basis points does the annual percentage rate of return decline? ©MC8b .

Numerical challengers

3. You forecast expenses of $5,800 payable in 4 months and $3,800 payable in 8 months. You make a deposit today of $8,350 that should perfectly finance the future expenditures. What annual percentage rate of return (compounded monthly) does the account earn? ©MC9 .

4. You forecast bills of $8,200 in one month, $5,100 in six months, and $4,750 in twelve months. You make a deposit today of $17,000 that should perfectly finance the bills. What annual percentage rate, compounded monthly, does the account earn? ©MC10 .

5. Here are two future expenses that you want to save for today: $2,700 payable in 4 years, and $6,100 payable in 9 years. You make an investment today that would perfectly finance the future expenses if the investment were to earn a target 7.5% average annual rate of return (compounded annually). Instead, however, the investment earns so much that after the last expense is made, your account still has $1,500 remaining. What was the actual annual percentage rate? ©MC11 .

ANSWERS TO CHAPTER 4 EXERCISES

EXERCISES 4.1

1. The percentage increase during the first period is larger than the percentage decline during the second. For example, if the stock price doubles to \$20 from \$10 during the first period, it is up 100%. But when it falls during the second period from \$20 to \$10 it is down only 50%. The arithmetic average will be positive, and larger than the geometric average rate of return of zero.
2. The statement is true. A column of, say, five prices makes a unique column of four rates of return. A column of four rates of return, however, can make an infinite number of columns of five prices. In this regard, prices carry more information than rates of return.
3. The geometric ROR is $(60 \div 40)^{0.5} - 1$, or 22.5% . The arithmetic ROR is {(19-40)/40 + (60-19)/19) ÷ 2, or 81.6%.
4. The answer is \$20(1–.15)(1+.65) = \$28.05
5. The cumulative ROR is (12.4 / 10.0) - 1, which is 24%. The average annual geometric ROR is $(12.4 / 10.0)^{1/3} - 1$, which is 7.43%.
6. The geometric average rate of return obviously is negative because you lost money. Hence, the broker must be correctly quoting the arithmetic average. The sum of two periodic rates of return, divided by two, equals 20 percent. Let P_1 represent the portfolio value one year ago. Then using the definition for the arithmetic ROR shows

$\{ [(P_1 - 10{,}000)/10{,}000] + [(9{,}150 - P_1)/P_1] \} \div 2 = 0.20$

Rearrange the above to obtain the following quadratic equation

$P_1^2 - 24{,}000 P_1 + 91{,}500{,}000 = 0$

The two roots are P_1 = \$4,754 or P_1 = \$19,245 . Both work and are legitimate answers.

EXERCISES 4.2A

1. Play with your calculator. Begin with PV = \$1,000 and N = 10 years and r = 0.10. For this baseline case, FV equals \$2,594. Double PV and FV doubles to \$5,187 (a 100% increase). Double N to 20 and FV rises to \$6,727 (a 159% increase). Double r to 20% and FV rises to \$6,192 (a 139% increase). For typical settings, doubling the investment horizon has the biggest impact. [N.B. Doubling N has a larger impact as long as the initial $N > \log(1+ 2r) \div \log(1 + r)$].
2. Find FV as $24(1.0625)^{375}$, or 179,293,000,000 . That is, \$179 billion.
3. Find FV as $\$1{,}000(1.055)^5$, or \$1,307. Total interest is \$307.
4. Interest-on-principal is \$5,000(0.12) per year, or \$600. Over 18 years the interest-on-principal sums to 18(\$600), or \$10,800. The total accumulation after 18 years is $\$5{,}000(1.12)^{18}$, or \$38,449. Of the total accumulation, \$5,000 is initial principal and \$10,800 is interest-on-principal. The rest, \$22,649, is interest-on-interest.
5. The balance at the beginning of the current year is the balance on which this year's interest is based. The balance after 3 years is $\$3{,}500(1.0875)^3$, or \$4,501. This year's interest is \$4,501(0.0875), or \$394.
6. Because \$2,500 = (0.0675)(beginning-of-year balance), the beginning-of-year balance is \$2,500/(0.0675), or \$37,037. The end of year balance therefore is \$37,037 plus \$2,500, which is \$39,537.
7. The sum of monthly returns divided by 12 yields the arithmetic average monthly rate of return of 43.75 percent. First use this number as r in the time value relation. The beginning wealth, PV, is \$12,500. The ending wealth after twelve months with r at 0.4375 is $\$12{,}500(1.4375)^{12}$, or \$973,205. Now find the geometric average monthly rate of return as the product of the twelve "one plus monthly ROR" raised to the one-twelfth, minus one, which is –1.41 percent. Use this as r and find FV is $\$12{,}500(1-0.0141)^{12}$, or \$10,542. There is a big-time difference between \$10,542 and \$973,205! The answer based on the geometric average is correct. Shareholders in Saf T Lok had quite a ride in 2525, and they ended up losing money.

EXERCISES 4.2B

1. When r > 0 then always PV < FV. Yet if rate of return r is negative then beginning wealth PV is less than ending wealth FV. Investments realize negative rates of return all too often.
2. Find that PV equals $19,500 ÷ (1.16)^4$, which is $10,770.
3. Find PV as $3,500 ÷ (1.0125)^8$, which is $3,169.
4. Find the amount of the deposit as $6,200 ÷ (1.0125)^{14}$, which is $5,210. That deposit compounds at 1.05 percent per month for 14 months. The ending balance is $5,210(1.0105)^{14}$, which is $6,031. You lack $169.
5. Find the beginning of this year's balance as $2,200 ÷ 0.0625, which is $35,200. To obtain an accumulation of $35,200 over 17 years at 6.25% per year means the initial deposit was $35,200 ÷ (1.0625)^{17}$, which is $12,559.

EXERCISES 4.2C

1. Find the N that satisfies the equation $6,000 = $2,400 (1.14)^N$, which is N equals 7 years.
2. Find the N that satisfies the equation $100,000 = $2,660 (1.12)^N$, which is N equals 32 years.
3. The beginning of year balance is $1,100 ÷ 0.0725, which is $15,172. Find the N that satisfies the equation $15,172 = $7,000 (1.0725)^N$, which is N equals 11 years. Because it is eleven years from the time of the deposit until one year ago, the initial deposit occurred 12 years ago.
4. The account balance today is $14,095. Find the N that satisfies the following equality: $7,000(1.0725)^N$ = $14,095 or N is 10 years.
5. The key factor is the length of time until the pay-off occurs. As long as the loan's principal plus accrued interest is less than $29,800 then the deal is profitable. Find the N that satisfies the equation $29,800 = $15,300 (1.0870)^N$, which is N equals 8. As long as the payoff occurs in less than 8 years the deal is profitable.

EXERCISES 4.2D

1. Find the r that satisfies $1,350(1 + r)^5$ = $2,750, which is r equals 15.29%.
2. Find the r that satisfies $900(1 + r)^8$ = $2,700, which is r equals 14.72%.
3. Find the r that satisfies $1(1 + r)^{24}$ = $2, which is r equals 2.93%.
4. The beginning of year balance was $8,200. For $8,200 to earn interest of $800 implies an annual interest rate of 9.76% (that is, $800/$8,200).
5. Find that the deposit equals $5,300 ÷ (1.0125)^{18}$, which is $4,238. Then find the rate r that satisfies $4,328(1 + r)^{18}$ = $5,080, or r equals 1.01%.
6. The decision depends on A's price. Find that the rate of return on B satisfies $6,800(1 + r)^7$ = $10,000 or r is 5.66%. For A to return 5.66% suggests it's price is $8,000 ÷ 1.0566^{11}$, or $4,364. The decision rule is that if A costs less than $4,364 then choose A for its higher rate of return. If A costs more than $4,364 then choose B for its higher rate of return.

EXERCISES 4.2E

1. At 12% doubling occurs every 6 years. A $10,000 deposit grows to $20,000 after 6 years, $40,000 after 12 years, etc. An investor at age 55 has a 15 year savings horizon. The will finance about two and a half doubling periods by the time age 70 is reached. So the ending wealth is approximately halfway between $40,000 and $80,000 or $60,000. An investor at age 25 has a 45 year investment horizon, encompassing about 7½ doubling periods at 12 percent. The ending wealth for this investor is about $1.7 million. Complete the other 4 table cells with similar logic. Obtain a table that approximately appears as below.

age at time of investment	8% ROR	12% ROR	Horizon length to age 70
25 years old	$320,000	$1,700,000	45 years
40 years old	$80,000	$320,000	30 years
55 years old	$35,000	$60,000	15 years
	9 years	6 years	Length of doubling period

Note that an exact table is easily found with a financial calculator. Nonetheless, simple in-the-head approximations perhaps forcefully convey a key insight. Investment horizon and rate of return are both extremely important. The market, not the investor, largely determines rates of return. The investor controls the length of the investment horizon. Save young, make wise investments, and wealth accumulates.

EXERCISES 4.3A

1. Find FV as $\$1{,}200(1 + .055/12)^{60}$, or \$1,579. Total interest is \$379.
2. Find PV as $\$15{,}000 \div (1 + .0625/2)^{20}$, or \$8,106.
3. Find PV as $\$4{,}200 \div (1 + .125/12)^{48}$, or \$2,554.
4. The target accumulation is \$5,650. Find N from $\$3{,}100\ (1 + .09/4)^{N} = \$5{,}650$, or N equals 27 quarters (6 3/4 years).
5. Find the amount you borrow as $\$6{,}200 \div (1 + .0825/12)^{14}$, or \$5,633. This loan of \$5,633 incurs interest such that the future sum is $\$5{,}633\ (1 + .1075/12)^{14}$, or \$6,382. You lack \$182.
6. Find the beginning of this quarter's balance as $\$350 \div (0.055 / 4)$, which is \$25,455. Thus, the end of quarter balance is \$25,805 (= \$350 + \$25,455). To obtain an accumulation of \$25,805 over 14 years at a 5.5% annual rate compounded quarterly means the initial deposit was $\$25{,}805 \div (1+ .055/4)^{4(14)}$, which is \$12,011.
7. The deposit equals $\$5{,}300 \div (1+ .0725/12)^{18}$, or \$4,755. Next find the monthly periodic rate from $\$4{,}755\ (1+ r)^{18} = \$5{,}080$, or r is 0.37%. Multiply the monthly rate by twelve to get the annual percentage rate of 4.41%
8. The balance at the beginning of the current month is the balance on which this month's interest is based. The balance after 41 months is $\$1{,}200(1+ .0775/12)^{41}$, or \$1,562. This month's interest is \$1,562 x 0.0775/12, or \$10.09.
9. This quarter's interest of \$620 equals (0.0525/4)x(B.O.P. Balance), so the beginning of period balance is \$47,238. Add this quarter's interest to the beginning balance to get the end of period balance of \$47,858.
10. This period's interest of \$608 equals (0.075/2)x(B.O.P. Balance), so the beginning of period balance is \$16,220. The end of period current balance therefore equals \$16,828. Find N from the following: $\$16{,}828 = \$9{,}000(1+ .075/2)^{N}$, or N is 17 semiannum (8½ years).
11. The account balance equals the initial principal plus total lifetime interest, or \$4,521. Find N from the following: $\$4{,}521 = \$3{,}200(1+ .065/12)^{N}$, or N is 64 months (5 years, 4 months).
12. Find the present value of costs. For example, with supplier X at a 4.4% discount payable on day 30 the present value of cost is $\$2{,}081 = \$2{,}200(1- .044)(1+ .128/365)^{-30}$. For supplier X at full price payable on day 140 the present value of cost is $\$2{,}095 = \$2{,}200(1+ .128/365)^{-140}$. Similar computations for supplier Z shows that the present values of the discounted and full prices equal \$1,965 and \$1,979. The overall lowest cost is purchase at the discounted price from supplier Z.

13a. First find the total accumulation if you stick with the original alternative. Find that $FV = \$40{,}000 \times (1 + 0.0770/2)^{14 \times 2}$; = \$115,197. Second find the total accumulation for the alternative investment if you immediately cash in the original deal, pay the \$250 penalty, and reinvest \$39,750 (= \$40,000 - \$250). Find that $FV = \$39{,}750 \times (1 + 0.0762/12)^{14 \times 12}$; FV = \$115,127. The original deal accumulates more so don't switch.

13b. Find the initial deposit for the new alternative that provides the same total accumulation as the original deal. Find that $115,197 = PV x (1 + 0.0762/12)^{14 \times 12}$; PV = $39,774. That is, the biggest penalty you can afford to pay for switching is $226 (= $40,000 - $39,774). A penalty smaller than $226 makes switching worthwhile and vice versa.

EXERCISES 4.3B

1. Savers want to receive as much interest as often as possible. Savers benefit from increased compounding frequency. Borrowers want to pay as little interest as infrequently as possible. Borrowers benefit from diminished compounding frequency.
2. The EAR equals $(1 + 0.1570/12)^{12} - 1$, or 16.88%.
3. The best investment has the highest EAR. For the monthly deal the EAR is $(1 + 0.0934/12)^{12} - 1$, or 9.75%. For the daily deal the EAR is $(1 + 0.0924/365)^{365} - 1$, or 9.68%. The monthly deal is better.
4. Find the r that satisfies $\$3,250\ (1 + r)^{32} = \$3,860$, which is r equals 0.54% per month. The APR is twelve times that, or 6.47%. The EAR is $(1 + 0.0054)^{12} - 1$, or 6.66%.

EXERCISES 4.4

1. The positive correlation between inflation and interest accentuates valuation errors that result from ignoring inflation. To see this accentuation examine two scenarios for accumulating $100 of real income over 10 years during which time the approximate real interest rate equals 4 percent. For scenario 1 suppose inflation and interest equal 2 percent and 6 percent, respectively. The ignorant saver discounts $100 for 10 years at 6 percent and believes that a deposit today of $55.84 (= $\$100 \div 1.06^{10}$) perfectly finances the purchase. Over 10 years, however, the purchase prices grows to become $121.90 (= $\$100 \times 1.02^{10}$). For the low inflation environment there is a shortfall of $21.90 (since only $100 accumulates). For scenario 2 suppose inflation and interest equal 12 percent and 16 percent, respectively. The ignorant saver deposits today $22.67 (= $\$100 \div 1.16^{10}$) and over 10 years the purchase prices grows to become $310.58 (= $\$100 \times 1.12^{10}$). For the high inflation environment there is a shortfall of $210.58. As percent of actual purchase price the shortfall equals 18% (=$21.90/$121.90) for the low inflation environment and 68% (=$210.58/$310.58) for the high inflation environment. Potential valuation errors accentuate with inflation because savers may incorrectly perceive more rapid wealth accumulation and therefore save less when, in fact, high inflation means future wealth simply won't buy as much!

2a. The precise real rate is $(1+.094) \div (1+.055) - 1$, or 3.70%.
2b. The actual house price is $\$168,000 \times (1+.055)^3$, or $197.273.
2c. The deposit is $\$197.273 \div (1+.094)^3$, or $150,666. This answer is identical to $\$168,000 \div (1+.037)^3$.

3a. Find that the actual house price is $\$148,000 \times (1+.017)^3$, or $155,677. Next use the lump-sum relation that $\$113,000 \times (1+r)^3 = \$155,677$ and therefore r equals 11.3%.
3b. With r = 11.3% and inflation at 1.7%, find the actual real rate as $1.113 \div 1.017$ or 9.4%.

EXERCISES 4.5A

1. Precise answers require numbers, of course. When shifting the same dollar amounts, however, it always is better to make the deposits as soon as possible. For example, ending wealth is greater with a strategy that invests $25 this period and $75 next period, instead of a strategy that invests nothing this period and $100 next period. In general, though, deferring deposits is better when the size of the subsequent deposit increases faster than the discount rate.
2. Consider first the ending wealth immediately after the last deposit. Today's deposit

contributes two years of interest to the ending wealth. Next year's deposit contributes one year of interest. The third deposit contributes no interest. The ending wealth therefore equals $\$3,200(1 + 0.125)^2 + \$4,500(1 + 0.125)^1 + \$5,000$. The account balance two years from today is \$14,112. Consider now the account balance one additional year later. The simplest procedure multiplies the balance at the beginning of this last year, \$14,112 by (1 + 0.125), which is \$15,877. This is identical to $\$3,200(1.125)^3 + \$4,500(1.125)^2 + \$5,000(1.125)^1$.

3. Find the ending wealth immediately after the last deposit, which is five months from today. Today's deposit contributes five months of interest to the ending wealth. Next month's deposit contributes four months of interest. The last deposit contributes no interest. The ending wealth therefore equals $\$900(1 + 0.015)^5 + \$500(1 + 0.015)^4 + \$750$, or \$2,250.

4. Today's deposit contributes twelve months of interest to the ending wealth. The subsequent deposit contributes three months of interest. The ending wealth therefore equals $\$1,000(1 + 0.075/12)^{12} + \$1,200(1 + 0.075/12)^3$, or \$2,300.27 .

EXERCISES 4.5B

1. Simply find the present value as $\$7,400/(1.15)^1 + \$9,000/(1.15)^2$, which is \$13,240.

2. The deposit equals the present value of $\$900/(1 + 0.075/12)^1 + \$500/(1 + 0.075/12)^6 + \$750/(1 + 0.075/12)^{12}$. The deposit is \$2,072.

3a. The investment equals the present value of $\$5,300/(1.108)^4 + \$7,000/(1.108)^9$. The investment is \$6,298.

3b. The investment of \$6,298 accumulates 10.8% interest for four years. The balance immediately before the first payment therefore is $\$6,298(1.108)^4$, or \$9,492. The first expense of \$5,300 is paid, thereby reducing the balance to \$4,192.

3c. ©MC5c The investment of \$6,298 accumulates 9.9% interest throughout the entire horizon (10.8% minus 90 basis points is 9.9%). The balance immediately before the first payment therefore is $\$6,298(1.099)^4$, or \$9,187. The first expense of \$5,300 is paid, thereby reducing the balance to \$3,887. This balance grows for five more years to reach $\$3,887(1.099)^5$, or \$6,232. Because your expense is \$7,000 there is a shortfall of \$768.

4. The cost of the investment is $\$1,000/(1 + 0.14/12)^9 + \$2,000(1 + 0.14/12)^{18}$, or \$2,524 . Find CF_2 by relating the cost to the discounted cash flows given the 12.5 percent actual rate of return:

$\$2,524 = \$1,000 / (1+ 0.125/12)^9 + CF_2 / (1+ 0.125/12)^{18}$,
or CF_2 is \$1,944.

5. First find the deposit that perfectly finances the expenses when the account earns the target APR of 8.0%. The first expense occurs in 24 quarters, the second in 36 quarters, and the quarterly periodic interest rate is 2.0% (= 8%/4). Thus, find PV = $\$4,900/1.02^{24} + \$7,300/1.02^{36}$ or PV = \$6,625. Make that deposit at the beginning. The actual APR that the account earns is less than 8.0% by 2.4% and equals 5.6%. Thus, the actual quarterly periodic rate is 1.40%. At the end of 24 quarters find that the total accumulation is FV = $\$6,625 \times 1.014^{24}$, which is FV = \$9,249. You withdraw \$4,900 to pay the first bill thereby leaving the account balance at \$4,349. This sum grows for 12 quarters, from time 24 to 36, and the total accumulation becomes FV = $\$4,349 \times 1.014^{12}$, or FV = \$5,139. Since you require \$7,300 and only have \$5,139 then the shortfall equals \$2,161.

EXERCISES 4.5C

1. Find the r that satisfies $\$12,000 = \$7,400/(1+r)^1 + \$9,000/(1+r)^2$, which is 22.8%.

2a. Find the r that satisfies the equality: $\$4,000 = \$900/(1 + r)^1 + \$1,000/(1 + r)^2 +$

$3,200/(1 + r)^3 . The monthly periodic rate, r, is 10.6%. The APR is twelve times that, or 126%.

2b. Find the r that satisfies the equality: $4,500 = $900/(1 + r)^1 + $1,000/(1 + r)^2 + $3,200/(1 + r)^3 . The monthly periodic rate, r, is 5.3%. The APR is twelve times that, or 63%. The higher price causes the APR to decline to 63% from the promised 126%, a decline of over 60 percent (that is, the decline exceeds 6,000 basis points).

3. Find the r that satisfies the equality: $8,350 = $5,800/(1 + r)^4 + $3,800/(1 + r)^8 . The monthly periodic rate, r, is 2.6%. The APR is twelve times that, or 30.6%.

4. Find the r that satisfies the equality: $17,000 = $8,200/(1 + r)^1 + $5,100/(1 + r)^6 + $4,750/(1 + r)^{12} . The monthly periodic rate, r, is 1.16%. The APR is twelve times that, or 13.9%.

5. Find the amount of the investment as $2,700/(1.075)^4 + $6,100/(1.075)^9 , or $5,203. Now find the r that satisfies the equality: $5,203 = $2,700/(1 + r)^4 + ($6,100+$1,500)/(1 + r)^9 . The annual percentage rate, r, is 9.6%.

CHAPTER 5: FUTURE AND PRESENT VALUES OF ANNUITIES

Combining cash flows at different points in time requires accounting for differences in time value. The general time value formula for mixed cash flows from the previous chapter (formula 4.11) properly handles all situations. That approach, however, is very general because it accommodates situations where the cash flow each period is possibly a different amount. For some financial situations the cash flow each period is exactly the same amount. Consumer and mortgage loans, for example, generally have a fixed payment that is exactly the same every month. Many investment or savings plans, too, stipulate a constant periodic cash flow. Procedures simplify when the cash flows are all the same amount. In this chapter we examine cash flow streams in which the cash flow each period is exactly the same.

1. The time value formula for constant annuities

Recall the previous chapter's general time value formula for mixed cash flow streams from formula 4.11:

$$PV = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} + \frac{FV}{(1+r)^N} .$$

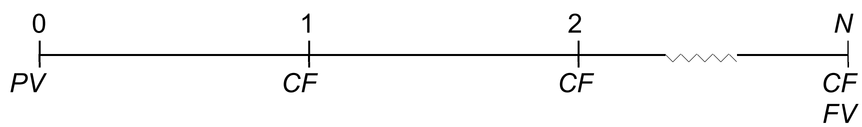
When $CF_1 = CF_2 = \ldots = CF_N$ the following simplification occurs:

FORMULA 5.1 Constant annuity time value formula

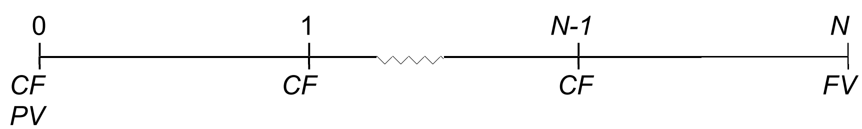
$$PV = \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots + \frac{CF}{(1+r)^N} + \frac{FV}{(1+r)^N}$$

$$= (CF)\left\{\frac{1-(1+r)^{-N}}{r}\right\} + FV(1+r)^{-N}$$

Equation 5.1 is the constant annuity time value formula. Variable definitions and cash flow timing are the same as before. *CF* is the periodic cash flow that occurs at times 1 through *N*. Each period *CF* is the same amount. There are *N* unique cash flows of amount *CF*. *PV* equals the beginning wealth one period before the first periodic cash flow. The ending wealth *N* periods later is *FV*. The last *CF* occurs at the same time as *FV.* The periodic interest rate *r* equals *APR* ÷ *m*, where *APR* is the annual percentage interest rate and *m* is the number of compounding periods per year. The time line below illustrates the essential timing of cash flows.



Some textbooks refer to cash flows consistent with the preceding time line as *ordinary annuities.* That perspective maintains that cash flows occur at the end-of-periods. An alternative scenario pertains to *annuities due* in which case the cash flows are said to occur at the beginning-of-periods. The time line below illustrates essential timing for *annuities due:*



With an *annuity due* the first *CF* is concurrent with *PV*, the last *CF* occurs one period before *FV*, and still there are *N* occurrences of *CF*. Most calculators allow setting whether cash flows occur at end or beginning-of-periods. Practically speaking, however, as far as a time line goes the end of one period is the beginning of the next and so this distinction is a little arbitrary and potentially confusing. The important fact is occurrence of the first and last *CF!* All lessons in this book avoid potential confusion by eliminating labels *ordinary annuities* and *annuities due*. Instead, the lessons explicitly specify timing of cash flows – all *Calculator Clues* assume that you keep your calculator set to end-of-period!

The most significant simplification inherent with formula 5.1 is elimination of the summation expression. For example, suppose a cash flow stream contains 360 monthly cash flows (*N*=360) and all are exactly the same amount, like a 30-year mortgage. Usage of the general time value formula in equation 4.11 involves summation of 360 different terms. The constant annuity time value formula in Equation 5.1 does not involve that summation. Instead, an exponent in one of the terms takes on the value 360.

Five variables appear in formula 5.1: *FV*, *PV*, *N*, *CF*, and *r*. When any four of the variables are set to numerical values, the fifth becomes an unknown that takes on a value satisfying the equation. Almost always the signs on *N* and *r* are positive and easy to interpret. The signs for *FV*, *PV*, and *CF,* however, may sometimes be positive and other times negative. Interpreting the signs on these variables is very important and sometimes complicated. The issue complicates further because different calculators sometimes adopt different rules regarding signage.

Here are three short lessons about variable signs for *FV*, *PV*, and *CF* in formula 5.1 (or any of its rearrangements shown in this chapter).

(1) Signage is simple to interpret when one of the three variables is zero. For example, if *PV* equals zero because there is no beginning wealth but simply there are deposits *CF* and ending wealth *FV* then signage is simple. Likewise in the lump-sum relation when *CF* is zero then the signs on *FV* and *PV* are easy to interpret.

(2) When *FV*, *PV*, and *CF* are all non-zero then remember the baseline scenario that formula 5.1 exemplifies. Beginning wealth *PV* flows into an account, periodic *CF* flow out of the account (like withdrawals), and ending wealth *FV* is the balance immediately after the last *CF*. For the preceding scenario all variables are positive. For scenarios that reverse the flow then reverse the sign. For example, when periodic deposits *CF* flow into the account assign in formula 5.1 a negative sign to *CF*.

(3) Usually there are two approaches for signing all variables. Whatever is positive in approach 1 is negative in approach 2, and vice versa. Both approaches lead to the same correct numerical answer. For example, the previous paragraph states that when *PV* and *FV* are positive then periodic deposits have negative signs. An alternative approach reverses signs: when *PV* and *FV* are both negative then assign a positive sign to periodic deposits. The choice of signs in a problem is a relative issue.

The preceding paragraphs apply to formula 5.1 or any of its rearrangements shown throughout this chapter. Calculators adopt their own unique rules. On the *BAII Plus©* financial calculator variable signs are easier to interpret by taking the perspective of one of the problem participants. Assign a positive sign to money flowing into your pocket such as withdrawals or stock dividends. Deposits, however, flow out of your pocket and into the asset account. They are leaving your pocket so give them a negative sign.

The sections below discuss scenarios that rely on the constant annuity time value formula.

EXERCISES 5.1

Concept quiz

1. Explain how inflation integrates into the constant annuity time value formula.

2. Future values of annuities

Suppose you make a series of identical deposits and want to know the ending balance. For this scenario, *FV* is the unknown variable. Rearrange and isolate *FV* on the left-hand-side:

FORMULA 5.2 Future value of a constant annuity stream

$$FV = PV(1+r)^N - CF\left\{\frac{(1+r)^N - 1}{r}\right\}$$

$$= PV(1+r)^N - CF\left\{FVIFA_{rate=r\,,\ periods=N}\right\}$$

Solving for *FV* requires assigning numerical values for *PV*, *N*, *CF*, and *r.*

The expression in curly brackets is the "future value interest factor for an annuity", abbreviated *FVIFA*. The expression depends only on *r* and *N*. The intuitive meaning of *FVIFA* is simply stated.

DEFINITION 5.1 Future value interest factor of annuities (FVIFA)

FVIFA is the future value of one dollar deposits made for N consecutive periods that earn the periodic discount rate *r*:

$$FVIFA_{r,N} = \frac{(1+r)^N - 1}{r} \ .$$

Bankers in an earlier era owned "time value books" containing *FVIFA* tables. The tables, similar to the one in Panel A of Appendix 1, list a different *N* for each row and a different periodic rate for each column. The tables simply compute the value of the expression in curly brackets. Looking at the *FVIFA* table with a periodic rate equal to 15 percent and *N* equal to 10, for example, shows a table entry equal to 20.3037 .

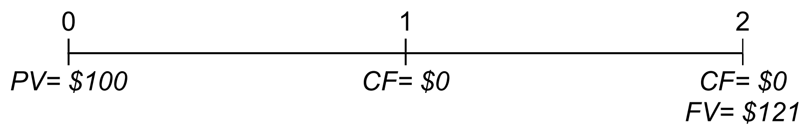
$$FVIFA_{rate=15\%,\ N=10} = \left\{\frac{(1+.15)^{10} - 1}{.15}\right\}$$

$$= 20.3037$$

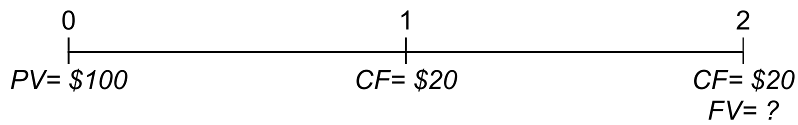
This means that if one dollar per year is deposited for ten years, and interest of 15 percent per year accrues, the account balance equals \$20.30 immediately after the last deposit. Because contributed principal equals \$10, the total market interest equals \$10.30.

FVIFA tables enable easy computation of future sums even though the deposit is different than one dollar. With a \$500 deposit, and the same rate of 15 percent for 10 years, the future value equals \$500 x 20.3037, or \$10,152. The tables are easy, but financial calculators and spreadsheets pretty much make the tables obsolete.

The variable signs in equation 5.2 deserve discussion. Begin with an example in which 10 percent interest compounds annually in a savings account for 2 years. With a beginning wealth *PV* of \$100, and *CF* of \$0, the ending *FV* wealth two periods later is \$121 (that is, \$121 = \$100x1.10^2). This lump-sum scenario is shown in the time line below:



Now extend the example. Suppose that $20 is withdrawn from the account at times 1 and 2; that is, *CF* = $20. This annuity scenario is shown in this time line:



Recall that formula 5.1 (and its rearrangement in 5.2) assumes that when *PV, CF,* and *FV* are all positive that *CF* represents a withdrawal, or return of cash flow. This problem fits that description. On the right-hand-side of formula 5.2 subtract the positive *CF* from the positive $PV(1+r)^N$. How much now is the ending balance at time 2? Substitute into equation 5.2 to find that:

$$FV = \$100(1.10)^2 - \$20\left\{\frac{(1.10)^2 - 1}{0.10}\right\}$$

$$= \$121 - \$42$$

$$= \$79$$

The $42 subtracted-out equals the future value of the withdrawal stream. The withdrawals naturally diminish the ending balance below $121; it falls to $79.

CALCULATOR CLUE 5.1 The figure above also is computable with the time value functions. On the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 1 ENTER 2nd CPT to enforce annual compounding. Then solve the preceding problem as follows:

100 PV 20 +/– PMT 2 N 10 I/Y CPT FV

The display shows $–79. The sign on the $100 initial deposit, PV , is positive because cash flows into the account to establish the balance. The sign on the two $20 withdrawals, PMT , is negative because cash flows out of the account. The sign on the ending wealth, FV, is negative because funds are available to flow out of the account.

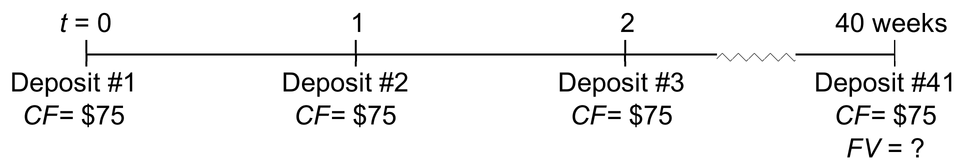
2.A. Ending wealth, FV, as the unknown variable

Regular savings plans typically involve a series of deposits, a known interest rate, and the unknown variable is ending wealth. Consider the example below.

EXAMPLE 1 Find savings plan accumulation. ©FV10a

You wish to save for the holiday season by starting a regular savings plan at the local bank. Deposits will be made every week, with the first one today. In 40 weeks, at the time you make the final deposit, you will withdraw all accumulated funds. If your deposits are $75 weekly, and the annual interest rate of 6.25 percent compounds weekly, how much will be available for the withdrawal?

SOLUTION
First get the time line right. Deposits commence right now, time zero, and conclude exactly 40 weeks from today.



Count the number of deposits. The first deposit occurs at time 0, the second deposit at time 1, etc., and finally the forty-first deposit occurs at time 40. Because *N* is the number of cash flows, *N* equals 41. This highlights that *N* is properly thought of as the number of cash flows, not as a time subscript.

Other variable settings include *r* at 0.0625 ÷ 52 (the APR, *i,* is 0.0625 and *m* is 52). *CF* definitely is $75, but is it positive or negative? Either approach is correct. Because the deposit flows out of your pocket, set *CF* to $–75. The beginning wealth, *PV*, equals zero because there is no account balance preceding the first cash flow. Notice that if a beginning balance were relevant for this problem, the timing requires the beginning wealth exactly one period before the first deposit.

Substitution of all settings into equation 5.2 shows:

$$FV = \$0 - (\$\text{-}75)\left\{\frac{\left(1+\dfrac{0.0625}{52}\right)^{41} - 1}{\dfrac{0.0625}{52}}\right\}.$$

$$= \$3{,}150$$

The account accumulates $3,150. The total contributed principal equals $3,075 (that is, $3,075 = 41 x $75). The remainder of the accumulation, $75, is total market interest.

CALCULATOR CLUE 5.2 For the algebraic solution to the preceding problem, compute and store in memory the value of the periodic rate. See the discussion in the previous chapter that storing variables in the calculator's memory reduces rounding error. Type
.0625 ÷ 52 = STO 1
Now compute the present value by typing
RCL 1 + 1 = y^x 41 – 1 = ÷ RCL 1 X 75 = .
The display shows $3,150.

The remainder of this chapter uses time value functions for solving problems. Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 52 ENTER 2nd CPT to enforce weekly compounding. Then type:
75 +/– PMT 41 N 6.25 I/Y CPT FV .

The display shows $3,150.

The signs are consistent with the earlier discussion. *CF* is negative because deposits represent monies flowing out of your pocket. *FV* takes on an opposite and positive sign, implying that at the end of the investment horizon monies are available to flow into your pocket. Notice, however, that all signs could have been reversed. If *CF* were positive then *FV* would be negative, but exactly the same outcome obtains.

Beware! Your calculator gives you wrong answers as well as right ones. It has no conscience! Therefore there is a definite advantage for scenarios when easy approximation of an answer is possible. When the approximation is relatively close to the precise number from your time value calculation then likely the precise answer is correct. Conversely, when the approximation and precise answer are miles apart then this signals a need to double-check. The Rule of 72 from the previous chapter provides approximations within the lump-sum time value framework. The rule modifies for approximating the future value of a constant annuity stream. The modified rule requires some multiplication and is prone to larger approximation errors yet, still, the modified rule may sometimes be useful.

RULE 5.1 The modified rule of 72 for constant annuities

The Rule of 72 for lump-sums states that the approximate number of years in which a sum of money doubles, *D*, equals 72 divided by the annual rate of return. The modified rule approximates total future value, *FV,* as

$$FV = N \times CF \times (1 + ½N/D),$$

where *N* is the number of years in the savings plan and *CF* is the constant annual deposit.

Intuition underlying the modified rule is that *N/D* is the savings horizon as a proportion of the doubling period. Thus, $CF \times (1 + ½N/D)$ represents the approximate average accumulation per year, which multiplied by number of years, equals total accumulation *FV.*

Suppose, for example, that you wish to approximate the future value of a stream of $1,200 annual deposits earning 6%. *CF* equals $1,200. Divide 72 by 6 and find that the doubling period *D* equals 12. All that remains is *N*, the number of years in the savings plan. Say that you save for 12 years, implying that *N/D* is 100%; you save for an entire doubling period. This is the approximation:

$$FV = 12 \times \$1,200 \times (1.5).$$

Average accumulation per year is about $1,800 (= $1,200 × 1.5); the first year's deposit doubles to become worth $2,400 and the last year contributes $1,200. The approximate answer is that *FV* equals $21,600 (= 12 × $1,800). The precise answer from the financial calculator is $20,244 (= $1,200 × $FVIFA_{r=6\%,N=12}$). The approximation overstates the precise value by about 7%. Bank on the precise number!

By saving for 6 years (implying *N/D* is 50%) then you have this approximation:

$$FV = 6 \times \$1,200 \times (1.25),$$

which is $9,000. The precise answer is $8,370 (= $1,200 × $FVIFA_{r=6\%,N=6}$). The overstatement is about 7½%.

The modified rule provides a method for checking the ball-park reasonableness of an answer even though its approximations tend to have more error than lump-sum approximations with the Rule of 72. Suppose, for example, that a savings plan deposits $200 monthly for 6 years (first deposit one month from now; last in exactly 6 years) at 12% compounded monthly. Approximate the answer with the *Modified Rule of 72*. The doubling period *D* is 6 years (= 72 ÷ 12) and *N/D* is 1 (= 6/6), and the annual cash flow *CF* is $2,400:

FV = 6 × $2,400 × (1.5),

The approximate answer equals $21,600. Use the financial calculator to find that the precise answer is $20,942 (= $200 × $FVIFA_{r=12\%\div12,N=72}$). That's pretty close!

EXERCISES 5.2A

Numerical quickies

1. Family friends of yours got a tax refund of $2,600 today. Instead of spending the money, they plan to deposit it into an account that earns 9.90% compounded annually. They expect to receive 10 same-sized annual tax refunds and to immediately deposit them into this account. Otherwise, they'll leave the account alone.

1a. Find the account balance after their last deposit. ©FV10a

1b. Find the amount of total interest that the account will earn. ©FV10b

2. Your parents contribute $50 monthly to a college savings plan for you that earns 9.80% compounded monthly. The first deposit was exactly 9 years ago. Find the account balance after today's monthly deposit and crediting of monthly interest. ©FV7

3. Your company contributes $1,250 each quarter to your college for setting up a scholarship fund. The account earns 6.50% compounded quarterly. The first deposit was exactly 15 years ago and no funds have thus far been withdrawn. Find the account balance and total amount of accumulated interest after today's quarterly deposit and crediting of quarterly interest. ©FV8

Numerical challenger

4. An account is today credited with its annual interest thereby bringing the account balance to $12,490. The interest rate is 5.70% compounded annually. You plan to make annual withdrawals of $1,450 each. The first withdrawal is in exactly one year and the last in exactly 9 years. Find the account balance immediately after the last withdrawal. ©FV5

5. An account is today credited with its monthly interest thereby bringing the account balance to $8,290. The interest rate is 6.40% compounded monthly. You plan to make monthly withdrawals of $70 each. The first withdrawal is in exactly one month and the last in exactly 12 years. Find the account balance immediately after the last withdrawal. ©FV9

2.B. Using the annuity and lump-sum formulas together

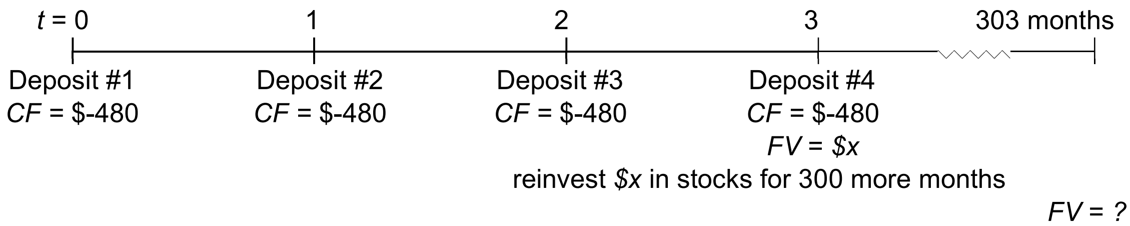
The example below illustrates that some financial scenarios require usage of both lump-sum and annuity time value formulas.

EXAMPLE 2 Save an annuity and subsequently compound as a lump-sum

With wages from a summer job you make a total of 4 deposits at $480 per month into a savings account earning 5.50% compounded monthly. At the time of the last deposit, you close the savings account and invest all the money in stocks with the intention of leaving the stocks alone for the subsequent 25 years (dividends automatically are reinvested). The stocks are expected to provide an average annual geometric return of 16% compounded monthly. When you finally sell the stocks, how much do you get?

SOLUTION

Examine the time line to ascertain clear understanding of the question.



The problem has two stages. The first stage requires finding the accumulation in stocks at the time of the last deposit (the time line shows this unknown amount as *$x*). This computation uses the annuity formula wherein *N* equals 4 (the number of deposits is 4), *CF* is $-480, and *r* is 0.0550 ÷ 12. There is no beginning balance before the first cash flow, so *PV* is zero. Substitute into formula 5.2 and solve for *FV*.

$$FV = (\$480)\left\{\frac{\left(1+\frac{0.0550}{12}\right)^{4}-1}{\frac{0.0550}{12}}\right\}.$$

$$= \$1{,}933$$

Four deposits plus total market interest accumulate $1,933. At the time of the last deposit, move all money out of the savings account and into the stock market. This occurs on the time line at *t*=3. Leave the money alone for 25 more years (300 months). The account accumulates market interest at a 16 percent annual percentage rate (compounded monthly). To find the ending wealth use the lump-sum time value formula, 4.8, and set *N* to 300; *r* to 0.16 ÷ 12 and *PV* to $1,933. Solve for *FV*.

$$FV = \$1{,}933\left(1+\frac{0.16}{12}\right)^{300},$$

$$= \$102{,}798$$

CALCULATOR CLUE 5.3 Solve the preceding problem by typing **2nd** **FV** to clear the time value memories, and **2nd** **I/Y** 12 **ENTER** **2nd** **CPT** to enforce monthly compounding. Then type:
480 **+/–** **PMT** 4 **N** 5.5 **I/Y** **CPT** **FV** .
The display shows that $1,933 is the accumulation at the time of the last deposit. While this sum still shows on the display, find the compound value of this sum after 300 additional months at 16 percent by typing:
PV 300 **N** 16 **I/Y** 0 **PMT** **CPT** **FV** .

The display shows $102,798.

Your summer job allows you to make four deposits totaling $1,920 (that is, $1,920 = 4 x $480). Your patience over the next 25 years allows the sum to grow to more than $100,000! Time, given a fair chance and rate of return, can create a lot of wealth.

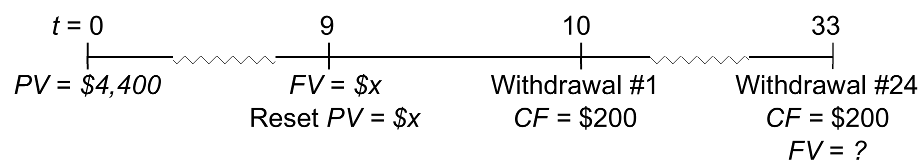
The preceding example applies the annuity formula to find an account balance at the conclusion of a savings annuity. Then the lump-sum formula compounds that balance into a future sum. The opposite timing pattern also occurs. That is, a lump-sum of money compounds into the future after which point the account is either increased or decreased by annuity activity. The example below illustrates this scenario.

EXAMPLE 3 Compound as a lump-sum and subsequently withdraw an annuity ©FV6

Today you inherit an account with a balance of $4,400. For a while you don't do anything with the account but it continues to accrue interest at a rate of 8.50% compounded monthly. Exactly 10 months from today you start withdrawing $200 monthly from the account. You make a total of 24 consecutive monthly withdrawals. Find the account balance immediately after the last withdrawal.

SOLUTION

Examine the time line to ascertain clear understanding of the question.



The problem has two stages. The first stage applies the lump-sum formula to find the balance *FV* one period before the first withdrawal:

$$FV = \$4{,}400\left(1+\frac{0.085}{12}\right)^{9},$$

$$= \$4{,}689$$

At conclusion of stage one *FV* equals $4,689. This represents *$x* on the graphic. The second stage uses the annuity formula wherein *N* equals 24 (the number of withdrawals), *CF* is $200, and *r* is 0.085 ÷ 12. Notice that the 1st withdrawal occurs 10 months from today, the 2nd in 11 months, …, and 24th in 33 months. *PV* represents the account balance one period before the first cash flow and equals $4,689. Substitute into formula 5.2 and solve for *FV*.

$$FV = \$4{,}689\left(1+\frac{0.085}{12}\right)^{24} - (\$200)\left\{\frac{\left(1+\frac{0.085}{12}\right)^{24}-1}{\frac{0.085}{12}}\right\},$$

$$= \$342$$

You inherited $4,400 and withdrew a total of $4,800. Immediately after the last withdrawal the account still had a balance of $342. Ain't *time value* wonderful!

CALCULATOR CLUE 5.4 Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 12 ENTER 2nd CPT to enforce monthly

compounding. Then type:
4400 PV 9 N 8.5 I/Y CPT FV .
The display shows $-4,689 and represents the accumulation one period before the annuity activity. While this sum still shows on the display reset *PV* and add the annuity variables by typing:
PV 24 N 200 PMT CPT FV .
The display shows $342. Notice that the PMT enter as positive numbers and are cash inflows for you. *FV* also displays as a positive number and is available for you.

EXERCISES 5.2B

Numerical quickies

1. With wages from a summer job you make a total of 5 deposits at $430 per month into a savings account earning 5.10% compounded monthly. At the time of the last deposit, you close the savings account and invest all the money in stocks with the intention of leaving the stocks alone for the subsequent 21 years (dividends automatically are reinvested). The stocks are expected to provide an average annual geometric return of 12.00% compounded monthly. When you finally sell the stocks, how much do you get? ©FV3

Numerical challengers

2. Today you inherit an account with a balance of $2,600. For a while you don't do anything with the account but it continues to accrue interest at a rate of 9.90% compounded monthly. Exactly 10 months from today you start an ambitious savings plan and deposit $230 monthly into the account. You make a total of 16 consecutive monthly deposits. Find the account balance immediately after the last deposit. ©FV11 .

3. Today you inherit an account with a balance of $2,200. For a while you don't do anything with the account but it continues to accrue interest at a rate of 10.00% compounded monthly. Exactly 10 months from today you start withdrawing $150 monthly from the account. You make a total of 16 consecutive monthly withdrawals. Find the account balance immediately after the last withdrawal. ©FV12

4. Today you inherit an account with a balance of $5,800. For a while you don't do anything with the account but it continues to accrue interest. Exactly 17 months from today you start an ambitious savings plan and deposit $220 into the account. You plan to deposit that much each month. Exactly 36 months from today you reconsider your plan, make your last deposit, and make no additional deposits. You nonetheless leave the account alone and it continues to accrue interest at a rate of 6.6% compounded monthly. You finally close the account exactly 7 years from today. How much is the total accumulation? ©FV6

5. You wish to accumulate a total of $4,400 for a special purpose. Today you open a new account and make your first deposit. You make the last deposit when you withdraw your target accumulation exactly 10 years from now. You make equal-size deposits quarterly such that if the savings rate is 7.10% compounded quarterly then you'll reach the target accumulation and your withdrawal will draw the account balance down to zero. After you go and make your target withdrawal, however, you are surprised to see that $780 is left-over in the account. Find the actual annual savings rate. ©FV19a

3. Present values of annuities

Present value represents the initial worth of a cash flow stream. Many decisions require finding the present worth of future expected returns. Finding present value also requires specifying the target rate of return, or market interest, that is subtracted from future returns. The general time value formula for constant annuities is equation 5.1, restated below:

$$PV = (CF)\left\{\frac{1-(1+r)^{-N}}{r}\right\} + FV(1+r)^{-N}$$

$$= CF\left\{PVIFA_{rate=r,\ periods=N}\right\} + FV(1+r)^{-N} .$$

Solving for *PV* requires assigning numerical values for *FV*, *N*, *CF*, and *r.*

The expression in curly brackets is the “present value interest factor for an annuity”, abbreviated *PVIFA*. The expression depends only on *r* and *N*. The intuitive meaning of *PVIFA* is simply stated.

DEFINITION 5.2 Present value interest factor of annuities (PVIFA)
PVIFA is the initial deposit earning interest at the periodic rate *r* that perfectly finances a series of *N* consecutive one dollar withdrawals:

$$PVIFA_{r,N} = \frac{1-(1+r)^{-N}}{r} .$$

The bankers’ time value books also contain *PVIFA* tables. The tables, similar to the one in Panel B of Appendix 1, list a different *N* for each row and a different periodic rate for each column. The tables simply compute the value of the expression in curly brackets. Looking at the *PVIFA* table with a periodic rate equal to 15 percent and *N* equal to 10, for example, shows a table entry equal to 5.0188 .

$$PVIFA_{rate=15\%,\ N=10} = \left\{\frac{1-(1+.15)^{-10}}{.15}\right\}$$

$$= 5.0188$$

This means a deposit of \$5.02 earning interest of 15 percent per year finances a withdrawal of one dollar per year for 10 years. The total withdrawals equal \$10, implying total market interest equals \$4.98 (that is, \$4.98 = \$10.00 – \$5.02).

PVIFA tables enable easy computation of present values even though withdrawals differ from one dollar. With a \$500 withdrawal, and the same rate of 15 percent for 10 years, the present value equals 500 x 5.0188, or \$2,509. Basically, the twenty-five hundred dollar deposit supports over the next ten years withdrawals totaling five thousand dollars. The tables are pretty obsolete, of course, because financial calculators are better.

3.A. Beginning wealth, PV, as the unknown variable

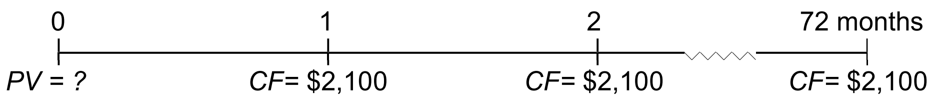
The following example specifies the cash flow stream and the target rate of return, and finds as the unknown variable the initial wealth supporting the stream.

EXAMPLE 4 What is the proper price for an annuity? ©PV8 ©PV9

You might invest in an asset that will return cash flow to you of $2,100 per month for 72 months. Your target annual rate of return on the investment is 15 percent, compounded monthly. How much should you pay for the investment?

SOLUTION

The time line illustrates the timing implicit in the problem:



Substitute settings into equation 5.1 that *CF* equals $2,100 and *N* equals 72. There is no ending balance because the account is drawn to zero, so *FV* equals $0. The monthly periodic rate *r* equals 0.15 ÷ 12, or 0.0125.

$$PV = (\$2,100)\left\{\frac{1-(1.0125)^{-72}}{.0125}\right\}$$

$$= \$99,314$$

CALCULATOR CLUE 5.5 Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Then type:

2100 PMT 72 N 15 I/Y CPT PV .

The display shows $99,314.

You get a 15 percent annual rate of return when you pay $99,314 for an investment and get back 72 monthly returns of $2,100 each.

Formula 5.1 allows finding beginning wealth (*PV*) when there is an annuity cash flow history as well as an ending balance (*FV)*. The annuity cash flows *CF* may be either deposits or withdrawals. The direction of the cash flows matter. When money flows out of the account (withdrawals) then in formula 5.1 *FV* and *CF* have the same sign. Conversely, for deposits the signs of *FV* and *CF* in formula 5.1 are opposites. The example below illustrates proper procedure for handling this type scenario.

EXAMPLE 5 Find *PV* given an ending balance plus a withdrawal history ©PV7

A friend received an inheritance 4 years ago and put all funds into an account earning 10.00% compounded quarterly. Exactly one quarter after establishing the account the friend started withdrawing $950 per quarter. Today quarterly interest will be credited to the account and she'll make another quarterly withdrawal and then the balance will be $13,104. How much was the friend's inheritance?

SOLUTION
With an APR of 10.0% and quarterly compounding the periodic rate *r* equals 2.5% (=10.0% ÷ 4). The annuity cash flow *CF* equals $950 and *N*, the number of withdrawals, is 16. Notice that *CF* flows out of the account and into your friend's pocket so assign it a positive sign. *FV* is the balance immediately after the last withdrawal and equals $13,104. *FV* represent funds available to your friend as an inflow. *CF* and *FV* have the same signs because they both flow out of the account and into your friend's pocket. Alternatively, if *CF* were a deposit (instead of a withdrawal) then its sign would be opposite the sign of *FV*. Always ascertain that cash flow timing considerations are consistent with formula 5.1: (a) *PV* occurs one period before the first *CF*; (b) *FV* occurs immediately after the last *CF*; and (c) there are *N* cash flows. Formula 5.1 is perfectly appropriate for this example. Substitute settings:

$$PV = (\$950)\left\{\frac{1-(1.025)^{-16}}{.025}\right\} + \$13,104(1025^{-16})$$

$$= \$21,229$$

State the solution another way. Deposit $21,229 into an account earning 10% compounded quarterly, then beginning one quarter later withdraw $950 and make withdrawals for 16 quarters, then immediately after making the last withdrawal the account balance is $13,104.

CALCULATOR CLUE 5.6 Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 4 ENTER 2nd CPT to enforce quarterly compounding. Then type:
950 PMT 16 N 10 I/Y 13104 FV CPT PV .

The display shows $-21,229. That's the answer. The negative sign means that, unlike the withdrawals and ending balance that flow out of the account and into your friend's pocket, the beginning wealth *PV* flows into the account.

The time value relation is extremely flexible. Formula 5.1 includes five variables (*PV, CF, FV, r* and *N)*. Supply numerical settings for any 4 and the 5th takes on a unique value called "the answer." Previous examples solve for *PV* and *FV*. Seldom does a situation call for finding *N* as unknown variable. Often, however, you may need to find the periodic rate of return *r*, as the two examples below illustrate.

EXAMPLE 6 Find *PV* for an annuity stream and then the actual ROR on counteroffer
You might invest in an asset that will return after-tax cash flow to you of $2,200 per month for 30 months (first cash flow one month from now), and after receiving the last cash flow you'll immediately receive after-tax net proceeds from liquidation equal to $15,000. You make an offer to buy the asset so that you'll get your "target" annual rate of return of 15.7% (compounded monthly). Instead, however, the seller makes a counteroffer that is $3,500 higher than your offer. Find your annual rate of return if you buy at the counteroffer price and receive the expected cash flows.

SOLUTION

With an APR of 15.7% and monthly compounding the periodic rate *r* equals 1.31% (= 15.7% ÷ 12). The 30 (= *N*) periodic cash flows *CF* of $2,200 are inflows for you, much like they are withdrawals from an asset account. Likewise, the liquidation proceeds of $15,000 (= *FV)* also is an inflow. In formula 5.1 assign *CF* and *FV* positive signs. Note that the cash flow timing considerations are consistent with formula 5.1. Substitute settings and solve for the offer price, *PV*:

$$PV = (\$2,200)\left\{\frac{1-(1.0131)^{-30}}{.0131}\right\} + \$15,000(10131^{-30}),$$

$$= \$64,455$$

You make an offer to buy the asset at $64,455 but the seller counteroffers at $67,955 (= $64,455 + $3,500). Now assign $67,955 to *PV* and solve for the monthly rate of return *r* from formula 5.1:

$$\$67,955 = (\$2,200)\left\{\frac{1-(1+r)^{-30}}{r}\right\} + \$15,000(1+r)^{-30}.$$

The formula does not have a "closed-form solution," meaning that you cannot isolate *r* by itself on the left. Your financial calculator, however, is pretty smart and, as *Calculator Clue 5.7* explains, finds that the APR for the actual rate of return is 11.95% (or 0.99% per month). The extra cost of $3,500 reduces your annual rate of return by 375 BP (= 15.70% – 11.95%).

CALCULATOR CLUE 5.7 Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Then type:
2200 PMT 30 N 15.7 I/Y 15000 FV CPT PV .
The display shows $-64,455. That's the offer price. The negative sign means that, unlike the periodic cash flows and liquidation proceeds that flow into your pocket and are positive, the beginning wealth *PV* flows out of your pocket. While the display still shows $-64,455 perform these steps to get the counteroffer price and solve for the annual rate of return:
+/- + 3500 = +/- PV CPT I/Y .
The display shows that the actual annual rate of return is 11.95%

EXAMPLE 7 Find *PV* for an annuity stream and then the actual ROR on counteroffer

You might pursue an investment that incurs a large up-front cost today. Furthermore, it requires payments of $7,500 per month for 24 months (first payment one month from now). Immediately after making the last payment, however, you will receive after-tax net proceeds of $310,000. You make an offer to buy the asset so that you'll get your "target" annual rate of return of 14.2% (compounded monthly). Instead, however, the seller makes a counteroffer that is $5,000 higher than your offer. Find your annual rate of return if you buy at the counteroffer price and receive the expected cash flows.

SOLUTION

This is similar to the previous example except for the signage. The 24 (= *N*) periodic cash flows *CF* of $7,500 are outflows for you, sort of like deposits into an asset account, so assign a negative sign. But *FV* of $310,000 is an inflow for you so in formula 5.1 make it positive. With a monthly periodic rate *r* equal to 1.18% (= 14.2% ÷ 12) solve for the offer price:

$$PV = (\$-7,500)\left\{\frac{1-(1.0118)^{-24}}{.0118}\right\} + \$310,000(1.0118^{-24}),$$

$$= \$77,846$$

Assign the counteroffer price of $82,846 (= $77,846 + $5,000) to *PV* and solve for the monthly rate of return *r* from formula 5.1:

$$\$82,846 = (\$-7,500)\left\{\frac{1-(1+r)^{-24}}{r}\right\} + \$310,000(1+r)^{-24}.$$

The financial calculator finds that the APR for the actual rate of return is 12.61%, or 159 BP less than your target (= 14.20% – 12.61%).

CALCULATOR CLUE 5.8 Solve the preceding problem by typing 2nd FV to clear the time value memories, and 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Then type:
7500 +/- PMT 24 N 14.2 I/Y 310000 FV CPT PV +/- + 5000 = +/- PV CPT I/Y .
The display shows that the actual annual rate of return is 12.61%.

EXERCISES 5.3A

Numerical quickies

1. You might invest in an asset that will return after-tax cash flow to you of $1,200 per year for 8 years (first cash flow one year from now), after that the asset probably will be worthless. You make an offer to buy the asset so that you'll get a 8.80% rate of return (compounded annually). Find the offer price. ©PV8

2. You're quite fortunate because this afternoon, just like this date in each of the past 9 years, you shall withdraw $1,600 from an account that your guardian angel established for you exactly 10 years ago. After the withdrawal the balance will equal zero. The account earns 6.30% per year (compounded annually, interest is being credited this morning). Except for your withdrawals the account has been untouched. Find the initial deposit that your guardian angel used to establish the account. ©PV5

3. You might invest in a security that will return after-tax cash flow to you of $1,600 per year for 9 years (first cash flow one year from now), after which the security likely can be sold immediately for $7,700 . You make an offer to buy the security so that you'll get a 8.50% rate of return (compounded annually). Find the offer price. ©PV9

Numerical challengers

4. A friend received an inheritance 6 years ago and put all funds into an account earning 8.50% compounded quarterly. Exactly one quarter after establishing the account the friend started a savings plan that deposits $650 per quarter. Today the quarterly deposit is due and quarterly interest will be credited to the account, thereby bringing the balance to $49,924. How much was the friend's inheritance? ©PV6

5. A friend received an inheritance 3 years ago and put all funds into an account earning 9.50% compounded quarterly. Exactly one quarter after establishing the account the friend started withdrawing $1,350 per quarter. Today quarterly interest will be credited to the account and she'll make another quarterly withdrawal and then the balance will be $11,403. How much was the friend's inheritance? ©PV7

6. You might invest in an asset that will return after-tax cash flow to you of $2,700 per month for 15 months (first cash flow one month from now), and after receiving the last cash flow you'll immediately receive after-tax net proceeds from liquidation equal to $95,600. You make an offer to buy the asset so that you'll get your "target" annual rate of return of 18.20% (compounded monthly).
6a. What is your offer price? ©PV10a
6b. Instead, however, the seller makes a counteroffer that is $9,600 higher than your offer. Find your annual rate of return if you buy at the counteroffer price and receive the expected cash flows. ©PV10b .

7. You might purchase an investment that incurs a large up-front cost today. Furthermore, it requires payments of $2,400 per month for 15 months (first payment one month from now). Immediately after making the last payment, however, you will receive after-tax net proceeds of $78,900. You make an offer to purchase the asset so that you'll get your "target" annual rate of return of 20.30% (compounded monthly).
7a. Find the up-front purchase price that you offer to pay today. ©PV11a .
7b. The seller makes a counteroffer that is $6,000 higher than your offered purchase price. Find your annual rate of return if you buy at the counteroffer price and receive the expected cash flows. ©PV11b .

3.B. The special case of perpetuities

Suppose an account with $1 million earns 10 percent interest compounded annually. Each year the account earns $100,000 of interest (that is, $100,000 = 0.10 x $1,000,000). Each year, too, suppose you withdraw $100,000 from the account. Now ask the question, when will the account balance draw down to zero? The answer is: *never*. The account balance never goes to zero. The preceding scenario describes a perpetuity.

DEFINITION 5.3 PERPETUITY
A *perpetuity* is an account that maintains a specified principal balance perpetually even in the absence of subsequent deposits.

The balance never diminishes because all withdrawals consist exclusively of interest, not principal. For example, in the preceding illustration the balance begins at $1 million. Exactly one year later, immediately before the first withdrawal, the account earns periodic interest of $100,000 and the balance rises to $1,100,000. Then the $100,000 withdrawal occurs, the balance falls back to $1 million. The cycle repeats perpetually.

The perpetuity is a special case of the constant annuity time value relation shown in formula 5.1. Mathematically speaking, with a perpetuity N goes to infinity. As N gets larger and larger, the expression $(1+r)^N$ gets larger and larger, too (as long as $r > 0$, actually). Dividing this ever larger number into a future sum causes the present value of that sum to vanish. Cash flows way out yonder have virtually zero effect on present value!

The perpetuity formula relates beginning wealth, periodic cash flow, and rate of return as follows:

FORMULA 5.3 Present value of a perpetual and constant stream

$$PV = \frac{CF}{r} \quad .$$

Variable definitions are the same as always. *PV* is the beginning balance, *CF* is the periodic cash flow. The periodic interest rate r equals the annual percentage rate i divided by m, the number of compounding periods per year.

The perpetuity relation has many useful applications. Endowment funds for non-profit foundations are perpetuities. The Ford Foundation, Annenberg Foundation, and many others, have huge balances that each year spin-off market interest. The foundations never consume their principal. Instead, the periodic market interest is the source of financing for grants that the foundation sponsors. These foundations hope to operate forever. As the example below shows, too, universities rely on perpetuities to finance many important functions.

EXAMPLE 8 What size deposit sets up the memorial fund?

You wish to establish a fund at your alma mater that finances a $5,000 scholarship twice each year. The account earns a 12 percent average annual rate of return, compounded semiannually. How much do you need to deposit to establish the endowment fund?

SOLUTION

For this scenario, *CF* equals $5,000 and *i/m* equals 0.12 ÷ 2, or 0.06. Substitution into the perpetuity formula shows:

$$PV = \frac{\$5{,}000}{0.06}$$

$$= \$83{,}333$$

Make a deposit of $83,000 and forevermore your monies will spin-off $5,000 scholarships twice each year.

EXERCISES 5.3B

Numerical quickies

1. Your unrealistic dream is to win the lottery, deposit the money into an account earning 7.5% interest compounded annually, and forevermore draw out $1 million per year. Find the amount you need to win. ©PV13 .

2. An alumni group wants to establish an endowment fund for paying expenses associated with hiring a distinguished professor of business. The annual expenses should run about $140,000 (payable in 12 monthly installments). Find the size of the requisite endowment if the account earns 8.8% compounded monthly. ©PV12 .

3. Your college has a fixed and constant endowment fund of $35 million that each year generates interest income for paying scholarships and faculty salaries. The interest rate has fallen from 10% a few years ago to 4% today (compounded annually). Find the decline in annual income that the college unfortunately faces. ©PV14 .

4. Cash flows connecting beginning and ending wealth

Many financial situations require finding the amount of each periodic cash flow that satisfies the constant annuity time value relation. Because the cash flows are all the same amount, equation 5.1 easily rearranges to get *CF* alone on the left-hand side. There are two equivalent rearrangements of the formula. The first rearrangement relies on the future value interest factor for an annuity found in *FVIVA* tables.

FORMULA 5.4 Cash flow as a function of *FVIFA*

$$CF = \left(PV(1+r)^N - FV\right) \Big/ \left\{\frac{(1+r)^N - 1}{r}\right\}$$

$$= \left(PV(1+r)^N - FV\right) \Big/ FVIFA_{rate=r,\ periods=N}$$

The second rearrangement relies on the present value interest factor for an annuity found in *PVIVA* tables.

FORMULA 5.5 Cash flow as a function of *PVIFA*

$$CF = \left(PV - FV(1+r)^{-N}\right) \Big/ \left\{\frac{1-(1+r)^{-N}}{r}\right\}$$

$$= \left(PV - FV(1+r)^{-N}\right) \Big/ PVIFA_{rate=r,\ periods=N}$$

Equations 5.4 and 5.5 are identical in every significant way. Using one instead of the other is arbitrary since they give the same answer and require the same information. For a given problem setup, however, one might be easier to use.

4.A. Cash flow, CF, as the unknown variable

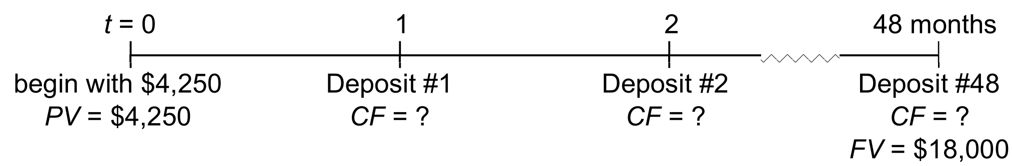
The examples below require finding the periodic cash flow that satisfies the time value formula. The first example specifies a future value and finds the unknown deposit. The second example specifies both present and future values.

EXAMPLE 9 Find the required deposit

Four years from today you expect to place a downpayment on your first house. You forecast the required downpayment at $18,000. The savings account earns 8.25 percent annual interest, compounded monthly. Your savings account balance today is $4,250 and you intend to make additional deposits each month. The first deposit is one month from today and the last is four years from today, immediately before you withdraw the entire accumulation of $18,000. How much is each monthly deposit?

SOLUTION

Use the time line to verify the timing of cash flows.



Use equation 5.2 to solve for *CF*. The annual percentage rate *i* equals 8.25 percent and compounding occurs monthly, so *m* equals 12. The periodic interest rate *r* is 0.006875 (that is, 0.006875 = .0825÷12).

$$CF = \left(\$4,250(1.006875)^{48} - \$18,000\right) \Big/ \left\{\frac{(1.006875)^{48} - 1}{0.006875}\right\}$$

$$= \$-213.54$$

CALCULATOR CLUE 5.9 Solve the preceding problem by typing **2nd** **FV** to clear the time value memories, and **2nd** **I/Y** 12 **ENTER** **2nd** **CPT** to enforce monthly compounding. Then type:

4250 **PV** 18000 **+/–** **FV** 48 **N** 8.25 **I/Y** **CPT** **PMT** .

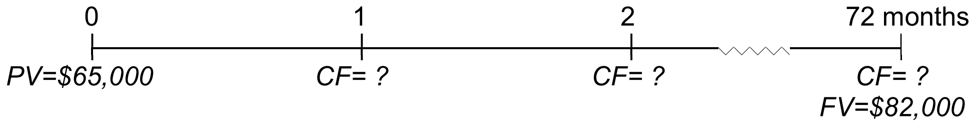
The display shows $213.54 .
Make 48 monthly deposits of $213.54 each and you'll accumulate the downpayment required to buy the house.

EXAMPLE 10 Find effect of ROR on monthly income

You are investing $65,000 today. You expect to receive monthly returns for 72 months with the first one exactly one month from today. At the time of the last monthly return you also expect to sell the investment for $82,000. You can either invest in a savings account earning 6 percent per annum or in a stock fund earning 18 percent per annum (compounded monthly). How much difference in monthly cash flow do the alternative investments provide?

SOLUTION

Inspect the time line for this problem.



The unknown variable is *CF*, the constant monthly cash flow. Known variables, however, include the beginning wealth, *PV*, at $65,000 and the ending wealth, *FV*, at $82,000. The number of cash flows, *N*, equals 72. The periodic rate differs according to the investment; *r* equals 0.06 ÷ 12 (that is, 0.005) for the savings account and 0.18 ÷ 12 (that is, 0.015) for the stock fund. In either case, substitute the known variables into equation 5.4 and solve for *CF*. For the savings account, substitution shows:

$$CF = \left(\$65{,}000(1.005)^{72} - \$82{,}000\right) \Big/ \left\{\frac{(1.005)^{72} - 1}{0.005}\right\}$$

$$= \$128$$

From the savings account the cash flowing into your pocket each month is $128.

The stock fund uses the higher periodic interest rate of 1.5 percent per month:

$$CF = \left(\$65{,}000(1.015)^{72} - \$82{,}000\right) \Big/ \left\{\frac{(1.015)^{72} - 1}{0.015}\right\}$$

$$= \$842$$

> CALCULATOR CLUE 5.10 To solve this problem with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Solve for the cash flow from the 6 percent savings account by typing:
>
> 65000 PV 82000 +/– FV 72 N 6 I/Y CPT PMT
>
> The display shows that periodic cash flow equals $–128.
>
> Obtain the cash flow for the 18 percent stock fund by entering only I/Y ; the other variables may remain the same. While the display still shows $–128, type
>
> 18 I/Y CPT PMT
>
> The display shows that periodic cash flow equals $–842.

The rate of return is three times larger on the stock fund than on the savings account (that is, 18% ÷ 6% = 3). The periodic cash flow from the stock fund, however, is more than six times larger than from the savings account (that is, $842 ÷ $128 = 6.6). The effect of compounding makes time value computations very sensitive to the rate of return.

EXERCISES 5.4A

Numerical quickies

1. You wish to accumulate $10,000 for a special purpose. You today open an account that earns 7.0% compounded monthly by making the first of many deposits, all the same

size. Your last monthly deposit is in exactly 4 years. After that last deposit and crediting of monthly interest your target balance is reached. Find the amount of each deposit. ©FV16 .

2. Exactly 5 years ago you made a deposit that opened an account earning 13.0% compounded monthly. Every month since that time you have made a deposit of exactly the same amount. After today's deposit and crediting of monthly interest the account balance is $12,000. Find the amount of each deposit. ©FV14 .

3. You inherit an account with $14,000 that earns 8.0% compounded quarterly. One quarter later you make the first of 20 quarterly withdrawals, all the same size, and draw down the account to zero. Find the amount much of each withdrawal. ©FV13 .

Numerical challengers

4. Exactly 5 years ago you inherited an account with balance of $16,000 that earns 10.30% compounded quarterly. One quarter later you made the first of many quarterly deposits, all the same size. After today's deposit and crediting of quarterly interest the account balance is $68,200. Find the amount of each deposit. ©FV15

5. You are investing $12,000 today. You expect to receive monthly returns for 216 months with the first one exactly one month from today. At the time of the last monthly return you also expect to sell the investment for $10,000. You can either invest in a savings account earning 5.30% percent per annum or a stock fund earning 15.00% percent per annum (compounded monthly). How much difference in monthly cash flow do the alternative investments provide? ©FV18

6. Today you open an account with a $16,000 deposit that earns 11.20% compounded annually. You've set a target for the account so that in exactly 5 years its balance will be $30,000. To reach the target you'll adjust the balance annually; each year's adjustment will be exactly the same amount and the first adjustment occurs exactly one year from now. After the last annual adjustment in exactly 5 years, and crediting of that year's interest, the account balance exactly equals the target. Describe the annual adjustment that you make each year. ©FV17

4.B. Other two-stage problems

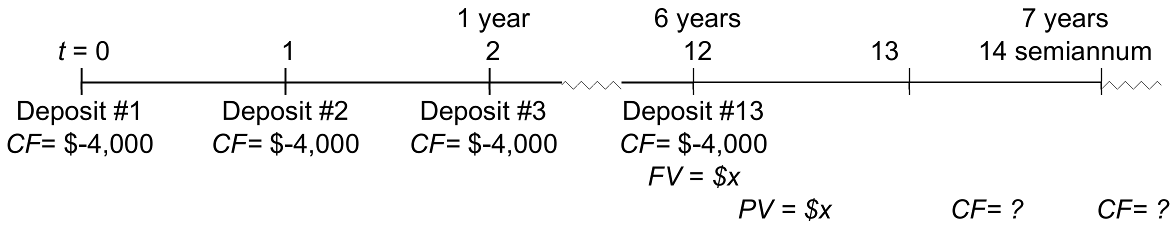
Many finance situations involve saving over time in order to finance withdrawals over time. Solving these problems often requires doing several sequential time value computations. Consider the examples below.

EXAMPLE 11 How big is the endowment's scholarships ©TS1b

You wish to establish an endowment fund that will provide student financial aid awards every semiannum, perpetually. You will make deposits semiannually equal to $4,000 each, with the first one today and the final one in 6 years. The first award is to be granted one semiannum after the last deposit. The savings rate always is 8.9% compounded semiannually. How much is each award?

SOLUTION

Examine the time line to ascertain a clear understanding of the words.



This problem has two stages: a deposit stage during which savings accumulate, and a withdrawal stage during which scholarships are withdrawn. The first deposit is today, time 0, and the final one occurs in six years. There are in total 13 deposits. A sum of money, say *$x*, will have accumulated by the time of the last deposit. That sum represents for the second stage of the problem a beginning wealth that finances perpetual withdrawals. The question asks, how much is each withdrawal?

The first stage involves finding the future value of thirteen $4,000 semiannual deposits that earn interest at an annual rate of 8.9% (that is, 4.45% compounded semiannually). Use the constant annuity time value formula that solves for *FV* (equation 5.2):

$$\text{FV} \;=\; (\$4{,}000)\left\{\frac{(1.0445)^{13}-1}{0.0445}\right\}.$$

$= \$68{,}423$

The account balance immediately after the final deposit equals $68,423. This represents *$x* on the timeline.

The scholarship exactly equals the interest that $68,423 earns per semiannum. With a semiannual rate of 4.45 percent, the cash flow financing the scholarship is

$CF \;=\; (0.0445)(\$68{,}423)$

$= \$3{,}045.$

CALCULATOR CLUE 5.11 To solve this problem with the time value functions on the *BAII Plus*© type 2nd FV to clear the time value memories. Type 2nd I/Y 2 ENTER 2nd CPT to enforce semiannual compounding. Then solve for the accumulation at the conclusion of the savings stage by typing:

4000 +/– PMT 13 N 8.9 I/Y CPT FV .

The display shows the account balance equals $68,423 immediately after the last deposit. Now use the arithmetic keys and multiply the value on the display by the periodic rate of 0.0445 to obtain the periodic interest of $3,045.

The savings deposits finance scholarships of $3,045 per semiannum perpetually. By donating thirteen deposits totaling $52,000 (that is, $52,000 = 13x$4,000) you endow an endless stream of students with resources for pursuing collegiate studies.

EXAMPLE 12 Find the ROR on a counter-offer for a two-stage annuity ©PV3c

You might invest in an asset that will return after-tax cash flow to you of $2,200 per month for 5 months (first payment one month from now), followed by $3,500 per month for 4 months . You make an offer to buy the asset so that you'll get your "target" annual rate of return of 15.7% (compounded monthly). Instead, however, the seller makes a counter-offer that is $950 higher than your offer. If you buy at the counter-offer price, and receive the expected cash flows, what is your annual rate of return?

SOLUTION
This problem sequences a 5-period annuity stream together with a 4-period stream to make a 9-period stream. The first step is to find how much you offer for this two-stage annuity. Find PV by discounting the cash flows with the target monthly periodic rate of 1.31% (= 15.7% ÷ 12):

$$PV = \frac{\$2,200}{1.0131^1} + \frac{\$2,200}{1.0131^2} + \ldots + \frac{\$2,200}{1.0131^5} + \frac{\$3,500}{1.0131^6} + \ldots + \frac{\$3,500}{1.0131^9}$$

$$= \$23,282$$

An alternative representation that obtains the identical answer is to apply PVIFA to the two annuities, and further apply the lump-sum relation to the latter stage:

$$PV = \$2,200\left\{\frac{1 - 1.0131^{-5}}{0.0131}\right\} + \frac{\$3,500\left\{\frac{1 - 1.0131^{-4}}{0.0131}\right\}}{1.0131^5}$$

$$= \$23,282$$

Regardless of how you view this two-stage problem, you offer \$23,282 and the seller counter-offers for \$950 more. The second step relates the counter-offer price of \$24,232 (= \$23,282 + \$950) to the cash flows, leaving as an unknown variable the annual percentage rate of return:

$$\$24,232 = \frac{\$2,200}{\left(1+\frac{APR}{12}\right)^1} + \ldots + \frac{\$2,200}{\left(1+\frac{APR}{12}\right)^5} + \frac{\$3,500}{\left(1+\frac{APR}{12}\right)^6} + \ldots + \frac{\$3,500}{\left(1+\frac{APR}{12}\right)^9}$$

$$\frac{APR}{12} = 0.5688\%$$

$$APR = 6.83\%$$

The price jump to \$24,232 from \$23,282 reduces your annual ROR to 6.83% from 15.7%

CALCULATOR CLUE 5.12 On the BAII Plus© type CF and clear unwanted numbers by typing 2nd CE/C . Now enter this problem's cash flow stream:
↓ 2200 ENTER ↓ 5 ENTER ↓ 3500 ENTER ↓ 4 ENTER
Now find the present value of the stream given the rate is 15.7 percent compounded monthly. Type:
NPV 15.7 ÷ 12 = ENTER ↓ CPT

The display shows \$23,282. You can't buy at that price, however, and must pay \$950 more. With \$23,282 still on the display, type:

+ 950 = +/– STO 1
Now plug that higher price into the cash flow stream and compute the periodic rate of return, which must be multiplied by 12 to obtain the annual return:
CF RCL 1 ENTER IRR CPT X 12 = .

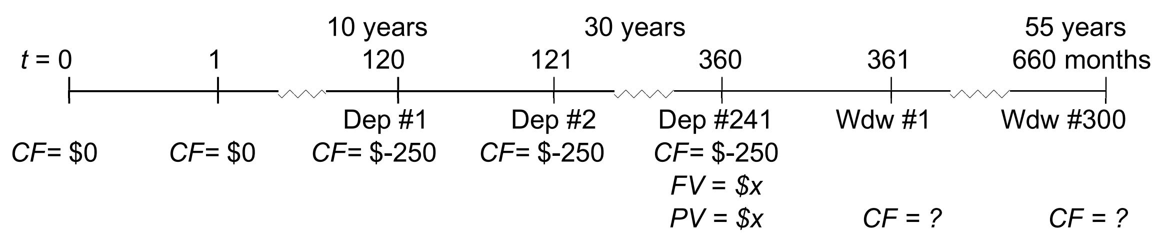
The display shows 6.83 percent.

EXAMPLE 13 Saving young versus saving later ©TS2a

Your two twin sisters, Prudence and Candy, are pursuing two different financial strategies for early retirement. Both sisters intend to retire exactly 30 years from today. Candy does not want to start saving until exactly 10 years from today, at which time she'll make her first monthly deposit of $250. She'll continue making monthly deposits for 20 years, so that Candy's final deposit occurs exactly 30 years from today when she retires. The other sister, Prudence, plans to deposit $250 per month for 10 years with the first deposit today and the last one exactly 10 years from today. Prudence will not deposit anything beyond that, but she will let interest continue to accrue. The annual savings rate always is 12% compounded monthly. Both sisters intend to draw down the savings accounts to zero by making monthly withdrawals during retirement for 25 years. The first withdrawal is one month after retirement commences. How much monthly income should each sister expect in retirement?

SOLUTION

First examine the time line for late-starter Candy. Her first deposit is exactly 10 years, or 120 months, from today. Her last deposit is exactly 30 years, or 360 months, from today. Then in month 361, Candy commences withdrawing money from the account (the time line abbreviates withdrawal as "Wdw").



Throughout twenty years Candy makes 241 deposits at $250 each. Find the future value of this accumulation with the following settings: the account begins at $0 so *PV* is zero, *N* equals the number of deposits and is 241, *CF* is $-250, and the monthly periodic interest rate *r* is 0.01. Use equation 5.2 to solve for *FV*.

$$\text{FV} = (\$250)\left\{\frac{1.01^{241} - 1}{0.01}\right\}.$$

$= \$250{,}037$

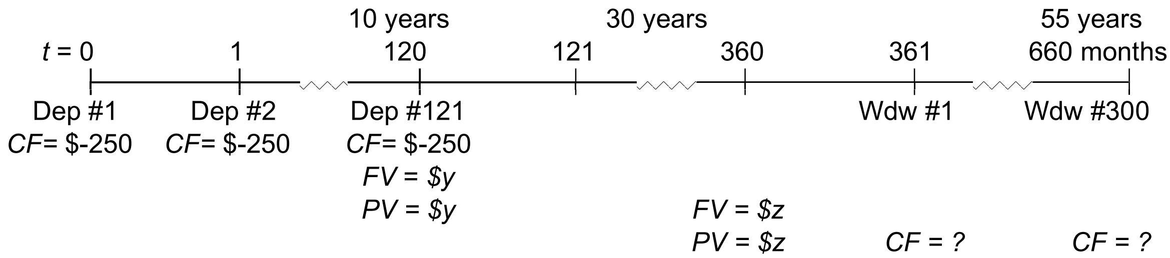
Candy has about one-quarter million dollars in her account immediately after making her final deposit. This future value is shown on the time line as *$x*. At that time Candy retires. For this retirement stage, the $250,037 represents a beginning wealth that finances a series of monthly withdrawals over 25 years (300 months). To find the amount of each withdrawal, use the constant annuity formula (equation 5.5) and set *N* to 300 and the monthly periodic rate to 0.01. Set *PV* to $250,037. Notice that this number was the answer for *FV* from the savings stage. Now this same number is *PV* for the retirement stage. Because the account is drawn down to zero, *FV* equals \$0. Solve for *CF*.

$$CF = \$250{,}037 \Big/ \left\{ \frac{1 - 1.01^{-300}}{.01} \right\}$$

$$= \$2{,}633$$

Candy makes wise investments, saving \$250 per month for twenty years enabling her to retire on monthly income of \$2,633 for twenty-five years. The power of compound interest is amazing!

Consider the case for Prudence. She saves immediately, as shown in the time line below.



Prudence makes 121 monthly deposits of \$250 each. Find the future value immediately upon making the last deposit: *N* is 121, *PV* is zero, *CF* is \$-250, and the periodic interest rate is 0.01:

$$FV = (\$250)\left\{ \frac{1.01^{121} - 1}{0.01} \right\}.$$

$$= \$58{,}335$$

After making the last deposit the account balance is \$58,335. This future value is shown on the time line as *\$y*. Prudence doesn't save anymore, but she lets the interest accrue for 20 years. During this time, her sister Candy is saving every month. Not Prudence, she is letting her money earn money. To find the balance in Prudence's account after 20 years, use the lump-sum time value formula. Set *N* to 240 and the monthly periodic interest rate to 0.01. Set *PV,* the beginning wealth, to \$58,335. Notice that this number was the answer for *FV* from the savings stage. Now this same number is *PV* for the retirement stage. There are no cash flows in the middle for Prudence:

$$FV = \$58{,}335(1.01^{240})$$

$$= \$635{,}415\ .$$

Prudence has almost two-thirds a million dollars when she retires. The time line shows this number as *\$z.* The \$635,415 represents a beginning wealth that finances a series of monthly withdrawals over 25 years (300 months). To find the amount of each withdrawal, use the constant annuity formula (equation 5.5) and set *N* to 300, the monthly periodic interest rate to 0.01, and *PV* to \$635,415. Because the account is drawn down to zero, *FV* equals \$0. Solve for *CF*.

$$CF = \$635{,}415 \Big/ \left\{\frac{1 - 1.01^{-300}}{.01}\right\}$$

= *$6,692*

Prudence makes wise investments, saving $250 per month for ten years enabling her to retire on monthly income of $6,692 for twenty-five years.

CALCULATOR CLUE 5.13 To solve this problem with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Solve for the accumulation for Candy upon retirement by typing:
250 +/– PMT 241 N 12 I/Y CPT FV .
The display shows the account balance equals $250,037 when Candy retires. Obtain the cash flow financed by this accumulation over the subsequent 300 months. While the display still shows $250,037 type
PV 0 FV 300 N CPT PMT .
The display shows Candy receives $2,633 per month during retirement.
Find the accumulation for Prudence at the conclusion of her savings stage by typing
250 +/– PMT 121 N 0 PV CPT FV .
The display shows Prudence accumulates $58,335. Compound this sum for 240 months. While the display still shows $58,335 type
PV 0 PMT 240 N CPT FV .
The display shows the account balance equals $635,415 when Prudence retires. Obtain the cash flow financed by this accumulation over the subsequent 300 months. While the display still shows $635,415 type
PV 0 FV 300 N CPT PMT .
The display shows Prudence receives $6,692 per month during retirement.

Prudence saves half as much principal as her sister Candy. Nonetheless, Prudence retires on two-and-a-half times as much income. Total contributed principal equals $30,250 for Prudence and $60,250 for Candy (that is, total contributed principal equals $250 x #deposits, Prudence makes 121 deposits and Candy makes 241). Total withdrawals sum to about $2 million for Prudence and $0.8 million for Candy (that is, the sum of withdrawals equals 300 x monthly income; Prudence gets $6,692 per month and Candy gets $2,633). Total accumulated interest, and not principal, represents almost all the money on which these retirees will live. The money made by their labor is less than the money made by their money! They needed to work and save, though, to earn the seed money for investing.

The preceding example vividly illustrates the big reward for saving early. The rule of 72 allows insight about why saving early is so important. At a 12 percent annual rate, the rule of 72 says that money doubles about every 6 years. Suppose a beginning sum is $30,000. The approximate value per doubling period is shown in the table below:

doubling period	number of years	wealth at end of doubling period
0	0	$ 30,000
1	6	$ 60,000
2	12	$120,000
3	18	$240,000
4	24	$480,000
5	30	$960,000

Wealth creation is not the same in every six-year period. The increase in wealth is $30,000 during the first six-year period and $480,000 during the last six year period. Roughly speaking, an individual may work and save for their entire career. And after 24 years perhaps their accumulation equals, say, $480,000. The wealth created, though, in one additional doubling period may easily exceed an entire lifetime of pension contributions. Finding an extra doubling period at the end of a career often is impossible. Saving early creates a huge difference in ending wealth. Through the magic of compound interest, your money can earn more money than your labor ever will. But you have got to give it time. Time itself is a source of value.

EXERCISES 5.4B

Numerical challengers

1. Your first monthly deposit of $170 is made today and the last one is 2 years from today. You then increase the amount of each deposit. From 2 years and one month from today until exactly 9 years from today, you deposit $280 monthly. Upon making the last deposit you close the account. The savings rate always is 5.10% compounded monthly.
1a. When you close the account, how much is the total accumulation? ©FV1a
1b. When you close the account, you withdraw the entire accumulation. How much total interest did you earn? ©FV1b .

2. You are considering two different strategies for a savings account that you intend to close when you retire exactly 26 years from today. For Strategy 1, deposit $270 per month for 5 years (first deposit today; last one exactly 5 years from today); no new deposits will be made after the end of the deposit period, but interest continues to accrue until the account is closed. For Strategy 2, you'll make your first monthly deposit exactly 5 years from today, each monthly deposit also equals $270, and you'll continue making monthly deposits for 21 years, so that you make the final deposit exactly 26 years from today when you close the account. The savings rate always is 4.60% compounded monthly. Compare the accumulations at time of retirement from the two alternative strategies. ©FV4a

3. You wish to establish an endowment fund that will provide students with a $1,900 scholarship every semiannum, perpetually. To finance the scholarships you will make a series of equal deposits into a savings account. The deposits will be made semiannually, with the first one today and the final one in 4 years. The first scholarship is to be awarded one semiannum after the last deposit. The savings rate is 7.90% compounded semiannually. How much is each deposit? ©TS1a

4. You wish to establish an endowment fund that will provide student financial aid awards every month, perpetually. To finance the scholarships you will make a series of equal deposits into a savings account. The monthly deposits will equal $2,800 each, with the first one today and the final one in 8 years. The first award is to be granted one

month after the last deposit. The savings rate is 7.10% compounded monthly. How much is each award? ©TS1b

5. Suppose an employee saves $235.17 per month for 34 years (each year there are 12 monthly deposits). The savings rate is 4.50% compounded monthly. The worker wishes to withdraw $2,090 per month, commencing exactly one month after making the last savings deposit. For how many months can they make withdrawals? ©TS2a

6. Suppose an employee saves $108.91 per month for 32 years (each year there are 12 monthly deposits). The savings rate is 6.00% compounded monthly. The worker wishes to withdraw the same amount each month for a total of 136 months, with the first withdrawal exactly one month after the last savings deposit. How much is each monthly withdrawal? ©TS2b

7. You might invest in an asset that will return after-tax cash flow to you of $3,000 per month for 5 months (first payment one month from now), followed by $3,800 per month for 8 months . You make an offer to buy the asset so that you'll get your "target" annual rate of return of 19.5% (compounded monthly). Instead, however, the seller makes a counter-offer that is $1,350 higher than your offer. If you buy at the counter-offer price, and receive the expected cash flows, what is your annual rate of return? ©PV3c

5. Amortization mechanics

Amortization means "spreading over time." Studying the nature of loans and repayment schedules involves investigating amortization mechanics. Loans represent perhaps the most useful application of time value principles. Most consumer loans and many business loans stipulate a payment that is the same each period. Each payment includes interest due that period, plus some repayment of principal. The loan payment remains constant throughout the entire life of the loan, and eventually the loan is repaid in full. The constant annuity formula in equation 5.1 governs loan mechanics. Restatement of that equation shows:

$$PV = (CF)\left\{\frac{1-(1+r)^{-N}}{r}\right\} + FV(1+r)^{-N}$$

Variable definitions are the same as before. They may be restated, however, in loan jargon.

FORMULA 5.6 Fixed payment amortized loans

$$\begin{pmatrix} Principal \\ outstanding \end{pmatrix} = \begin{pmatrix} periodic \\ loan \\ payment \end{pmatrix}\left\{\frac{1-(1+r)^{-N}}{r}\right\} + \begin{pmatrix} Balloon \\ payment \end{pmatrix}(1+r)^{-N}.$$

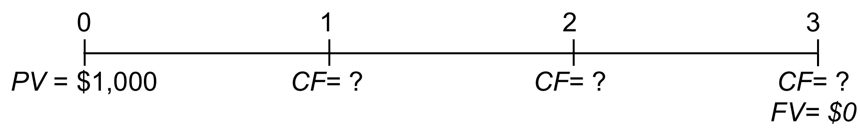
PV represents the principal that remains to be repaid with the *N* remaining payments. When a loan is new then *N* represents the number of payments over the complete loan term. *CF* represents the periodic loan payment and always is the same amount with a "fixed payment amortized loan." *FV* represents a balloon payment, that is, an extra surcharge due at the time of the last payment. The loan's periodic interest rate *r* equals the annual percentage rate divided by *m,* the compounding and payment frequency.

5.A. Partitioning the payment into principal and interest

©AM6

A loan to you by the bank is a lot like the savings account story told in the previous sections. The bank, however, makes a deposit in you. And every payment by you to the bank is just like the bank withdrawing money from its account. A loan is an asset for the lender. The asset requires an up-front cost, but generates a stream of future returns. A loan is a liability for the borrower. Nonetheless, the cash flows and balances satisfy the time value relations discussed previously.

Consider first a simplistic story in which a loan of $1,000 carries an 8 percent annual interest rate, and is repaid over three years with constant payments. The timeline below illustrates cash flows for this scenario.



The bank effectively makes a deposit in the borrower. The borrower pays 8 percent interest per year on the deposit. After one year, the loan balance of $1,000 plus accrued periodic interest of $80 (that is, $80 = 0.08 x $1,000) sums to $1,080. This sum equals the "payoff amount," also called "P&I" for principal and interest. The borrower might choose to completely cancel the loan by paying the bank $1,080. Most consumer loans provide borrowers the right to cancel loans by giving the bank the payoff amount. Thirty-year home mortgages, for example, are quite often paid off before thirty years because the homeowner-borrower sells the house, relocates, and repays the loan early. Some loans allow early repayment without penalty but some do not.

For our simplistic story, however, the loan is not repaid early. Instead, the borrower sends the bank a loan payment. The payment certainly includes the periodic interest of $80. But how much principal is repaid? The constant annuity formula provides an answer to this question. The payment, *CF*, is found by setting *r* to 0.08, *N* to 3, and *PV* to $1,000.

$$\$1,000 = (CF)\left\{\frac{1-(1.08)^{-3}}{0.08}\right\}$$

$$CF = \$1,000 / 2.5771$$

$$= \$388.03$$

A payment of $388.03 exactly repays the loan in three years.

CALCULATOR CLUE 5.14 To find the loan payment with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 1 ENTER 2nd CPT to enforce annual compounding. Then solve for the loan payment by typing: 1000 PV 3 N 8 I/Y CPT PMT

The display shows that periodic cash flow for the loan payment is $–388.03 .

The table below shows more detail about the scenario.

	t = 0	t = 1	t = 2	t = 3
beginning of period balance	$0.00	$1,000.00	$691.97	$359.30
periodic interest expense	$0.00	$80.00	$55.36	$28.73
new balance, “payoff amount”	$1,000.00	$1,080.00	$747.33	$388.03
end of period cash flow	$0.00	$–388.03	$–388.03	$–388.03
end of period balance	$1,000.00	$691.97	$359.30	$0.00

The payoff amount at the end of the first year is $1,080. The payment of $388.03 reduces the balance to $691.97 (that is, $691.97 = $1,080.00 - $388.03).

The bank partitions the loan payment into two parts. First, part of the payment pays the periodic interest expense. This part of the payment goes on the bank’s financial statements as income. The remainder of the payment, as shown in the graphic below, repays principal.

1st payment
$388.03
/ \
$80.00 $308.03
interest principal

The principal repaid with the first payment equals $308.03. The outstanding principal immediately before the first payment is $1,000. The first payment reduces outstanding principal to $691.97 (that is, $691.97 = $1,000.00 – $308.03).

Periodic interest during the second year equals the beginning of year balance ($691.97) times the 0.08 interest rate, or $55.36. The payment still equals the same $388.03, and is allocated as follows:

2nd payment
$388.03
/ \
$55.36 $332.67
interest principal

The periodic interest expense in the second payment naturally is less than in the first payment because with each passing period the outstanding principal declines. Conversely, the proportion of a payment allocated to repaying principal increases throughout loan life.

The beginning of the third year finds the outstanding principal at $359.30. Interest of $28.73 accrues so that at the end of the third year the P&I equals $388.03. The final payment perfectly repays the loan.

3rd payment
$388.03
/ \
$28.73 $359.30
interest principal

Notice that payments over the life of the loan sum to $1,164.09 (that is, $1,164.09 = 3 x $388.03). The amount borrowed equals $1,000 so total lifetime interest equals $164.09.

Very few loans actually have one payment per year. Most consumer loans stipulate monthly payments with monthly compounding of interest. A large number of corporate loans stipulate semiannual payments and semiannual compounding. The example below figures the payoff amount.

EXAMPLE 14 Find the loan's payoff amount midway through the first billing cycle ©AM10a

You borrow $15,000 at 10.30% over 4 years (monthly payments) to finance your dream pre-owned car. Exactly 20 days after taking out the loan, you want to pay it off in-full because a better financing deal appears. There is no pre-payment penalty and 31 days are in the first monthly billing cycle. Find the payoff amount (principal plus interest).

SOLUTION

Since you have not yet made any payments then the outstanding principal equals $15,000, the original amount borrowed. The monthly periodic interest rate is 0.86% (= 10.30% ÷ 12). Thus, the total interest for the entire first month equals $128.75 (= $15,000 × 10.30% ÷ 12). You only owe interest for 20 days, however, not the entire month. The bank computes intraperiod accrued interest by multiplying total monthly interest times the proportion of the month for which the loan is outstanding; in this case that proportion is 20/31st. Thus, the payoff amount is found as follows:

$$\begin{pmatrix} payoff \\ amount \end{pmatrix} = \$15{,}000 + \$15{,}000 \left(\frac{0.1030}{12}\right)\left(\frac{20}{31}\right)$$

$$= \$15{,}083.06$$

Pay the bank $15,083.06 on the twentieth day after getting the loan and the obligation is repaid in-full.

Everyday of the loan-life interest accrues. The example below shows that for many typical situations the amount of lifetime interest may actually exceed the amount borrowed.

EXAMPLE 15 Find the loan's lifetime interest

Your uncle borrows $185,000 at 10.5% to be repaid monthly over 20 years. How much total interest is paid over the life of the loan?

SOLUTION

Total interest equals the difference between amount repaid and amount borrowed. To find the total amount repaid, first compute the payment with formula 5.6 by setting N to 240 (that is, 240 months = 20 years x 12 months/year), PV to $185,000, and the monthly periodic interest rate r to 0.105 ÷ 12, or 0.0088. FV is zero because the last payment repays the loan in full.

$$\$185,000 = (CF)\left\{\frac{1-(1.0088)^{-240}}{0.0088}\right\}$$

$$CF = \$185,000 / 100.1623$$

$$= \$1,847$$

The sum of 240 payments at $1,847 is $443,280 (that is, $443,280 = 240 x $1,847). Thus, the amount repaid equals $443,280 and the amount borrowed equals $185,000. The difference is interest:

total lifetime interest = *$443,280* – *$185,000*

= *$258,280* .

CALCULATOR CLUE 5.15 To solve this problem with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Solve for the loan payment by typing:
185000 PV 240 N 10.5 I/Y CPT PMT .
The display shows the loan payment is $–1,847.

Find the total payments by multiplying the figure on the display by 240, and subsequently subtract the principal. While $–1,847 is still on the display, type
X RCL N = +/– – RCL PV = .

The display shows total lifetime interest is $258,280.

On many mortgages the lifetime interest may be larger than the amount borrowed. Still, however, most individuals feel very fortunate to live in an economy where borrowing is possible. In undeveloped economies it may be impossible to borrow for purchasing a large asset, such as a house. Financial markets create possibilities.

EXAMPLE 16 What is this payment's allocation to P&I ©AM3d

This morning you mailed your 18th monthly payment on your car loan. The original loan was for $12,500 at 9.25% for 48 months. Here are three questions. (i) How much of this morning's payment is interest? (ii) What is the outstanding balance after this morning's payment is credited to the loan? (iii) How much interest to-date has been paid?

SOLUTION

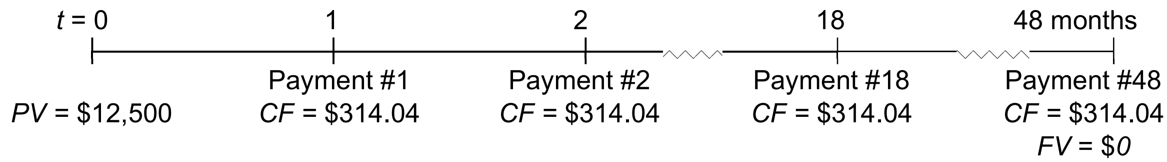
Again the first step is to find the payment. *N* equals 48, the monthly rate *r* is 0.095 ÷ 12 or 0.0079, *PV* is $12,500 and *FV* is zero.

$$\$12,500 = (CF)\left\{\frac{1-(1.0079)^{-48}}{0.0079}\right\}$$

$$CF = \$12,500 / 39.8039$$

$$= \$314.04$$

The timeline below shows the lifetime cash flows for the loan.



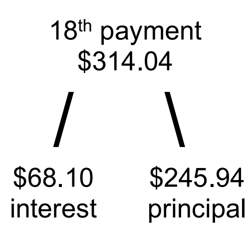
To determine how much of the eighteenth payment is interest requires multiplying the monthly interest rate times the beginning of month balance. In other words, the first step is to find the loan balance at the beginning of the eighteenth month. One ridiculous procedure for determining the outstanding principal is to work through the amortization table for 17 months. This approach works because each of the 17 payments is partitioned into principal and interest. Each month the outstanding principal is marked down by the amount of repaid principal. This backward-looking approach, though, is not the best way.

A forward-looking approach posits that the beginning of the eighteenth month occurs immediately after the seventeenth payment is made. At this point in time, 31 payments remain (that is, 31 = 48 – 17). Discounting these 31 remaining payments removes the interest component, and the present value equals the outstanding balance. Find the outstanding principal when 31 payments remain by using equation 5.6 wherein *CF* equals $314.04, *N* equals 31, and *r* equals 0.095 ÷ 12, or 0.0079. Compute *PV*.

$$PV = (\$314.04)\left\{\frac{1-(1.0079)^{-31}}{0.0079}\right\}$$

$$= \$8{,}602.63$$

The outstanding principal equals just over eighty-six hundred dollars immediately after the seventeenth payment is made, that is, when there remain 31 payments. The periodic interest accruing during the subsequent month is $68.10 (that is, $68.10 = $8,602.63 x 0.095 ÷ 12). Partitioning of the eighteenth payment looks as follows:



CALCULATOR CLUE 5.16 To solve this problem with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Then solve for the payment by typing:
12500 PV 48 N 9.5 I/Y CPT PMT .
The display shows the loan payment is $–314.04. Find the principal outstanding by setting N to the number of payments remaining at the beginning of the eighteenth period. While $–314.04 is still on the display, type
31 N CPT PV .
The display shows the outstanding principal is $8,602.63 . Use the arithmetic keys and multiply the value on the display by the periodic rate to obtain the periodic interest. While $8,602.63 is still on the display, type

X .095 ÷ 12 = .
The display shows the periodic interest is $68.10.

The second question asks for the outstanding balance after the eighteenth payment. The $8,602.63 beginning of month balance goes down by $245.94 to equal $8,356.69 at the end of the month. The forward-looking approach that obtains the same answer finds the present value of the 30 remaining payments (that is, 30 = 48 – 18):

$$PV = (\$314.04)\left\{\frac{1-(1.0079)^{-30}}{0.0079}\right\}$$

$$= \$8,356.69$$

CALCULATOR CLUE 5.17 Find the principal outstanding when 30 payments remain by typing
30 N CPT PV .
The display shows the outstanding principal is $8,356.69 .

The third question asks how much interest to-date has been paid. Recognize that the original loan balance is $12,500 and after eighteen payments the balance is $8,356.69. The total principal repaid therefore equals $4,143.31 (that is, $4,143.31 = $12,500 – $8,356.69). Eighteen payments sum to $5,652.71 (that is, $5,652.72 = 18 x $314.04). The interest to-date is the difference between total payments and principal repaid, or $1,509.41 (that is, $1,509.41 = $5,652.71 – $4,143.31).

CALCULATOR CLUE 5.18 While $8,356.69 is still on the display, find the total principal repaid, and then total interest, by typing:
+/– + 12500 = +/– + RCL PMT +/– X 18 = .
The display shows total interest-to-date is $1,509.

5.B. Re-pricing loans: book versus market value

The loan's outstanding principal is its book value. The book value may be computed by either of two procedures discussed previously. Procedure one relies on amortization mechanics to partition each payment into periodic interest and principal repayment. This procedure then steps through each payment and writes-down the outstanding balance by the amount of principal repaid. This first procedure is backward-looking and inefficient. The second procedure relies on the constant annuity time value formula to discount remaining payments. The discounting process, that is division by one plus the rate raised to an exponent, takes out the interest from all remaining cash flows. With this forward-looking procedure the easy to compute sum of discounted cash flows equals the outstanding principal.

Loans are assets, and in today's sophisticated financial markets selling loans is commonplace. Banks and other mortgage originators lend money to prospective homeowners. The homeowner signs a legal document describing the rights and obligations of each party. The homeowner gets the money and buys the house. The lender gets the legal loan document. The loan document entitles the loan's owner to collect payments of an agreed upon amount for the next fifteen or thirty years.

The lender often holds on to the loan and receives payments for the next few decades. More commonly, however, the lender sells the loan document. Sometimes the loan sells at its book value. Sometimes it doesn't. Competitive forces determine the loan's market price. The constant annuity time value formula describes the relation between the market price and expected cash flows. Consider the following example.

EXAMPLE 17 Loan's book and market value ©AM4c

This morning your bank received the 26th monthly payment on a $250,000 thirty-year 9 percent mortgage. Your bank may continue to receive payments for the next 28 years or so. The bank, however, has an immediate need to raise cash because demand for local construction loans is booming. They thus choose to sell the loan in order to raise cash. Investors in the national market that buy similar mortgages are willing to accept a 7.75 percent annual rate of return. What are the loan's book value and market value?

SOLUTION

First we need to specify the loan's payment stream. At the original conditions *N* equals 360 (that is, 360 months = 30 years x 12 months/year), the monthly rate *r* is 0.09 ÷ 12 or 0.0075, *PV* is \$250,000 and *FV* is zero.

$$\$250,000 = (CF)\left\{\frac{1-(1.0075)^{-360}}{0.0075}\right\}$$

$$CF = \$250,000 / 124.2819$$

$$= \$2,011.56$$

The initial conditions stipulate that 360 payments of \$2,011.50 completely repay the 9 percent \$250,000 loan.

To find the outstanding principal immediately after receipt of the 26th payment, find the present value of the remaining payments. Set *N* to 334 (that is, 334 = 360 – 26) and solve formula 5.1 for *PV* when *r* is 0.0075 and *CF* is \$2,011.56:

$$PV = (\$2,011.56)\left\{\frac{1-(1.0075)^{-334}}{0.0075}\right\}$$

$$= \$246,095.81$$

Note in passing that the borrower has repaid a little less than \$4,000 even though the bank has received payments totaling more than \$52,000 (that is, 26 x \$2,011 > \$52,000). That's a lot of interest!

The book value of the loan is its outstanding principal and equals \$246,095.81. The market value is found by discounting the cash flow stream with the rate of return that investors are willing to accept. Today's investors are willing to accept 7.75 percent on this loan. This suggests that interest rates today are lower than 26 months ago when they were 9 percent. As everyone knows, interest rates rise and fall over time.

Find the present value by slightly modifying the preceding equation. Keep *N* at 334 and *CF* at \$2,011.56. Change the rate, however, so that *r* equals 0.0775 ÷ 12, or 0.0065:

$$PV = (\$2,011.56)\left\{\frac{1-(1.0065)^{-334}}{0.0065}\right\}$$

$= \$275,191.22$

CALCULATOR CLUE 5.19 To solve this problem with the time value functions on the *BAII Plus©* type 2nd FV to clear the time value memories. Type 2nd I/Y 12 ENTER 2nd CPT to enforce monthly compounding. Solve for the payment by typing:
250000 PV 360 N 9 I/Y CPT PMT .
The display shows the loan payment is $–2,011.56. While $–2,011.56 is still on the display, find the principal outstanding after 26 payments are made by typing
RCL N – 26 = N CPT PV .
The display shows the loan's book value is $246,096 . Find the loan's market value with an annual market discount rate of 7.75 percent by typing
7.75 I/Y CPT PV .

The display shows the loan's market value is $275,191.

The investor is willing to purchase the loan from the bank for more than $275 thousand dollars. The investor is not necessarily crazy, either, to pay this much for a loan that was originally $250,000. Purchasing the loan, after all, entitles the loan's owner to receive 334 payments of $2,011.56; the sum of payments is $671,859 (that is, $671,859 = 334 x $2,011.56). If a 7.75 percent rate of return represents a good deal in the current economic setting, then maybe the investment makes a lot of sense. The mortgage investor should exercise caution, however, because the borrower might come in the door tomorrow and payoff the loan's outstanding principal of $246,095.81 in which case the investor suffers an immediate $29K loss.

EXERCISES 5.5

Concept quiz

1. Is it generally better for a household to borrow with a short-term or with a long-term loan (assume that fees, interest rates, and all else are equal)?

2. A homeowner borrowing $185,000 at 10.5% repayable monthly over 20 years pays lifetime interest of $258,280. Is this situation so ridiculous that the government should enact legislation limiting to something reasonable the amount of interest that households pay?

Numerical quickies

3. How much is the monthly interest on a 10.30% (compounded monthly) loan with beginning of month outstanding balance equal to $9,900? ©AM1

4. How much is the payment for a loan of $139,000 with an annual interest rate of 8.10% (compounded annually) repayable over 30 years with payments due annually? ©AM2

5. You borrow $20,000 at 9.60% over 3 years (monthly payments) to finance your dream pre-owned car. Exactly 15 days after taking out the loan, you want to pay it off in-full

because a better financing deal appears. There is no pre-payment penalty and 31 days are in the monthly billing cycle. Find the payoff amount (principal plus interest). ©AM10a

6. Your search for a new car surely depends on the monthly payment that you can afford. The absolute maximum income that you can allocate to your car payment is $325 per month. If the loan's annual percentage rate is 8.10%, find the most that you can afford to borrow. ©AM6

7. You have just bought a house by borrowing $260,000 at a 7.90% annual interest rate (compounded monthly) repayable with fixed payments over 35 years. When finally in the far-off future you make your last payment, how much of that last payment will be principal and how much will be interest? ©AM9c

8. Your friend is taking out a mortgage for $153,000 at 9.80% repayable with monthly payments over 25 years. She respects your financial expertise and asks "how many payments will I have to make before I reduce the principal balance by half its original amount." You pull out your calculator, and tell her the number of payments she'll make to reduce the balance by half is: ©AM5a

9. The Bank issued a home mortgage of $160,000 at 8.70% repayable monthly over 25 years. Today the bank received payment number 125 and, as a result, the Bank properly records the loan's book value equal to the outstanding balance. In order to raise cash, however, the Bank intends to sell the loan for the highest price it can get. The selling price of the loan, its market value, is set so that the loan offers the buyer a rate of return equal to 9.70%; this is slightly greater than the prevailing interest rate on new and similar loans. How does the loan's book value compare to its market value? ©AM4c

Numerical challengers

10. You borrow $30,000 at 7.50% over 4 years (monthly payments) to finance your dream pre-owned car. Exactly 25 days after making payment number 3, you want to pay it off in-full because a better financing deal appears. There is no pre-payment penalty and 31 days are in the monthly billing cycle. Find the payoff amount (principal plus interest). ©AM10c

11. Suppose to purchase a car you borrow $20,000 repayable monthly over 4 years at an annual percentage interest rate of 9.80%. Contrast the interest paid during the first and second halves of the loan life. ©AM8

12. The Company borrowed $170,000 at 9.60% to be repaid monthly over 15 years. They just remitted payment number 78.
12a. How much interest-to date has been paid? ©AM3d
12b. How much total interest is scheduled to be paid over the life of the loan? ©AM3c
12c. How much of the next monthly payment is principal repayment? ©AM3i

ANSWERS TO CHAPTER 5 EXERCISES

EXERCISES 5.1

1. Rule 4.3 from the previous chapter applies to all time value formulas. When the cash flows are in current dollars then the proper discount rate is the nominal rate. When the cash flows are in constant dollars then the proper discount rate is the precise real rate. Consider what this means when each cash flow is exactly the same number.

The numbers represent current dollars when each period exactly the same amount is paid (or received). This occurs, for example, when someone deposits $125 monthly into a savings account, or when a loan requires a $400 monthly payment, etc. Because these are current dollars the proper discount rate is the nominal rate. Any computations of *PV* or *FV* represent current dollar answers.

The numbers represent constant dollars when each period the purchasing power of the cash flows is exactly the same. This occurs, for example, in a long-run plan when someone expects that deposits may rise over time due to effects of inflation and a rising salary. Maybe deposits begin at $125 monthly but next year, assuming 4 percent inflation and salary-hike, deposits will be 4 percent higher. Current dollar deposits rise but constant dollar deposits are constant. When entering these constant dollar deposits as *CF* then the proper discount rate is the precise real rate. Any computations of *PV* or *FV* represent constant dollar answers. To convert a constant dollar answer into a current dollar answer requires inflating the number by the inflation rate, exactly like inflating a current price to find the future price.

EXERCISES 5.2A

1a. ©FV10a Apply formula 5.2 and find FV = \$2,600 × (1.0990^{10} – 1) ÷ 0.0990, which equals \$41,239.
1b. *Total contributed principal* equals \$26,000 (= 10 × \$2,600) so therefore *Total interest* equals \$15,239 (= \$41,239- \$26,000).

2. ©FV7 If the first deposit were exactly one year ago and last one today there would have been 13 deposits (first deposit 12 months ago, 2nd 11 months ago, …, 12th one month ago, 13th today). With first deposit 9 years ago there have been 109 total deposits (= 12 × 9 + 1). The monthly periodic rate is 0.82% (= 9.80% ÷ 12). Apply formula 5.2 and find FV = \$50 × (1.0082^{109} – 1) ÷ 0.0082, which equals \$8,735 (reduce rounding errors on your calculator by storing settings or using the time value memories).

3. With first deposit 15 years ago there have been 61 total deposits (= 4 × 15 + 1). The quarterly periodic rate is 1.625% (= 6.50% ÷ 4). Apply formula 5.2 and find FV = \$1,250 × (1.01625^{61} – 1) ÷ 0.01625, which equals \$128,709. The total contributed principal is \$76,250 (= 61 × \$1,250), which means that total interest equals \$52,459 (= \$128,709 – \$76,250).

4. This problem is just like the one from the opening paragraph of Section 2 (preceding subsection 2A). Apply formula 5.2 where *PV* is \$12,490 and *CF* is \$1,450 and *r* is 0.0570 and *N* is 9. Find FV = {\$12,490 × 1.0570^{9} } – {\$1,450 × (1.0570^{9} – 1) ÷ 0.0570}; which is \$4,113. Read *Calculator Clue 5.1* to read more about signage for this problem.

5. The monthly periodic rate is 0.53% (= 6.40% ÷ 12). There are 144 withdrawals (= 12 × 12). Apply formula 5.2 to find FV = {\$8,290 × 1.0053^{144} } – {\$70 × (1.0053^{144} – 1) ÷ 0.0053}; which is \$2,725.

EXERCISES 5.2B

1. The monthly periodic rate r is 0.425% (= 5.1% ÷ 12). *PV* equals $0 because there is no on-going balance prior to the first deposit. There are 5 deposits, *CF,* equal to $430. Apply formula 5.2 to find the account balance immediately after the last deposit: *FV* = $430 × ($1.00425^5$ – 1) ÷ 0.00425; which is $2,168. Now leave that sum alone while it compounds for 252 months (= 12 × 21) at 1% per month (= 12% ÷ 12). Use the lump-sum formula: *FV* = $2,168 × 1.01^{252} which is $26,614.

2. The monthly periodic rate r is 0.825% (= 9.90% ÷ 12). *PV* initially equals $2,600. There are 16 deposits, *CF,* equal to $230 and the 1st one occurs 10 months from today, the 2nd one in 11 months, 3rd one in 12 months, …, and 16th in 25 months. Note that *PV* compounds for 9 months to become $2,800 (=$2,600 × 1.00825^9) exactly one period before the first annuity cash flow. Apply formula 5.2 to find *FV* = {$2,800 × 1.00825^{16} } – {$–230 × ($1.00825^{16}$ – 1) ÷ 0.00825}; which is $3,193 + $3,917; which is $7,110. Note that both *PV* and *CF* increase *FV.*

3. The monthly periodic rate r is 0.833% (= 10% ÷ 12). *PV* initially equals $2,200. There are 16 withdrawals, *CF,* equal to $150 and the 1st one occurs 10 months from today, the 2nd one in 11 months, 3rd one in 12 months, …, and 16th in 25 months. Note that *PV* compounds for 9 months to become $2,371 (=$2,200 × 1.00833^9) exactly one period before the first annuity cash flow. Apply formula 5.2 to find *FV* = {$2,371 × 1.00833^{16} } – {$150 × ($1.00833^{16}$ – 1) ÷ 0.00833}; which is $2,707 – $2,556; which is $151. Note that both *PV* and *CF* are positive. *PV*, however, increases *FV* whereas *CF* is subtracted away and decreases *FV.*

4. ©FV6 The monthly periodic rate r is 0.55% (= 6.6% ÷ 12). *PV* initially equals $5,800. The 1st deposit occurs 17 months from today, the 2nd in 18 months, 3rd in 19 months, …, and in 36 months the 20th deposit occurs (=36 – 17 + 1). Note that *PV* compounds for 16 months to become $6,332 (=$5,800 × 1.0055^{16}) exactly one period before the first annuity cash flow. Apply formula 5.2 to find that *FV,* the balance immediately after the last deposit is $11,704 (={$6,332 × 1.0055^{20} } – {$–220 × ($1.0055^{20}$ – 1) ÷ 0.0055}). Note that *PV* and *CF* have opposite sign yet both increase *FV.* Apply the lump-sum relation to the balance for another 48 months (= 84 – 36) and find that in 7 years *FV* equals $15,229 (={$11,704 × 1.0055^{48}).

5. First find the amount of each deposit that accumulates $4,400 when the APR is 7.10%. The quarterly periodic rate is 1.78% (= 0.0710/4). The number of deposits is 41 (= 4 x 10 + 1). Find that $4,400 = Dep x {($1.0178^{41}$ - 1)/0.0178}, or Dep = $73.87. Next find the actual APR given that the actual accumulation is $5,180 (= $4,400 + $780). From $5,180 = $73.87 x {(($(1+APR/4)^{41}$ - 1)/(APR/4)}, find that APR = 10.0%.

EXERCISES 5.3A

1. ©PV8 Apply formula 5.1 in which *CF* equals $1,200 and *N* is 8. The periodic rate r is 8.8%. *FV* is the value of the asset immediately after delivering the last cash flow and equals zero. Solve and find that *PV* equals $6,691 (= $1,200 × (1 – 1.088^{-8}) ÷ 0.088).

2. Apply formula 5.1 in which *CF* equals $1,600 and *N*, the number of cash flows, is 10. The periodic rate r is 6.3%. *FV* is the balance after the last cash flow and equals zero. Notice that *PV* occurs one period before the first cash flow and therefore formula 5.1 is perfectly appropriate. Solve and find that *PV* equals $11,611 (= $1,600 × (1 – 1.063^{-10}) ÷ .063).

3. ©PV9 Apply formula 5.1 in which *CF* equals $1,600 and *N* is 9. The periodic rate r is 8.5%. *FV* is the value of the asset immediately after delivering the last cash flow and

equals $7,700. Solve and find that *PV* equals $13,486 (= [$1,600 × (1 – 1.085^{-9}) ÷ 0.085] + [$7,700 × 1.085^{-9}]).

4. Apply formula 5.1 in which the periodic rate *r* is 2.125% (=8.5% ÷ 4). *CF* flows out of your friend's pocket and into the account and equals $-650. The number of deposits *N* is 24. *FV* is the balance after the last deposit and equals $49,924. *FV* represent funds available to your friend as an inflow; *CF* and *FV* have opposite signs. Note also that *PV* occurs one period before the first cash flow and therefore formula 5.1 is perfectly appropriate. Solve and find that *PV* equals $18,018 {*PV* = [$-650 × (1 – 1.02125^{-24}) ÷ .02125] + ($49,924 × 1.02125^{-24})}. Find on the financial calculator with *CF*=$-650 and *FV*=$+49,925 that *PV* = $-18,018. On the financial calculator *CF* and *PV* are both positive because both flow into the account whereas *FV* flows out and its sign is negative.

5. ©PV7 Apply formula 5.1 in which the periodic rate *r* is 2.375% (=9.5% ÷ 4). *CF* equals $1,350 and *N*, the number of withdrawals, is 12. *FV* is the balance after the last withdrawal and equals $11,403. *FV* represent funds available to your friend as an inflow; *CF* and *FV* have the same signs. Note also that *PV* occurs one period before the first cash flow and therefore formula 5.1 is perfectly appropriate. Solve and find that *PV* equals $22,557 {= [$1,350 × (1 – 1.02375^{-12}) ÷ .02375] + ($11,403 × 1.02375^{-12})}. Find on the financial calculator with *CF*=$650 and *FV*=$+11,403 that *PV* = $-22,557. Sign of *PV* is opposite *CF* and *FV*.

6a. Apply formula 5.1 in which the periodic rate *r* is 1.52% (=18.2% ÷ 12). *CF* equals $2,700 (assign it a positive sign) and the number of cash flows *N* is 15. *FV,* also positive, is $95,600. Solve with formula 5.1 and find that *PV* equals $112,259 (= [$2,700 × (1 – 1.0152^{-15}) ÷ .0152] + ($95,600 × 1.0152^{-15})}).

6b. The counteroffer equals $121,859 (= $112,259 + $9,600). Solve with your financial calculator this equation, $121,859 = [$2,700 × (1 – $(1+r)^{-15}$) ÷ *r*] + ($95,600 × $(1+r)^{-15}$)}, and find that the annual rate of return equals 10.37%.

7a. Apply formula 5.1 in which the periodic rate *r* is 1.69% (=20.3% ÷ 12). *CF* equals $2,400 (assign it a negative sign) and the number of cash flows *N* is 15. *FV,* assign it a positive sign, is $78,900. Solve with formula 5.1 and find that *PV* equals $29,786 (= [$–2,400 × (1 – 1.0169^{-15}) ÷ .0152] + ($78,900 × 1.0169^{-15})}).

7b. The counteroffer equals $35,786 (= $29,786 + $6,000). Solve with your financial calculator this equation, $35,786 = [$–2,400 × (1 – $(1+r)^{-15}$) ÷ *r*] + ($78,900 × $(1+r)^{-15}$)}, and find that the annual rate of return equals 10.26%.

EXERCISES 5.3B

1. Apply formula 5.3 to find that *PV*, the amount of the deposit one period before the first withdrawal, equals $13.33 million (= $1 million ÷ 0.075).

2. The periodic monthly cash flow *CF* equals $11,667 (= $140,000 ÷ 12) and the periodic interest rate is 0.7333% (= 8.8% ÷ 12). Apply formula 5.3 to find that *PV*, the amount of the requisite endowment, equals $1.59 million (= $11,667 ÷ 0.0073).

3. With *PV* equal to $35 million and *r* at 10% the annual interest *CF* was $3.5 million *(CF = r × PV)*. It has fallen to $1.4 million (= .04 × $35 million), a decline of $2.1 million. Falling interest rates are not good for everybody.

EXERCISES 5.4A

1. Apply formula 5.4 in which the periodic rate *r* is 1.08% (=7.0% ÷ 12). The number of deposits *N* is 49 (= 4 × 12 + 1). Beginning wealth *PV* is zero. *FV* occurs immediately after the last cash flow and therefore formula 5.4 (or 5.5) is perfectly appropriate. *FV* equals $10,000. Solve and find that *CF* equals $-177 {= [$0 – $10,000] ÷ [($1.0058^{49}$ – 1) ÷ 0.0058]}. Recall that in formula 5.1, as in 5.4 which is simply a rearrangement, when *PV* and *FV* are positive then a positive *CF* signals a withdrawal whereas a negative *CF* signals a deposit.

2. Apply formula 5.4 in which the periodic rate *r* is 1.08% (=13.0% ÷ 12). The number of deposits *N* is 61 (= 5 × 12 + 1). Beginning wealth *PV* is zero. *FV* occurs immediately after the last cash flow and therefore formula 5.4 (or 5.5) is perfectly appropriate. *FV* equals $12,000. Solve and find that *CF* equals $-140 {= [$0 – $12,000] ÷ [($1.0108^{61}$ – 1) ÷ 0.0108]}.

3. Apply formula 5.4 in which the periodic rate *r* is 2.0% (=8.0% ÷ 4). The number of withdrawals *N* is 20. Notice that beginning wealth *PV* occurs one period before the first cash flow, that *FV* is the balance after the last withdrawal and equals $0, and therefore formula 5.4 (or 5.5) is perfectly appropriate. Solve and find that *CF* equals $856 {= $14,000 × 1.02^{20} ÷ [(1.02^{20} – 1) ÷ 0.02]}.

4. Apply formula 5.4 in which the periodic rate *r* is 2.58% (=10.3% ÷ 4). The number of deposits *N* is 20 (= 5 × 4). Notice that beginning wealth *PV* occurs one period before the first cash flow, that *FV* occurs immediately after the last cash flow, and therefore formula 5.4 (or 5.5) is perfectly appropriate. *PV* equals $16,000. *FV* is ending wealth and equals $68,200. Solve and find that *CF* equals $-1,616 {= [$16,000 × 1.0258^{20} – $68,200] ÷ [($1.0258^{20}$ – 1) ÷ 0.0258]}. The negative *CF* (given *PV* and *FV* are positive) means that *CF* are deposits. On the financial calculator assign *PV* a positive sign since that money flows into the account, assign *FV* a negative sign since this is available to flow out of the account, and compute that *PMT* is positive – it too flows into the account.

5. Apply formula 5.4 in which the periodic rate *r* is 0.44% (=5.3% ÷ 12) for the savings account and 1.25% (=15.0% ÷ 12) for the stock fund. The number of withdrawals *N* is 216. Notice that beginning wealth *PV* occurs one period before the first cash flow, that *FV* occurs immediately after the last cash flow, and therefore formula 5.4 (or 5.5) is perfectly appropriate. *PV* equals $12,000. *FV* is ending wealth and equals $10,000. Solve and find that *CF* for the savings account equals $59 {= [$12,000 × 1.0044^{216} – $10,000] ÷ [($1.0044^{216}$ – 1) ÷ 0.0044]} and for the stock fund is $152 {= [$12,000 × 1.0125^{216} – $10,000] ÷ [($1.0125^{216}$ – 1) ÷ 0.0125]}. The positive *CF* (given *PV* and *FV* are positive) means that *CF* are withdrawals. On the financial calculator assign *PV* a positive sign since that money flows into the account, assign *FV* a negative sign since this is available to flow out of the account, and compute that *PMT* is negative – it too flows out of the account.

6. ©FV17 Apply formula 5.4 in which the periodic rate *r* is 11.20%. The number of withdrawals *N* is 5. Notice that beginning wealth *PV* of $16,000 occurs one period before the first cash flow, that *FV* of $30,000 occurs immediately after the last cash flow, and therefore formula 5.4 (or 5.5) is perfectly appropriate. Solve and find that *CF* equals $-447 {= [$16,000 × 1.1120^{5} – $30,000] ÷ [($1.1120^{5}$ – 1) ÷ 0.1120]}. The negative *CF* (given *PV* and *FV* are positive) means that *CF* are deposits. On the financial calculator assign *PV* a positive sign since that money flows into the account, assign *FV* a negative sign since this is available to flow out of the account, and compute that *PMT* is positive – it too flows into the account.

EXERCISES 5.4B

1a. The periodic rate *r* always is 0.42% (= 5.10% ÷ 12). The first stage is the annuity deposit stream of $170. The number of deposits *N* is 25 (= 2 × 12 + 1). *PV* at this point is zero, and *CF* is $-170; recall that for the formulas when *PV* and *FV* are positive then deposits are negative. Apply formula 5.2 to find the accumulation at time of last deposit for the first stage. Find that *FV* equals $4,474 {= $0 – $-170 × [$1.0042^{25}$ – 1] ÷ 0.0042]}. That sum becomes the beginning balance one period before commencement of the second annuity deposit stream of $280. The number of deposits *N* in the second stage is 84 (= 12 × (9 – 2)). *PV* at this point is $4,474 and *CF* is $-280. Solve with formula 5.2 and find that *FV* equals $34,583 {= $4,474($1.0042^{84}$) – $-280 × [$1.0042^{84}$ – 1] ÷ 0.0042]}.

1b. You contributed principal of $4,250 (= 25 × $170) in the first stage and $23,520 (= 84 × $280) in the second stage. The bank added interest to your account of $6,813 (= $34,583 – $4,250 – $23,520).

2. The periodic rate *r* always is 0.38% (= 4.60% ÷ 12). For the first stage of Strategy 1 is the number of deposits *N* is 61 (= 5 × 12 + 1). *PV* at this point is zero, and *CF* is $-270; recall that for the formulas when *PV* and *FV* are positive then deposits are negative. Apply formula 5.2 to find the accumulation at time of last deposit for the first stage of Strategy 1. Find that *FV* equals $18,515 {= $0 – $-270 × [$1.0038^{61}$ – 1] ÷ 0.0038]}. Now apply the lump-sum relation to find the accumulation after 21 additional years (252 months): *FV* =$18,515 × 1.0038^{252} ; = $48,557. Strategy 2 has only one stage. The number of deposits *N* is 253 (= 21 × 12 + 1) and *CF* is $-270. Solve with formula 5.2 and find that *FV* equals $114,993 {= $0 – $-270 × [$1.0038^{253}$ – 1] ÷ 0.0038]}. After 26 years Strategy 1 accumulates $48,557 and Strategy 2 accumulates $114,993. Strategy 2 accumulates $66,436 more than Strategy 1.

3. The periodic rate *r* always is 3.95% (= 7.90% ÷ 2). Use perpetuity formula 5.3 to find the necessary account balance one period before the first scholarship. *CF* equals $1,900 so *PV* equals $48,101 (= $1,900 ÷ 0.0395). Now apply formula 5.4 to find the deposit history that accumulates an ending wealth *FV* of $48,101. *PV* at this point is zero and the number of deposits *N* is 9 (= 4 × 2 + 1). Find that *CF* equals $-4,555 {= ($0 – $48,101) ÷ [($1.0395^{9}$ – 1) ÷ 0.0395]}. The negative sign signals that these are deposits.

4. ©TS1b The periodic rate *r* always is 0.59% (= 7.10% ÷ 12). Apply formula 5.2 to find the accumulation *FV* that results from 97 monthly deposits *N* (= 8 × 12 + 1) when *CF* equals $-2,800. *PV* equals $0. Find that *FV* equals $365,439 {= $0 – $-2,800 × [($1.0059^{97}$ – 1) ÷ 0.0059]}. Use perpetuity formula 5.3 to find the amount of monthly interest that this accumulation spins off. *CF* equals $2,162 (= $365,439 × 0.0059). The endowment fund will provide student financial aid awards of $2,162 every month forevermore.

5. ©TS2a The periodic rate *r* always is 0.38% (= 4.50% ÷ 12). Apply formula 5.2 to find the accumulation *FV* that results from 408 monthly deposits *N* (= 34 × 12) when *CF* equals $-235.17. *PV* equals $0. Find that *FV* equals $226,076 {= $0 – $-235.17 × [($1.0038^{408}$ – 1) ÷ 0.0038]}. Now apply formula 5.1 in which *PV* equals $226,076 and ending wealth *FV* equals $0 and *CF* equals $2,090. Substitution shows:

$226,076 = ($2,090) × [(1 – 1.0038^{-N}) ÷ 0.0038]}.

Use the financial calculator (or take logarithms) to find the *N* equals 139. The savings plan supports 139 monthly withdrawals of $2,090; that's about 12 ½ years – hopefully the employee planned for inflation!

6. The periodic rate *r* always is 0.50% (= 6.0% ÷ 12). Apply formula 5.2 to find the accumulation *FV* that results from 384 monthly deposits *N* (= 32 × 12) when *CF* equals $-108.91. *PV* equals $0. Find that *FV* equals $126,083 {= $0 – $-108.91 × [($1.0050^{384}$ – 1)

÷ 0.0050]}. Now apply formula 5.4 to find the amount *CF* that a beginning wealth *PV* of $126,083 supports when *N* is 136 and *FV* equals $0. Find that *CF* equals $1,280 {= ($126,083 – $0) ÷ [(1.0050^{136} – 1) ÷ 0.0050]}. The positive sign signals that these are withdrawals.

7. ©PV3c Use the CF calculator worksheet to find with a monthly periodic rate of 1.63% (= 19.5% ÷ 12) that PV = $40,397 {= $3,000(1 - 1.0163^{-5})/0.0163 + $3,800(1 - 1.0163^{-8})/(0.0163 x 1.0163^{5})}. Add $1,350 to get the actual cost of $41,747. Now find the actual APR from $41,747 = {= $3,000(1 - (1 + APR/12)$^{-5}$)/(APR/12) + $3,800(1 - (1 + APR/12)$^{-8}$)/((APR/12) x (1 + APR/12)^{5})}, or APR = 13.9%. See *Calculator Clue 5.12* for the keystrokes.

EXERCISES 5.5

1. In general, whether a short-term loan is better than a long-term loan depends on the specific situation. One thing is absolutely certain: you pay more interest with a long-term loan than with a short-term loan (all else equal). But sometimes, paying the extra interest might be better.

Collapse the story to these essentials: Say that you borrow $100 at 10%. Is it better (a) to repay $110 after one period or (b) to repay $121 after two periods? Definitely total lifetime interest is $10 with the 1-period loan and $21 with the 2-period loan. In both cases the geometric average periodic rate of return is 10%. But whether the 1-period loan is better than the 2-period loan largely depends on whether the marginal utility of money is constant. Marginal utility of money depends on what you do with the money.

The marginal utility of money is very high when income is insufficient for covering needs. The pauper on the street has very high marginal utility for money because $1 may represent the difference between life (food & shelter) and death (starvation & exposure). Millionaires generally have relatively low marginal utility for money. For most households, however, life cycle factors (see table 9.4) affect marginal utility for money. Generally, marginal utility of money is highest during early and mid-career stages. Deferring loan payments into the remote future is worth the cost of interest when in the near term it means being able to afford baby food, furniture, and gas money to get to work. Marginal utility of money generally is lower during late-career stage, however, because income is highest and expenses are declining with downsizing households. Using a longer-term loan to defer loan payments until the remote future may indeed be an intelligent strategy even though lifetime interest is higher.

Determining for on-going well-established complex companies whether a short-term loan is better than a long-term loan is even more complicated because of interaction between risk and rates of return. We need more lessons about the structure of finance before considering that issue.

2. The short answer is NO. Interest rates are determined by competitive forces of supply and demand in financial markets between borrowers and lenders. Households may avoid paying any interest at all by paying cash for everything. In many lesser developed countries the financial markets are so incomplete that loans for buying houses and cars just don't exist. These cash-only societies really dampen wealth accumulation. Financial markets allow households to capitalize future income; get the house now by borrowing in financial markets and repay the loan with salary that you'll earn later. Borrowing and investing and paying interest are a lot better than the alternative.

3. The monthly interest is $84.97 (= $9,900 x 0.1030 ÷ 12).

4. Use formula 5.6 and find $139,000 = *PMT* x [(1 – 1.0810^{-30}) ÷ 0.0810]; or *PMT* equals $12,464.

5. ©AM10a The accrued interest for 15 days is $77.42 {= $20,000 x (0.0960 ÷ 12) x (15 ÷ 31)}. The payoff amount therefore equals $20,077.42 (= $20,000 + $77.42).

6. ©AM6 The periodic payment declines as the loan term increases (all else equal). Compute the payment for different terms, say 3 years, 4 years, and 5 years. Use formula 5.6 with a monthly periodic rate *r* of 0.67% (= 8.10% ÷ 12) and 36 payments (*N* for 3 years) and find: *PV* = $325 x [(1 – 1.0067^{-36}) ÷ 0.0067]; or *PV* = $10,356. For 48 months you can afford to borrow $13,287 {= $325 x [(1 – 1.0067^{-48}) ÷ 0.0067]}. For 60 months you can afford to borrow $15,991 {= $325 x [(1 – 1.0067^{-60}) ÷ 0.0067]}.

7. ©AM9c The monthly periodic rate *r* is 0.66% (= 7.9% ÷ 12) and the number of payments *N* is 420 (= 35 × 12). First find the payment by using formula 5.6: $260,000 = *PMT* x [(1 – 1.0066^{-420}) ÷ 0.0066]; or *PMT* equals $1,827.82. Second find the principal outstanding when only 1 payment remains: *PV* = $1,827.82 × [(1 – 1.0066^{-1}) ÷ 0.0066]; or *PV* equals $1,815.87. The principal repaid with the last payment is $1,815.87. The interest during the last month is $11.95 (= $1,827.82 – $1,815.87), which also may have been computed as $1,815.87 × 7.9% ÷ 12.

8. ©AM5a The monthly periodic rate *r* is 0.82% (= 9.8% ÷ 12) and the number of payments *N* is 300 (= 25 × 12). First find the payment by using formula 5.6: $153,000 = *PMT* x [(1 – 1.0082^{-300}) ÷ 0.0082]; or *PMT* equals $1,368.80. Second, use the formula to find *N* when outstanding principal equals $76,500 (= $153,000 ÷ 2): $76,500 = 1,368.80 x [(1 – 1.0082^{-N}) ÷ 0.0082]; or *N* equals 75 (use the financial calculator to solve for *N*, or take logarithms). When 75 payments remain the principal is at half its original amount. That point in time occurs after making 225 payments (= 300 – 75).

9. ©AM4c The monthly periodic rate *r* is 0.725% (= 8.7% ÷ 12) and the number of payments *N* is 300 (= 25 × 12). First find the payment by using formula 5.6: $160,000 = *PMT* x [(1 – 1.00725^{-300}) ÷ 0.00725]; or *PMT* equals $1,310. Second find the principal outstanding after 125 payments were made and 175 remain: *PV* = $1,310 × [(1 – 1.00725^{-45}) ÷ 0.00725]; or *PV* equals $129,650. Third find the *PV* when the new periodic rate equals 0.808% (= 9.7% ÷ 12) *PV* = $1,310 × [(1 – 1.00808^{-45}) ÷ 0.00808]; or *PV* equals $122,452. The loan's book value is $129,650 and its market value is $122,452.

10. The monthly periodic rate *r* is 0.63% (= 7.5% ÷ 12) and the number of payments *N* is 48 (= 4 × 12). First find the payment by using formula 5.6: $30,000 = *PMT* x [(1 – 1.0063^{-48}) ÷ 0.0063]; or *PMT* equals $725.37. Second find the principal outstanding at beginning of the fourth month, that is after three payments were made and 45 remain: *PV* = $725.37 × [(1 – 1.0063^{-45}) ÷ 0.0063]; or *PV* equals $28,376. Third find that the accrued interest for 25 days is $143 {= $28,376 × 0.0063 × (25 ÷ 31)}. The payoff amount therefore equals $28,519 (= $28,376 + $143).

11. The monthly periodic rate *r* is 0.82% (= 9.8% ÷ 12) and the number of payments *N* is 48 (= 4 × 12). First find the payment by using formula 5.6: $20,000 = *PMT* x [(1 – 1.0082^{-48}) ÷ 0.0082]; or *PMT* equals $505.33. Now find the principal outstanding when 24 payments remain: *PV* = $505.33 × [(1 – 1.0082^{-24}) ÷ 0.0082]; or *PV* equals $10,973. The principal repaid during the first half of the loan life is $9,027 (= $20,000 – $10,973) and during the second half is $10,973. The total payments during either half sum to $12,128 (= $505.33 × 12). Total interest equals $3,101 (= $12,128 – $9,027) during the first half of the loan life and $1,155 (= $12,128 – $10,973) during the second half.

12a. ©AM3d The monthly periodic rate *r* is 0.80% (= 9.6% ÷ 12) and the number of payments *N* is 180 (= 15 × 12). First find the payment by using formula 5.6: $170,000 = *PMT* x [(1 – 1.0080^{-180}) ÷ 0.0080]; or *PMT* equals $1,785.45. Now find the principal outstanding after 78 payments have been made, that is, when 102 payments remain: *PV* = \$1,785.45 × [(1 – 1.0080^{-102}) ÷ 0.0080]; or *PV* equals \$124,171. The principal repaid to-date equals \$45,829 (\$170,000 – \$124,171). The sum of all payments remitted to-date equals \$139,265 (= 78 × \$1,785.45). The interest to-date equals \$93,436 (\$139,265 – \$45,829).

12b. The sum of lifetime payments equals \$321,382 (= 180 × \$1,785.45). The lifetime interest equals \$151,382 (\$321,382 – \$170,000).

12c. Since already we know that the principal outstanding after the 78^{th} payment is \$124,171 the problem is straightforward. Next month's periodic interest equals \$993.46 (\$124,171 × 0.096 ÷ 12). Notice therefore that next month's principal repayment is \$792.09 (\$1,785.45 – \$993.46).

CHAPTER 6: TIME VALUE APPLICATION 1, CAPITAL BUDGETING BASICS

Principles for computing present values have direct and important applications. Companies and individuals, for example, might compute present values to decide whether planned expenditures are advisable. Real estate investors might compute present values to identify profitable units. Securities investors might compare present values and costs in order to identify courses of action that create wealth. Selection of the discount rate for computing present values is an important step in these analyses and lessons in later chapters focus on determination of relevant discount rates. Chapter 6 takes financing rates as given and focuses on application of present value techniques to capital budgeting analyses.

1. Alternative assessment measures and rules

The decision of whether or not a company should buy a new machine, a new building, develop and launch a new product, or perhaps even buy a new subsidiary, is always difficult. The decision-makers must decide which type of information they want to collect. Then they might collect the information by making phone calls, conducting surveys, reading reports, and doing a lot of legwork (or hiring MBAs to do it!). Ultimately, though, they must process the information and rely on keen intuition to interpret the signals.

The importance of intuition for capital budgeting decisions cannot be overstated. Two different individuals can look at the same scenario and reasonably come up with two different and opposite assessments. Some people obviously are gifted with special intuition. A lot of intuition, however, comes from experiential learning. Learning techniques for processing information, and using those techniques in situation after situation, develops keen intuition. Nonetheless, given a cash flow stream and the relevant discount rate then capital budgeting decisions often rely on 4 common assessment measures:

- average accounting rate of return
- payback period
- internal rate of return
- net present value

The average accounting rate of return suffers from several well-known biases and problems. Consequently, an in-depth discussion is not offered. In a nutshell, however, the average accounting rate of return relates a project's expected contribution to net income and its historical cost. The measure is okay over a short investment horizon when net income approximates cash flow. As the length of the project's time period increases, and as project cash flow deviates from its contribution to net income, there is a decline in the information content of the average accounting rate of return. Discussions below offer insights about the other assessment measures.

1.A. Payback period

The strength of this measure is computational ease.

DEFINITION 6.1 Payback period
The payback period is the length of time required to recover an investment's cost.

EXAMPLE 1 Simplest payback period problem
A project costs $3,000 and returns after-tax cash flow of $1,000 per year for 5 years. What is the project's payback period?

SOLUTION
The payback period is 3 years because this is the length of time it takes to recover the cost.

The preceding example illustrates the simplicity of computing the payback period. The example below enables a little more insight about the measure.

EXAMPLE 2 Payback period for uneven cash flow stream
A capital investment costs $46,000. Your data suggests the investment generates after-tax cash flow of $2,500 per month throughout the first year, $1,500 per month throughout the second year, and $500 per month throughout the third year. What is the project's payback period?

SOLUTION
The payback period is the number of months required to recover the $46,000 cost. Examine the table below

month	monthly after-tax cash flow	cumulative cash flow
1	$2,500	$2,500
2	$2,500	$5,000
...		
11	$2,500	$27,500
12	$2,500	$30,000
13	$1,500	$31,500
…		
22	$1,500	$45,000
23	$1,500	$46,500

After 22 months the cash flows sum to $45,000. The cost almost, but not quite, has been recovered. By the end of the 23rd month, the cost is fully recovered.

The precise numerical value for the payback period depends on some information not given. There are two possibilities. (1) If the monthly cash flow occurs as a lump sum at the end of the month, then the payback period is 23 months. (2) If the cash flows accumulate uniformly throughout the month then the payback period is greater than 22 but less than 23 months. To compute the fractional month:

fractional period = cost not yet recovered ÷ periodic cash flow
= (46,000 - 45,000) / $1,500
= 0.67 .

The payback period would be 22.67 months.

The preceding example highlights some weaknesses of the payback period assessment measure. Decision-makers feed their intuition with the knowledge that if they spend $46,000 they will recover the cost within 23 months. But is this good or bad?

One problem with the payback period is that it doesn't render a "good vs. bad" decision rule. It is true, however, that *a short payback period is better than a long payback period (all else equal)*. As the length of the payback period increases there probably is an increase in the likelihood that something won't go as planned. An investment recovering its cost within 23 months almost surely is less risky than one recovering its cost in 10 years (all else equal).

Another problem with the payback period is that it ignores the time value of money. This shortcoming is easily corrected by computing and accumulating discounted values. This alternative measure is called the "discounted payback period."

EXAMPLE 3 Discounted payback period

For the data in example 2 above, what is the discounted payback period if the financing rate is 9.50% compounded monthly?

SOLUTION

Add a column to the table that computes the discounted value of each monthly cash flow.

month	monthly cash flow	discounted monthly cash flow	cumulative discounted cash flow
1	$2,500	$2,480	$2,480
2	$2,500	2,460	$4,540
...			
24	$1,500	1,242	44,074
25	$500	411	44,485
...			
28	$500	401	45,697
29	$500	398	46,095

The present value of $2,500 received in one month when discounted by a monthly periodic rate of .095/12ths is $2,480 (= $2,500 ÷ 1.00792). The present values for all other monthly cash flows are found analogously. Subsequently, add together all discounted cash flows to find the cumulative sum each month. The discounted payback period is about 29 months (if the cash flow accumulates daily then the discounted payback is 28.76 months (0.76 = (46,000-45,697)÷ 398).

The difference between discounted and regular payback periods for short horizons, such as 1 or 2 years, is qualitatively trivial. The difference becomes more significant as the time horizon increases.

Perhaps the biggest problem with the payback period, discounted or not, is that it ignores anything that happens beyond the payback period. For example, what would happen to the payback period for the above example if the cash flow did not terminate with conclusion of the third year. Suppose, for example, the investment were also to deliver $500,000 per month during months 37 through 48. How does this change in the cash flow stream affect the computation of the payback period? The answer: absolutely not at all! The payback period still would be about 23 months because that is how long it takes to recover the cost of $46,000. A big shortcoming of the payback period, especially when cash flows are not always the same size, is that its computation is independent of all on cash flows occurring after the payback point is reached. Significant cash flows might occur after the payback period elapses, however, and a good analysis should account for them.

Because the payback period is easy to apply, many rules-of-thumb are based on it. Some companies allow workgroups to make purchasing decisions, for example, when payback periods are less than one year. Payback periods longer than a year, however, might require approval from management higher-up. Regardless, more sophisticated assessment measures often are needed.

1.B. Internal rate of return ("IRR")

The *IRR* is the most popular capital budgeting assessment measure in use. It has a strong intuitive foundation and relies on a solid theoretical framework. This measure was originally introduced by legendary John Maynard Keynes in his best seller *The General Theory of Employment, Interest, and Money* (1936). Although Keynes called this measure the "marginal efficiency of capital", we know it today as the *IRR*. The measure is so widely used that most financial calculators name a button after it!

The *IRR* is the discount rate at which the present value of cash inflows equals the present value of cash outflows. The formula used for computing the *IRR* is

FORMULA 6.1 Internal rate of return, expanded version

$$\sum_{t=0}^{N} \frac{CF_t^{outflows}}{(1+IRR)^t} = \sum_{t=0}^{N} \frac{CF_t^{inflows}}{(1+IRR)^t}$$

Typically, the cash flow streams are known numbers, and the *IRR* is the only unknown variable in the equation. Quite often all the outflows occur at the beginning of a project *(t=0)* and simply equal the investment's cost, in which case the *IRR* definition is

FORMULA 6.2 Internal rate of return, concise version

$$cost = \sum_{t=0}^{N} \frac{CF_t^{inflows}}{(1+IRR)^t}$$

The *IRR* usually cannot be isolated by itself on the left-hand-side. Instead, calculator or spreadsheet programs use a trial-and-error algorithm to find a value for the *IRR* satisfying the equality. While calculators and computers efficiently solve for the *IRR*, solving by-hand is often impractical.

The following decision rule guides usage of the *IRR*.

RULE 6.1 When the IRR says good deal!
The *internal rate of return* is the discount rate at which the present value of cash inflows equals the present value of cash outflows. A project creates economic profit when its *IRR* exceeds the financing rate.

Projects with a high *IRR* can be financed profitably at normal rates. At low financing rates, of course, they make even more money.

EXAMPLE 4 Simplest *IRR*
Reconsider the project that costs $3,000 and returns after-tax cash flow of $1,000 per year for 5 years. What is the project's *IRR*?

SOLUTION
The *IRR* is the number that satisfies the following equality.

$$\$3,000 = \frac{\$1,000}{(1+IRR)^1} + \frac{\$1,000}{(1+IRR)^2} + \frac{\$1,000}{(1+IRR)^3} + \frac{\$1,000}{(1+IRR)^4} + \frac{\$1,000}{(1+IRR)^5}.$$

The solution for the *IRR* is tedious to obtain by hand. A financial calculator, however, easily finds the *IRR* is 19.86%. The project is profitable as long as it can be financed at a rate less than 19.86%.

CALCULATOR CLUE 6.1 The *BAII Plus* calculator solves Example 4 with the CF function. Hit CF and 2nd CLR WORK to clear any numbers previously stored in the cash flow memories. Now enter the cash flow stream for the problem. Type 3000 +/- ENTER to set the initial cash flow, CF0, equal to $–3,000. Now hit ↓, the down-arrow. This moves you to the subsequent cash flow. Hit 1000 ENTER to set CF1, cash flow one, at $1,000. Again hit ↓. Because you will receive $1,000 for five months, hit 5 ENTER to set the frequency for the first cash flow at five periods. The display shows F01=5. To compute the internal rate of return for the cash flow stream stored in the CF memories, hit IRR CPT . The answer of 19.86% appears on the display.

Because the cash flow each period is always the same number the problem also can be solved with the time value functions. Type 2nd FV to clear the time value memories, and 2nd I/Y 1 ENTER 2nd CPT to set for annual compounding. Solve the preceding problem by typing:
3000 +/- PV 1000 PMT 5 N CPT I/Y

The answer of 19.86% appears on the display.

EXAMPLE 5 *IRR* for an uneven cash flow stream
Reconsider the investment that costs $46,000 and generates after-tax cash flow of $2,500 per month throughout the first year, $1,500 per month throughout the second year, and $500 per month throughout the third year. What is the project's *IRR*?

SOLUTION
The *IRR* is the number that satisfies the following equality

$$\$46,000 = \frac{\$2,500}{(1+IRR)^{1}} + \frac{\$2,500}{(1+IRR)^{2}} + \ldots + \frac{\$2,500}{(1+IRR)^{12}} + \frac{\$1,500}{(1+IRR)^{13}} + \ldots + \frac{\$1,500}{(1+IRR)^{24}}$$

$$+ \frac{\$500}{(1+IRR)^{25}} + \ldots + \frac{\$500}{(1+IRR)^{36}}$$

A financial calculator quickly solves the above equation and finds that the monthly IRR equals 1.27%. On an annual basis, this is equivalent to 15.24% annual percentage rate (multiply by 12). The project is profitable as long as the financing rate is less than a stated rate of 15.24% (compounded monthly).

CALCULATOR CLUE 6.2 Hit CF and 2nd CLR WORK to clear any numbers previously stored in the cash flow memories. Now enter the cash flow stream for the problem. Type 46000 +/- ENTER to set the initial cash flow, CF0, equal to $–46,000. Now hit ↓, the down-arrow. This moves you to the subsequent cash flow. Hit 2500 ENTER to set CF1, cash flow one, at $2,500. Again hit ↓. Because you will receive $2,500 for twelve months, hit 12 ENTER to set the frequency for the first cash flow at twelve periods. Hit ↓ 1500 ENTER to set CF2=$1,500. Hit ↓ 12 ENTER to set F02=12 periods. Hit ↓ 500 ENTER to set CF3=$500 and ↓ 12 ENTER to set F03=12 periods. To compute the internal rate of return for the cash flow stream stored in the CF memories, hit IRR CPT. The display shows 1.2729, the periodic internal rate of return. Multiply by twelve to get the annual percentage rate of 15.24 percent. This problem cannot be solved with the time value functions because the cash flow differs across periods.

The *IRR* results in a definite yes or no decision about whether the project is profitable. The measure properly reflects the time value of money. Also, it reflects the entire cash flow stream, not simply the front-end. These tremendous strengths of the measure undoubtedly contribute to its popularity.

The *IRR* has subtle shortcomings. First, a cash flow stream might have more than one *IRR*. This occurs especially when the cash flows switch signs several times; for example, outflows followed by inflows, followed by outflows, followed by inflows, etc. With multiple *IRR*, it is not clear which one is "right". Second, the *IRR* does not reveal how much wealth an investment creates. For example, an investment whose *IRR* is 20% for a company whose financing rate is 12% certainly creates wealth. It is entirely possible, though, that an alternative project whose *IRR* is 15% creates more wealth than the one with the 20% *IRR*. In other words, the *IRR* correlates imperfectly with wealth creation. Undoubtedly, the best investments are the ones that create the most wealth. A more sophisticated assessment measure is needed.

1.C. Net present value ("NPV")

NPV equals the amount of capitalized economic profit that an investment creates. Positive *NPV* investments should be pursued because they create wealth in excess of economic costs of production. Negative *NPV* investments should be avoided because they incur economic losses. Compute *NPV* as the present value of cash inflows minus the present value of outflows:

FORMULA 6.3 Net present value, expanded version

$$NPV = \sum_{t=0}^{N} \frac{CF_t^{inflows} - CF_t^{outflows}}{(1+r)^t} \ .$$

Quite often all the outflows occur at the beginning of a project *(t=0)* and simply equal the investment's cost. For this typical scenario,

FORMULA 6.4 Net present value, concise version

$$NPV = \sum_{t=0}^{N} \frac{CF_t^{inflows}}{(1+r)^t} - cost \ .$$

EXAMPLE 6 Simplest *NPV*

Reconsider the project that costs $3,000 and returns after-tax cash flow of $1,000 per year for 5 years. What is the project's *NPV* if the financing rate is 9.5% compounded annually?

SOLUTION

The following equation depicts the *NPV* computation

$$NPV = \frac{\$1,000}{(1+.095)^1} + \frac{\$1,000}{(1+.095)^2} + \frac{\$1,000}{(1+.095)^3} + \frac{\$1,000}{(1+.095)^4} + \frac{\$1,000}{(1+.095)^5} - \$3,000$$

$$= \$3,839.71 - \$3000 \qquad .$$

$$= \$839.71$$

The *NPV* is a positive number. Thus, this project creates economic profit and is a feasible choice for the company.

EXAMPLE 7 *NPV* for uneven cash flow stream

Reconsider the investment that costs $46,000 and generates after-tax cash flow of $2,500 per month throughout the first year, $1,500 per month throughout the second year, and $500 per month throughout the third year. What is the project's *NPV* if the financing rate is 9.5% compounded monthly?

SOLUTION

The following equation specifies the project's *NPV*.

$$NPV = \frac{\$2,500}{(1+.095/12)^1} + \frac{\$2,500}{(1+.095/12)^2} + \ldots + \frac{\$2,500}{(1+.095/12)^{12}} + \frac{\$1,500}{(1+.095/12)^{13}}$$

$$+ \ldots + \frac{\$1,500}{(1+.095/12)^{24}} + \frac{\$500}{(1+.095/12)^{25}} + \ldots + \frac{\$500}{(1+.095/12)^{36}} - \$46,000$$

$$= \$48,793.20 - \$46,000$$

$$= \$2,793.20$$

A financial calculator quickly solves the above equation for the *NPV*. This project creates economic profit and is a feasible choice for the company.

CALCULATOR CLUE 6.3 To clear any numbers previously stored in the cash flow memories type CF 2nd CLR WORK . Now enter the cash flow stream for the problem: 46000 +/- ENTER ↓ 2500 ENTER ↓ 12 ENTER ↓ 1500 ENTER ↓ 12 ENTER ↓ 500 ENTER ↓ 12 ENTER .

To compute the net present value for the cash flow stream stored in the CF memories, hit NPV . The calculator prompts for the periodic financing rate. Enter the monthly rate by typing 9.5 ÷ 12 = ENTER . The display shows 0.7917, the monthly periodic financing rate. Now hit ↓ CPT . The display shows the net present value is $2,793.20. To compute NPV at any other rate, say 18 percent compounded monthly for example, type ↓ 18 ÷ 12 = ENTER ↓ CPT . The display shows NPV is $–1,231.75. The project is not profitable with an 18 percent annual financing rate.

A company making capital expenditures secures internal and external financing to make it happen. The company management surely expect that the present value of cash inflows from the future will surpass the huge *Capex*. It's a strategic, gutsy decision. The U.S. railroad industry set all time records for industry capital expenditures in 2014. CSX Corporation runs railroads nearly everywhere from North Dakota to the Gulf coast and Atlantic. CSX appears below in a snippet from table 2.1 ranked 213th sorted on *Total assets*.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
CSX	$31,782	31	$1,864	$12,026	$29,025
IEP	$31,761	60	1,025	20,661	12,681
IP	$31,528	69	1,395	29,080	21,394
JCI	$31,518	170	1,178	42,730	28,408
EOG	$30,574	3	2,197	14,233	45,835
COST	$30,283	184	2,039	105,156	48,869

SNIPPET from table 2.1 in chapter 2: CSX

CSX has $31.7 billion *Total assets* in 2013. Capital expenditures totaled $9.4 billion for fiscal years 2011 to 2014. That is a very high ratio of *Capex* to *Total assets*. During the same time period the CSX stock price rose is up 61%. Apparently the stock market agreed with company management that economic profit existed. For comparative purposes, consider the bottom row of the snippet, COST, the Costco retailer with 184,000 employees, comparable *Total assets*, a $48.9 billion market cap company. For Costco, *Capex* totaled $6.8 billion for fiscal years 2011 to

2014. That, too, represents a huge growth rate. During the same period the Costco stock price was up 70%!

EXERCISES 6.1

Concept quiz

1. Methods for assessing the profitability of a cash flow stream seem so straightforward that capital budgeting decisions seem mechanical. Explain possible reasons, then, that “the importance of intuition for making capital budgeting decisions cannot be overstated.”

Numerical quickies

2. A company pursues a cost-cutting initiative that costs $20,000 to implement. Thereafter, however, the initiative reduces after-tax costs by $5,500 per year perpetually. How long, in years, is the payback period? ©CB15 .

3. A company pursues a cost-cutting initiative that costs $23,000 to implement. Thereafter, however, the initiative reduces after-tax costs by $6,000 per year perpetually. The project financing rate is 14.7% compounded annually. How long, in years, is the <u>discounted</u> payback period? ©CB16a .

4. The Company pays $22,000 for an asset that is expected to generate after-tax cash flows at a rate of $900 per month for the first year, $800 per month for the second year, and $700 per month for the third year. How long, in months, is the investment’s payback period? ©CB1 .

5. A company pursues a cost-cutting initiative that costs $20,000 to implement. Thereafter, however, the initiative reduces after-tax costs by $3,500 per year perpetually. The project financing rate is 14.7% compounded annually. Find the project’s net present value and internal rate of return. ©CB17a .

6. The company buys an asset that costs $16,800 and returns net cash flow of $1,900 per year for 4 years, followed by $2,200 per year for 8 additional years. Find the asset’s internal rate of return. ©CB11 .

7. The company buys an asset that costs $14,300 and returns net cash flow of $1,800 per year for 5 years, followed by $2,700 per year for 5 additional years. The company financing rate is 7.4% compounded annually. Find the asset’s net present value. ©CB17b .

8. The company considers investing in an asset that costs $11,900 and returns after-tax net cash flow of $170 per month for 3 years, followed by $280 per month for an additional 3 years. The company financing rate is 10.0%. According to the decision rule for the “internal rate of return”, is this project a wise investment for the company? ©CB12 .

9. The company buys an asset that costs $11,200 and returns net cash flow of $160 per month for 3 years, followed by $270 per month for 4 additional years. The company financing rate is 10.9% compounded monthly. Find the asset’s internal rate of return and net present value. ©CB19b .

Numerical challengers

10. The bank issues a $168,000 mortgage for 25 years (monthly payments) at an annual rate of 10.40%. How long for the bank is the payback period? ©CB7 .

11. The Company pays $28,000 for an asset that is expected to generate after-tax cash flows at a rate of $1,100 per month for the first year, $1,300 per month for the second year, and $1,000 per month for the third year. The project financing rate is 15.3% compounded monthly. How long, in months, is the investment's discounted payback period? ©CB14 .

12. The bank issued a $148,000 15-year mortgage (monthly payments) with an annual interest rate of 10.40%. They just received payment number 74 and have decided to sell the loan. The buyer of the loan expects to receive an annual rate of return equal to 11.50%. For the original bank that issued the loan, what was the internal rate of return? ©CB8 .

2. Significance of NPV

Wealth creation is an incredibly important objective. Individuals, companies, and governments, all pursue this objective. Recall the discussion from early in the book about the corporate cash flow cycle. Financial market transactions capitalize economic profit when capitalists provide financing for entrepreneurs pursuing positive *NPV* investments. Positive *NPV* investments, especially ones representing expenditures for *PP&E*, exist when there is relatively strong demand for the goods and services the assets produce. Economic sectors where positive *NPV* investments exist are like "hot spots" and they attract financing. Entrepreneurs flock to the opportunity for economic profit. In short, the wants and needs of society create the potential for wealth creation — the financial markets allow entrepreneurs to capitalize it.

NPV is an important concept that merits more discussion.

2.A. NPV represents economic profit

This lesson illustrates wealth creation by a positive *NPV* investment. Recall the example above when an investment costs $3,000 and returns after-tax cash flow of $1,000 per year for 5 years. With a financing rate of 9.5% the *NPV* is $839.71. Suppose the entrepreneur finances the entire $3,000 cost of the project, and also borrows an additional $839.71. The total loan amount, that is the source of financing, therefore equals $3,839.71. The entrepreneur uses the funds as follows:

total financing
$3,839.71

/ \

$3,000 buy project | $839.71 put in pocket

The entrepreneur pays $3,000 to the vendor in order to purchase the investment. The remainder of the money is "pocketed." Perhaps the money is held in a company cash account, perhaps it is paid-out as a dividend to shareholders or as a Christmas bonus to employees, etc. The important point is that the company has this extra money to use as it sees fit. The extra money equals NPV and represents economic profit.

Suppose the lender agrees to accept loan payments equal to the amount of the project cash flows. On all outstanding principal, however, the entrepreneur must pay 9.5 percent interest. The table below illustrates amortization mechanics for this loan.

year	beginning-of-year outstanding principal	end-of-year principal & accumulated interest	project cash flow & loan payment	end-of-year outstanding principal
1	$3,839.71	$4,204.48	$1,000	$3,204.48
2	$3,204.48	$3,508.91	$1,000	$2,508.91
3	$2,508.91	$2,747.25	$1,000	$1,747.25
4	$1,747.25	$1,913.24	$1,000	$913.24
5	$913.24	$1,000.00	$1,000	$0.00

At the end of the first year, the principal plus accumulated interest equals $4,204.48 (=1.095x$3,839.71). The company receives project cash flows of $1,000 and applies it all to the loan. The outstanding principal therefore drops to $3,204.48 (=$4,204.48–$1,000). Each year additional interest accumulates, and the company uses the $1,000 project cash flow to pay down the loan. At the beginning of the fifth year, the outstanding balance of $913.24 incurs interest expense of $86.76 (=.095x$913.25), so that principal and accumulated interest is $1,000. The company receives the final project cash flow and completely repays the loan.

The project cash flows completely repay the loan. The loan is larger than the project cost, however. The economic profit equals the difference between the loan amount ($3,839.71) and the project cost ($3,000). The investment's net present value of $839.71 is economic profit. For the scenario illustrated above, the company receives the new wealth up-front by borrowing more than the investment cost. The company is said to "capitalize" the economic profit. Alternatively, if the company borrows a lesser amount, say just enough (or less) to pay for the investment, then project cash flows would exceed the loan payments. The economic profit accrues in the future and the company is said to "amortize" the wealth. Regardless of whether the company amortizes or capitalizes the wealth, undertaking positive *NPV* investments creates wealth. Financial markets capitalize the wealth when announcement of the event occurs.

2.B. Relation between NPV and IRR

The *IRR* relates quite simply to *NPV*.

RULE 6.2 Identity between NPV and IRR
The IRR for a project is the financing rate at which the project's NPV is zero.

No economic profit is created when a project is financed at a rate equal to its *IRR*. Still, however, zero *NPV* investments are worthwhile because they allow the company to capture transformation value by combining factors of production and receiving revenues equal to economic production costs. If the financing rate were lower, the project would create economic profit for the company because *NPV* is positive when the financing rate is less than the *IRR*. Conversely, *NPV* is negative and economic losses accrue when the financing rate exceeds the *IRR*.

In most situations the *NPV* and *IRR* assessment measures lead to identical qualitative decisions about profitability. Perhaps more decision-makers use the *IRR* instead of *NPV* because individuals easily compare rates of return. The *IRR* is a percentage rate measure whereas *NPV* is a dollar measure. Both assessment tools require specification of the cash flow stream, too. They both use the financing rate, although in different order. Computing *NPV* requires up-front knowledge about the financing rate. The *IRR* procedure, conversely, computes the *IRR* number without up-front knowledge of the financing rate. Yet determining whether the computed *IRR* is good or bad requires comparison to the financing rate. Thus, information requirements of the two approaches are identical. The discussion below explains, however, that the *NPV* measure quite often is more useful than *IRR*. *NPV* is therefore the most sophisticated assessment measure of profitability.

B1. The NPV profile

The "NPV Profile" is a graph showing the relation between financing rate and *NPV*. The horizontal axis spans the domain of possible financing rates. The vertical axis measures *NPV*.

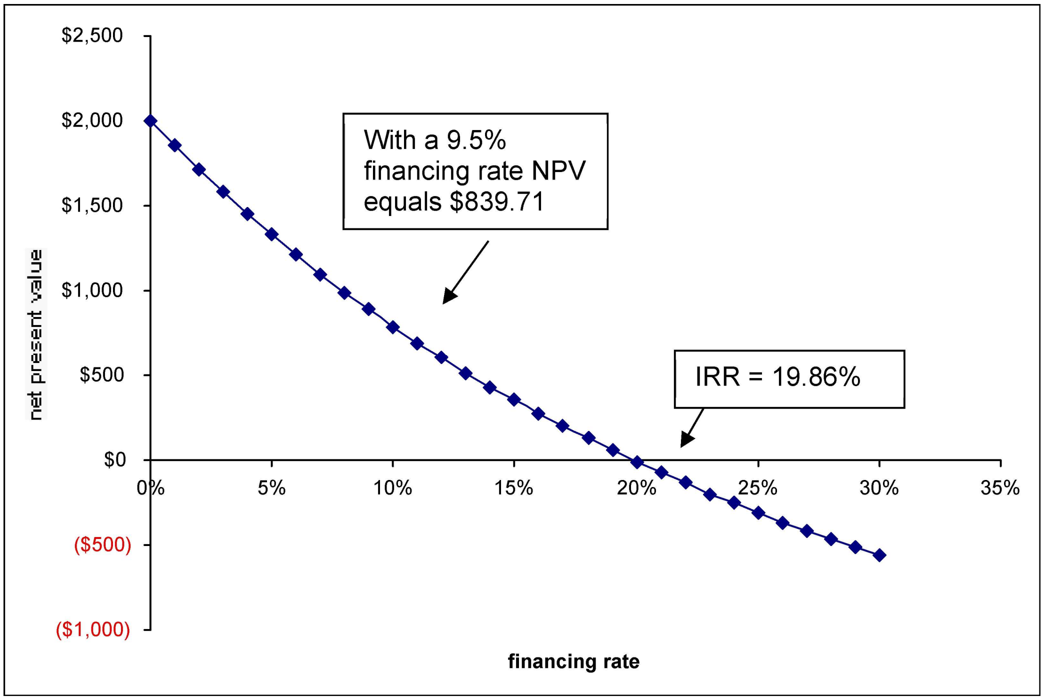


FIGURE 6.1 NPV profile for a capital budgeting project

Each dot on the line represents the net present value for a given financing rate. The maximum wealth that the project could possibly create would occur if the financing rate were zero. This, of course, is extremely unlikely. Nonetheless, the coordinates at the vertical intercept include a financing rate of zero and an *NPV* of $2,000. Notice that computing *NPV* for a zero financing rate is easy. Simply add together the cash inflows and subtract the cost:

$$NPV = \frac{\$1,000}{(1+0.0)^1} + \frac{\$1,000}{(1+0.0)^2} + \frac{\$1,000}{(1+0.0)^3} + \frac{\$1,000}{(1+0.0)^4} + \frac{\$1,000}{(1+0.0)^5} - \$3,000$$

$$= \$1,000 + \$1,000 + \$1,000 + \$1,000 + \$1,000 - \$3000$$

$$= \$2,000$$

The *NPV* declines as the financing rate increases. This occurs because rising financing rates consume the cash flows, thereby leaving less residual wealth for the company. As the chart shows, the *NPV* is $839.71 with the 9.5% financing rate.

The point where the line crosses the horizontal axis is special. The coordinates at the horizontal intercept include an *NPV* of $0 and a financing rate of 19.86%. Because the IRR equals the financing rate when *NPV* is zero, the following rule is always true.

RULE 6.3 Significance of the horizontal intercept in the NPV profile
The rate at which the *NPV* profile crosses the horizontal axis is the *IRR*.

For all financing rates less than the *IRR*, that is to the left of the horizontal intercept, *NPV* is a positive number. Conversely, *NPV* is negative for all financing rates greater than the *IRR*.

B2. Ranking competing investments

. ©CB2a .

The *IRR* formula links all cash flows within a time value framework. One problem with the *IRR*, however, occurs when comparing *IRR* for alternative projects. The ranking of *IRR* does not necessarily correspond to the ranking of *NPV*. Always the project with the biggest *NPV* creates the most wealth. But the project with the biggest *IRR* does not necessarily create the most *NPV*. Sometimes it does, sometimes it doesn't. The *IRR* correlates imperfectly with economic profit. Plotting two *NPV* profiles on the same graph easily illustrates why *IRR* ranking does not correlate with *NPV* ranking.

Compare these two projects. Project Steady costs $3,000 and returns $1,000 per year for five years. The previous section illustrates the *NPV* profile for Project Steady. Project Latebloom offers the following cash flow stream:

0	1	2	3	4	5
$–2,700	+200	+600	+1,000	+1,400	+1,800

Use the financial calculator to compute coordinates for Latebloom's *NPV* profile, and overlay it onto the one already obtained for Project Steady.

CALCULATOR CLUE 6.4 To clear any numbers previously stored in the cash flow memories type CF 2nd CLR WORK . Now enter the cash flow stream for the problem: 2700 +/- ENTER ↓ 200 ENTER ↓ ↓ 600 ENTER ↓ ↓ 1000 ENTER ↓ ↓ 1400 ENTER ↓ ↓ 1800 ENTER .

To compute the internal rate of return for the cash flow stream stored in the CF memories, hit IRR CPT . The display shows 18.21 percent. The horizontal intercept of Latebloom's NPV profile therefore includes coordinates of a zero NPV with an 18.21 percent financing rate. It intersects the horizontal axis a little to the left of Steady's NPV profile. Find the vertical intercept by adding Latebloom's cash flows and subtracting the cost. Alternatively, hit NPV and enter a rate of zero, then hit ↓ CPT . The display shows the net present value with a zero financing rate is $2,300.

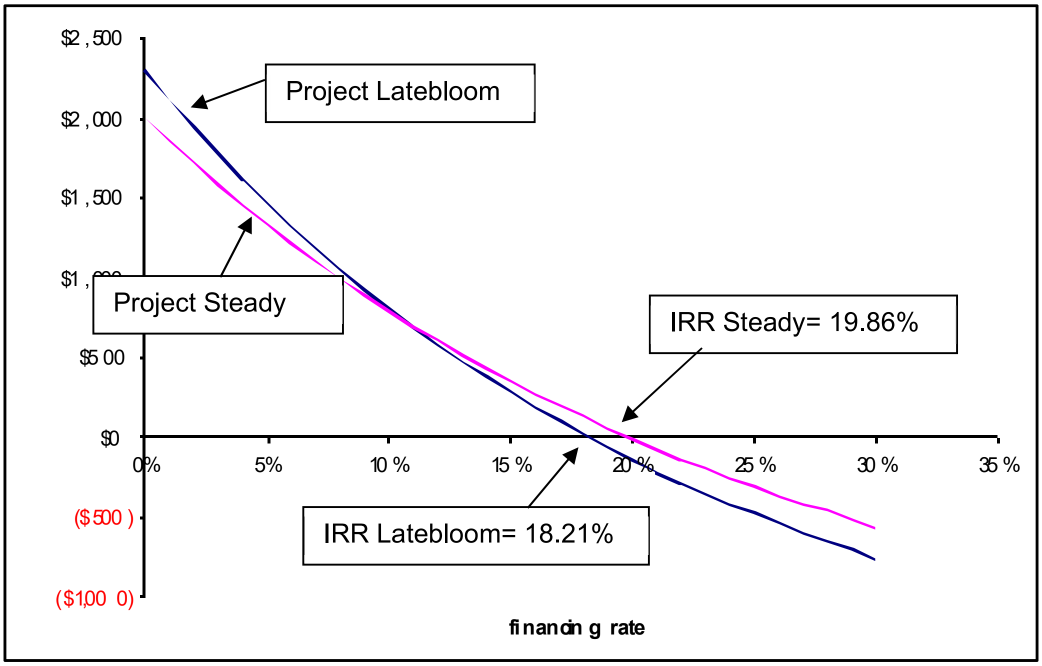


FIGURE 6.2 NPV profiles for two projects

The IRR is bigger for Project Steady than for Project Latebloom. Ranking project profitability by *IRR* is wrong, however. Always Project Steady has the biggest *IRR*. But only sometimes does it create the most wealth.

Inspect the *NPV* profile and observe that at a 5 percent financing rate the *NPV* is bigger for Project Latebloom than for Project Steady. Latebloom's line at 5 percent is above Steady's line. For a given financing rate the project whose line is on top creates the most wealth. The lines cross so the one on top changes. At low financing rates Latebloom is on top and Latebloom's *NPV* exceeds Steady's. At high financing rates, say 15%, Steady's line is on top and Steady creates more wealth.

NPV is the amount of economic profit and *NPV* depends on the financing rate. The sensitivity of a cash flow stream to a change in financing rate varies among projects. In the *NPV* profile above, for example, moving rightward along the graph shows a decline in *NPV* for both Latebloom and Steady. This decline is natural because a rising rate reduces the discounted value of future cash inflows. But Latebloom's *NPV* declines more than Steady's. Latebloom's cash flow stream is more sensitive than Steady's to the financing rate.

Differential sensitivity of cash flow streams to financing rates creates an intriguing economic situation that runs counterintuitive to first impression. People tend to think that because one cash inflow stream is better than the other in a 15 percent interest rate environment, it should be better in a 5 percent environment, too. After all, the inflows are not changing, only the financing rate is changing. Yet the preceding *NPV* profile clearly shows project profitability, and project ranking, depend explicitly on the financing rate. An entrepreneur with access to low-cost financing might conclude Project Latebloom is better than Project Steady. Another entrepreneur with access only to high-cost financing might conclude Project Steady is best. And they both are right, even though they reach opposite conclusions!

Cross-over rates

Draw two lines on a graph and they intersect unless they are parallel. *NPV* profiles for two different projects never will be parallel except in the special case where one is a linear transformation of the other. For example, the profiles are parallel when one's cash flows always equals twice the other's. That rarely happens. So two *NPV* profiles almost surely cross-over.

When *NPV* profiles cross-over there is a reversal in profitability rank.

The illustration shows three possible cross-over scenarios. In all scenarios the horizontal axis measures the financing rate and vertical axis measures the *NPV* of a project at that financing rate.

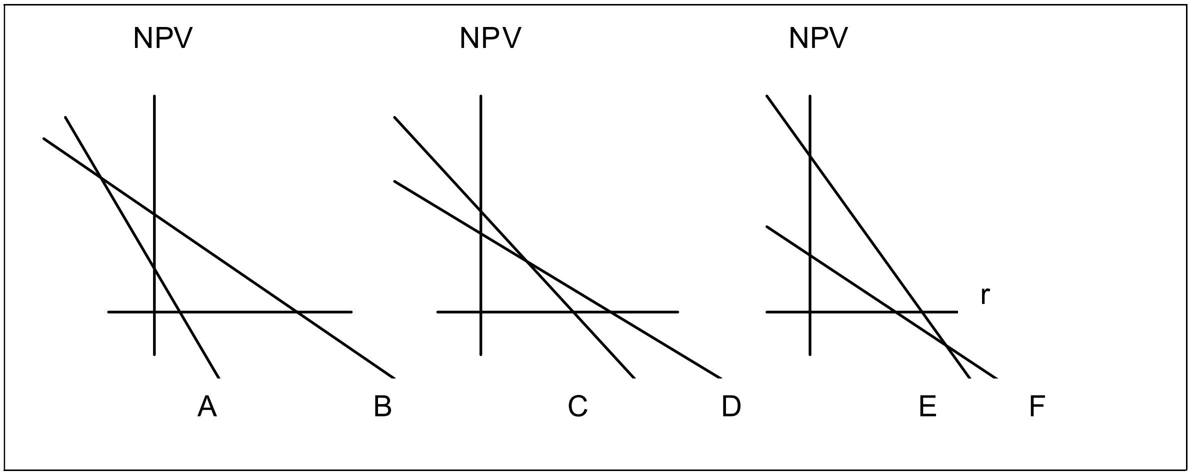


FIGURE 6.3 Potential configurations for cross-over rates

In the left-most figure the cross-over point occurs to the left of the vertical axis. The financing rate in that quadrant is negative. Financing rates cannot be negative. For all reasonable financing rates, therefore, the *NPV* is bigger for project B than *A*. *B* is more profitable than *A* for every plausible financing rate.

The center figure shows the scenario illustrated by the example about projects Steady and Latebloom. The cross-over point occurs at a rate for which the *NPV* is positive for both projects *C* and *D*. The most profitable project is *C* at relatively low financing rates. Project *D* is most profitable at rates somewhat higher than the cross-over point. Once financing rates get too high, however, both projects *D* and *C* have negative *NPV*.

In the right-most figure the cross-over point occurs at a relatively high financing rate. During the range in which *NPV* is positive, however, one fact is clear. If either project is profitable, then *E* is more profitable than *F*.

The cross-over point occurs when the *NPV* of the two projects are equal. The financing rate at the cross-over point is called the cross-over rate "*COR*". Use formula 6.4 to set the *NPV* of two projects, call them *A* and *B*, equal to each other. Then rearrange to obtain the following formula for finding the *COR*.

FORMULA 6.5 The cross-over financing rate *"COR"*

$$\text{cost}^{A} - \text{cost}^{B} = \sum_{t=1}^{N} \frac{CF_t^{\text{inflows for A}} - CF_t^{\text{inflows for B}}}{(1+COR)^t} \ .$$

In a typical problem all the cash flows and costs are known numbers. Finding the *COR* requires a trial and error algorithm identical to the one for finding the *IRR*. The following example shows that financial calculators easily find the cross-over rate.

EXAMPLE 8 Find *COR* for projects Steady and Latebloom . ©CB2a .

The table below summarizes cash flow streams for the two projects illustrated in the *NPV* profiles previously. Exactly what is the cross-over financing rate?

t =	0	1	2	3	4	5
Steady	$–3,000	1,000	1,000	1,000	1,000	1,000
Latebloom	$–2,700	200	600	1,000	1,400	1,800

SOLUTION

Substitute cash flows into formula 6.5 to obtain the following equation.

$$\$3{,}000\text{-}\$2{,}700=\frac{1{,}000-200}{(1+COR)^1}+\frac{1{,}000-600}{(1+COR)^2}+\frac{1{,}000-1{,}000}{(1+COR)^3}+\frac{1{,}000-1{,}400}{(1+COR)^4}+\frac{1{,}000-1{,}800}{(1+COR)^5}$$

$$\$300=\frac{800}{(1+COR)^1}+\frac{400}{(1+COR)^2}+\frac{0}{(1+COR)^3}+\frac{-400}{(1+COR)^4}+\frac{-800}{(1+COR)^5}$$

The solution for *COR* is found on a financial calculator as 10.62%.

CALCULATOR CLUE 6.5 To solve Example 8 clear any numbers previously stored in the cash flow memories. Type CF 2nd CLR WORK . Now enter the cash flow stream from the preceding equation:
300 +/- ENTER ↓ 800 ENTER ↓ ↓ 400 ENTER ↓ ↓ 0 ENTER ↓ ↓ 400 +/- ENTER ↓ ↓ 800 +/- ENTER .

These cash flows constitute the incremental cash flow stream associated with switching from Project Latebloom to Project Steady. That is, suppose you already were to own Project Latebloom. Now figure the effects of switching to Project Steady. At time zero pay an extra $300 to buy Project Steady (CF_0 = –300). Because of the switch you receive an extra 800 at time 1 (CF_1 = 800), an extra 400 at time 2 (CF_2 = 400), nothing extra at time 3 (CF_3 = 0), at time 4 receive 400 less (CF_4 = -400), and at time 5 receive 800 less (CF_5 = -800). To compute the internal rate of return for the cash flow stream stored in the CF memories, hit IRR CPT . The display shows 10.62 percent. This is *COR,* the financing rate at the cross-over point.

At a financing rate of 10.62% Projects Steady and Latebloom create exactly the same amount of wealth. The amount of *NPV* is found by taking the present value of either project's cash flow stream. Computations show *NPV* at the cross-over point is $731.38. A summary of profitability assessments at different financing rates yields the following table.

for this financing rate	*obtain this profitability assessment*
financing rate < 10.62%	both projects create wealth Latebloom creates more wealth than Steady
10.62% < financing rate < 18.21%	both projects create wealth Steady creates more wealth than Latebloom
18.21% < financing rate < 19.86%	Steady creates wealth Latebloom destroys wealth
19.86% < financing rate	both projects destroy wealth Latebloom destroys more wealth than Steady

EXERCISES 6.2

Concept quiz

1. True or false: A company should pursue Project L costing $100 that returns $110 in one period and is financed at 10% instead of pursuing Project H that costs $100, returns $115, and is financed at 15%.

2. Discuss how net present value from a capital budgeting project relates to wealth creation and the economic profit shown as the droplet in the cash flow cycle in chapter 1 (figure 1.3).

Numerical quickies

3. Should a company pursue Project L costing $100 that returns $110 in one period and is financed at 10% or Project H that costs $100, returns $116, and is financed at 15%. ©CB20

4. Your company is looking at an investment that today costs $4,600 and returns after-tax cash flow exactly one year, two years, and three years from today, respectively, equal to $2,000; $2,500; and $2,900 . The company intends to finance the investment at a rate of 14.2% and to repay the loan (principal and interest) with the investment cash flows as they occur. How much wealth will the investment create? ©CB4 .

5. The company must invest in either project X or Y. The company knows that X and Y would have identical net present values if the financing rate were 7.5%. The internal rate of return is 14.6% for project X and 20.2% for project Y. What is the likely relation between the company financing rate and ranking of project NPV. ©CB6 .

Numerical challengers

6. Consider the following cash flows for two mutually exclusive investments:

	t=0	t=1	t=2	t=3
A	($660)	$472	$281	$112
B	($780)	$88	$250	$738

Your boss claims that projects A and B represent exactly the same net present value for your company. You politely point out that, because of differences in cash flow timing, the only way these projects have the same net present value is if your company's actual financing rate equals one specific number. Find that rate. ©CB2b .

7. Consider the following cash flows for two mutually exclusive investments:

	t=0	t=1	t=2	t=3
A	($670)	$498	$309	$128
B	($890)	$104	$303	$919

Discuss the project ranking for various financing rates. ©CB2a .

8. Consider the following cash flows for two mutually exclusive investments:

	t=0	t=1	t=2	t=3
A	($620)	$457	$280	$115
B	($760)	$88	$255	$768

Under very special circumstances, the two projects offer exactly the same net present value. How much is that net present value? ©CB2c .

3. Incremental cash flow streams

Quite often sound financial decisions require comparison of alternative outcomes. A company trying to decide whether a new machine should replace an old machine, for example, must compare two different cash flow streams: the status quo stream if they do nothing versus the new stream if they pursue replacement. The incremental cash flow equals the change in cash flow resulting from a financial decision.

FORMULA 6.6 Incremental cash flows, general version

$$\Delta CF_t = \text{change in cash flow resulting from a decision}$$
$$= CF_t^{\text{new situation}} - CF_t^{\text{old situation}}$$

Profitability assessments of incremental cash flow streams rely on tools discussed in the previous sections.

EXAMPLE 9 Assess the profitability of refinancing a mortgage ©CB3c

Awhile ago you took out a 30-year mortgage (monthly payments) for \$130,000 at 9.25% and payment number 45 was paid this morning. You are deciding whether this afternoon you should refinance the outstanding principal by borrowing at today's lower rate of 6.75% an amount that just pays off the old loan. The new loan is for 30 years as of today. The total fees for getting the new loan equal 3% of the borrowed principal, and you will pay the fees today with funds from your savings account. Find the payback period, the *IRR*, and the *NPV* of the incremental cash flow stream resulting from refinancing the mortgage.

SOLUTION

First find the cash flow stream for the original loan from today forward. Compute the amount of each payment with the original loan from formula 5.6:

$$\$130,000 = (CF)\left\{\frac{1-\left(1+0.0925/12\right)^{-360}}{0.0925/12}\right\}$$

$$CF = \$1,069.48$$

The 45th payment is due this morning. This means that 315 payments remain. The cash flow stream for the status quo, that is if you do not refinance, appears like this:

t=0	1	2	...	315
\$0	*\$1,069.48*	*\$1,069.48*		*\$1,069.48*

Notice that the payment made this morning is an irrelevant sunk cost and is not on the time line.

Now specify the cash flow stream resulting from refinancing. The amount of the new loan equals the outstanding principal for the old loan. Find the principal outstanding when 315 payments remain from the following formula.

$$PV = \$1,069.48 \left\{ \frac{1 - \left(1 + {0.0925}/{12}\right)^{-315}}{{0.0925}/{12}} \right\}$$

$$= \$126,391.20$$

The fees equal 3% of the outstanding balance, or $3,791.74.

For this problem you choose to pay the fees with funds from your savings account. Some problems in Exercises 6.3 pay the fees by borrowing with the new loan enough to pay off the old loan plus pay the fees. That situation incurs no up-front out-of-pocket costs and doesn't require money in the savings account. That common strategy *amortizes* the fees over the life of the loan. In practice, homeowners also often *capitalize* home equity that may have accrued from inflating house prices by borrowing even more than the original loan's initial balance. They use the money for renovations, vacations, reducing credit card balances, etc.

Compute the payment for the new 30-year loan at 6.75% as follows.

$$\$126,391.20 = (CF) \left\{ \frac{1 - \left(1 + {0.0675}/{12}\right)^{-360}}{{0.0675}/{12}} \right\}$$

$$CF = \$819.77$$

The monthly payments for the new loan, $819.77, represent a significant reduction in the monthly mortgage payment. This motivates refinancing. The cash flow stream attached to the new loan includes the fees paid today and 360 monthly payments.

t=0	1	2	360
\$–3,791.74	*\$–819.77*	*\$–819.77*	*\$–819.77*

Incremental cash flows are relevant to profitability assessments. Use formula 6.6 to compute incremental cash flows.

for t = 0:	$\Delta CF_0 = \$-3{,}791.74 - 0;$	$= \$-3{,}791.74$
for t = 1 to 315:	$\Delta CF_t = \$-819.77 - (\$-1{,}069.48);$	$= \$249.71$
for t = 316 to 360:	$\Delta CF_t = \$-819.77 - 0;$	$= \$-819.77$

This time line summarizes the incremental cash flow stream for the refinancing decision.

t = 0	1	315	316	360
–3,791.74	*249.71*	*249.71*	*–819.77*	*–819.77*

The result of refinancing is an immediate cost of $3,791.74, followed by 315 monthly savings of $249.71, followed by an additional 45 monthly payments of $819.77. Notice that without refinancing there would be no payments in months 316 through 360. Refinancing extends the length of time payments must be made.

The traditional payback period is the length of time required to recover the up-front fees.

Divide \$3,791.74 by \$249.71 to compute the payback period as 15.2 months. The cost is fully recovered when the 16th payment is made. A traditional rule of thumb proclaims that mortgage refinancing is a good idea if the payback period is less than 24 months. According to that rule, refinancing makes sense.

The annual *IRR* for the refinancing decision satisfies this equation:

$$\$3,791.74 = \frac{\$249.71}{(1+IRR/12)^{1}} + \frac{\$249.71}{(1+IRR/12)^{2}} + \ldots + \frac{\$249.71}{(1+IRR/12)^{315}} + \frac{\$-819.77}{(1+IRR/12)^{316}}$$

$$+ \ldots + \frac{\$-819.77}{(1+IRR/12)^{360}}$$

Calculator Clue 6.6 finds that the annual *IRR* equals –5.79%. It conveys no economic meaning, thereby highlighting one of the subtle problems inherent with the *IRR*. Solution of the same problem in a spreadsheet finds that the annual *IRR* of +79.0% also satisfies the equation. A cash flow stream has multiple *IRR* whenever the stream switches back and forth between positives and negatives.

This is a fairly typical and realistic problem. The incremental cash flow stream, however, contains several sign reversals. The initial cash flow is negative, then 315 positive cash flows follow, and 45 negative cash flows conclude the stream. This atypical pattern throws a wrench into the mathematical process and renders the *IRR* less useful. The *NPV* is considered a more reliable assessment tool.

Assess the *NPV* of the preceding incremental cash flow stream with this equation.

$$NPV = \frac{\$249.71}{(1+\frac{APR}{12})^{1}} + \frac{\$249.71}{(1+\frac{APR}{12})^{2}} + \ldots + \frac{\$249.71}{(1+\frac{APR}{12})^{315}} + \frac{\$-819.77}{(1+\frac{APR}{12})^{316}}$$

$$+ \ldots + \frac{\$-819.77}{(1+\frac{APR}{12})^{360}} - \$3,791.74$$

The *APR* is the annual percentage rate for computing the present value. The financial calculator easily computes that with *APR* = 6.75% the *NPV* is \$27,461 and with *APR* = 9.25% the *NPV* is \$22,953. *NPV* is positive for any reasonable discount rate. This refinancing creates substantial personal wealth.

CALCULATOR CLUE 6.6 To solve this problem on the *BAII Plus©* type **2nd** **FV** to clear the time value memories. Type **2nd** **I/Y** 12 **ENTER** **2nd** **CPT** to enforce monthly compounding. Solve for the payment for the original loan by typing:
130000 **PV** 360 **N** 9.25 **I/Y** **CPT** **PMT** .
The display shows the loan payment is \$–1,069.48. While \$–1,069.48 is still on the display, hit **STO** **1** to store the original loan payment in memory 1.

Find the principal outstanding when 315 payments remain by typing 315 **N** **CPT** **PV**
The display shows the loan's book value is \$126,391.20 . While this number is on the display, compute the fees by typing **x** .03 **=** . The display shows fees equal \$3,791.74. Hit **STO** **2** to store the fees in memory 2.

Find the loan payment for the new loan by typing
360 **N** 6.75 **I/Y** **CPT** **PMT** .

The display shows the new loan payment is $–819.77. While $–819.77 is still on the display, hit STO 3 to store the new loan payment in memory 3.

Compute the amount of the monthly savings by typing

RCL 3 – RCL 1 =

The display shows the monthly savings is $249.71. While $249.71 is still on the display, hit STO 4 to store the monthly savings in memory 4.

Compute the payback period by typing

RCL 2 ÷ RCL 4 =

The display shows the payback period is 15.2 months.

Now enter the incremental cash flow stream in order to find *IRR* and *NPV.* To clear any numbers previously stored in the cash flow memories type CF 2nd CLR WORK . Now type:

RCL 2 +/- ENTER ↓ RCL 4 ENTER ↓ 315 ENTER ↓ RCL 3 ENTER ↓ 45 ENTER IRR CPT

The display shows the monthly *IRR* is a nonsensical –0.45 percent.

Find *NPV* at a 6.75% financing rate by typing

NPV 6.75 ÷ 12 = ENTER ↓ CPT .

The display shows the *NPV* is $27,461.

3.A. Choosing the proper stream to analyze

Most business capital budgeting decisions involve standard financial statement analyses. Correct analyses assess profitability by focusing on cash flow instead of net income. Recall discussions from the cash flow section of chapter 3. There are as many different cash flow streams as there are accounts for money to flow into: $CF^{\text{to shareholders}}$, $CF^{\text{to creditors}}$, $CF^{\text{from assets}}$, $CF^{\text{from operations}}$, and *Cash surplus*. Suppose that for a proposed project a business correctly forecasts all the various cash flow streams. On which stream does the company base profitability assessments? This rule summarizes the answer.

RULE 6.4 The relevant cash flow stream for profitability assessments

Capital budgeting analyses assess the profitability of incremental cash flows from assets.

Cash flow from assets identically equals the cash flow to capitalists (see formula 3.8). Consequently the proper discount rate for computing the present value of $CF^{\text{from assets}}$ for a project is the average financing rate that the company is required to pay all capitalists, creditors and shareholders alike. The company *cost of capital* is the discount rate for finding net present value of company investment opportunities and is the focus of lessons in chapter 11. For now, however, manipulate formula 3.10 specifying cash flow from assets and obtain the following formula for incremental cash flow.

FORMULA 6.7 Incremental cash flow from assets, precise version

$$\Delta CF^{\text{from assets}} = (\Delta EBITDA)x(1\text{- tax rate}) + (\text{tax rate})x(\Delta \text{Depreciation deduction}),$$

where *EBITDA* equals "earnings before interest, taxes, depreciation and amortization" (that is, sales minus variable costs, which is the same as sales minus cost-of-goods sold minus selling, general, and administrative expenses; see the discussion following formula 3.9.)

Formula 6.7 measures periodic incremental cash flow from assets, $\Delta CF^{\text{from assets}}$, that results from undertaking a project. First right-hand-side term, *ΔEBITDA,* represents change before taxes and financing costs of cash revenues minus cash costs. Multiplication of *ΔEBITDA* by *(1 – tax rate)* implies that every dollar of *EBITDA* incurs taxes equal to the tax rate. In reality, however, the company claims tax deductions. Every dollar of tax deduction reduces *Taxes due* by an amount equal to the tax rate. For example, consider a simplistic scenario in which *ΔEBITDA* is $100, the tax rate is 30%, and depreciation deductions are $25. Substitute into formula 6.7:

$$\Delta CF^{\text{from assets}} = \$100(1 - 0.30) + 0.30(\$25),$$

$$= \$100 - \$30 + \$7.50,$$

$$= \$77.50 .$$

EBITDA of $100 incurs proportional taxes of $30. In reality, though, the company does not pay $30 in taxes because they claim deductions of $25. The depreciation tax shield reduces the tax bill by $7.50; *Taxes due* equal $22.50 (= $30 – $7.50). Hence, the incremental cash flow for the company is actually $77.50 (= $100 – $22.50). Obtain the same answer of $77.50 by an alternate route, namely subtract *Depreciation* from *EBITDA* to get *EBIT* and then subtract *Taxes due* and add back in the non-cash *Depreciation* deduction (this alternate also appears in formula 3.10):

$$\Delta CF^{\text{from assets}} = \Delta EBIT x(1\text{- tax rate}) + \Delta Depreciation$$

$$= (\$100 - \$25)(1 - 0.30) + \$25,$$

$$= \$77.50 .$$

The alternative is equivalent to formula 6.7.

Depreciation deductions reduce taxable income and consequently are an important tax shield. Another important tax shield is *Interest expense*. Companies deduct *Interest expense* in order to compute *Taxable income* (see Table 2.5, for example). Right now, however, the lesson properly focuses on the role of depreciation deductions in determining incremental cash flow from assets and ignores the interest tax shield. The NPV profitability assessment procedure removes financing costs by discounting, and division by $(1 + r)^t$ is tantamount to subtraction of financing costs. The NPV procedure accounts for the interest tax shield through the company discount rate, or cost of capital, as chapter 11 discusses.

Recall that the company uses funds on *Capital expenditures* yet the IRS disallows immediate deduction of the entire expenditure for computing that year's *Taxable income*. A million dollars spent on wages is immediately subtracted from *Sales* for computing *Taxable income*, but a million dollars spent on *PP&E* is not. Instead, the government requires that companies spread the deductions over time. After all, the asset provides returns over many time periods. When U.S. corporate income taxes were first collected in 1913 the government required that businesses allocate deductions from *Capital expenditures* along a straight-line pattern over a tax life roughly equal to the asset service life. In 1945 the government completed a major assessment of how long things last: tax lives were set for typewriters, tractors, tools and dies and thousands of other assets based on engineering studies and surveys. In 1954 the government introduced alternatives to straight-line patterns, namely sum-of-years digits and declining-balance schedules. In 1981 a significant simplification to tax depreciation policy occurred when the government severed the direct link between a specific asset-type and its unique tax-life. Instead, a half-dozen "classes" were created, each with its own tax life, and each class encompasses many different types of assets.

Persistent on-going tinkering of tax depreciation policies by Congress attests to importance of this tax shield to politicians in Washington and to companies everywhere. Actual tax depreciation laws are more complex today than in 1981 because there are more asset classes and more exceptions. Furthermore, sometimes Congress enacts "temporary" policies.

For our lessons all problems suppose either straight-line schedules or the *Modified Accelerated Cost Recovery System* (“MACRS”) in Table 6.1.

	Examples of asset types in the Recovery class
3-year	Tractor units for over-the-road use.
5-year	Automobiles, taxis, buses, and trucks. Computers and peripheral equipment. Office machinery such as typewriters, calculators and copiers. Any property used in research and experimentation. Breeding cattle and dairy cattle. Appliances, carpets, furniture, etc., used in a residential real estate rental activity
7-year	Office furniture and fixtures. Agricultural machinery and equipment. Any property that does not have a class life and has not been designated by law as being in any other class.
10-year	Vessels, barges, and tugs. Any single purpose agricultural or horticultural structure Any tree or vine bearing fruits or nuts.
15-year	Certain improvements made directly to land or added to it such as shrubbery, fences, roads, and bridges.
20-year	Farm buildings (other than single purpose agricultural or horticultural structures).
27½ year	Residential rental property.
39 year	Non-residential rental property.

Tax depreciation deduction (%weight) for the Recovery class

year of use	3-year	5-year	7-year	10-year	15-year	20-year
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

TABLE 6.1 Tax depreciation schedules for the Modified Accelerated Cost Recovery System (MACRS).

Publication 946, U.S. Internal Revenue Service.

Obtain the periodic tax depreciation deduction by multiplying the "basis" times the percentage weight from Table 6.1 (or a straight-line weight of *1/L*, where *L* is the tax life). The basis represents the amount being depreciated over time. Compute the basis as the asset cost, that is the *Capital expenditure*, plus amounts paid for items such as sales tax, freight charges, and installation and testing fees. The basis does not depend on whether the asset is bought with cash from a checking account or with funds borrowed from debt or equity. The basis, in other words, reflects the use of funds not the source. The example below computes and discounts tax depreciation deductions.

EXAMPLE 10 Contrast MACRS and straight-line discounted tax depreciation deductions

The company is buying, shipping, and installing a lot of computer equipment this year. The company may choose to depreciate the basis along either the 5-year MACRS schedule or with a 5-year straight line schedule (each deduction is 1/5 the basis). Which choice is most advantageous for the company?

SOLUTION

The answer for this problem is very general irrespective of whether the cost is \$100 or \$100,000 and irrespective of whether the financing rate is 5% or 15% and irrespective of whether the tax rate is 28% or 34%. The most advantageous choice offers the highest present value of tax savings; and that is the one with the highest present value of tax depreciation deductions.

Say for simplicity that the basis is \$100 and the financing rate is 15%. With the straight-line schedule the deduction equals \$20 per year for 5 years (= \$100 × 1/5). Find the present value of the straight-line tax depreciation deductions with the constant annuity formula:

$$PV = \$20 \times \left\{ \frac{1 - 1.15^{-5}}{0.15} \right\}$$

$$= \$67.04 .$$

The basis of \$100 depreciated with a 5-year straight-line schedule provides discounted tax depreciation deductions of \$67.04 (that is, 67 cents per dollar of basis). With a 34 percent tax rate the depreciation tax shield provides discounted tax savings of \$22.79 (= 0.34 × \$67.04; that is, 22.8 cents per dollar of basis).

Compute the 5-year MACRS discounted deductions by using the weights from Table 6.1. The weight during the first year of use is 20% so the deduction is \$20. During second year of use the weight is 32% so the deduction is \$32; etc. Find the present value of the 5-year MACRS tax depreciation deductions given the 15% discount rate:

$PV = (\$20 \times 1.15^{-1}) + (\$32 \times 1.15^{-2}) + (\$19.20 \times 1.15^{-3}) + (\$11.52 \times 1.15^{-4}) + (\$11.52 \times 1.15^{-5}) + (\$5.76 \times 1.15^{-6})$,

$= \$69.02$.

The basis of \$100 depreciated with a 5-year MACRS class provides discounted tax depreciation deductions of \$69.02 (that is, 69 cents per dollar of basis). With a 34 percent tax rate the depreciation tax shield provides discounted tax savings of \$23.47 (= 0.34 × \$69.02; that is, 23.5 cents per dollar of basis).

Preceding computations show that the 5-year MACRS schedule provides more discounted tax savings than the 5-year straight-line schedule. There is an important *caveat* about this. Actual tax law generally stipulates a "half-year convention" for the first year of use because, on average, usage probably begins halfway through the year. Thus, the depreciation deduction for the first year by straight-line actually is $10 (= $100 × 1/5 × ½). Deductions in years 2 through 5 equal $20 each. Then during the sixth year of use the remaining $10 deduction is taken. This diminishes discounted tax savings by straight-line (to $62.67) and makes the *MACRS* choice even better. For simplicity, problems herein ignore the half-life convention when applying straight-line tax depreciation schedules. Table 6.1 already reflects the half-year convention for *MACRS* and weights apply as listed.

Incremental cash flows include exclusively the change induced by this project. Quite often the changes are very complex and often hard to perfectly predict. The next section considers complications. For now, however, consider examples showing a simple scenario in which an asset has one up-front cost, delivers a stream of cash flows shielded from taxes by depreciation deductions, and the asset has zero salvage value at the end of the project life.

EXAMPLE 11 Assess profitability of expansion project with straight-line, no salvage value ©CB10a .

A proposed two-year project has expected sales that begin at $28,000 during the first year and rise 8% during the second year. Start-up costs are $18,000 and variable costs equal 60% of sales. The start-up costs are depreciated to zero by straight-line over a two-year tax life. No salvageable assets remain beyond the project life. The company's tax rate is 39%, and the project's average financing rate is 14%. What are the *IRR* and *NPV* of the project's incremental cash flow stream?

SOLUTION

We are told that CF_0 = -18,000. Use formula 6.7 to find the change in cash flows caused by the project during the first and second years. For this story, the cash flows if the project is not undertaken equal zero. Thus, $\Delta CF^{\text{from assets}}$ simply equals the cash flow that the project generates.

$$\Delta CF_1^{\text{from assets}} = 28{,}000x(1\text{-}.6)x(1\text{-}.39) + (.39)(18{,}000/2) = 10{,}342$$

$$\Delta CF_2^{\text{from assets}} = 28{,}000x(1.08)x(1\text{-}.6)x(1\text{-}.39) + (.39)(18{,}000/2) = 10{,}889$$

Solve for the *IRR* from the following equation:

$$\$18{,}000 = \$10{,}342/(1+IRR)^1 + \$10{,}889/(1+IRR)^2$$

Compute that the IRR is 11.64%. Therefore, the investment is profitable as long as the firm's financing rate is less than 11.64%.

Solve for *NPV* from the following equation:

$$NPV = \$10{,}342/(1+.14)^1 + \$10{,}889/(1+.14)^2 - \$18{,}000$$

The *NPV* of the inflows is $–550, confirming that the project destroys wealth at a 14 percent financing rate.

EXAMPLE 12 Assess profitability of cost reductions with *MACRS*, no salvage value

A proposed project is expected to reduce pretax operating costs by $75,000 per year for 4 years. The project requires incremental expenses of $200,000. The company is eligible to depreciate the entire expense within the 3-year *MACRS* class. No salvageable assets remain beyond the project life. The company's tax rate is 34%, and the project's average financing rate is 12%. Find the *IRR* and *NPV* of the project's incremental cash flow stream.

SOLUTION

We are told that CF_0 = $-200,000. Use formula 6.7 to find the change in cash flows caused by the project during years 1-to-5. Notice that *ΔEBITDA* equals $75,000 per year and that the *MACRS* weights are from table 6.1.

$\Delta CF_1^{\text{from assets}} = \$75{,}000 \times (1-.34) + (.34) \times (\$200{,}000 \times 0.3333)$

$= \$49{,}500 + \$22{,}664$

$= \$72{,}164.$

$\Delta CF_2^{\text{from assets}} = \$49{,}500 + (.34) \times (\$200{,}000 \times 0.4445); = \$79{,}726$

$\Delta CF_3^{\text{from assets}} = \$49{,}500 + (.34) \times (\$200{,}000 \times 0.1481); = \$59{,}571$

$\Delta CF_4^{\text{from assets}} = \$49{,}500 + (.34) \times (\$200{,}000 \times 0.0741); = \$54{,}539.$

Solve for the *IRR* from the following equation:

$$\$200{,}000 = \$72{,}164/(1+IRR)^1 + \$79{,}726/(1+IRR)^2 + \$59{,}571/(1+IRR)^3 + \$54{,}539/(1+IRR)^4$$

Compute that the IRR is 13.26%. The investment is therefore profitable as long as the firm's financing rate is less than 13.26%.

The problem states that the financing rate is 12%. Solve for *NPV*:

$$NPV = \$72{,}164/1.12^1 + \$79{,}726/1.12^2 + \$59{,}571/1.12^3 + \$54{,}539/1.12^4 - \$200{,}000$$

The *NPV* is $5,051 confirming that the project creates economic profit with a 12 percent financing rate.

3.B. Complications to initial and terminal cash flows

Typically formula 6.7 properly specifies incremental cash flow from assets during the middle periods of the project. Complications often arise, however, during beginning and ending periods.

initial cash flow: CF_0 definitely includes the capital expenditure. Quite often, too, installation and shipping costs associate with the project. Include these additional costs in CF_0. These additional costs also increase the depreciable base on which deductions are computed. Another cost to include in CF_0 is requisite increases in *Net working capital*. Perhaps, for example, a project may require that the company increase standing inventory, receivables, or payables. Include this one time increase of *NWC* in CF_0. This cost is not included in the depreciable basis, however.

terminal cash flow: CF_N includes the incremental cash flow from assets that formula 6.7 specifies. Additionally, however, this cash flow in the last period includes the after-tax liquidation value of any assets. Also include funds released due to declines in *Net working capital* occurring at the conclusion of the project.

The following example uses the preceding principles.

EXAMPLE 13 Assess profitability of an expansion project with salvage value and working capital requirements

The Company is considering a short-term expansion into a new product line making commemorative plates for the Olympics which are scheduled to occur 4-years henceforward. The following factors weigh in the decision:

a. the plate presser costs $32,000 and may be depreciated for tax purposes along a 7-year MACRS class (weights equal 14.29%, 24.49%, 17.49%, 12.49%, 8.93%, 8.92%, 8.93%, and 4.46%)
b. installation and shipping cost $4,000
c. the project requires an increase in *Net working capital* of $3,000
d. product development and market study fees of $6,000 were spent developing the plan
e. commemorative plates sell for $30 each and variable costs are $16 per plate
f. projected sales over the next 4 years are 500 plates, 800 plates, 1000 plates, and 2000 plates (thereafter sales would be zero).
g. the plate presser loses half its market value for each year of use; it will be sold after the 4th year
h. the financing rate is 14% and the tax rate is 34%

What are the *IRR* and *NPV* of the project's incremental cash flow stream?

SOLUTION

Initial cash flow for this project equals the capital expenditure plus installation and shipping costs plus the increase in *Net working capital*. Notice the market study fees are an irrelevant sunk cost and do not contribute to the initial cash flow.

$CF_0 = -(\$32,000 + 4,000 + 3,000)\,; \quad = \$-39,000$

Computing cash flow from assets during subsequent periods requires specification of depreciation deductions. The depreciation deduction equals the MACRS weight times the depreciable basis. The weights are given in the problem set-up. The depreciable basis is given below.

depreciable basis = capital expenditure + shipping and installation costs

= $32,000 + 4,000

= $36,000

The table summarizes components of the cash flow from assets

period	*MACRS weight*	*depreciation deduction*	*sales – variable costs*
1	0.1429	$5,143	$7,000
2	0.2449	8,816	11,200
3	0.1749	6,297	14,000
4	0.1249	4,498	28,000

CALCULATOR CLUE 6.7 Obtain the deductions by multiplying $39,000 times the MACRS weights. Alternatively, the *BAII Plus* calculator solve for these deductions in a special worksheet. On the calculator, hit 2nd DEPR to enter the depreciation worksheet. Hit 2nd SET until the display shows DBX. This is the tax depreciation schedule that MACRS uses (“declining balance method with a switch to straight-line at mid-life.”) Hit ↓ 7 ENTER to set the tax life (LIF) at 7 years. Hit ↓ 7 ENTER to set the month placed in service (M01) at mid-year. Hit ↓ 36000

ENTER to set the depreciable basis (CST) at $36,000. Hit ↓ and leave salvage value (SAL) at its default value of zero. Hit ↓ 1 ENTER to set the tax year to 1, and hit ↓ and observe the display shows that the first year's depreciation deduction is $5,143. Hit ↑ 2 ENTER to set the tax year to 2, and hit ↓ to observe the second year's deduction. Repeat for years 3 and 4.

Application of formula 6.7 to the preceding numbers yields the following.

$\Delta CF_1^{\text{from assets}}$ = *7,000x(1-.34) + (.34)(5,143) = $6,369*

$\Delta CF_2^{\text{from assets}}$ = *11,200x(1-.34) + (.34)(8,816) = $10,389*

$\Delta CF_3^{\text{from assets}}$ = *14,000x(1-.34) + (.34)(6,297) = $11,381*

$\Delta CF_4^{\text{from assets}}$ = *28,000x(1-.34) + (.34)(4,498) = $20,009*

Adjust the cash flow during year 4 to reflect the release of *Net working capital* ($3,000) and the after-tax proceeds from selling the plate press. To find the latter, compute first the selling price of the asset. Because its market value declines by half each year,

Selling price = (1/2)($32,000)*(1/2)*(1/2)*(1/2)*

= $2,000

The net proceeds to the company of the asset sale take into account tax effects that occur when an asset is sold:

Net proceeds from sale = selling price – recapture taxes

Recapture taxes are computed as:

Recapture taxes = tax rate x (selling price – Remaining Book Value),

where

Remaining Book Value = original basis – accumulated deductions

Notice that if the selling price equals the Remaining Book Value then recapture taxes equal zero. Alternatively, the company must pay extra taxes if the asset is sold for more than its book value. The company receives tax relief if the asset is sold for less than its book value. Compute the asset's remaining book value after the 4^{th} deduction has been taken as follows:

Remaining Book Value = $36,000 – ($5,143 + $8,816 + $6,297 + $4,498)

= $11,246

(NOTE: The *BAII Plus* automatically computes Remaining Book Value. After finding the 4^{th} depreciation deduction as described in the preceding Calculator Clue, hit the down arrow to see the value for RBV is $11,246). Compute recapture taxes as:

recapture taxes = 0.34 x ($2,000 - $11,246),

= $–3,143

This represents tax relief. The company sells the asset for less than its book value, and this capital loss saves them $3,143 in taxes. Finally, find the net proceeds from the sale:

Net proceeds from sale = *$2,000 – ($-3,143)*

= *$5,143*

Compute the terminal cash flow during the fourth year as follows.

ΔCF_4 = *$20,009* + *$3,000* + *$5,143*

= *$28,152*

The three terms represent the cash flow from assets, the release of net working capital, and the net proceeds from the sale of the asset.

The following time line summarizes the project's incremental cash flow stream.

0	1	2	3	4
-39,000	+6,369	+10,389	+11,381	+28,152

The *IRR* for the preceding stream equals 12.9 percent and satisfies this equation.

$$\$39{,}000 = \frac{\$6{,}369}{(1+IRR)^1} + \frac{\$10{,}389}{(1+IRR)^2} + \frac{\$11{,}381}{(1+IRR)^3} + \frac{\$28{,}152}{(1+IRR)^4}.$$

Compute from this equation that *NPV* with a 14 percent financing rate equals $-1,069.

$$NPV = \frac{\$6{,}369}{(1.14)^1} + \frac{\$10{,}389}{(1.14)^2} + \frac{\$11{,}381}{(1.14)^3} + \frac{\$28{,}152}{(1.14)^4} - \$39{,}000.$$

EXERCISES 6.3

Concept quiz

1. Suppose a company plans to make a zero net present value investment in a real capital asset. Unexpectedly the federal government announces adoption of a new tax policy that, instead of spreading depreciation deductions over a lengthy tax life, instead allows an immediate deduction for the entire capital expenditure. Discuss likely effects of the policy change on the optimal company plan for capital expenditures.

Numerical quickies

2. The company has *Capital expenditure* of $40,000 that is being depreciated along the 3-year MACRS class (weights = 33.33%, 44.45%, and 14.81%, and 7.41%). They face a 30% tax rate and 9.5% financing rate. Compute the present value of tax savings resulting from the depreciation deductions for this *Capital expenditure*. ©CB23 .

3. Companies X and Y both spend $100,000 on heavy equipment. They each face a 35% tax rate and 12% financing rate. Company X elects to take straight-line tax depreciation deductions

over the next 4 years. Company Y elects to use the 3-year MACRS class for tax taking depreciation deductions (weights = 33.33%, 44.45%, and 14.81%, and 7.41%). Compute for each company the present value of tax savings resulting from the deductions. ©CB21 .

4. Two years ago the company had *Capital expenditure* of $32,000 that it is depreciating along the 3-year MACRS class (weights = 33.33%, 44.45%, and 14.81%, and 7.41%). The company has just taken their second annual deduction. They today are selling the asset for $5,300. Find the net proceeds from the sale given that they face a 35% tax rate and otherwise have substantial taxable income. ©CB24 .

Numerical challengers

5. The company invests $70,000 in an asset that should increase pre-tax revenue by $15,400 per year perpetually. The asset is depreciated for tax purposes by straight-line over 5 years. If the company faces a 30% tax rate and 13.9% financing rate, what is the investment's Net Present Value? ©CB13 .

6. Your company is analyzing purchase of a machine costing $4,000 today. The investment promises to add $10,000 to sales one year from today, $12,500 two years from today, and $14,500 three years from today. Incremental cash costs should consume 80% of the incremental sales. The tax rate is 30% and the company's financing rate is 12.9%. The investment cost is depreciated to zero over a 3-year straight-line schedule. Find the project's net present value and internal rate of return. ©CB10a .

7. A proposed project is expected to reduce pretax operating costs by $63,000 per year for 4 years. The project requires incremental expenses of $137,900. The company is eligible to depreciate the entire expense within the 3-year *MACRS* class (weights = 33.33%, 44.45%, and 14.81%, and 7.41%). No salvageable assets remain beyond the project life. The company's tax rate is 35%, and the project's average financing rate is 9.7%. Find the *IRR* and *NPV* of the project's incremental cash flow stream. ©CB25b

8. The Company is considering a short-term expansion into a new product line making commemorative plates for the Olympics which are scheduled to occur 4-years henceforward. From the following factors that may weigh in the decision, find the net present value and internal rate of return for the project. ©CB22a .

a. the plate presser costs $140,600 and may be depreciated for tax purposes along a 7-year MACRS class (weights equal 14.29%, 24.49%, 17.49%, 12.49%, 8.93%, 8.92%, 8.93%, and 4.46%)
b. installation and shipping costs equal $3,800
c. the project requires an increase in *Net working capital* of $5,600
d. product development and market study fees of $4,100 already have been spent developing the plan
e. commemorative plates sell for $20.40 each and variable costs are $16.50 per plate
f. projected sales over the next 4 years are 6,000 plates, 9,000 plates, 13,000 plates, and 18,000 plates (thereafter sales would be zero).
g. the plate presser loses half its market value for each year of use; it will be sold after the 4th year
h. the financing rate is 14.4% and the tax rate is 30%

9. You took out a 30-year mortgage (monthly payments) for $275,000 at 8.0% and payment number 77 is due today. You are deciding whether you should refinance the outstanding principal by borrowing at today's lower rate of 6.25% an amount that just pays off the old loan. The new loan is for 20 years as of today. The total fees for getting the new loan equal 2.8% of the borrowed principal, and you will pay the fees with money in your savings account.

9a. What is the payback period for recovering the cost of the fees? ©CB3em
9b. How much would you save in interest expense over the life of the loan? ©CB3d
9c. What is the net present value of the refinancing venture if your “personal discount rate” is 9%. ©CB3c
9d. Find the internal rate of return for the financing venture.

10. You took out a 20-year mortgage (monthly payments) for $125,000 at 7.25% and payment number 55 is due today. You are deciding whether you should refinance the outstanding principal by borrowing at today’s lower rate of 5.75% an amount that pays off the old loan. The new loan is for 20 years as of today. The total fees for getting the new loan equal 2.3% of the borrowed principal, and you will amortize the fees over the life of the new loan.
10a. What is the payback period for recovering the cost of the fees?
10b. How much would you save in interest expense over the life of the loan? ©CB3b
10c. What is the net present value of the refinancing venture if your “personal discount rate” is 12%. ©CB3a
10d. Find the internal rate of return for the financing venture.

ANSWERS TO CHAPTER 6 EXERCISES

EXERCISES 6.1

1. First understand exactly what constitutes a capital budgeting decision: find the best real asset-increasing use for money. There is a rich academic literature attesting to the complexity of the theory of capital investment. Several factors that complicate capital budgeting decisions are discussed below.

(1) Cash flow estimates usually are imprecise. Later lessons in this book teach that when a range of possible outcomes exist then statistical ideas of risk and uncertainty become relevant. Imagine, for example, a situation in which a capital investment most likely returns net cash flow in one year of $10,000 but, of course, there is no guarantee. Other possible outcomes may be that it loses $1,000 or gains $15,000. Assessment measures of profitability that this section introduces do not fully accommodate a range of uncertain outcomes. That’s one place where intuition of the decision-maker may make a difference.

(2) The relevant financing rate often is difficult to identify. Later lessons in this book teach that financing rates are dynamic. That is, a financing rate generally depends on the use to which the money is put. Maybe a company or household can borrow at 6% for one purpose, but for other purposes the rate may differ. Obviously this complicates application of the IRR and NPV decision rules because they require knowledge of the relevant financing rate. Decision-maker intuition once again may make a difference.

(3) Changing circumstances require flexibility, yet making a capital budgeting commitment in one direction may inhibit subsequent flexibility. For example, a later lesson in this chapter explains that refinancing a mortgage from a high interest rate, say 10%, down to a lower rate, say 7%, saves the borrower money. Yet refinancing is costly. Surely dropping a rate from 10% to 7% is worth the cost. But once the move to 7% has been made then the advantage of subsequent moves diminishes. That is, if one refinances at 7% and rates subsequently bottom at 6% then it’s unlikely refinancing from 7% to 6% is profitable. It may have been better for the decision-maker to not refinance at 7% but rather to wait until 6% (but that was an uncertain outcome). Making a capital budgeting decision today typically opens some subsequent real options while closing others. Later lessons in this book teach that options are valuable. Once again, intuition of the decision-maker potentially makes a difference!

2. The payback period equals the length of time required to recover $20,000. That is 3.6 years (= $20,000 ÷ $5,500).

3. Discounted revenue for the first year is \$5,231 (= \$6,000 × 1.147^{-1}). The second year contributes discounted cash flow of \$4,561 (= \$6,000 × 1.147^{-2}) and brings the cumulative total to \$9,792 (= \$5,231 + \$4,561). Continue and find that discounted cash flows for the first six years sums to \$22,891. The discounted payback period is a tad longer than 6 years. An easy way to find this answer is with the time value functions on the financial calculator. Set *PV* = -23000; *PMT* = 6000; *I/Y* = 14.7 (*P/Y* = 1), and compute that *N* = 6.04.

4. ©CB1 The first 12 months pay back \$10,800 (= \$900 × 12) of the \$22,000 cost, implying payback period exceeds one year. The cost remaining to be recovered beyond one year is \$11,200 (= \$22,000 – \$10,800). The second year contributes \$9,600 (= \$800 × 12), meaning that cost of \$1,600 (= \$11,200 – \$9,600) still remains to be recovered. The number of months in the third year required for recovering \$1,600 is 2.3 months (= \$1,600 ÷ \$700). The payback period is 26.3 months (= 2 × 12 + 2.3), or 27 months if cash flow occurs as an end-of-month lump-sum.

5. Use perpetuity formula 5.3 to find the present value of \$3,500 per year forever: *PV = \$3,500 ÷ 0.147;* or *PV* equals \$23,810. Likewise, use the perpetuity formula to solve for the discount rate that equates discounted cash flows to cost: *r = \$3,500 ÷ \$20,000*; or *r* equals 17.5%. Note that *NPV>0* and *IRR>financing rate* so the rules say that this project creates wealth.

6. The annual IRR satisfies formula 6.2:
\$16,800 = \$1,900 × $\{(1+IRR)^{-1} + \ldots + (1+IRR)^{-4}\}$ + \$2,200 × $\{(1+IRR)^{-5} + \ldots + (1+IRR)^{-12}\}$
Use the financial calculator to find that the IRR is 6.63%.

7. The NPV is found with formula 6.4:
NPV = \$1,800 × $\{1.074^{-1} + \ldots + 1.074^{-5}\}$ + \$2,700 × $\{1.074^{-6} + \ldots + 1.074^{-10}\}$ – \$14,300.
The NPV equals \$667. The positive NPV means the project is economically profitable.

8. The IRR is found with formula 6.2:
\$11,900 = \$170 × $\{(1+IRR/12)^{-1} + \ldots + (1+IRR/12)^{-36}\}$ + \$280 × $\{(1+IRR/12)^{-37} + \ldots + (1+IRR/12)^{-72}\}$
Use the financial calculator to find that the monthly periodic rate is 0.79%. The periodic rate equals *IRR/12*. Therefore, the annual IRR is 9.47% (= 0.79% × 12). The IRR decision rule says that when the IRR is less than the financing rate (9.47% < 10% for this problem) then the project incurs economic losses – this is a bad deal!

9. The monthly periodic rate *r* is 0.91% (= 10.9% ÷ 12). The NPV is found with formula 6.4:
NPV = \$160 × $\{1.0091^{-1} + \ldots + 1.0091^{-36}\}$ + \$270 × $\{1.0091^{-37} + \ldots + 1.0091^{-84}\}$ – \$11,200.
The NPV equals \$1,253. It creates wealth. The IRR is found with formula 6.2:
\$11,200 = \$160 × $\{(1+IRR/12)^{-1} + \ldots + (1+IRR/12)^{-36}\}$ + \$270 × $\{(1+IRR/12)^{-37} + \ldots + (1+IRR/12)^{-84}\}$
Use the financial calculator to find that the annual IRR is 14.0% (= 1.16% × 12).

10. ©CB7 . The monthly periodic rate *r* is 0.87% (= 10.4% ÷ 12) and the number of payments *N* is 300 (= 25 × 12). First find the payment by using formula 5.6: \$168,000 = *PMT* × [(1 – 1.0087^{-300}) ÷ 0.0087]; or *PMT* equals \$1,574.24. Now find that the number of months required for the bank to collect \$168,000 is 106.7 months (= \$168,000 ÷ \$1,574.24). The payback period equals 107 months (about 9 years).

11. Use the *CF* worksheet on the financial calculator to solve this problem. Set *CF0* = 0 and *CF01* = 1100 and *F01*= 12. Hit *NPV* and set *I* = 1.275% (=15.3 ÷ 12). Compute that *NPV* equals \$12,168. Total discounted cash flow for the first year sums to \$12,168 and has not yet recovered the cost. Hit *CF* (don't clear existing entries) and scroll down to set *CF02* = 1300 and *F02* = 12. Hit *NPV* (keep *I* = 1.275%) and compute that *NPV* equals \$24,520. The discounted payback period is longer than 2 years. The discounted cost that remains to be recovered during the third year is \$3,480 (= \$28,000 – \$24,520). Find the number of months into the third year required for recovering this amount as follows. An easy way to find the remaining months is with the time

value functions on the financial calculator. Set *PV* = -3480; *PMT* = 1000; *I/Y* = 15.3 (*P/Y* = 1), and compute that *N* = 5.4. The discounted payback period equals 29.4 months (= 24 + 5.4).

12. ©CB8 The monthly periodic rate *r* is 0.87% (= 10.4% ÷ 12) and the number of payments *N* is 180 (= 15 × 12). First find the payment by using formula 5.6: \$148,000 = *PMT* × [(1 – 1.0087^{-180}) ÷ 0.0087]; or *PMT* equals \$1,627. Now find the present value of the outstanding payments when discounted with a monthly periodic rate of 0.96% (= 11.5% ÷ 12) and 106 payments remain (= 180 – 74): *PV* = \$1,627 × [(1 – 1.0096^{-106}) ÷ 0.0096]; or *PV* equals \$107,989. Now find the annual *IRR* for a project that costs \$148,000 and returns \$1,627 for 74 months and, upon receiving the last monthly cash flow, also returns a balloon payment of \$107,989. Use formula 6.2:

$\$148,000 = \$1,627 \times \{(1+IRR/12)^{-1} + \ldots + (1+IRR/12)^{-74}\} + \$107,989 \times (1+IRR/12)^{-74}\}$

Find with the financial calculator that the annual *IRR* equals 10.0% (or 0.83% monthly).

EXERCISES 6.2

1. The net present values of L (= \$110/1.10^1 – \$100) and H (= \$115/1.15^1 – \$100) are equal at zero. Hence, the capitalized values of economic profit equal zero for both projects. Rule 1.1 states that the goal of the company is to pursue policies that maximize capitalized value of wealth creation so projects L and H are equivalent, neither is better, and the statement is false. There are, however, two important underlying issues that merit mention.

(1) Later lessons teach that market equilibrium drives net present values to zero in both real asset and financial markets. Hence, for projects L and H to attract different financing rates yet still offer zero NPV means that something else about these projects differ. That difference, as later lessons explain, is risk. In an equilibrium setting the problem setup implies that high-returning project H has higher risk than low-returning project L.

(2) The future values of incremental wealth differ. Project H returns \$15 to capitalists whereas L returns only \$10. In an equilibrium setting they both offer zero NPV to the company; they both offer zero NPV to capitalists providing financing. One project is not better than the other and company as well as capitalist are indifferent between L and H. Still, however, future values differ because more wealth accumulates for capitalists at 15% than at 10%; and capitalists include shareholders as well as creditors.

2. Net present value is identical to economic profit. The existence of positive net present value attracts entrepreneurial activity. At equilibrium economic profit and net present value equal zero. Occurrence of zero net present value does <u>not</u> mean that wealth creation equals zero. Companies capture transformation value even though economic profits may equal zero. Thus, economic equilibrium does <u>not</u> imply absence of wealth creation. To the contrary, consider a case in which the monetary inflation rate is zero, the nominal financing rate is 4%, and a company makes a zero net present value investment. The project generates sufficient revenues that pay for economic costs of production which include a 4% real return to financing sources. A zero net present value investment creates transformation value that in actuality is shared by all stakeholders and all capitalists in accordance with their respective bargaining position under the umbrella of principal-agent relations.

3. The net present values of L (= \$110/1.10^1 – \$100) and H (= \$116/1.15^1 – \$100) are \$0, and \$1, respectively. Hence, the capitalized value of created wealth is greater for Project H. Rule 1.1 states that the goal of the company is to pursue policies that maximize capitalized value of wealth creation so project H is better.

4. NPV is found with formula 6.4:

$NPV = \$2,000 \times 1.142^{-1} + \$2,500 \times 1.142^{-2} + \$2,900 \times 1.142^{-3} - \$4,600.$

The NPV equals \$1,015. The project creates wealth. The company could capitalize the wealth by borrowing \$5,615 (= \$4,600 + \$1,015). Instead, however, the problem states the company only borrows \$4,600 and pays as it goes. The future value of the wealth remaining for the company after the third cash flow and financing payment equals the future value of NPV. Use the lump-sum relation to find that the future value of \$1,015 at 14.2% for 3 years is \$1,512 (= \$1,015 × 1.142^3).

5. The problem states that the cross-over rate equals 7.5% and that $IRR_Y > IRR_X$. Sketch this onto an NPV Profile and see that $NPV_Y > NPV_X$ when the financing rate exceeds 7.5%. Most likely for a financing rate less than 7.5% then $NPV_Y < NPV_X$.

6. Apply formula 6.5 and subtract cash flows A from B. Then use the financial calculator to solve for the COR from:
$\$120 = \$\text{-}384 \times (1+COR)^{-1} + \$\text{-}31 \times (1+COR)^{-2} + \$626 \times (1+COR)^{-3}$.
The cross-over rate is 7.49%. At that rate the NPVs are equal.

7. . ©CB2a . Sketch the NPV profile for each project by finding the two intercepts for each project. That is, use the financial calculator to solve for the IRR from:
$\$670 = \$498 \times (1+IRR)^{-1} + \$309 \times (1+IRR)^{-2} + \$128 \times (1+IRR)^{-3}$.
The IRR for A is 24.0%. That's the horizontal intercept for project A. Its vertical intercept equals its NPV given a discount rate of zero. That NPV is:
$NPV_{@r=0\%} = \$498 + \$309 + \$128 - \$670$; $= \$265$.
Do analogous computations for B and find that its IRR equals 16.7% and its NPV at 0% financing rate is \$436. The profiles cross. Use formula 6.5 to find the cross-over rate:
$\$220 = \$\text{-}394 \times (1+COR)^{-1} + \$\text{-}6 \times (1+COR)^{-2} + \$791 \times (1+COR)^{-3}$.
The cross-over rate is 10.9%. Answer the question as follows:

$NPV_A < NPV_B$ when $r < 10.9\%$.
$NPV_A > NPV_B$ when $r > 10.9\%$.

Note that NPV_B goes negative when *r* exceeds 16.7% and NPV_A goes negative when *r* exceeds 24.0%.

8. Apply formula 6.5 and subtract cash flows A from B. Then use the financial calculator to solve for the COR from:
$\$140 = \$\text{-}339 \times (1+COR)^{-1} + \$\text{-}25 \times (1+COR)^{-2} + \$653 \times (1+COR)^{-3}$.
The cross-over rate is 9.45%. At that rate the NPVs are equal. Find the NPV of either project with formula 6.4 at 9.45%:
$NPV = \$457 \times 1.0945^{-1} + \$280 \times 1.0945^{-2} + \$115 \times 1.0945^{-3} - \$620$.
The NPV equals \$119.

EXERCISES 6.3

1. Before the policy change the investment had zero net present value. Say it cost a million dollars. Before the policy change the company was splitting the million dollars of tax depreciation deductions across several years in accordance with the asset's tax depreciation schedule. Then the government declared that the company could take the entire one million dollar deduction immediately. The present value of the deductions to the company definitely increases; discounted tax savings increase, too. This fiscal stimulus by the government (at least in the short-run) transforms the zero NPV project into a positive NPV project (and probably some projects that were negative NPV now become positive NPV). The policy change creates economic profit, a so-called windfall, and probably stimulates capital investment. In the absence of any response by capitalists and stakeholders then surely the windfall accrues to equity. The capital goods supplier, however, sees the droplet forming (see figure 1.3) and competes for his share, too, perhaps by raising the price of the capital asset.

2. The deductions equal the weight times the *Capital expenditure*. The tax savings equals the tax rate times the deduction. Find the present value of tax savings resulting from the depreciation deductions from this:
$PV = 0.30 \times \$40{,}000 \times \{0.3333 \times 1.095^{-1} + 0.4445 \times 1.095^{-2} + 0.1481 \times 1.095^{-3} + 0.0741 \times 1.095^{-4}\}$,

which equals $10,073.

3. Each dollar of tax deductions reduces taxable income by $1 and saves the company 35 cents of taxes (the tax rate times $1). For company X the deduction is $25,000 per year for 4 years. The discounted tax savings equal

$PV = 0.35 \times \$25{,}000 \times \{ 1.12^{-1} + 1.12^{-2} + 1.12^{-3} + 1.12^{-4} \}$,

which is $26,577. For company Y the deduction equals $100,000 times the MACRS weight. The discounted tax savings equal

$PV = 0.35 \times \$100{,}000 \times \{0.3333 \times 1.12^{-1} + 0.4445 \times 1.12^{-2} + 0.1481 \times 1.12^{-3} + 0.0741 \times 1.12^{-4} \}$,

which is $28,155. The discounted tax savings are bigger for company Y so they are better off than company X. Whenever a company *accelerates* tax depreciation deductions by taking them sooner rather than later, the company is better off.

4. The net proceeds from the sale equal the sale price of $5,300 adjusted for any recapture taxes or credits associating with the sale. Computing recapture taxes requires finding the remaining book value for the asset, which in turn equals the deductions that have not yet been taken:

remaining book value = $32,000 × (0.1481 + 0.0741); = $7,114.

They sell the asset for less than its book value so effectively the company may claim a capital loss. Every dollar of loss saves 35 cents of taxes. The net proceeds from the sale equal:

net proceeds from the sale = $5,300 + 0.35 × (7,114 – $5,300); = $5,935.

5. The deduction each year for 5 years is $14,000 (= $70,000 ÷ 5). The depreciation tax savings therefore equals $4,200 each year (= $14,000 x 0.30). The incremental cash flow for the first 5 years is the same as in year 1:

$$\Delta CF_1 = [\$15{,}400 \times (1 - 0.30)] + [0.30 \times \$70{,}000 \div 5], \\ = [\$10{,}780] + [\$4{,}200] \\ = \$14{,}980.$$

From year 6 and thereafter the incremental cash flow each year is $10,780 because the depreciation tax shield will have expired. Find the NPV with a 13.9% financing rate as:

$NPV = \$14{,}980 \times \{1.139^{-1} + \ldots + 1.139^{-5}\} + \{\$10{,}780 \times 1.139^{-5} \div 0.139\} - \$70{,}000.$

NPV is $22,008. Note the same answer is found by finding the perpetuity value of $10,780 and then adding back in the annuity value of the depreciation tax savings:

$$NPV = \$10{,}780 \div 0.139 + \$4{,}200 \times \{1.139^{-1} + \ldots + 1.139^{-5}\} - \$70{,}000 \\ = \$22{,}008.$$

6. ©CB10a . The implication is that in one year *Sales* of $10,000 minus cash costs of $8,000 equals *EBITDA* of $2,000. The company would pay 30%, or $600, in proportional taxes except that a depreciation deduction of $1,333 (= $4,000 ÷ 3) provides tax savings of $400 (= $1,333 × 0.30). The company therefore actually pays taxes of only $200. Subtracting $200 from *Sales* minus cash costs leaves the company with incremental cash flow of $1,800. Quickly get the same number by applying formula 6.7:

$$\Delta CF_1 = [\$10{,}000 \times (1 - 0.80) \times (1 - 0.30)] + [0.30 \times \$4{,}000 \div 3], \\ = [\$10{,}000 \times 0.14] + [\$400] \\ = \$1{,}800.$$

Note that for cash flows 2 and 3 you can quickly compute:

$$\Delta CF_2 = [\$12{,}500 \times 0.14] + [\$400] \\ = \$2{,}150$$

and $$\Delta CF_3 = [\$14{,}500 \times 0.14] + [\$400] \\ = \$2{,}430$$

The annual IRR satisfies formula 6.2:

$\$4{,}000 = \$1{,}800 \times (1+IRR)^{-1} + \$2{,}150 \times (1+IRR)^{-2} + \$2{,}430 \times (1+IRR)^{-3}$

Use the financial calculator to find that the IRR is 26.0%. The NPV is found with formula 6.4 and a financing rate of 12.9%:

$NPV = \$1{,}800 \times 1.129^{-1} + \$2{,}150 \times 1.129^{-2} + \$2{,}430 \times 1.129^{-3} - \$4{,}000.$

The NPV equals $970. It's economically profitable.

7. Initial cash flow at time 0 equals the asset cost of $137,900. Find $CF^{from\ assets}$ for years 1 through 4 as follows:
CF_1 = $63,000 × (1 – 0.35) + 0.35 × $137,900 × 0.3333;
= $40,950 + $48,265 × 0.3333;
= $57,037.
CF_2 = $40,950 + $48,265 × 0.4445; = $62,404.
CF_3 = $40,950 + $48,265 × 0.1481; = $48,098.
CF_4 = $40,950 + $48,265 × 0.0741; = $44,526.
The IRR satisfies:
$137,900 = $57,037 × $(1+IRR)^{-1}$ + $62,404 × $(1+IRR)^{-2}$ + $48,098 × $(1+IRR)^{-3}$ + $44,526 × $(1+IRR)^{-4}$,
The *IRR* of 20.0% exceeds the 9.7% average financing rate. Find NPV as:
NPV = $57,037 × 1.097^{-1} + $62,404 × 1.097^{-2} + $48,098 × 1.097^{-3} + $44,526 × 1.097^{-4} – $137,900;
or *NPV* equals $33,129. The project creates economic profit and is good.

8. Initial cash flow at time 0 equals the asset cost of $140,600 plus installation and shipping costs of $3,800 plus *Net working capital* of $5,600. That sum of $150,000 is a cash outflow. Note that the basis for tax depreciation deductions of $144,400 equals the asset cost of $140,600 plus installation and shipping costs of $3,800. Each plate generates *EBITDA* of $3.90 (= $20.40 – $16.50). Total *EBITDA* equals $3.90 times number of plates sold. Apply formula 6.7 for cash flows 1 to 3:
CF_1 = $3.90 × 6,000 × (1 – 0.30) + 0.30 × 0.1429 × $144,400; = $22,570.
CF_2 = $3.90 × 9,000 × (1 – 0.30) + 0.30 × 0.2449 × $144,400; = $35,179.
CF_3 = $3.90 × 13,000 × (1 – 0.30) + 0.30 × 0.1749 × $144,400; = $43,067.
Cash flow 4 adjusts for release of *Net working capital* and *Net proceeds from sale*:
CF_4 = $3.90 × 18,000 × (1 – 0.30) + 0.30 × 0.1249 × $144,400 + $5,600 + *Net proceeds from sale*;
= $60,151 + *Net proceeds from sale*.
The *Net proceeds from sale* equal the sale price of $17,575 (=$140,600 × ½ × ½ × ½) adjusted for recapture taxes or credits associating with the sale. Computing recapture taxes requires finding the remaining book value for the asset, which in turn equals the deductions that have not yet been taken:
Remaining book value = $144,400 × (0.0893 + 0.0892 + 0.0893 + 0.0445); = $45,111.
They sell the asset for less than its book value so the company pays recapture taxes on the capital gain. Thus:
Net proceeds from the sale = $17,575 + 0.30 × (45,111 – $17,575); = $25,836.
and
CF_4 = $60,151 + $25,836; = $85,986.
Now find NPV:
NPV = $22,570 × 1.144^{-1} + $35,179 × 1.144^{-2} + $43,067 × 1.144^{-3} + $85,986 × 1.144^{-4} – $150,000.
NPV equals $-24,423 and indicates this project is NOT profitable. The IRR is:
$150,000 = $22,570 × $(1+IRR)^{-1}$ + $35,179 × $(1+IRR)^{-2}$ + $43,067 × $(1+IRR)^{-3}$ + $85,986 × $(1+IRR)^{-4}$,
or IRR equals 7.6% and is less than the financing rate.

9a. First find the payment on the original loan given the monthly periodic rate of 0.67% (= 8% ÷ 12) and term of 360 months:
$275,000 = PMT × { [1 – $(1.0067)^{-360}$] ÷ 0.0067 }.
Find that PMT = $2,018. If you sent in 77 payments then there remain 283 payments (= 360 – 77). Solve for principal outstanding PV in this equation.
PV = $2,018 ×{ [1 – $(1.0067)^{-283}$] ÷ 0.0067 }.
Find that PV= $256,511. That's the original loan's outstanding principal. The fees for the new loan equal $7,182 (= $256,511 × 0.028) and are paid from your savings account. The new loan

exactly pays off the original loan and, with a term of 240 months (= 20 × 12) and monthly periodic rate of 0.52% (= 6.25% ÷ 12), satisfies this:
$\$256,511 = PMT \times \{ [1 - (1.0052)^{-240}] \div 0.0052 \}$.
or PMT = $1,875. The monthly payment declines to $1,875 from $2,018 and represents monthly savings of $143. It takes 51 months to recover the fees (= $7,182 ÷ $143), that's the payback period (about 4 ¼ years).

9b. Use results from the preceding part and find that the original loan's outstanding principal equals $256,511 and the payment is $2,018. If you send in an additional 283 payments of $2,018 each that represents total expenditure of $571,052 (= 283 × $2,018) and total interest of $314,541 (=$571,052 - $256,511).

For the new loan the original principal equals $256,511 and the payment is $1,875. Submitting 240 payments of $1,875 each represents total expenditure of $449,979 (=240 × $1,875) and total interest of $193,467 (=$449,979 - $256,511).

The refinancing saves you $121,074 of interest over the life of the loan (= $314,541 - $193,467). That's a lot, but comparing dollars across different time periods requires accounting for time value effects. You really should be evaluating with NPV!

9c. ©CB3c Use results from the preceding parts and find that the incremental cash flow equals $7,182 at time 0 because you pay fees up front from your savings account. Then for 240 months the incremental cash flow for you is $143 (= $-1,875 – $-2,018) because your payments are lower if you refinance. Then for the 43 months (= 283 – 240) after that your incremental cash flow equals $2,018 because you'll be making zero payments for the new loan but would still be paying off the old loan; you are saving money during these months if you refinance. The NPV for you with a periodic rate of 0.75% (= 9% ÷ 12) satisfies this formula:
$NPV = \$143 \times \{ [1 - (1.0075)^{-240}] \div 0.0075 \} + \$2,018 \times (1.0075)^{-240} \times \{ [1 - (1.0075)^{-43}] \div 0.0075 \} - \$7,182$
which is $21,008. This creates wealth for you. Note that using the monthly market interest rate of 0.52% (= 6.25% ÷ 12) for discounting shows NPV is $34,667. Wealth creation either way is substantial.

9d. The annual IRR for you satisfies this formula:
$\$7,182 = \$143 \times \{(1+IRR/12)^{-1} + \ldots + (1+IRR/12)^{-240}\} + \$2,018 \times \{(1+IRR/12)^{-241} + \ldots + (1+IRR/12)^{-283}\}$
The monthly periodic rate found by the financial calculator is 2.09%, or about 25% per annum. If you apply the IRR rule it too says refinance.

10a. First find the payment on the original loan given the monthly periodic rate of 0.60% (= 7.25% ÷ 12) and term of 240 months:
$\$125,000 = PMT \times \{ [1 - (1.0060)^{-240}] \div 0.0060 \}$.
Find that PMT = $988. If you sent in 55 payments, then there remain 185 payments (= 240 – 55). Solve for principal outstanding PV in this equation.
$PV = \$988 \times \{ [1 - (1.0060)^{-185}] \div 0.0060 \}$.
Find that PV= $109,868. That's the original loan's outstanding principal. The fees for the new loan equal $2,527 (= $109,868 × 0.023) and are paid by increasing the amount of the loan to pay for the fees. The new loan equals $112,395 (= $109,868 × 1.023) with a term of 240 months and monthly periodic rate of 0.48% (= 5.75% ÷ 12), satisfies this:
$\$112,395 = PMT \times \{ [1 - (1.0048)^{-240}] \div 0.0048 \}$,
or PMT = $789. The monthly payment declines to $789 from $988 and represents monthly savings of $199. It takes 13 months to recover the fees (= $2,527 ÷ $199); that's the payback period.

10b. Use results from the preceding part and find that the loan's outstanding principal equals $109,868 and the payment is $988. If you send in an additional 185 payments of $988 each that represents total expenditure of $182,774 (= 185 × $988) and total interest of $72,906 (=$182,774 - $109,868).

For the new loan the original principal equals \$112,395 and the payment is \$789 . If you send in 240 payments of \$789 each that represents total expenditure of \$189,386 (=240 × \$789), and total interest of \$76,991 (=\$189,386 - \$112,395).

The refinancing actually costs you an extra \$4,085 over the life of the loan (= \$72,906 - \$76,991) wrongly suggesting the refi is bad. You really should be evaluating with NPV, it sends a more accurate dynamic signal!

10c. ©CB3d Use results from the preceding parts and find that the incremental cash flow equals \$0 at time 0 (because you amortize the fees then you don't pay anything up front). Then for 185 months the incremental cash flow for you is \$199 (= \$-789 – \$-988) because your payments are lower if you refinance. Then for the 55 months after that your incremental cash flow equals \$-789 because you'll be making payments for the new loan but would not have paid anything these months with the old loan. The NPV for you with a periodic rate of 1% (= 12% ÷ 12) satisfies this formula:

$NPV = \$199 \times \{1.01^{-1} + \ldots + 1.01^{-185}\} + \$\text{-}789 \times \{1.01^{-186} + \ldots + 1.01^{-240}\}$

which is \$11,465. This creates wealth for you. Note that using the monthly market interest rate of 0.48% (= 5.75% ÷ 12) for discounting shows NPV is \$8,657. Wealth creation either way is substantial. Use NPV instead of incremental lifetime interest to make decisions.

10d. The annual IRR for you satisfies this formula:

$0 = \$199 \times \{(1+IRR/12)^{-1} + \ldots + (1+IRR/12)^{-185}\} + \$\text{-}789 \times \{(1+IRR/12)^{-186} + \ldots + (1+IRR/12)^{-240}\}$

The monthly periodic rate found by the financial calculator is 0.14%, or about 1.6% per annum. This number is hogwash because if you apply the IRR rule it says don't refinance. That's wrong! This problem exemplifies a scenario in which the IRR is not trustworthy because the cash flows switch signs from positive to negative. NPV always is reliable.

CHAPTER 7: TIME VALUE APPLICATION 2, BOND VALUATION

1. Bond basics: Notation, quotation, and cash flow
2. Relation between price and yield-to-maturity
3. Bond price movements
   - 3.A. Constant interest rates and scientific amortization
   - 3.B. Horizon analysis and changes in interest rate
   - 3.C. Riding the yield curve

A bond is an "IOU" that a company or government issues in order to borrow money. The bond represents a legal contract that defines specific obligations to which the issuer agrees. The most significant obligation is the schedule of cash flows that the issuer promises to pay the lender. In essence, when the lender gives money to the issuer, and in return the issuer gives the lender the bond, the lender is purchasing the bond; the lender is a bond investor. The investor probably buys the bond because they want the cash flows that the bond promises.

Analysis of bond cash flows relies on time value techniques. Bond analyses offer an excellent opportunity for practicing and developing intuition about present value procedures. The analysis of bonds also is interesting, and relevant, because almost every individual at some point owns bonds, even if only indirectly through pension savings. Chapter 9 presents details about different types of financial securities in the bond and other credit markets. For now, however, lessons focus on important time value characteristics of bonds.

1. Bond basics: Notation, quotation, and cash flow

The bond market is much larger than the stock market. Companies issue both bonds and stocks. While corporate issuers certainly are important participants in the bond market, the largest issuers are governments – state, local and federal governments and their sponsored agencies issue a lot of bonds. Table 7.1 offers perspective on relative size of trading activity in U.S. bond markets. Column 1 shows that trading in U.S. Treasury securities during 2013 averages more than $500 billion per day. Column 6 shows for comparative purposes that stock trading on the New York Stock Exchange during 2013 averages $46 billion per day, less than 1/10th the trading in Treasuries. Media attention on stocks in the popular press is huge. Media attention on bonds is subdued perhaps because the volatility and variety of stories underlying company stocks are more exciting than the staid but rich bond markets. Around the globe the bond markets always are much larger than stock markets – primarily because governments don't issue stock but apparently love to borrow with bonds.

The table also lists daily trading in U.S. Agency securities issued by organizations such as the Federal Home Loan Bank Board; municipal securities issued by hospitals, airports, transportation authorities, and a slew of other public enterprises; and column 4 lists average daily trading volume for long-term corporate bonds. Even though, as chapter 9 discusses, these different securities have unique institutional characteristics their time value characteristics are nearly identical.

	Average Daily Trading Volume ($ billions)					
	U.S. Treasury Securities - 1 -	U.S. Agency Securities - 2 -	Municipal Securities - 3 -	Long-Term Corporate Securities - 4 -	Mortgage backed securities - 5 -	NYSE Stocks - 6 -
1991	$128	$6	not available	not available		$6
1992	152	6				7
1993	174	9				9
1994	191	16				10
1995	193	24				12
1996	204	31			38	16
1997	212	40			47	23
1998	227	48			70	29
1999	187	55			67	35
2000	207	73			69	43
2001	298	90	9		112	41
2002	366	82	9	19	154	40
2003	434	82	11	21	206	36
2004	501	77	12	21	207	46
…					Peak$_{2008}$ 345	
2013	554	8	12	18	222	46

TABLE 7.1 Average daily trading volume of selected financial securities.
Compiled by author

Some corporate bonds trade on the New York Stock Exchange. On the NYSE, however, bond transactions are modest relative to stock transactions. Most bonds trade in private markets set-up among institutions like brokerage houses, banks, pension funds, investment bankers, etc. Participants in the private market communicate through sophisticated telecommunication and computer links. The Bloomberg Financial Network and Bondmarkets.com are among the most widely used computer networks for the private bond market.

Private market transactions are not subject to the same scrutiny as, say, transactions occurring on an organized exchange like the NYSE. Trades on organized exchanges are guaranteed by the exchange's "clearinghouse." For example, suppose an investor bids and pays $1 million for bonds. If the trade were on the NYSE then the investor is assured they will take delivery of the bonds. If the other party were to "take the money and run", the NYSE would step in, deliver the bonds, and pursue the fleeing party. In the private market, however, buyer beware – there is substantial "counter-party risk". The exchange clearinghouse guarantees that investors execute the transaction faithfully, but the clearinghouse does not guarantee that the bond issuer will make the promised payments. The bond owner bears the risk that the issuer might go bankrupt.

A few companies, such as Moody's or Standard & Poor's, rate the "quality" of bonds by analyzing the financial health of the issuer. Moody's and Standard & Poor's make money by selling the ratings information to traders or anyone else that might want it. By looking at the bond rating, the potential investor gets an idea about the likelihood that the bond issuer can pay the scheduled cash flows. Ratings such as “AAA” suggest the issuer is strong and reliable; “AA” is still strong, but less so. Bonds in the “BBB” to “B” category carry a little more risk than A-rated bonds. The riskiest rating of all is in the “C” category. These bonds often are referred to as junk bonds.

Most bond trades occur between one investor and another. Recall that only once is the issuer involved in a primary market transaction with selling a bond. Thereafter, however, the bond may change hands from one investor to another in secondary market transactions. The trades might happen for many reasons; perhaps investors have different expectations about future interest rates, or maybe they have different cash requirements. Regardless, the bond issuer promises to pay the bond owner, whoever it may be, a specified schedule of cash flows.

Cash flows from corporate and government bonds generally differ from the cash flows that attach to consumer loans. Car loans and home mortgages, for example, consist of periodic payments that include both interest and principal; some of these are fixed payment loans. Corporate and government bonds, however, generally are “balloon loans.” Payments consist only of interest, until the last payment, at which time one huge payment, the *balloon*, repays the principal in-full. It is very common for large companies and institutions to have many different bond issues outstanding at the same time.

Glean insight about bond cash flows and terminology by examining the table below showing standard price quotes.

BOND QUOTES

Issuer	Current Yield	Volume	Close	Net Change
AMR 8.10s38	7.9	3	102 3/8	–5/8
ATT 4 1/2 s36	4.8	55	98½	…
ATT 8 5/8s 31	8.1	82	111½	+1/2
Motrla zr33	…	30	73¾	–1
Unisys 8s35	cv	73	97 7/8	–3/8

The table shows quotations for 5 bonds. For all bonds some traits are always the same. All bonds, for example, promise to pay their annual interest in two semi-annual installments. Also, the face value for each bond, that is its outstanding principal, is $1,000.

The first entry on each row is the bond issuer. Issuer for the first bond is the AMR company, the owner of American Airlines. AMR borrowed $1,000 at sometime in the past, we can’t tell how long ago simply by looking at this quote. Regardless, this bond promises interest payments at a rate of 8.10% per year. This is the bond’s *coupon rate*; the coupon rate determines the periodic interest payment that the bond owner receives. The interest payment is referred to as the “coupon” because long ago the bonds were issued with detachable coupons that the owner mailed to the company in order to request their interest payment. The coupon equals the face value times the coupon rate divided by two.

FORMULA 7.1 Semiannual coupon

The interest payment for a bond is paid semiannually and is called a coupon. The semiannual coupon is computed as

$$coupon = face\ value \times annual\ coupon\ rate \div 2$$

The AMR bond pays total interest per year of $81 (= $1,000 x 0.081), payable in two semiannual payments of $40.50 each

The two digits following the coupon rate, 38, represent the year that the bond matures and repays its principal. AMR promises to pay the owner of this bond $1,000 in 2038. We can't tell the exact day of the year that the bond matures simply by looking at this quote. Regardless, the bond investor has a good idea about the bond cash flows because of the identifier "8.10s38". The letter "s" in the bond quote has no meaning; it simply separates the different numbers. When traders discuss this bond they refer to it as the "AMR eight-point-tens of thirty-eight"; pronounce the "s".

The column with label "Close", 102 3/8, implies that the last trade of the day was at one-hundred-two and three-eighths per cent of face value.

FORMULA 7.2 Bond price

Bond prices in the U.S.A. typically are quoted as a percent of par. The dollar price of the bond is computed as

bond price = face value x quoted percentage price

For the AMR bond quoted at 102 3/8, the price is

AMR bond price = $1,000 x 102.375%

= $1,023.75

The investor purchasing the bond pays $1,023.75 in order to receive a coupon of $41.50 every six months, and upon receiving the final coupon in 2038, the owner receives an additional balloon payment of $1,000. Beyond 2038, the AMR company has no obligations because this loan is paid-in-full.

The column with label "Current yield" shows that for the AMR eight-point-tens of thirty-eight, the current yield is 7.9%. *Current yield* equals the annual interest payment divided by the current price.

FORMULA 7.3 Current yield

current yield = face value x annual coupon rate ÷ bond price

The current yield for the AMR 8.10s38 equals the $81 annual coupon divided by the price of $1,023.75, which equals 7.9%.

Many investors compare the current yield from bonds to the interest rate paid on a savings account. For example, an investor pays $1,023.75 for this bond and receives $81 interest per year, representing an effective interest rate of 7.9%. Suppose the bank savings account rate is 5 percent. The bond's current yield obviously is higher than the savings rate at the bank, but then again, this bond is a little riskier than a savings account! Investors decide whether the extra 290 basis points of return from the bond is worth the extra risk.

The Motorola bond in row 4 is different from the others because the Motorola's coupon rate is zero percent; the "zr" in the bond quote means "zero". This bond never pays interest at all! A natural question is why any investor would be willing to purchase a bond that doesn't pay interest. The answer is that investors buy it for a low price. Notice that to receive the $1,000 face value Motorola promises to pay in the year 2033, an investor need pay only $737.50 to purchase the bond. The price for the Motorola bond is substantially less than the other bond's prices. Investors for a zero coupon bond earn their rate of return by paying a small price.

The last row shows that the Unisys issue is a "convertible bond" (cv). This bond provides its owner, under certain circumstances, the right to convert the bond into stock.

The convertible bond price tends to be affected by factors affecting both bond and stock markets. Chapter 9 provides more details about convertible bonds.

2. Relation between price and yield-to-maturity

The yield-to-maturity ("*YTM*") is the discount rate that equates the present value of promised cash flows to the bond price, as shown in formula 7.4.

FORMULA 7.4 Yield-to-maturity and bond cash flows

$$bond\ \ price = \frac{coupon_1}{\left(1+YTM/2\right)^1} + \frac{coupon_2}{\left(1+YTM/2\right)^2} + \ldots + \frac{coupon_N}{\left(1+YTM/2\right)^N} + \frac{face\ \ value}{\left(1+YTM/2\right)^N}$$

$$= \ coupon\left\{\frac{1-\left(1+YTM/2\right)^{-N}}{YTM/2}\right\} \ + \ \frac{face\ \ value}{\left(1+YTM/2\right)^N}$$

N is the number of expected coupons. The right-hand-side sums *N* + 1 terms. The first *N* terms equal the present value of *N* expected coupons. They sum to the coupon multiplied by $PVIFA_{YTM/2\,,\,N}$. The last term is the present value of the principal repayment (i.e., the face value). The investor receives the face value at the same time that the final coupon is paid so the two last terms on the right-hand-side are both discounted *N* periods. The yield-to-maturity is an annual percentage rate. The formula divides the yield-to-maturity by two because interest compounds semiannually.

Formula 7.4 is analogous to the constant annuity time value equation from formula 5.1. Only the names have been changed. Bond price on the left-hand-side represents the present value of expected cash flows. The yield-to-maturity represents the internal rate of return for the bond investment. The *YTM* is the discount rate that equates the bond price to the present value of the cash flow stream. The *YTM* and *IRR* are conceptually equivalent. The *YTM* nomenclature exclusively applies to bond cash flows, whereas the *IRR* nomenclature appears in discussions about almost any type of investment.

For most applications, known variables for the preceding equation include the *coupon*, *face value*, and *N*. Of the remaining two variables, *YTM* and bond price, either one might be the known variable which means the other is the unknown. Sometimes a trader might be given the price and want to know the implied yield-to-maturity, whereas in other cases the trader might know the desired yield-to-maturity and wants to know the implied price. A financial calculator easily solves for any variable, as long as there only is one unknown.

EXAMPLE 1 Find YTM and counteroffer price

Suppose that for the "AT&T eight-and-five-eighths of 31" listed in row 3 there remain 39 semiannual coupons before the bond is retired in year 2031. (a) Find the yield-to-maturity at the quoted price of 111½. (b) You wish to make a bid on this bond so that your yield-to-maturity equals 7.72%. How much should you bid?

SOLUTION

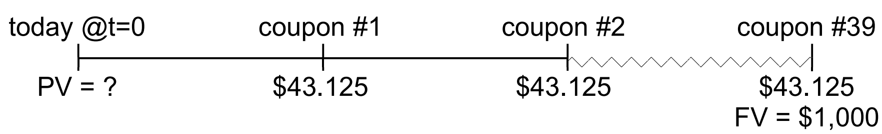
The coupon rate of 8 5/8 implies that the annual interest payment for the bond equals \$86.25. Each semiannual coupon therefore equals \$43.125 (the payments actually alternate between \$43.12 one time and \$43.13 the next time, but ignore this technicality.) Substitute in formula 7.4 that *coupon* = \$43.125, *N* = 39, *face value* = \$1,000 , and set the initial bond price at \$1,115. Solve for the unknown *YTM* from this setup.

$$\$1,115 = \frac{\$43.125}{\left(1+YTM/2\right)^{1}} + \frac{\$43.125}{\left(1+YTM/2\right)^{2}} + \ldots + \frac{\$43.125}{\left(1+YTM/2\right)^{39}} + \frac{\$1,000}{\left(1+YTM/2\right)^{39}}$$

$$\$1,115 = \$43.125 \left\{ \frac{1-\left(1+YTM/2\right)^{-39}}{YTM/2} \right\} + \frac{\$1,000}{\left(1+YTM/2\right)^{39}}$$

$$YTM = 7.49\%$$

The accompanying *Calculator Clue* describes steps for finding that the yield-to-maturity at the quoted price equals 7.49%. Consider now the second part of the question. You wish to make a bid to buy the bond such that the yield-to-maturity is 7.72%. The time line illustrates the scenario.



The unknown *bond price* represents the present value at which the known cash flows provide the known target rate of return.

$$bond\ price = \frac{43.125}{\left(1+.0772/2\right)^{1}} + \frac{43.125}{\left(1+.0772/2\right)^{2}} + \ldots + \frac{43.125}{\left(1+.0772/2\right)^{39}} + \frac{1,000}{\left(1+.0772/2\right)^{39}}$$

$$= \$1,090.46$$

The annual rate of return is 7.72% if you pay \$1,090.46 and receive 39 semiannual coupons of \$43.125 each, along with \$1,000 at the same time the last coupon is received.

CALCULATOR CLUE 7.1 Solve the preceding problem by typing [2nd] [FV] to clear the time value memories, and [2nd] [I/Y] 2 [ENTER] [2nd] [CPT] to enforce semiannual compounding. Then type:
43.125 [PMT] 1000 [FV] 39 [N] 1115 [+/-] [PV] [CPT] [I/Y] .
The display shows the annual yield-to-maturity is 7.49%. Notice that on the *BAII Plus* that FV and PMT have the same sign and are opposite to the sign of PV. For the second part of the question find the bid price with these steps:
7.72 [I/Y] [CPT] [PV] .

The display shows the present value is \$1,090.46.

The preceding example shows that if you buy the AT&T eight-and-five-eighths of 31 for \$1,090.46 and receive the stipulated cash flows through the maturity date in 2031 that you'll get your 7.72% target rate of return. Suppose, however, that you buy the bond today and in six months receive the first coupon of \$43.125 and sell the bond? What is the annual rate of return from that venture?

The answer requires the bond price six months from now. Preceding discussion states that bond price implies yield-to-maturity, and vice versa. That is, knowing one means knowing the other. For this question suppose that the yield-to-maturity remains constant. Six months from now the bond price equals the discounted sum of all remaining cash flows. Only difference is that one coupon will have been received and so only 38 coupons remain. Substitute settings into formula 7.4 just as in Example 1 and re-compute the bond price prevailing six months from now when N = 38.

$$bond\ \ price = \frac{\$43.125}{\left(1+.0772/2\right)^{1}} + \frac{\$43.125}{\left(1+.0772/2\right)^{2}} + \cdots + \frac{\$43.125}{\left(1+.0772/2\right)^{38}} + \frac{\$1,000}{\left(1+.0772/2\right)^{38}}$$

$$= \$1,089.43$$

The bond price declines to \$1,089.43 in six months from \$1,090.46 today. That's a capital loss of \$1.03. Find the annual rate of return *ROR* for the scenario shown in this time line

t=0	1
PV = \$1,090.46	PMT = \$43.125 FV = \$1,089.43

and that satisfies the general time value formula 5.1

$$\overbrace{\$1,090.46}^{PV} = \overbrace{\frac{\$43.125}{\left(1+ROR/2\right)^{1}}}^{CF_1} + \overbrace{\frac{\$1,089.43}{\left(1+ROR/2\right)^{1}}}^{FV}$$

$$\left(1+ROR/2\right) = \frac{\$1,132.56}{\$1,090.46}$$

$$ROR = 7.72\%\ .$$

The annual rate of return for this deal, even though there is a capital loss, equals the yield-to-maturity of 7.72%. Rule 7.1 summarizes this lesson.

RULE 7.1 Actual *ROR* equals *YTM* whenever interest rates remain constant

The actual *ROR* for a bond investment equals the yield-to-maturity whenever the bond is held to maturity or, if sold early, the yield-to-maturity is the same as when bought.

The yield-to-maturity also is called the *promised yield*. Buy the bond, receive the coupons, and if you sell the bond at anytime then, as long as the yield-to-maturity is the same as when purchased, the investment's actual internal rate of return equals the promised yield-to-maturity. The nearby *Calculator Clue* validates this rule when you buy the bond in the second row of the quote table: the AT&T four-and-one-halfs of 36; assume that initially 18 coupons remain; assume you receive a dozen coupons and sell the bond and yield-to-maturity remains constant.

CALCULATOR CLUE 7.2 These are the steps to find the actual annual rate of return for the AT&T four-and-one-halfs of 36 at ninety-eight and one-half after receiving a dozen coupons (initially say 18 coupons remain; *YTM* remains constant). Set-up the calculator by typing [2nd] [FV] to clear the time value memories and [2nd] [I/Y] 2 [ENTER] [2nd] [CPT] to enforce semiannual compounding.

Set the initial coupon and find the yield-to-maturity.
45 [÷] 2 [=] [PMT] 1000 [FV] 18 [N] 985 [+/-] [PV] [STO] [1] [CPT] [I/Y].
The display shows the *YTM* is 4.71%. Notice the keystrokes store the initial price of $985 in memory 1 because the number is used again later. Now use these steps find the price after receiving a dozen coupons.
[RCL] [N] [-] 12 [=] [N] [CPT] [PV].
The display shows the price of the bond is $994.29 when 6 coupons remain and the *YTM* equals 4.71%. Now use these steps to compute the actual rate of return when the cost is $985, the cash inflows equal $22.50 every six months for six years, and immediately after the twelfth periodic cash flow there is an additional inflow of $994.29:
[+/-] [FV] [RCL] [1] [+/-] [PV] 12 [N] [CPT] [I/Y].
The display shows that the actual rate of return equals 4.71%, just as expected.

Preceding discussions show that *YTM*, the promised yield, properly measures the total rate of return even though returns accrue two different ways. Some return comes from coupons and other return comes from capital gains (or losses). This formula shows that the *YTM* partitions total return into these two sources.

FORMULA 7.5 Components for bonds of the promised yield-to-maturity

$$yield-to-maturity = \left(\frac{coupon}{bond\ \ price} \quad + \quad \%\Delta(bond\ \ price) \right) \times 2$$

$$= \begin{pmatrix} current \\ yield \end{pmatrix} + \begin{pmatrix} capital \\ gains \\ yield \end{pmatrix}$$

The total return from a bond investment has two sources. A current income component provides immediate cash flow in the form of semiannual coupons while a changing price component causes capital gains or losses. Table 7.2 contrasts characteristics for these two components.

Current yield (*coupon / price*)	Capital gains yield (*%Δprice*)
realized cash flow	accrued cash flow
immediately taxable	taxes are deferred
relatively predictable & more certain	very unpredictable & more uncertain
relatively large and usually the main reason for buying the bond	relatively small and for stable interest rate settings is not a significant decision variable (except for zero coupon bonds)

TABLE 7.2 Component characteristics for the total rate of return

The table shows the coupon is a realized and taxable cash flow that is fairly predictable. Bond investors usually purchase the bond for the primary purpose of receiving the coupons. The expected capital gain (or loss), conversely, is an accrual that is usually relatively small and is not taxable until the security is sold. Two alternative bonds may provide the same expected total rate of return. They may not be equally appealing, however, if the partitioning of their total returns into coupons and capital gains varies.

Zero coupon bonds are an exception to the preceding discussion. Their current yield equals zero because they pay no coupons. The entire total return for a zero coupon bond therefore occurs as capital gains. The Internal Revenue Service has special laws requiring bondholders of zero coupon bonds to pay taxes on capital gains as they accrue. Can't escape the IRS!

Electric companies raise huge sums of money by selling bonds. Examine the snippet below from table 2.1 showing SO sorted 101st by *Total assets* in the list of 11,000 U.S. companies circa beginning of year 2014.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
CMA	$65,227	9	$ 541	2,610	$ 8,667
ICE	64,818	4	254	1,795	25,866
SO	64,546	26	1,710	17,087	36,468
FCX	63,473	36	2,658	20,921	39,174
APA	61,637	5	2,232	16,402	34,013
DE	59,521	67	3,537	37,347	30,592

SNIPPET from table 2.1 in chapter 2: SO

The Southern Company has one common stock, ticker symbol SO, on the liability side its balance sheet. The *Total assets* of $64.5 billion, however, includes wholly owned subsidiaries Alabama Power Co., Georgia Power Co., Gulf Power in Florida, Mississippi Power Company, several other companies, and many other special investment vehicles. Long-term debt of $21.3 billion on the liability side of the SO holding company balance sheet is the financing source for about one-third of the SO *Total assets.* Mostly the long-term debt carries the subsidiary name on the debt securities. A quick check of the FINRA website shows Alabama Power with 28 different bond issues. The longest term bond matures in nearly 30 years. Some mature and repay the principal value of $1,000 very soon. The biggest interest rate that SO is paying on its Alabama Power bonds is 7 3/4 percent, the lowest is ½ percent. An Alabama Power *5.65s of 35* is priced at par. There is huge diversity in financial securities even within one company's balance sheet.

EXERCISES 7.2

Concept quiz

1. A company intends to issue $1 million of bonds with coupon rate of 6.25% for financing *Capital expenditures*. They must choose between 5-year or 20-year bonds. Otherwise, the bonds are identical. Discuss possible considerations that might influence the decision.

Numerical quickies

2. The company today issues a 10-year $1,000 bond that carries a 6.5 percent coupon rate. Find the total interest that the company expects to pay over the lifetime of the bond. ©BD9 .

3. Today is a day in December 2525 and a zero coupon bond that matures in June 2538 has an annual yield-to-maturity of 7.00% (semiannual compounding). Find the bond price. ©BD10a .

4. Today is a day in June 2525 and a zero coupon bond that matures in December 2539 has a quoted price of 26.00 percent of par (semiannual compounding). Find the annual yield-to-maturity. ©BD10b .

5. A 10-year bond with a 4.40% coupon rate was issued with a 5.37% yield to maturity. Find the bond price at time of issue. ©BD7a

6. A 20-year bond with a 7.80% coupon rate was issued at a price of $1,130. Find the bond yield to maturity at time of issue. ©BD7b

7. Today is a day in June 2525 and a bond with annual coupon rate of 12.40% just yesterday paid a coupon. The bond matures in June 2545 and its annual yield-to-maturity equals 8.80% (semiannual compounding). Find the bond price. ©BD11a .

8. Today is a day in June 2525 and a bond with annual coupon rate of 2.90% just yesterday paid a coupon. The bond matures in June 2540 and its quoted bond price is 71.21 percent of par (semiannual compounding). Find the annual yield-to-maturity. ©BD11b .

Numerical challengers

9. Today is a day in March 2525 and a bond with annual coupon rate of 3.80% just yesterday paid a coupon. The bond matures in September 2536 and its quoted bond price is 74.47 percent of par (semiannual compounding). Find the current yield and capital gains yield. ©BD12a

10. Today is a day in November 2525 and a bond with annual coupon rate of 5.40% just yesterday paid a coupon. The bond matures in May 2537 and its quoted bond price is 74.53 percent of par (semiannual compounding). You wish to make a bid such that your promised rate of return is 30 basis points greater than the quoted annual yield-to-maturity. Find the price as percent of par that you offer for the bond. ©BD13a

11. A bond with a coupon rate of 7.30% has a price that today equals $868.92 . The $1,000 bond pays coupons every 6 months, 30 coupons remain, and a coupon was paid yesterday. Suppose you buy this bond at today's price and hold it so that you receive 20

coupons. You sell the bond upon receiving that last coupon. Find the selling price if the bond's yield-to-maturity remains constant. ©BD14

3. Bond price movements

Bond prices, like stock prices, change with time. The time value principles previously presented govern the relationships, however. Inspection of bond prices in the preceding quote table shows a range from 73 3/4 to 111 ½ percent of par. Rearrangement and experimentation of formula 7.4 reveals that the relation between coupon rate and yield to maturity determines the level of the bond price.

RULE 7.2 Relation between yield-to-maturity, coupon rate, and bond price

$$\textit{Bond price} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \textit{\$1,000 when coupon rate} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \textit{YTM}.$$

The bond price exceeds face value and the bond is said to sell at a *premium* when coupon rate > yield-to-maturity. Conversely, bond price is less than face value and the bond is said to sell at a *discount* when the coupon rate < *YTM*. When coupon rate and yield to maturity are equal the bond price equals face value and the bond is said to sell at *par*.

Borrowers usually set coupon rates so that the bonds sell in the primary market at a price near face value. The coupon rate is printed on the bond and is unchanging. The overall level of interest rates, on the other hand, rises and falls with economic factors such as inflation. Yield-to-maturity for any particular bond correlates highly with the overall level of rates. Thus, a bond "born" in a 10% interest rate environment probably has a 10% coupon rate and sells at par. If interest rates subsequently drop to 7% then its likely price rises. New bonds of that era tend to have 7% coupon rates and sell at par. The old bond with 10% coupon rate generates higher coupons than the new 7% bonds and, hence, the price of the old 10% bond rises to reflect its advantageous cash flow.

A 10% bond in a 7% world sells for more than $1,000! The bond sells at a premium. Conversely, a 10% bond in a 13% world sells for less than $1,000. This bond sells at a discount to face value. The preceding discussion exemplifies Rule 7.3:

RULE 7.3 Inverse relation between bond price and interest rate movements

$$\textit{Existing bond prices} \begin{Bmatrix} \textit{rise} \\ \textit{fall} \end{Bmatrix} \textit{when subsequent interest rates} \begin{Bmatrix} \textit{fall} \\ \textit{rise} \end{Bmatrix}.$$

Rule 7.3 is strictly true for exclusively the relation between a particular bond's price and yield-to-maturity. Because a particular *YTM* generally rises or falls with the overall level of interest rates, though, the rule is generally true. A violation may occur if, for example, a particular bond suddenly becomes riskier due to financial distress of the issuing company. For that scenario the particular bond price falls and yield-to-maturity rises irrespective of whether overall interest rates are rising or falling. Still, rule 7.3 is a useful generalization.

3.A. Constant interest rates and scientific amortization

Suppose interest rates remain constant. What happens to the bond price over time? The clear answer is that over time the bond price converges to face value (as long as the bond does not default). Imagine that a bond maturing tomorrow delivers cash flow of $1,000. The market value of the bond today is a smidgeon away from $1,000 irrespective of whether the coupon rate is 1% or 20%. Figure 7.1 illustrates this phenomenon.

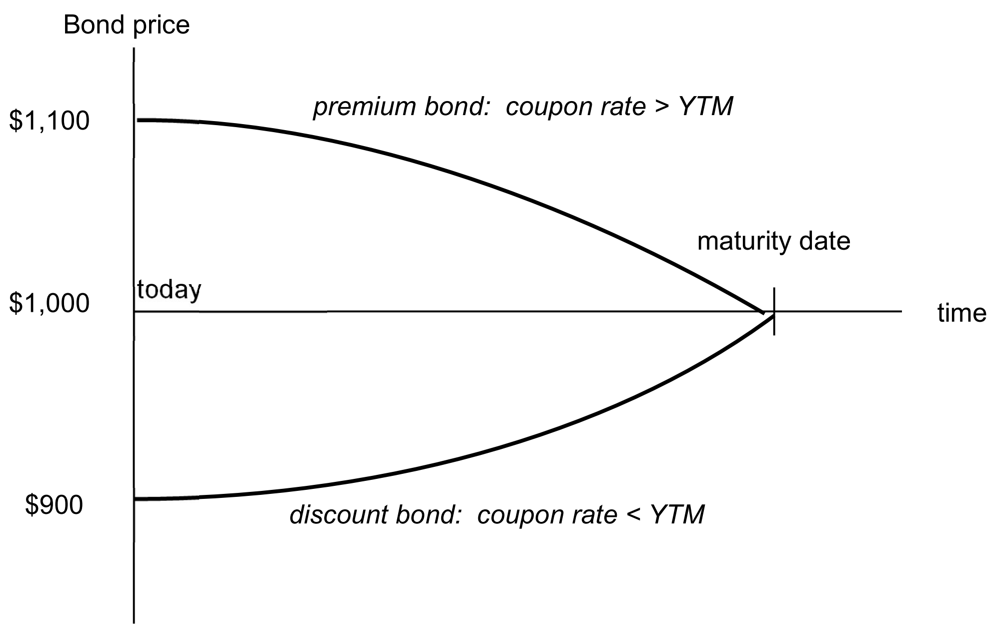


FIGURE 7.1 Evolution of bond price over time given constant yield-to-maturity
The premium bond with $1,100 price today converges by the maturity date to its face value of $1,000. The discount bond with price of $900 also converges to face value.

Scientific amortization refers to the evolutionary path of bond price from its current price toward face value given yield-to-maturity remains constant. The path is predictable and depends exclusively on diminishing *N*, number of coupons, in formula 7.4. Beneath the horizontal axis in Figure 7.1 is the evolution of price for a discount bond with current price about $900. The price certainly rises by the maturity date to $1,000. Suppose that the maturity date is 20 years from today. If the price were to rise by straight-line from $900 up to $1,000 then in 10 years the price would equal $950. Notice, however, there is slight curvature in the scientific amortization time-path. The price on the curve halfway between today and the maturity date is below $950.

Likewise, the curve above the axis illustrates motion for a premium bond as its price declines from today's $1,100 to the $1,000 maturity price. The price for the premium bond at a point in time halfway between today and the maturity date is somewhat above $1,050.

Even though both bonds possibly provide the same total rate of return there is a huge difference in relative importance of current yield versus capital gains yield. The discount bond generates a total capital gain of $100; the premium bond a capital loss of $100. Still, as the example below shows, the promised rate of return can be identical.

EXAMPLE 2 Find coupon rates at which a premium and discount bond have equal *YTM*

Two bonds have identical 10% yield-to-maturity and 20 years (40 coupons) remain until maturity. One is a discount bond with price of $900. The other is a premium bond with price of $1,100. (a) Compare their coupon rates. (b) Compare according to scientific amortization their current yields expected in 12 years.

SOLUTION

Use formula 7.4 in which *Face value* = $1,000, *N* = 40, and yield-to-maturity is 10%. Set the bond price and find the unknown periodic coupon. For the discount bond solve this formula.

$$\$900 = coupon \left\{ \frac{1 - \left(1 + 0.10/2\right)^{-40}}{0.10/2} \right\} + \frac{\$1{,}000}{\left(1 + 0.10/2\right)^{40}}$$

$$coupon = \$44.17$$

The discount bond pays $88.34 interest per year [= $44.17 ×2] implying an annual coupon rate of 8.83% [= $88.34 ÷ *Face value*] and current yield of 9.82% [= $88.34 ÷ $900]. Because total rate of return equals 10 percent, by the way, the annual capital gains yield today for the discount bond equals 18 basis points [= 10% - 9.82%].

Similar computations for the premium bond with price of $1,100 show that its semiannual coupon equals $55.83; annual coupon rate equals 11.16%; current yield is 10.15%, and annual capital gains yield is -15 basis points.

Summary of the solution for part (a) is that in an economic environment when yield-to-maturities equal 10% then a 20-year bond with coupon rate of 8.83% has price of $900. A 20-year bond with coupon rate of 11.16% has price of $1,100. They both provide the same total rate of return of 10% even though the discount bond provides relatively small coupons plus capital gains whereas the premium bond provides relatively large coupons plus capital losses.

Scientific amortization presumes that the coupon rate and yield-to-maturity remain constant. Changing with time, however, are number of remaining coupons, price, and partitioning of *YTM* into current yield and capital gains yield.

For part (b) find the price prevailing when 8 years remain to maturity (*N* = 16). For the discount bond find the price from this formula:

$$bond\ \ price = \$44.17 \left\{ \frac{1 - \left(1 + 0.10/2\right)^{-16}}{0.10/2} \right\} + \frac{\$1{,}000}{\left(1 + 0.10/2\right)^{16}}$$

$$= \$936.84$$

The annual interest of $88.34 remains constant, of course, but the price rises to $936.84 in twelve years from $900 today. The current yield therefore declines to 9.43% in twelve years [= $88.34 ÷ $936.84] from 9.82% today. The current yield is declining and capital gains yield is increasing. In figure 7.1 the capital gains yield relates to the slope of the curve showing the bond price. For the discount bond on the curve beneath the horizontal axis the line is getting steeper as time to maturity approaches. The slope becomes a bigger positive number and capital gains become an increasingly larger proportion of total return (but the capital gain still remains much smaller than the coupon.)

Twelve years from today the price of the premium bond equals $1,063 and current yield increases to 10.50% from 10.15% today. Notice that the curve for the premium bond turns steeply downward as maturity nears. The capital gains yield is negative, implying capital losses, and over time becomes even more negative. The relatively large coupon and current yield, however, assure the total rate of return equals the 10% yield-to-maturity, just as promised. Scientific amortization shows that even with constant yield-to-maturity the current yield over time rises for premium bonds and falls for discount bonds.

CALCULATOR CLUE 7.3 Solve example 2 with these steps. Set-up the calculator by typing 2nd FV to clear the time value memories and 2nd I/Y 2 ENTER 2nd CPT to enforce semiannual compounding.

Find coupon rate and current yield for the 20-year discount bond with price of $900 and 10% yield-to-maturity:

1000 FV 40 N 900 +/- PV 10 I/Y CPT PMT .

The display shows the semiannual coupon equals $44.17. Find the annual interest

x 2 = ,

equals $88.34, thereby implying the 8.834% annual coupon rate. Find today's current yield:

÷ RCL PV +/- = .

The display shows that the annual current yield today equals 9.82%. Now use these steps to find the price and current yield twelve years from today:

RCL N - 24 = N CPT PV RCL PMT x 2 = ÷ RCL PV +/- = .

The display shows the current yield equals 9.43% when 16 coupons remain. Easily perform the same steps for the premium bond.

3.B. *Horizon analysis and changes in the interest rate*

Analysts devote substantial resources trying to predict whether future interest rates, like stock prices, will rise or fall. Changing interest rates affect bond prices as shown in rule 7.3. Time value relations perfectly specify how the total rate of return for a bond investment relates to interest rate predictions. *Horizon analysis* for bonds finds the total rate of return resulting from a predicted change in interest rates. The example below illustrates horizon analysis.

EXAMPLE 3 Horizon analysis for a 6-month holding period

Today is a day in February 2525 and a bond with annual coupon rate of 7.50% and price of $950 just yesterday paid a coupon. The bond matures in August 2534 (semiannual compounding). Suppose you buy the bond at today's price, hold it 6 months, receive a coupon, and sell the bond. Contrast the annual rate of return from the investment if the bond yield-to-maturity 6 months from now (a) is lower by 125 basis points, or (b) is higher by 125 basis points.

SOLUTION

Find first the promised yield-to-maturity with formula 7.4. To find *N* count the number of semiannual periods between February 2525 and August 2534. There are 8 complete years from 2526 to 2533 inclusively (2533 – 2526 + 1) with two coupons each, that's 16, plus two coupons in 2534 and one remaining in 2525 bringing the total to 19. With *Face value* of $1,000 and price of $950 and coupon of $37.50 solve this formula:

$$\$950 = \$37.50 \left\{ \frac{1 - \left(1 + YTM/2\right)^{-19}}{YTM/2} \right\} + \frac{\$1{,}000}{\left(1 + YTM/2\right)^{19}}$$

$$\text{or} \qquad YTM = 8.27\% \, .$$

The promised yield of 8.27% equals the actual rate of return if the *YTM* were to remain constant, but it doesn't. For scenario (a) the *YTM* falls by 125 basis points to become 7.02%. Solve for the price existing 6-months from today given that *N* diminishes to 18 and *YTM* equals 7.02%

$$\text{bond price} = \$37.50 \left\{ \frac{1 - \left(1 + 0.0702/2\right)^{-18}}{0.0702/2} \right\} + \frac{\$1{,}000}{\left(1 + 0.0702/2\right)^{18}}$$

$$= \$1{,}031.62 \, .$$

Cash flows for this deal include purchase of a bond today for $950 and receipt in 6 months of a $37.50 coupon plus $1,031.62 selling price. Total ending wealth over the six month horizon equals $1,069.12 [= $37.50 + $1,031.62]. Find the annual rate of return *ROR* as:

$$ROR = \left(\frac{\$1{,}069.12 - \$950}{\$950} \right) \times 2$$

$$= 25.08\% \, .$$

Cast these cash flows within a "savings account scenario:" deposit $950 into the account and six months later withdraw $1,069.12 and, given interest compounds semiannually and the account balance after the withdrawal is zero, the account pays interest at a 25.08% annual percentage rate.

The return is much larger than the promised yield of 8.27% because by end of the investment horizon the interest rate falls. According to rule 7.3 a falling interest rate causes bond prices to rise. The bondholder benefits.

Scenario (b) with a rising interest rate is not so favorable for the investor because *YTM* rises and bond price falls. With new *YTM* of 9.52% [= 8.27% + 1.25%] the bond price in 6 months when *N* = 18 is $879.68. The *ROR* equals -6.91%. Existing bondholders lose money when interest rates rise.

CALCULATOR CLUE 7.4 Solve example 3 with these steps. Set-up the calculator by typing [2nd] [FV] to clear the time value memories and [2nd] [I/Y] 2 [ENTER] [2nd] [CPT] to enforce semiannual compounding.

Find yield-to-maturity at the beginning:
1000 [FV] 19 [N] 950 [+/-] [PV] [STO] [1] 37.50 [PMT] [CPT] [I/Y] .
The display shows the original *YTM* equals 8.27%. Find the price in 6 months when *N* =

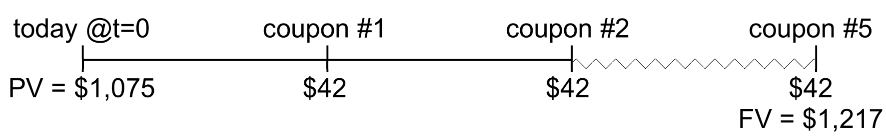
18 and *YTM* falls by 1.25%:
[RCL] [N] [-] 1 [=] [N] [RCL] [I/Y] [-] 1.25 [=] [I/Y] [CPT] [PV] .
The price of $1,031.62 represents ending wealth occurring at the same time as the last (and only) payment of $37.50. Find the annual *ROR* throughout the 1-period investment horizon:
[+/-] [FV] [RCL] [1] [PV] [1] [N] [CPT] [I/Y] .
The display shows that the annual *ROR* equals 25.08%. Perform the latter steps to obtain the answer for scenario (b).

EXAMPLE 4 Horizon analysis for an *N*-year holding period ©BD19b

A bond with annual coupon rate of 8.40% and price of $1,075 just yesterday paid a coupon. A total of 24 coupons remain to be paid. Suppose you buy the bond at today's price, hold it 2½ years, receive 5 coupons, and then sell the bond. Find the annual rate of return throughout the investment horizon if at the time you sell the bond its yield-to-maturity has fallen a total of 200 basis points.

SOLUTION

This is very similar to example 3 except that the horizon is 5 periods (instead of 1). With *N* = 24 and *Face value* = $1,000 and *bond price* = $1,075 and *Coupon* = $42 compute that the promised yield-to-maturity equals 7.44%. After 5 coupons have been received there remain 18. With *N* = 19 and the yield-to-maturity of 5.44% [= 7.44% - 2%] the new price equals $1,217. The annual rate of return pertains to this timeline:



and satisfies constant annuity formula 5.1:

$$\$1{,}075 = \$42 \left\{ \frac{1 - \left(1 + ROR/2\right)^{-5}}{ROR/2} \right\} + \frac{\$1{,}217}{\left(1 + ROR/2\right)^{5}}$$

$$\text{or} \qquad ROR = 12.5\% .$$

The falling interest rate pushes the actual *ROR* above the promised yield.

CALCULATOR CLUE 7.5 Solve example 4 with these steps. Set-up the calculator by typing [2nd] [FV] to clear the time value memories and [2nd] [I/Y] 2 [ENTER] [2nd] [CPT] to enforce semiannual compounding.

Find beginning yield-to-maturity:
1000 [FV] 24 [N] 1075 [+/-] [PV] [STO] [1] 42 [PMT] [CPT] [I/Y] .
Find the price in 2½ years when *N* = 19 and *YTM* falls by 2%:
[RCL] [N] [-] 5 [=] [N] [RCL] [I/Y] [-] 2 [=] [I/Y] [CPT] [PV] .
Find the annual *ROR* throughout the 5-period investment horizon:
[+/-] [FV] [RCL] [1] [PV] [5] [N] [CPT] [I/Y] .
The display shows that the annual *ROR* equals 12.5%.

EXERCISES 7.3B

Concept quiz

1. The IRS requires that an investor in a zero coupon bond declare as taxable income the price change occurring along a standard amortization path. Discuss whether an investor generally may prefer to compute the price change with a straight-line path or with a scientific amortization path.

Numerical quickies

2. Bond X has annual coupon rate of 6.8%, 20 coupons remain until maturity, and its price is $920. Premium bond Z with price of $1,060 also has 20 coupons remaining. Under what condition is the yield-to-maturity larger for bond Z than for bond X? ©BD18 .

3. Today is a day in August 2525 and a bond with annual yield-to-maturity of 7.00% just yesterday paid a coupon. The bond matures in February 2530 and its quoted bond price is 83.64 percent of par (semiannual compounding). Find the annual coupon rate and today's current yield. ©BD16b .

Numerical challengers

4. A bond with yield-to-maturity of 7.40% and 17 coupons remaining until maturity has a price today of $1,170. Find the coupon rate, today's current yield, and current yield expected after 11 coupons have been received (assume scientific amortization and constant *YTM*). ©BD15 .

5. Today is a day in June 2525 and a bond with annual yield-to-maturity of 11.20% just yesterday paid a coupon. The bond matures in June 2540 and its quoted bond price today is 77.72 percent of par (semiannual compounding). Contrast the annual capital gains yield today with the annual capital gains yield for the six months that conclude with June 2540 (assume scientific amortization and constant *YTM*). ©BD17b .

6. A bond with annual coupon rate of 5.10% and price of $1,090 just yesterday paid a coupon. A total of 23 coupons remain to be paid. Suppose you buy the bond at today's price, hold it and receive 8 coupons, and then sell the bond. If at the time you sell the bond its yield-to-maturity has decreased a total of 50 basis points find the bond selling price and the annual rate of return throughout the investment horizon. ©BD19b .

3.C. Riding the yield curve

Will future interest rates rise or fall? Interest rate movements often affect economic activity. The question is therefore important for main street consumers deciding about house and car purchases, for Wall Street investors deciding about bond portfolios, and for Washington policymakers deciding on national growth and employment objectives. Besides the issue of rising or falling, however, there is another subtlety: short-term and long-term interest rates likely differ!

The *yield curve* is a snap-shot of yield-to-maturities on a sample of bonds identical in every way except for term to maturity. Figure 7.2 illustrates a *normal* yield curve.

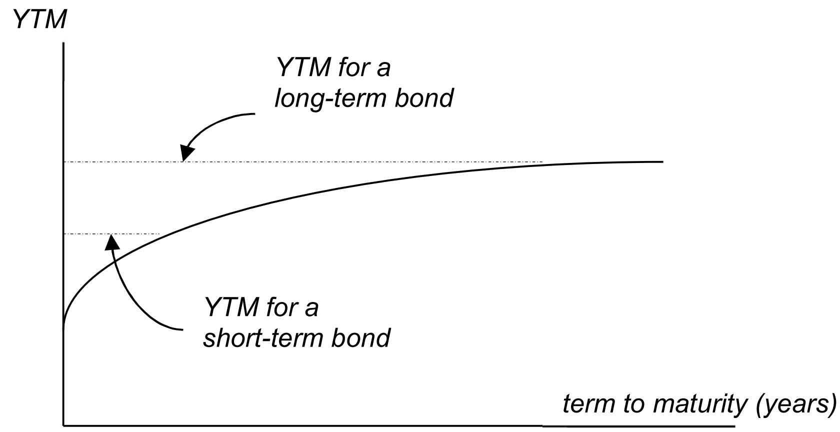


FIGURE 7.2 Normal yield curve
Notes Each point on the curve represents a bond identical to all other points except for difference in term to maturity. The yield-to-maturity is normally lower for short-term rates and higher for long-term rates. The yield curve shape changes, however, rising, falling, flattening, and sometimes inverting so that short-term rates are biggest. The normal yield curve is the normal shape, however.

Lessons in chapter 11 explain factors shaping the yield curve. For now, however, focus on time value relationships inherent with bonds. Already Rule 3 instructs that bond price moves inversely with yield-to-maturity. Inspection of figure 7.2 shows that *YTM* normally falls as maturity nears. Surely if one holds a long-term bond until maturity then the actual *ROR* equals promised *YTM*. But if the bond is sold early then perhaps the investor benefits from a strategy known as *riding the yield curve*. The example below illustrates the strategy.

EXAMPLE 5 Riding the yield curve ©BD5a .

You expect that, for a given term, interest rates will remain unchanged. As is normal, the yield curve is sloped upward and yield increases with the term of the bond. The yield-to-maturity for a 2-year zero-coupon bond is 8.25%. For a 4-year zero-coupon bond, the yield to maturity is 8.45%. You enhance your returns for a 2-year investment horizon by following a strategy called "riding the yield curve". You buy the long-term bond and sell it after its yield has fallen to the short term rate. What is your rate of return from riding the yield curve?

SOLUTION

The price of the 2 year zero-coupon bond is:

$$PV = \$1{,}000 / (1 + .0825/2)^4$$

$$= \$850.71 .$$

If you buy the bond now and it matures at $1000 two years (4 half-years) from now, your ROR is 8.25%.

Instead of buying the 2-year bond and holding it to maturity, however, you buy the 4-year bond and hold it two years. The price of the 4 year bond today is determined from the following present value relation:

$PV = \$1,000 / (1 + .0845/2)^8$

$= \$718.17.$

Two years from now its yield-to-maturity will fall to 8.25% because the yield curve remains unchanged. Thus, its price will be

$PV = \$1,000 / (1 + .0825/2)^4$

$= \$850.71 .$

In other words, today's 4-year bond in 2 years will have the same price as today's 2-year bond! If you buy the 4-year bond and hold it for two years, your annual ROR (compounded semiannually) is determined from the following relation:

$\$718.17 = \$850.71 / (1 + ROR/2)^4$

Use algebra or your calculator to find that *ROR* equals 8.65% . This strategy increases your return by 40BP above the 2 year bond rate!

CALCULATOR CLUE 7.6 Solve example 5 with these steps. Set-up the calculator by typing [2nd] [FV] to clear the time value memories and [2nd] [I/Y] 2 [ENTER] [2nd] [CPT] to enforce semiannual compounding.

Find and store in memory the prices for the 2-year bond:
1000 [+/-] [FV] 4 [N] 8.25 [I/Y] [CPT] [PV] [STO] [1]
and for the 4-year bond
8 [N] 8.45 [I/Y] [CPT] [PV] [STO] [2]
Now find the annual *ROR* throughout the 4-period investment horizon:
[RCL] [1] [+/-] [FV] [RCL] [2] [PV] [4] [N] [CPT] [I/Y] .
The display shows that the annual *ROR* equals 8.65%. This is 40 basis points bigger than the 2-year *YTM*.

EXERCISES 7.3C

Numerical quickies

1. The yield-to-maturity for a zero coupon bond is 10.30% for a 1-year bond and 11.36% for a 2-year bond. You wish to make a 1-year investment and obviously can buy the 1-year bond and hold it to maturity. Suppose, however, that you think the yield curve will remain the same throughout the future. You can pursue an alternative strategy of buying a 2-year bond, holding it for 1 year, and selling it when it has 1 year remaining to maturity. How does this alternative strategy enhance your average annual rate of return? (Assume, if necessary, that you can buy fractions of bonds.) ©BD5b .

2. The yield-to-maturity for a zero coupon bond is 11.80% for a 1-year bond, 12.70% for a 2-year bond, and 13.26% for a 3-year bond. You wish to make a 2-year investment

and obviously can buy the 2-year bond and hold it to maturity. Suppose, however, that you think the yield curve will remain the same throughout the future. You can pursue an alternative strategy of buying a 3-year bond, holding it for 2 years, and selling it when it has one year remaining to maturity. Relative to the 2-year yield-to-maturity, by how many basis points does this alternative strategy enhance your average annual rate of return? (Assume, if necessary, that you can buy fractions of bonds.) ©BD5a .

Numerical challenger

3. The yield-to-maturity for a zero coupon bond is 11.20% for a 1-year bond, 11.97% for a 2-year bond, and 12.31% for a 3-year bond. You think the yield curve will remain the same throughout the future. You wish to make a 1-year investment, that is, buy a bond today and sell it in one year. You can pursue three alternative strategies, call them S1, S2, and S3. For strategy S1, you buy the 1-year bond and hold it to maturity, in which case your annual rate of return obviously is 11.20%. For S2, buy a 2-year bond today and sell it when it has 1 year remaining to maturity. For S3, buy a 3-year bond today and sell it when it has 2 years remaining to maturity. What are your average annual rates of return for strategies S2 and S3? (Assume, if necessary, that you can buy fractions of bonds.) ©BD5c .

ANSWERS TO CHAPTER 7 EXERCISES

EXERCISES 7.2

1. Rule 1.1 states that the goal of the company is to pursue policies that maximize capitalized value of wealth creation. Just like most households are “price-takers” in their consumer goods marketplace, so too most companies are “rate-takers” in the financing sources marketplace. This means that most companies cannot capture incremental economic profit from their financing source. Instead, they capitalize incremental wealth by transforming factors of production from stakeholder markets into goods and services that customers want. To the extent that competitive financing sources are willing to lend the company $1 million at 6.25%, irrespective of whether the term is 5-years or 20-years, suggests they both offer zero NPV to the company; they both offer zero NPV to capitalists providing financing. In such a case one loan-term is not better than the other and company as well as capitalist are indifferent between the two. There are, however, important underlying issues that merit mention. Also, re-read answers for conceptual questions #1 in Sections 5.5 and 6.2 because they too relate to this question.

(a) Later lessons teach that equilibrium interest rates typically depend on loan-term. Usually 5-year interest rates are less than 20-year rates. Hence, interest *per year* usually is less for a 5-year than a 20-year loan. Cash flow differences typically translate into risk differences. Regardless of rate difference, however, the 20-year loan may be riskier for the company due to its lengthier obligation period and this may translate into higher financing rates *for equity*. Later lessons consider interaction between risk and equilibrium financing rates.

(b) Often companies and households match the term of financing sources with the length of the expected service stream from underlying real capital assets. In other words, finance short-term assets with short-term liabilities and long-term assets with long-term liabilities. Introducing a *gap* between implied maturities of assets and liabilities generally increases overall risk for equity.

(c) Future growth opportunities for the company affect its marginal utility of money. The marginal utility of money is very high when the company has unusually large short-term cash-needs (as in the land-rich cash-poor balance sheet scenario). Deferring repayment of principal into the remote future may be worth the cost of interest when in the near term it means being able to pay unusually large research and development costs thereby allowing the company to survive. And if the company expects to invent and launch a blockbuster product sometime down the road then long-term financing may indeed be an intelligent strategy even though lifetime interest is higher.

2. ©BD9 The company pays $65 interest per year (two equal semiannual installments of $32.50) for 10 years for a total of $650 interest.

3. Biggest difficulty with this problem is counting the number of semiannual periods between December 2525 and June 2538. There are 12 complete years from 2526 to 2537 inclusively (2537 – 2526 + 1) with two coupons each, that's 24, plus 1 in 2538 and none remaining in 2525, bringing the total to 25. Use formula 7.4 wherein *N*= 25, coupon = $0, *YTM* = 7.00%, face value = $1,000 and solve for the unknown bond price.
$\textit{bond price} = \$1{,}000 \times (1+0.07/2)^{-25}$; or *bond price = $423.*

4. Count the number of semiannual periods between June 2525 and December 2539. There are 13 complete years from 2526 to 2538 inclusively (2538 – 2526 + 1) with two coupons each, that's 26, plus two in 2539 plus one remaining in 2525 bringing the total to 29. Use formula 7.4 wherein *N*= 29, coupon = $0, bond price = $260, face value = $1,000 and solve for the unknown *YTM*.
$\$260 = \$1{,}000 \times (1+YTM/2)^{-29}$; or *YTM = 9.51%.*

5. Bond cash flows equal 20 coupons of $22 each plus the $1,000 repayment of principal. Find the bond price with formula 7.4 when the annual yield-to-maturity equals 5.37%:
$\textit{bond price} = \$22 \times \{1 - (1+0.0537/2)^{-20}\} \div 0.0537/2 + \$1{,}000 \times (1+0.0537/2)^{-20}$
The bond price at time of issue was $926. This means that the issuing company in a primary market transaction received $926 and promised to pay the bond investor the cash flows listed above.

6. ©BD7b Bond cash flows equal 40 coupons of $39 each plus the $1,000 repayment of principal. Find the annual yield-to-maturity with formula 7.4 when the bond price equals $1,130:
$\$1{,}130 = \$39 \times \{1 - (1+YTM/2)^{-40}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-40}$
Use a financial calculator to find that the yield-to-maturity at time of issue was $6.62%.

7. Count the number of semiannual periods between June 2525 and June 2545. There are 19 complete years from 2526 to 2544 inclusively (2544 – 2526 + 1) with two coupons each, that's 38, plus one in 2545 plus one remaining in 2525 bringing the total to 40. Use formula 7.4 wherein *N*= 40, coupon = $62, *YTM* = 8.80%, face value = $1,000 and solve for the unknown bond price.
$\textit{bond price} = \$62 \times \{1 - (1+0.0880/2)^{-40}\} \div 0.0880/2 + \$1{,}000 \times (1+0.0880/2)^{-40}$
or bond price equals $1,336.

8. Count the number of semiannual periods between June 2525 and June 2540. There are 14 complete years from 2526 to 2539 inclusively (2539 – 2526 + 1) with two coupons each, that's 28, plus one in 2540 plus one remaining in 2525 bringing the total to 30. Use formula 7.4 wherein *N*= 30, coupon = $14.50, bond price = $712.10, face value = $1,000 and solve for the unknown *YTM*.
$\$712.10 = \$14.50 \times \{1 - (1+YTM/2)^{-30}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-30}$
or *YTM* equals 5.8%.

9. Count the number of semiannual periods between March 2525 and September 2536. There are 10 complete years from 2526 to 2535 inclusively (2535 – 2526 + 1) with two

coupons each, that's 20, plus two in 2536 plus one remaining in 2525 bringing the total to 23. Use formula 7.4 wherein N= 23, coupon = \$19.00, bond price = \$744.70, face value = \$1,000 and solve for the unknown *YTM*.
$\$744.70 = \$19.00 \times \{1 - (1+YTM/2)^{-23}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-23}$
or *YTM* equals 7.08%.

Formula 7.5 shows that the yield-to-maturity equals the current yield plus capital gains yield. Annual current yield equals 5.10% [= \$38/\$744.70] and capital gains yield therefore must equal 1.98% [= 7.08% - 5.10%].

10. Count the number of semiannual periods between November 2525 and May 2537. There are 11 complete years from 2526 to 2536 inclusively (2536 – 2526 + 1) with two coupons each, that's 22, plus one in 2537 and none remaining in 2525 bringing the total to 23. Use formula 7.4 wherein N= 23, coupon = \$27.00, bond price = \$745.30, face value = \$1,000 and solve for the unknown *YTM*.
$\$745.30 = \$27.00 \times \{1 - (1+YTM/2)^{-23}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-23}$
or *YTM* equals 9.00%.

You wish to get a 9.30% return [= 9.00% + 0.30]. Get the counteroffer price:
$price = \$27.00 \times \{1 - (1+0.0930/2)^{-23}\} \div 0.0930/2 + \$1{,}000 \times (1+0.0930/2)^{-23}$
or price equals \$728.03. Your offer is 72.8 percent of par.

11. Use formula 7.4 wherein N= 30, coupon = \$36.50, bond price = \$868.92, face value = \$1,000 and solve for the unknown *YTM*.
$\$868.92 = \$36.50 \times \{1 - (1+YTM/2)^{-30}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-30}$
or *YTM* equals 8.90%.

After 20 coupons are received there remain 10. Find the new price:
$price = \$36.50 \times \{1 - (1+0.0890/2)^{-10}\} \div 0.0890/2 + \$1{,}000 \times (1+0.0890/2)^{-10}$
or price equals \$936.54.

EXERCISES 7.3B

1. The financial incentive of a tax payer is of course following the law. Beyond that, however, it's generally better to pay less than more and to pay later than sooner. The total price change over the life of the bond, and hence total taxes, is the same irrespective of the amortization path. The path affects the timing of the tax payments; the timing affects the present value of tax liabilities. Investors prefer to reduce present value of tax liabilities.

Inspection of figure 7.1 reveals that the scientific amortization path for a discount bond lays beneath the straight-line connecting "today's price" with "\$1,000 maturity face value." The straight-line path accelerates the price change and increases the present value of taxes. The scientific amortization path delays price change and reduces present value of tax liabilities.

2. ©BD18 Quick computation with the financial calculator shows that bond X has a 7.98% yield-to-maturity. Z has *YTM* of 7.98% when its semiannual coupon equals \$44.29. That equals an annual coupon rate of 8.86%. When the coupon rate for Z exceeds 8.86% then Z's yield-to-maturity exceeds X's.

3. ©BD16b . Count the number of semiannual periods between August 2525 and February 2530. There are 4 complete years from 2526 to 2529 inclusively (2529 – 2526 + 1) with two coupons each, that's 8, plus one in 2530 and none remaining in 2525 bringing the total to 9. Use formula 7.4 wherein N= 9, *YTM*= 7%, bond price = \$836.40, face value = \$1,000 and solve for the unknown coupon.
$\$836.40 = coupon \times \{1 - (1+0.07/2)^{-9}\} \div 0.07/2 + \$1{,}000 \times (1+0.07/2)^{-9}$

or coupon equals \$13.50. Double that to get the interest per year equal \$27.00, implying a coupon rate of 2.70%. Divide annual interest by price to obtain today's current yield of 3.23%.

4. Use formula 7.4 wherein N = 17, YTM = 7.4, , bond price = \$1,170, face value = \$1,000 and solve for the unknown coupon.
$\$1{,}170 = \text{coupon} \times \{1 - (1+0.074/2)^{-17}\} \div 0.074/2 + \$1{,}000 \times (1+0.074/2)^{-17}$
or coupon \$50.65. Double that to get the interest per year equal \$101.30, implying a coupon rate of 10.13%. Divide annual interest by price to obtain today's current yield of 8.66%.

Find the price after receiving 11 coupons by setting N = 6 and compute that price is \$1,072. The current yield at that time is 9.45%.

5. ©BD17b Count the number of semiannual periods between June 2525 and June 2540. There are 14 complete years from 2526 to 2539 inclusively (2539 – 2526 + 1) with two coupons each, that's 28, plus one in 2540 and one remaining in 2525 bringing the total to 30. Use formula 7.4 wherein N= 30, YTM= 11.20%, bond price = \$777.20, face value = \$1,000 and solve for the unknown coupon.
$\$777.20 = \text{coupon} \times \{1 - (1+0.112/2)^{-30}\} \div 0.112/2 + \$1{,}000 \times (1+0.112/2)^{-30}$
or coupon equals \$40.50. Double that to get the interest per year equals \$81.00, implying a coupon rate of 8.10%. Divide annual interest by price to obtain today's current yield of 10.42%. Subtract from YTM to get today's annual capital gains yield of 0.78%.

For the last semiannual period find the bond price as
$price = (\$1{,}000 + \$40.50) \div (1+0.112/2); = \$985.32$.
The bond price during that period rises to \$1,000. The annual capital gains yield is 2 × (\$1,000 - \$985.32) ÷ \$985.32 which is 2.98%. Note also that the annual current yield during that semiannum equals 8.22% [= \$81 ÷ \$985.32]. Capital gains yield plus current yield equals YTM.

6. Use formula 7.4 wherein N= 23, coupon = \$25.50, bond price = \$1,090, face value = \$1,000 and solve for the original and unknown YTM.
$\$1{,}090 = \$25.50 \times \{1 - (1+YTM/2)^{-23}\} \div YTM/2 + \$1{,}000 \times (1+YTM/2)^{-23}$
The original YTM equals 4.11%. It decreased to become 3.61% and N decreases to become 15. Solve for the selling bond price:
$price = \$25.50 \times \{1 - (1+0.0361/2)^{-15}\} \div 0.0361/2 + \$1{,}000 \times (1+0.0361/2)^{-15}$
The selling price of the bond is \$1,097. Solve for the annual ROR throughout the 8 period investment horizon:
$\$1{,}090 = \$25.50 \times \{1 - (1+ROR/2)^{-8}\} \div ROR/2 + \$1{,}097 \times (1+ROR/2)^{-8}$
The annual ROR equals 4.83%.

EXERCISES 7.3C

1. ©BD5b . Find the prices today for the bonds:
$P_1 = \$1{,}000 \div (1+.103/2)^2 ; = \$904.44;$
$P_2 = \$1{,}000 \div (1+.1136/2)^4 ; = \$801.73.$
The bond that today has 2 half-years remaining to maturity and promising a 10.3% yield has price of \$904.44. The bond with 4 half-years to maturity and promising an 11.36% yield has price of \$801.73. Buy the 2-year bond today for \$801.73 and sell it next year (after two semiannual periods) when it has 1 year remaining to maturity and forecast price of \$904.44. That ROR is found with the lump-sum time value formula 4.6:
$\$904.44 = \$801.73 (1 + ROR/2)^2 ;$
or ROR for that 2-semiannum investment equals 12.43%. Buy a 1-year bond and get 10.30% or buy a 2-year bond and hold it one year and, given constant yield curve, make 12.43%. The difference is 213 basis points.

2. Find the prices today for the bonds:

P_1 = *\$1,000 ÷ (1+.1180/2)* ^{2} *; = \$891.68;*
P_2 = *\$1,000 ÷ (1+.1270/2)* ^{4} *; = \$781.72*.
P_3 = *\$1,000 ÷ (1+.1326/2)* ^{6} *; = \$680.34*.
Buy the 3-year bond today for \$680.34 and sell it in two year (after four semiannual periods elapse) when it has 1 year remaining to maturity and forecast price of \$891.68. That *ROR* is found with the lump-sum time value formula 4.6:
\$891.68 = \$680.34 (1 + ROR/2) ^{4} ;
or *ROR* for that 4-semiannum investment equals 13.99%. Buy a 2-year bond and get 12.70% or buy a 3-year bond and hold it two years and, given constant yield curve, make 13.99%. The difference is 129 basis points.

3. Find the prices today for the bonds:
P_1 = *\$1,000 ÷ (1+.1120/2)* ^{2} *; = \$896.75;*
P_2 = *\$1,000 ÷ (1+.1197/2)* ^{4} *; = \$792.54*.
P_3 = *\$1,000 ÷ (1+.1231/2)* ^{6} *; = \$698.81*.

For S2 find the *ROR* from buying a 2-year bond today for \$792.54 and selling it in one year (after two semiannual periods elapse) when it has 1 year remaining to maturity and forecast price of \$*896.75*. That *ROR* is found with the lump-sum time value formula 4.6:
\$896.75 = \$792.54 *(1 + ROR/2)* ^{2} ;
or *ROR* for that 2-semiannum investment equals 12.74%.

For S3 find the *ROR* from buying a 3-year bond today for \$698.81 and selling it in one year (after two semiannual periods elapse) when it has 2 years remaining to maturity and forecast price of \$792.54. That *ROR* is found with the lump-sum time value formula 4.6:
\$792.54 = \$698.81 (1 + ROR/2) ^{2} ;
or *ROR* for that 2-semiannum investment equals 12.99%.

S1 promises 11.20%. S2 promises 12.74%. And S3 promises 12.99%. A plot of these points is known as the *rolling yield curve for a 1-year horizon.*

CHAPTER 8: TIME VALUE APPLICATION 3, STOCK VALUATION

The financial media focus on stock markets. When stock prices plummet, headlines tout all types of causes. When an election is forthcoming, prognosticators proclaim likely effects on stock prices. When financial gurus advertise self-help revivals and hawk newsletters, *prima facie* evidence is superior stock picking performance. Predicting stock price movements sometimes seems to some the mother of all mysteries.

Even though access to information and stock market statistics today are better than ever, the basic notion about forces that drive stock prices is about the same as three-quarters a century ago. Famous books from the 1930s, *Security Analysis* by Benjamin Graham and David Dodd, *The Theory of Investment Value* by John Burr Williams, and *The General Theory of Employment, Interest, and Money* by John Maynard Keynes, recognize two factors determining stock prices.

- The speculative factor relates stock price movements to market psychology and investor trading behavior.
- The entrepreneurial factor relates stock price movements to consensus expectations about the discounted value of company cash flows.

Both factors are important determinants of stock prices and merit discussion. This chapter proceeds in section 1 to contrast the speculative and entrepreneurial factors. Section 2 discusses stock valuation by discounting expected dividends. Section 3 investigates the most commonly used approach for analyzing stocks: price multiples.

1. Technical versus fundamental analysis

The most significant difference between speculative and entrepreneurial factors pertains to timing. Stock prices in the long-run require support from sales, profit, and cash flow. The entrepreneurial factor explains the long-run sustenance of stock prices. Fundamental analyses relate stock prices to entrepreneurial factors. Stock prices in the short-run appear sometimes to vibrate randomly, as if driven by animal spirits. The speculative factor explains short-term price movements. Technical analyses relate stock prices to speculative factors.

Specific sectors or stocks somehow become market favorites and investors flock to them like a herd. Potential profits or losses are huge when the herd is on the move. Technical analyses decipher market psychology by using where the stock price has been in order to predict where the price is going.

The oldest and most common technical analysis trading strategy employs a moving average stock price. A simple *N*-period moving average stock price equals the sum of *N* consecutive prices divided by *N*

FORMULA 8.1 Simple moving average stock price

$$(N - period\ moving\ average\ stock\ price)_v = \frac{\sum_{t=0}^{N-1} price_{v-t}}{N}$$

Consider the daily stock prices in Table 8.1.

day - 1 -	closing price - 2 -	long – run moving average (5 – day) - 3 -	short – run moving average (signal) (1 – day) - 4 -	(2 – day) - 5 -
1	$40.00	…		
2	$41.50	…		
3	$42.25	…		
4	$40.75	…		
5	$42.50	$41.40	$42.50 (buy)	$41.63 (buy)
6	$41.00	$41.60	$41.00 (sell)	$41.75 (buy)
7	$40.25	$41.35	$40.25 (sell)	$40.63 (sell)
8	$39.85	$40.87	$39.85 (sell)	$40.05 (sell)
9	$41.50	$41.02	$41.50 (buy)	$40.68 (sell)
10	$40.25	$40.57	$40.25 (sell)	$40.88 (buy)

TABLE 8.1 Computing moving average stock prices

The company stock price for the last trade of the first day is $40.00. The financial media report this number as the closing stock price. The table does not present the highest price of the day, the lowest price of the day, nor the beginning price of the day. Those prices exist and often are available. Many technical analysts may use these additional numbers to personalize trading strategies. For our purposes, however, let's say we are interested in computing the 5-day simple moving average closing stock price. The computation requires five daily prices. At the end of day 5 the stock closes at a price of $42.50 and finally enough data exists to compute the 5-day moving average from formula 8.1:

$$\left(5-day\ moving\ average\ stock\ price\right)_5 = \frac{\sum_{t=0}^{4} price_{5-t}}{5}$$

$$= \frac{\$40.00 + 41.50 + 42.25 + 40.75 + 42.50}{5}$$

$$= \$41.40$$

The 5-day moving average stock price on day 5 equals $41.40.

This technical trading strategy computes and compares two moving averages of different length. The comparison generates a buy or sell signal according to the following rule:

RULE 8.1 The moving average trading strategy
A buy signal results when the short-run moving average becomes bigger than the long-run moving average. Conversely, a sell signal results when the short-run moving average becomes smaller than the long-run moving average.

An important academic study in the prestigious *Journal of Finance* (William Brock, Josef Lakonishok, and Blake LeBaron, 1992) reports that the most popular lengths are 200 days for the long-run moving average and 1 day for the short-run moving average. The study concludes that throughout the past century this technical trading strategy significantly outperforms overall market returns.

The data in table 8.1 enable a simple illustration for implementing the strategy. Consider first the case where the long-run average depends on five daily prices, and the short-run average uses one day. On day five the short-run 1-day average equals the closing price of $42.50 (an average computed from one number simply equals that number!). The long-run 5-day average for this day equals $41.40. The signal is “buy” because the short-run 1-day average surpasses the long-run 5-day moving average. The trading strategy suggests that if you don’t own the stock then buy it; if you already own it then hold it.

On day 6 the closing price is $41.00 . Calculations show that the 5-day moving average becomes $41.60:

$$\left(5-day\ moving\ average\ stock\ price\right)_6 = \frac{\sum_{t=0}^{4} price_{6-t}}{5}$$

$$= \frac{41.50 + 42.25 + 40.75 + 42.50 + 41.00}{5}$$

$$= \$41.60$$

On day 6 the signal is "sell" because the short-run 1-day average of $41.00 is smaller than the long-run 5-day moving average of $41.60. The trading strategy suggests that if you own the stock then sell it; if you don't own it then don't buy it. A more aggressive strategy involves short-selling the stock. A later chapter discusses short-sales.

Scan down column 4 of Table 8.1 and observe that the signal from the 1-day moving average remains "sell" until day 9. At that time, the signal switches to buy because the closing stock price of $41.50 surpasses the 5-day moving average of $40.57. This signal reverses the next day, however, indicating a rather quick in-and-out position on the stock.

Figure 8.1 charts the data for the previous example. Notice that a signal reversal occurs whenever the short-run moving average crosses the long-run moving average.

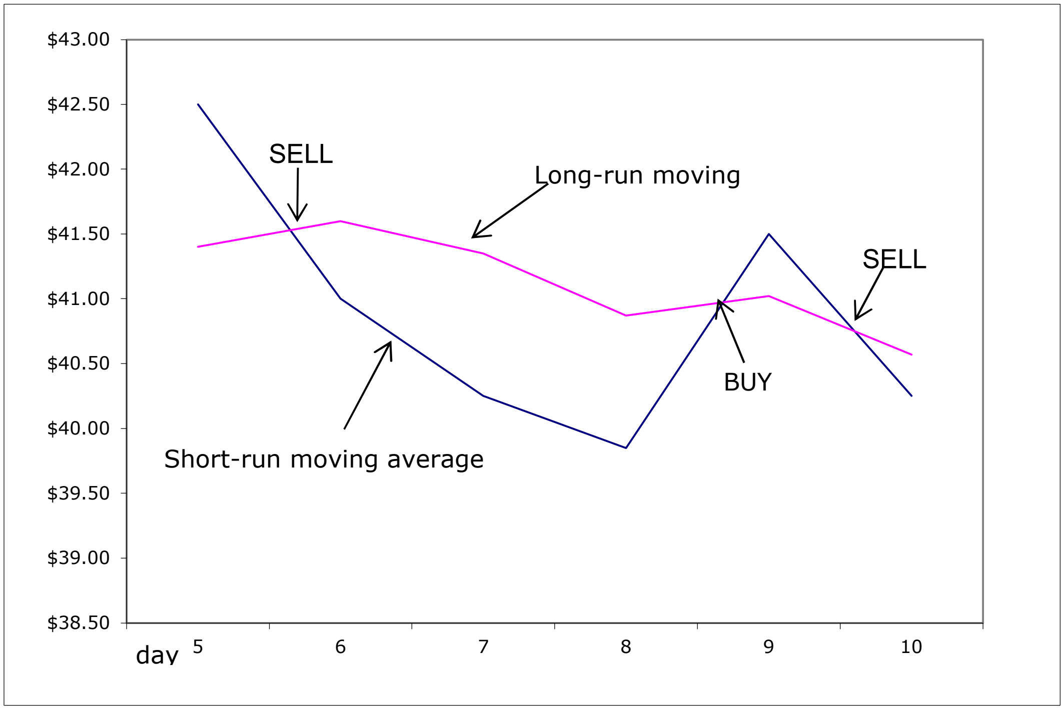


FIGURE 8.1 Moving average trading strategy based on data in table 8.1

At the far left the short-run moving average is above the long-run moving average. It crosses from above, heading down, and causes a sell signal. Later toward the right, the short-run moving average crosses from below (heading up) causing a buy signal, only to reverse itself immediately.

The short-run moving average is not restricted to equal one day. Column 5 of Table 8.1 lists the 2-day moving average. On day 6, for example, the 2-day moving average equals the sum of the two prices, $41.00 and $42.50, divided by two. Comparison of the 2-day and 5-day moving averages generates a buy signal: the 2-day moving average ($41.75) exceeds the 5-day moving average ($41.60). The signal on day 6 depends on which short-run moving average is used. Comparison of the long-run average with a 1-day moving average generates a sell signal, but comparison with a 2-day average generates a buy signal. Additional contradictions may arise if instead of 5 days the long-run moving average were 50 days, 100 days, or 200 days. Ambiguities abound with technical trading rules. Comparison of columns 4 and 5 shows several instances when signals conflict. Nonetheless, the study by Brock, Lakonishok, and LeBaron (*op cit.*) finds that the moving average rule, irrespective of length, performs pretty good.

The signal reverses from buy-to-sell, or vice versa, whenever the short-run moving average crosses the long-run moving average. The cross-over price for a signal reversal is found from the following formula:

FORMULA 8.2 Cross-over price for a signal reversal, general version

$$\begin{pmatrix} cross-over\ \ price \\ for\ \ a\ \ signal\ \ reversal \end{pmatrix}_v = \frac{S\left(L \times MA_{v-1}^{L} - price_{v-L}\right) - L\left(S \times MA_{v-1}^{S} - price_{v-S}\right)}{L - S}$$

where L and S equals the number of periods in the long-run and short-run moving averages, respectively, MA equals the respective moving average at the end of the prior period, and $price_{-t}$ equals the stock price t periods previously.

The example below illustrates the usefulness of the preceding formula.

EXAMPLE 1 Find the cross-over price for a 5-day v. 30-day moving average strategy

At the market close yesterday the 30-day moving average share price for the company stock was $22.50 and the 5-day moving average was $21.75. The share prices 30 and 5 days ago were $18.75 and $23.25, respectively. According to a trading rule that generates a signal when the 5-day and 30-day moving averages cross, what would be today's closing stock price that generates a signal reversal?

SOLUTION

Realize that at the close of market yesterday a "sell" signal prevails because the short-run average of $21.75 is less than the long-run average of $22.50. The investor following this trading rule consequently does not own this stock. The point is to find the price that the stock would have to climb to today so that the signal becomes "buy." Substitute the numbers for this problem into formula 8.2 and compute:

$$\begin{pmatrix} cross-over\ \ price \\ for\ \ a\ \ signal\ \ reversal \end{pmatrix} = \frac{5\left(30 \times \$22.50 - \$18.75\right) - 30\left(5 \times \$21.75 - \$23.25\right)}{30 - 5}$$

$$= \frac{\$716.25}{25}$$

$$= \$28.65$$

For this stock to get on the buy-list today its price would have to rise to $28.65.

Formula 8.2 simplifies when the length of the short-run moving average is one day. The simplification occurs because yesterday's one-day moving average is yesterday's price, and the second term in the numerator disappears:

FORMULA 8.3 Cross-over price for a signal reversal when the short-run is 1-day

$$\begin{pmatrix} \text{cross} - \text{over price} \\ \text{for a signal reversal} \end{pmatrix}_v = \frac{L \times MA^L_{v-1} - price_{v-L}}{L-1}$$

The preceding formula enables computation of a cross-over price which, when compared to the most recent stock price, provides a percentage change required for a signal reversal.

EXAMPLE 2 Use the 1-day v. 60-day moving average to find required percentage price change

At the market close for company stock yesterday the 60-day moving average and closing share prices were $38.50 and $43.75, respectively. The share price 60 days ago was $35.25. The trader uses a rule that generates a signal when the current closing price crosses the 60-day moving average. How much would today's price have to change relative to yesterday's close for this stock to get on the sell-list?

SOLUTION

Yesterday's closing price exceeds the 60-day moving average, which explains why the trader already owns the stock. Find the cross-over price by substitution into formula 8.3:

$$\begin{pmatrix} \text{cross} - \text{over price} \\ \text{for a signal reversal} \end{pmatrix} = \frac{60 \times \$38.50 - \$35.25}{60-1}$$

$$= \$38.56$$

The stock goes on the sell-list if the price falls to $38.56. This represents a decline of 11.9% relative to yesterday's close of $43.75; that is, –11.9% = ($38.56 – $43.75) ÷ $43.75.

The moving average trading rule modifies easily to accommodate the style and beliefs of any investor. The proper length of moving averages, as discussed above, is debatable. Furthermore, one investor might insist that a signal reversal be confirmed for 3 subsequent periods before executing a trade. Another investor might insist that the short-run average surpass the cross-over price by 3 percent before executing a trade. Ambiguities abound with technical trading rules.

Financial science offers no insights about proper construction of technical trading rules. Bookstores nonetheless contain dozens of titles that technically analyze short-run price movements. The books often proclaim golden rules for deciphering market psychology. Yet human behavior and crowd psychology are difficult to predict. A later chapter discusses why technical trading rules should not consistently provide market-beating returns. Most knowledgeable investors and finance professors believe that the stock price history cannot predict where the price is going next. Predicting the effect of the speculative factor on stock prices seems as imprecise as predicting which fads or sayings will catch-on, or predicting the next turn in direction for a flock of flying birds.

> "For this reason, training in speculation, however intelligent and thorough, is likely to prove a misfortune to the individual, since it may lead him into market activities which, starting in most cases with small successes, almost invariably end in major disaster."
> Benjamin Graham and David Dodd. *Security Analysis,* 1934, page 12.

Fundamental analysis relates stock prices to entrepreneurial factors. Entrepreneurial factors shape company cash flows. Expectations about future cash flows support today's stock price. Time value principles fundamentally link present values to future cash flows.

EXERCISES 8.1

Numerical quickies

1. The 20-day moving average share price for the company stock at the close of market yesterday was \$26.00. The share price 20 days ago was \$21.25. According to a trading rule that generates a signal when the share price crosses the 20-day moving average, what would be today's cross-over price that generates a signal reversal? ©TK1 .

2. At the close of market yesterday the 20-day and 2-day moving average share prices for the company stock were \$25.25 and \$23.00 , respectively. The share prices 20 and 2 days ago were \$20.50 and \$23.50 , respectively. According to a trading rule that generates a signal when the 2-day moving average crosses the 20-day moving average, what would be today's cross-over stock price that generates a signal reversal? ©TK3 .

3. At the close of market yesterday the 5-day and 2-day moving average share prices for the company stock were \$39.75 and \$36.25, respectively. The share prices 5 and 2 days ago were \$32.25 and \$37.00, respectively. According to a trading rule that generates a signal when the 2-day moving average crosses the 5-day moving average, what would be today's cross-over stock price that generates a signal reversal? ©TK4 .

2. Intrinsic value for stocks

Fundamental analysis generates a buy or sell signal by relating today's stock price to an estimate of the stock's current intrinsic value. In the 1930s Graham and Dodd forcefully explored analysis of security values from company fundamentals and J.B. Williams introduced intrinsic value for common stocks as present value of expected dividends. Intrinsic value is a subjective estimate of a stock's "true value" and equals the discounted sum of an investment's expected cash flows.

FORMULA 8.4 Intrinsic value, general version

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t}$$

The intrinsic value relation contains three components: a cash flow stream, a discount rate, and the stock value. Given any two components, the third is pre-determined. The stock value represents intrinsic value when the known variables are the cash flow stream and discount rate. The stock value represents the actual stock price when either the discount rate or cash flow stream is the unknown variable.

Intrinsic value is a subjective assessment because typically the analyst estimates either the cash flow stream or discount rate. Rarely are these variables known with 100 percent confidence. Different analysts likely use different numbers. Computations of intrinsic value consequently differ between analysts. Intrinsic value, like beauty, often is in the eye of the beholder.

Comparison of intrinsic value with the actual trading price enables insight about whether the stock is under or overvalued. Rule 8.2 summarizes the investment strategy.

RULE 8.2 The intrinsic value trading strategy

The signal a fundamental analysis generates is that:

$$\text{if } \begin{pmatrix} \text{intrinsic} \\ \text{value} \end{pmatrix}_t \begin{Bmatrix} > \\ < \end{Bmatrix} \begin{pmatrix} \text{actual} \\ \text{price} \end{pmatrix}_t \text{ then } \begin{Bmatrix} \text{buy} \\ \text{sell} \end{Bmatrix}.$$

When intrinsic value exceeds the actual price the asset is undervalued. The asset is worth more than it costs. Always buy undervalued assets. When intrinsic value is less than the actual price then the asset is overvalued.

The example below reinforces the notion that basic fundamental analysis is an application of time value principles.

EXAMPLE 3 Find intrinsic value given specific future cash flows

Your best assessment is that a particular share will pay dividends of $1.40 one year from now, $1.75 in 2 years, and $2.00 in 3 years. You believe that the share could be sold in 3 years for $30. You won't undertake the investment for an expected rate of return less than 16%. What to you is the share's intrinsic value today?

SOLUTION

Use formula 8.4. The cash flow stream and discount rate are known. Stock value is the unknown variable and represents intrinsic value:

$$\begin{pmatrix} \text{intrinsic} \\ \text{value} \end{pmatrix} = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t}$$

$$= \frac{\$1.40}{1.16} + \frac{\$1.75}{(1.16)^2} + \frac{\$2.00 + \$30.00}{(1.16)^3}$$

$$= \$23.00$$

The stock's intrinsic value according to your convictions is $23. If the actual price is $23 or less then put the stock on your buy-list. State the implication differently. If $23 is deposited in a savings account and one year later $1.40 is withdrawn, two years later $1.75 is withdrawn, three years later $32 is withdrawn, and after the last withdrawal the account balance is zero, then the account pays an interest rate of 16 percent. At a price of $23 the stock satisfies your criterion of a 16 percent rate of return.

The preceding example demonstrates that intrinsic value identically equals the present value of a discounted cash flow stream. The next example shows that formula 8.4 usefully finds the rate of return, too.

EXAMPLE 4 Find the rate of return on a counter-offer

You offer $23 for the share that you expect to return dividends of $1.40 one year from now, $1.75 in 2 years, $2.00 in 3 years, and also in 3 years you believe the share could be sold for $30. Your offer is rejected and instead a counter-offer of $24 is accepted. What is the rate of return if you buy at the counter-offer price and receive the expected cash flows?

SOLUTION

Use formula 8.4 and set stock value equal to the actual stock price. Solve for *r*:

$$\$24.00 = \frac{\$1.40}{(1+r)^1} + \frac{\$1.75}{(1+r)^2} + \frac{\$2.00 + \$30.00}{(1+r)^3}$$

Use the calculator to find that *r*, the geometric average annual rate of return, equals 14.29%.

CALCULATOR CLUE 8.1 You must use the advanced calculator functions to solve this problem because it does not have an algebraic solution. On the BAII Plus© type CF and clear unwanted numbers by typing 2nd CE/C. Now enter this problem's cash flow stream as follows:

24 +/– ENTER ↓ 1.40 ENTER ↓ ↓ 1.75 ENTER ↓ ↓ 32 ENTER

Now find the periodic rate of return that satisfies this time value formula. Hit:

IRR CPT

The display shows 14.29 percent.

Before continuing lessons on stock valuation, learn a little about distribution and acquisition of equities in the U.S.A.

STREET-BITE Distribution and acquisition of U.S. equities

Who owns stocks and how do they get them? A survey of U.S. households by the Investment Company Institute (see www.ici.org) and the Securities Industry Association (http://www.siaonline.org/) estimated that 1-out-of-2 U.S. households in 2002 own equity securities (52.7 million households or 49.5% of total). This is substantially higher than the 1-out-of-5 households owning stocks in 1983. Stock ownership in the U.S. definitely spread from Wall Street toward Main Street. Since 2002, however, reports suggest that household ownership of stocks is in decline. Federal Reserve statistics, for example, show that direct individual stock and indirect fund stock ownership fell to 48.8% in 2013 from 53.2% in 2007. Table 8.2 discusses a few survey results.

	Millions of households (% of total)		
	Total	Inside employer retirement plans	Outside employer retirement plans
Own any type of equity	52.7 (49.5%)	36.2 (34.0%)	35.9 (33.7%)
Type: stock mutual funds	47.0 (44.2%)	33.2 (31.2%)	28.7 (27.0%)
individual stock	25.4 (23.9%)	8.8 (8.3%)	21.0 (19.7%)

TABLE 8.2 Number of U.S. households owning equities.
Notes: Ownership inside employer-sponsored retirement plans includes 401k, 403b, federal, state, or local plans, SEP-IRAs, SAR-SEP-IRAs, and SIMPLE IRAs. Roth IRAs and traditional IRAs are not employer-sponsored plans. Excludes stock options. Source: *Equity Ownership in America,* by the Investment Company Institute and the Securities Industry Association.

There are two types of equity securities that households may own: stock mutual funds or individual stocks. Chapter 9 explains that mutual funds collect money from many investors and pool the funds together to buy diversified portfolios of individual stocks. The middle row of the first column shows that 47 million households (44.2%) own stock mutual funds. The bottom row of column 1 shows that ownership of individual stocks is much lower (23.9%). Some households, of course, own both types.

There are two settings within which households acquire equity securities: inside an employer-sponsored retirement plan or independently. Chapter 9 explains that retirement pension plans, especially 401k defined contribution plans, have grown phenomenally over the past two decades. Comparison of row 1, columns 2 and 3, shows that ownership of any equity is about the same inside or outside of pension plans. Row 2 affirms that similarity. Row 3, however, shows that of the 25.4 million households owning individual stocks fully 21.0 million own stocks outside pension plans. 8.8 million households own individual stocks inside employer-sponsored retirement plans and nearly three-quarters of those (6.0 million) own employer stock.

The bottom-right cell containing 21.0 million households represents individual stock investors – about 1-out-of-5 households at the time. Among these hardcore investors, however, less than half (46%) conducted a stock transaction during the preceding year.

The survey also reports that equity investors generally have equity portfolios of moderate value. Of 52.7 million households owning any equity nearly half have securities (mutual funds and individual stocks) totaling less than $50,000; only 7 percent exceed $500,000. Among households owning any equity the median number of equity investments is 4 unique securities or mutual funds. This is a modest number.

Once an investor decides to buy or sell a stock then trade execution in the U.S.A. generally occurs in any one of these different market settings.

1. *New York Stock Exchange (NYSE)* is the focus of a chapter 3 *Street-bite* and many facts about the NYSE appear there. The NYSE lists stocks for about 2,750 large

capitalization companies. Total NYSE market cap is about $16.6 trillion in 2014 with average daily trading volume near 1,500 million shares worth $46 billion. It is the *big board.*

2. *Nasdaq* began as a public service initiative of the National Association of Securities Dealers (NASD). The Securities Act of 1933 established NASD in response to public outcry over the stock market crash of 1929 and the ensuing Great American Depression. Today all brokers or companies working with securities and the public must join NASD, obtain NASD licenses, and follow NASD regulations (see www.nasd.com). This self-regulatory organization in 1971 began collecting and distributing hard-to-get stock price quotations for many equities that were not listed on a stock exchange. The NASD Automated Quotation system (Nasdaq) allowed for market-makers to carry inventory of stocks and post bid prices for buying or ask prices for selling stocks. Nasdaq does not have a physical trading floor but instead is a telecommunications network with thousands of market-makers trading stocks for about 3,300 companies. Nasdaq is technically *not* a stock exchange but rather is a network of market-makers. Total market cap for Nasdaq stocks in 2014 is $8.5 trillion and daily trading volume averages 1,400 million shares worth $27 billion. In 1998 Nasdaq acquired the American Stock Exchange. In January 2000 NASD announced intentions to separate from Nasdaq. That separation was complete by mid-2002 and today Nasdaq is a publicly traded profit-making stock exchange (see www.nasdaq.com). That is, investors can buy stock with the Nasdaq name on it and receive profits that the company earns from fees and services provided to stock traders. In the meantime, the NASD was reorganized in 2007 as FINRA, the Financial Industry Regulatory Authority, empowered to concentrate on their self-regulatory Congressionally mandated mission of bringing integrity to the markets and confidence to investors.

3. *National Stock Exchange (NSX)* is the third largest stock market in the U.S.A. NSX was founded in 1885 as the Cincinnati Stock Exchange. In 1995 they moved to Chicago and in 2003 changed name to the *National Stock Exchange* (see www.nsx.com). In 2011 the NSX was acquired by the CBOE Stock Exchange (Chicago) and in 2015 NSX was spun-off to NSX Holdings, Inc., and today is headquartered in Jersey City, NJ. In 1980 NSX replaced their physical trading floor and became the first all-electronic stock exchange in the country. The majority of NSX trades are for stocks that also trade on Nasdaq. About one-third of all trades involving Nasdaq-listed stocks occur at the NSX instead of through Nasdaq market-makers. Similarly, about 15% of Amex-listed stock trades occur at NSX. The NSX also trades about 20 million shares per day of NYSE-listed stocks.

4. *Pacific Stock Exchange (PCX)* in San Francisco was founded in 1862. In 1999 PCX became the first U.S. stock exchange to demutualize, meaning switch from a member-owned organization like the NYSE used to be and instead operate as a for-profit publicly traded company. In 2002 the Pacific Exchange closed its equities floors and migrated stock trading to the Archipelago Exchange (*ArcaEx)*, an *electronic communications network* ("ECN"). ArcaEx trades Nasdaq-listed equity securities and exchange listed equity securities, including those traded on the New York Stock Exchange and American Stock Exchange. ArcaEx and other ECNs offer corporate issuers and investors the advantages of meeting directly, without intermediaries, within a fully electronic and totally transparent environment. ArcaEx began trading operations in March 2002. In 2005 the NYSE became a publicly traded company after acquiring ArcaEx. In 2015 the NYSE and ARCA are owned by ICE, the Intercontinental Exchange Holding Co.

5. *American Stock Exchange (AMEX)* organized at beginning of the 20th century in New York City and primarily traded stocks for companies too small to satisfy listing

requirements on the NYSE. The AMEX in the early 1990s introduced *exchange traded funds* that had a total market cap of about $200 billion. AMEX at one time listed equities for about 800 companies. They were acquired in 1998 by NASDAQ. In 2008 AMEX was acquired by the NYSE-Euronext and today operates as part of the Intercontinental Exchange group (ICE).

6. Three regional exchanges: The *Philadelphia Stock Exchange (PHLX)* was founded in 1790 and is the nation's oldest. In 2004 the PHLX demutualized and became a publicly traded for-profit company. Daily volume for stocks traded at PHLX in January 2005 averaged about 9 million shares. The PHLX had a stronger market position trading 3,600 equity options, 19 sectors index options, and currency options and futures. In 2008 PHLX was acquired by the NASDAQ. The *Boston Stock Exchange (BSE)* was founded in 1834 and average daily volume was about 50 million shares worth about $1.4 billion when the BSE was acquired by NASDAQ in 2007. The *Chicago Stock Exchange (CHX)* was founded in 1882. In 1949 CHX merges with the exchanges of St. Louis, Cleveland and Minneapolis/St. Paul to form the Midwest Stock Exchange and in 1959 they absorbed the New Orleans Stock Exchange. In 1993 they changed their name back to the Chicago Stock Exchange. In 2005 daily trading volume averaged about 65 million shares worth about $1.8 billion.

7. Over-the-counter stocks (OTC) in nearly 10,000 small and new companies are not traded on any of the preceding markets. Instead, they are bought and sold over the telephone or by computer on one of 3 tiered marketplaces - OTCQX, OTCQB, and OTC Pink. Many OTC stocks are called *penny stocks* because they cost less than a dime. When you pay a penny for a stock it doesn't take much to double your money, but then again *buyer beware!* Companies with securities on OTCQX must meet continuing financial and disclosure requirements and be qualified by a 3rd party professional investment bank or attorney advisor. OTCQB is the Venture Stage Marketplace with Current Reporting Companies. Companies are required to report to a US regulator such as the SEC or FDIC. OTC Pink is the Open Marketplace and has no disclosure requirements. The Penny Stock Reform Act of 1990 mandated that the SEC establish an alternate quotation system for OTC stocks in accordance with the Securities Exchange Act of 1934. In June 1990 the *OTC Bulletin Board* (OTCBB) began operation to provide transparency in the OTC equities market. The OTCBB operates today from the FINRA site. OTCBB daily trading in 2005 averaged 2,400 million shares worth $240 million for an average price of ten cents a share.

Preceding discussion highlights two tendencies about the U.S. equity market. First, the market is very competitive with many layers and alternative routes for trade execution. Second, in the past two decades the equity market has had significant changes in ownership structure. Another huge trading forum is Instinet (see www.instinet.com and www.inet.com). Daily volume of U.S. equities for these Reuters subsidiaries averages 800 million shares. They also trade foreign company stocks.

EXERCISES 8.2

Numerical quickies

1. The stock for a start-up company probably will pay no dividends until exactly 8 years from today. At that time it will pay $6.80 per year forever. You assess the intrinsic value

of the stock with a 10.3% discount rate. Find the stock's intrinsic value today. ©ST20
.

2. A company you are interested in just declared their annual dividend of $2.20. You expect the annual dividend next year will be larger by 4.4%, at which time you expect the stock could sell for $24. What is the intrinsic value today that would provide you with a 12.2% annual rate of return from investing in the stock? ©ST6 .

3. The company stock paid a dividend this morning of $3.00 and its current stock price is $84 . You think that a year from now the dividend will be $3.60 and the stock price will be $89. Find the rate of return if you buy the stock now at the current price, and in one year you receive the expected dividend and then sell the stock at the expected price. ©ST11 .

4. A stock you are buying today promises no dividends for a long time. In exactly 10 years you expect the stock will pay its first annual dividend of $3.70. At that time, you also believe that the stock could be sold for $34.00. If today you can buy the stock for $13.64, what is the expected annual rate of return on the stock investment? ©ST12 .

Numerical challenger

5. A stock you are buying today promises no dividends for a long time. In exactly 5 years you expect the stock will pay its first annual dividend of $5.60 which you expect will be paid annually forever. If today you can buy the stock for $40.43, what is the expected annual rate of return on the stock investment? ©ST21 .

2.A. Preferred stocks with constant dividends

Preferred stock has characteristics of both bonds and common stocks. Preferred stock, like a bond, stipulates the amount of cash flow that the investor receives. The periodic cash flow for preferred stock is called a dividend. Companies promise to pay preferred dividends, like common dividends, every calendar quarter when financial circumstances permit. If the company cannot pay a dividend, however, the investor has no legal recourse to force payment or bankruptcy. Most preferred stocks have covenants stipulating that when a company misses a preferred dividend they cannot pay any common dividends until preferred shareholders get all missed dividends. Preferred stocks, like common stocks, never mature. The company promises to pay dividends forever.

Preferred stocks trade on exchanges by the same procedures as common stocks. The preferred dividend for a particular stock is constant. The preferred stock for AMR (American Airlines) in the table above, for example, promises to pay $6.40 per annum forever. What do you think would happen to the price of AMR preferred stock if American Airlines suddenly won major flight contracts with every government in Europe and South America? The answer: probably nothing! Maybe the price of the common stock would skyrocket as investors suddenly realize future profits will be larger than previously expected. The preferred dividend, however, is fixed at $6.40 per year forever. The price of preferred consequently responds more to discount rate changes than to the good fortunes of the company. Because the dividend is constant, the formula for value relation simplifies to the perpetuity formula:

FORMULA 8.5 Intrinsic value, stocks with constant dividends

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \sum_{t=1}^{\infty} \frac{dividend}{(1+r)^t}$$

$$= \frac{dividend}{r}$$

As before, when stock value is the unknown variable it represents intrinsic value. When stock value is the actual stock price then the unknown variable is probably *r*. Even though dividends actually are paid quarterly (the AMR pays $1.60 four times a year), computations in this book assume dividend payments occur annually. This is strictly for convenience.

The following example gleans insight about pricing preferred stocks.

EXAMPLE 5 Find the preferred stock's intrinsic value

A share of AMR preferred stock paid a $6.40 annual dividend yesterday. You would buy the preferred stock if its return is 275 basis points more than the bank interest rate on 36-month Certificates of Deposit ("CD"). The 36-month CD rate today is 5.25 percent, and the preferred stock price is $74.50. What to you is the stock's intrinsic value, and should you buy it?

SOLUTION

There are two ways to view this problem. Both lead to the same qualitative answer, both use formula 8.5, and each is valid by itself. First find the stock's intrinsic value:

$$\begin{pmatrix} intrinsic \\ value \end{pmatrix} = \frac{\$6.40}{(0.0525 + 0.0275)}$$

$$= \$80.00$$

Comparison shows that the intrinsic value of $80 exceeds the stock price of $74.50 and a buy signal exists.

The second way to reach the same conclusion uses formula 8.5 with the price and dividend as known values. Solve for the rate of return the preferred stock promises:

$$\$74.50 = \frac{\$6.40}{r}$$

$$r = 8.59\%$$

The preferred stock promises 334 basis points more than the CD (= 0.0859 – 0.0525). The investment satisfies the criterion for a buy signal.

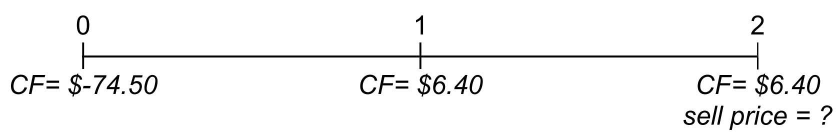
Solid blue-chip companies with little default risk typically issue preferred stocks. Investors in preferred stocks typically compare the preferred stock's rate of return with alternative investments, such as bank CDs. The example below shows that preferred stocks, even in the absence of significant default risk, still are riskier than CDs.

EXAMPLE 6 Find the rate of return upon conclusion of a preferred stock investment

You buy today for $74.50 a share of AMR preferred stock that yesterday paid a $6.40 annual dividend. The preferred stock promises a risk premium of 334 basis points relative to the 5.25 percent 36-month CD rate. You own the stock and collect the subsequent two annual dividends. You sell the preferred stock upon receipt of the second dividend. The CD rate at the time of the sell is 3 percent higher than today, while the risk premium on the preferred stock is constant. What is the average annual rate of return for the investment?

SOLUTION

The time line below shows the investment cash flows.



Use formula 8.5 with *r* equal to 6.34 percent to find the price at time of sell:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_2 = \frac{\$6.40}{(0.0525 + 0.0334 + 0.0300)}$$

$$= \$55.22$$

The $6.40 dividend plus the $55.22 sell price brings CF_2 to $61.62. The rate of return from the investment satisfies the following equation:

$$\$74.50 = \frac{\$6.40}{(1+r)^1} + \frac{\$61.62}{(1+r)^2}$$

Solve on the calculator and find that *r*, the geometric average annual rate of return, equals –4.66%. The preferred stock investment performs miserably. The CD may promise a lower rate of return but preferred stock carries the risk of a falling share price in response to rising interest rates.

CALCULATOR CLUE 8.2 You may solve example 6 on the BAII Plus© with either the CF menu or the time value keys. Use time value keys by typing 2nd FV and 2nd I/Y 1 ENTER CE/C to clear memories and enforce annual compounding. Now enter the dividend and compute selling price as follows:
74.5 +/– PV 6.40 PMT ÷ (.0525 + .0334 + .03) = FV 2 N CPT I/Y .

The display shows –4.66%.

EXERCISES 8.2A

Numerical quickies

1. You notice that the local electric company pays a $6 annual dividend on its preferred stock. The current price of the stock is $78. Find the promised rate of return for the preferred stock. ©ST23 .

2. The company preferred stock pays a $3.75 annual dividend. The local bank pays 4.5% interest on 5-year CDs. You consider the preferred stock an attractive investment if its *ROR* is 175 basis points more than the CD rate. Find your assessment of the preferred stock intrinsic value. ©ST22 .

3. The company preferred stock yesterday paid $6.25 annual dividend and today's stock price is $101.50. The local bank pays 4.65% interest on CDs. You consider the preferred stock an attractive investment if its *ROR* is 200 basis points more than the CD rate. Find the actual risk premium and is this stock a *buy* or a *sell*? ©ST24 .

Numerical challenger

4. The company preferred stock just yesterday paid its annual dividend of $5.00 per share. Today's share price is $58.10 . You believe the dividend yield is abnormally high but that, over the next two years, it will revert to its normal value of 6.50%. Your strategy is to buy the stock today and receive annual dividends for two years. Upon receiving the last dividend you expect the dividend yield will be normal, and your strategy is to sell the stock at that time. Compute the expected annual rate of return for the strategy. ©ST7 .

5. The company preferred stock just yesterday paid its annual dividend of $6.00 per share. Today's share price is $52.25 . You believe the dividend yield is abnormally high but that it will revert to its normal value of 7.0%. Your strategy is to buy the stock today and receive annual dividends for 4 years. Upon receiving the last dividend you expect the dividend yield will be normal. Your strategy is to sell the stock at that time. Compute the expected annual rate of return for the strategy. ©ST25 .

2.B. Common stocks with growing dividends

The intrinsic value relation in formula 8.4 broadly applies to almost any investment decision. Finding an asset's true worth, however, requires specific assumptions about future cash flows. With common stocks the situation is amazingly complex because, unlike preferred stocks, future cash flows are totally unspecified. Some analysts compute common stock intrinsic value by discounting operating cash flow. Other analysts discount dividends. Some analysts forecast cash flows growing rapidly in nearby years and more slowly in remote years. Other analysts specify smooth growth at a constant geometric rate. Fundamental analysis, like technical analysis, abounds with ambiguities. Fundamental analysis, however, rests on strong logical and theoretical foundations: present values relate to the discounted sum of expected cash flows.

Investors receive common stock cash flows from two sources. First, the investor receives dividend distributions from the company. Second, the investor receives a cash inflow upon selling the stock. Example 3 computes intrinsic value as the discounted sum of dividends and expected sale proceeds. The sale proceeds represent a transfer of wealth from the buyer to the seller. What cash flow does the subsequent investor expect to receive from purchasing the stock? Dividends and sale proceeds. Stretching into infinity the only cash flow the stock ever will generate is dividends and sale proceeds. Specifying that investors possess rational expectations about future dividends leads to an inference that sale proceeds vanish into the remote future. Today's common stock intrinsic value equals the discounted sum of the infinite dividend stream:

FORMULA 8.6 Intrinsic value is the discounted sum of the infinite dividend stream

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \sum_{t=1}^{\infty} \frac{dividend_t}{(1+r)^t}$$

B1. <u>The dividend growth rate</u>

Computing intrinsic value with formula 8.6 requires numbers for the discount rate and each and every one of the infinite dividends. It's pretty hard to come up with so many numbers. The problem is made tractable by restricting the shape of the dividend stream. Interesting insights, for example, occur in the special case when dividends grow at a constant rate. Smooth dividend growth at rate *g* satisfies the following equality:

FORMULA 8.7 Constant growth rate for dividends

$$dividend_s = dividend_{s-t}(1+g)^t$$

Smooth dividend growth occurs when one period's dividend <u>always</u> is *g* percent bigger than the previous period's dividend.

EXAMPLE 7 Find the dividend growth rate from two observations

A share of company stock paid a dividend today of \$4.60. Five years ago the dividend was \$2.90. Suppose the dividend grows smoothly at a constant rate. Find (i) the dividend growth rate, and (ii) next year's dividend.

SOLUTION

Use formula 8.7 to find (i) the dividend growth rate:

$$\$4.60 = \$2.90(1+g)^5$$

$$g = \left(\frac{\$4.60}{\$2.90}\right)^{1/5} - 1$$

$$g = 9.67\%$$

A dividend that today is \$4.60 and five year ago was \$2.90 grows at a geometric average annual rate of 9.67 percent. Find (ii) the dividend next year that is 9.67 percent larger than this year's dividend:

$$dividend_1 = dividend_0\,(1+g)^1$$

$$= \$4.60\,(1.0967)$$

$$= \$5.04$$

Inspection of company reports shows that dividends seldom grow smoothly. Table 8.3 shows a typical history of annual dividends.

year t - 1 -	annual $dividend_t$ - 2 -	% change from year t-1 - 3 -	g from time t to time 7 - 4 -
1	\$1.80	. . .	10.63%
2	\$1.80	0.00%	12.89%
3	\$2.60	44.44%	6.14%
4	\$2.20	−15.38%	14.47%
5	\$2.70	22.73%	10.55%
6	\$3.20	18.52%	3.12%
7	\$3.30	3.12%	. . .

TABLE 8.3 A hypothetical dividend history

Each year's percentage change varies from as high as 44 percent to as low as −15 percent. Dividends definitely do not change by exactly the same amount each year. Dividend growth definitely is not smooth.

Suppose the company just paid the \$3.30 annual dividend at time 7 and you wish to estimate from the dividend history the growth rate with formula 8.7. Choosing the proper beginning year is problematic. The geometric average annual percentage change from year 6 to 7 equals 3.12 percent. From year 5 through year 7, however, the estimate of *g* equals 10.55 percent (column 4, row 5; $(3.30/2.70)^{1/2} - 1$). From year three's dividend of \$2.60 until year seven's dividend of \$3.30 the *g* is 6.14 percent (that is, $0.0614 = (3.30/2.60)^{1/4} - 1$). Forecasting future dividends depends a lot on estimating *g*, and estimates of *g* from the dividend history are ambiguous.

Statistical procedures come to the rescue. We can estimate *g* from the data in table 8.3 by assuming that formula 8.7 is generally true but that random white noise causes vibration in observable growth rates. Rewrite formula 8.4 as

$$\log(dividend_t) = \log\left(dividend_0 (1+g)^t\right)$$

Apply simple math rules and add the white noise and find

$$\log(dividend_t) = \log(dividend_0) + t\log(1+g) + noise_t$$

Another important point about the preceding formula is that *our calculator* uses the formula to estimate growth rate *g*! This example offers a lesson about that handy tool.

EXAMPLE 8 Find the best estimate of the dividend growth rate from Table 8.3

Yesterday the company paid its annual dividend of $3.30 as shown in row 7 of Table 8.3. Assume that dividends grow smoothly in accordance with constant growth formula 8.7. Each year, however, there may be random white noise errors. Compute the best estimate of the dividend growth rate and plot the function that shows dividends growing smoothly. Also, find the best estimate for next year's dividend (t = 8).

SOLUTION

Follow the steps in this *Calculator Clue* to perform the statistical analysis.

CALCULATOR CLUE 8.3 The BAII Plus© calculator contains a spreadsheet that enables easy estimation of *g* for Example 8 that best fits formula 8.7. On the BAII Plus© type 2nd DATA (the data key is the same as the 7 key). The calculator opens a worksheet that contains two columns for holding data. The calculator refers to the two columns of data as column X and column Y. For this problem column X contains the period number and column Y contains the dividend amount. Clear unwanted numbers already stored in the data worksheet by typing 2nd CE/C. Enter this problem's data by typing

1 ENTER ↓ 1.80 ENTER
↓ 2 ENTER ↓ 1.80 ENTER
↓ 3 ENTER ↓ 2.60 ENTER
↓ 4 ENTER ↓ 2.20 ENTER
↓ 5 ENTER ↓ 2.70 ENTER
↓ 6 ENTER ↓ 3.20 ENTER
↓ 7 ENTER ↓ 3.30 ENTER

The preceding keystrokes set X_1 = 1 and Y_1 = 1.80, … , X_7 = 7 and Y_7 = 3.30. Note that the X column also could contain 1995, 1996, …, 2001 and exactly the same estimate of *g* would result. Find the statistical estimate of the growth rate from the data in memory by typing 2nd STAT . Now hit 2nd SET repeatedly until the display says "EXP". With this setting the calculator finds the exponential growth rate that best fits the Y and X columns. The statistics course calls this procedure an ordinary least squares estimate of the log(Y) on a trendline X.

To find the growth rate estimate hit ↓ repeatedly (probably 9 times) until the display shows "b = 1.1134" . The number equals one plus the growth rate. For the data in table 8.3, the best estimate of the dividend growth rate equals 11.34 percent. This growth rate is an important input for fundamental analyses of common stock intrinsic value.

Next find the dividend expected at time 8. Hit ↓ two more times until the display shows "X' = " . Type

8 ENTER ↓ CPT .

The display shows that Y'_8, the dividend expected at time 8, equals $3.76.

Figure 8.2 portrays findings for Example 8. Dots labeled div_t represent the actual dividend history from table 8.3. The solid line represents the path of expected dividends implied by the statistical analysis. Each triangular marked point on the line represents an expected dividend. Plug into your calculator that "X' = 7" and compute that expected div_7 = \$3.38. Actual div_7 of \$3.30 was less than expected by 8 cents. The white noise this period, $noise_7$, was minus 8 cents. The figure illustrates the noise at a given point in time as the distance between the actual dividend (the dot) minus the expected dividend (the triangle).

Project the line to time 8 and find that expected div_8 equals \$3.76. In reality the actual dividend probably won't exactly equal \$3.76 because some noise will occur. Still, the expected value of the noise according to the statistical model is zero. As far as we can trust that formula 8.7 is correct and that noise is white then the best estimate of next period's dividend is \$3.76.

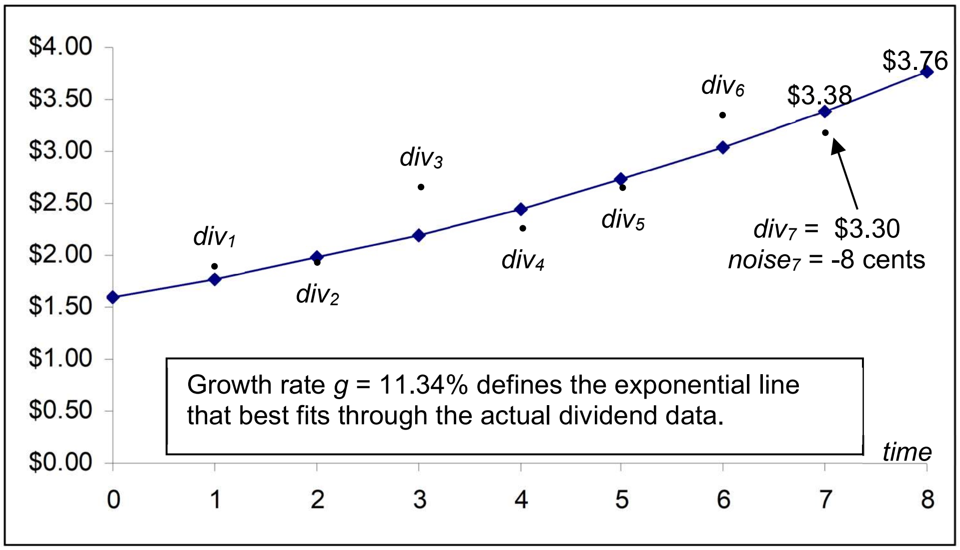


FIGURE 8.2: Plot of the dividend history from table 8.3 and the line of best fit for Example 8

B2. The constant growth dividend valuation model

Restricting the dividend cash flow stream to change through time at a constant rate g allows simplification of the intrinsic value relation. Stock value relates to the discounted sum of the perpetual and smoothly growing dividend stream as follows:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{dividend_1}{(1+r)^1} + \frac{dividend_2}{(1+r)^2} + \frac{dividend_3}{(1+r)^3} + \ldots \text{ to } \infty$$

$$= \frac{dividend_1}{(1+r)^1} + \frac{dividend_1(1+g)^1}{(1+r)^2} + \frac{dividend_1(1+g)^2}{(1+r)^3} + \ldots \text{ to } \infty$$

The left-hand-side of the above formula equals either of two metrics. "Stock value" represents intrinsic value when all right-hand-side values are specified. Alternatively, "stock value" represents the observed stock price when one of the right-hand-side variables is the unknown.

The right-hand-side represents the discounted sum of the infinite dividend stream. Summing all terms could be rather time consuming because there always is at least one more number to add. For the realistic case when $g < r$ the sum is convergent. That is, the numerator rises slower than the denominator. One-plus-r raised to the fiftieth power is much larger than one-plus-g to the fiftieth. The terms far to the right vanish. Stated differently, the contribution to present value of a dividend received 50-years from now is nil.

Formula 8.8 shows the discounted sum of the infinite dividend stream when $g < r$:

FORMULA 8.8 Intrinsic value, constant growth dividend valuation model

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{dividend_1}{r - g}$$

$$= \frac{dividend_0 \, (1 + g)}{r - g}$$

The two lines in formula 8.8 present algebraically equivalent representations of the constant growth dividend valuation model (also known as the Gordon dividend growth model). The cash flow timing implicit with formula 8.8 is that the stock valuation occurs at time *0.* The seller just received $dividend_0$ whereas the buyer expects to receive $dividend_1$ in exactly one period. The last line of formula 8.8 recognizes that with constant growth $dividend_1$ equals $dividend_0$ times $(1 + g)$.

EXAMPLE 9 Constant growth dividend valuation, simplest example

A share of company stock paid its annual dividend of $1.16 exactly 4 years ago. You expect that next year's dividend will equal $1.60. You believe the stock represents a fair investment if it were to offer a risk premium of 750 basis points relative to the current Treasury yield of 4.90 percent. The stock's current price is $20. According to the constant growth dividend valuation model: (i) sketch the expected dividend stream; (ii) compute the stock's current intrinsic value; and (iii) should you buy the stock?

SOLUTION

There are 5 years between "4 years ago" and "next year". Use formula 8.7 to find the dividend growth rate:

$$\$1.60 = \$1.16\,(1 + g)^5$$

$$g = 6.64\%$$

Each year's dividend is 6.64 percent larger than the previous year's. Sketch (i) the dividend stream as follows

t = -4	-3	0	1	4	5	50
div_{-4}	div_{-3}	div_0	div_1	div_4	div_5	div_{50}
\$1.16	\$1.24	\$1.50	\$1.60	\$1.94	\$2.07	\$37.39

The time line assumes that four years ago occurs at time –4, that right now is time 0, and that next year is time 1.

Find (ii) the stock's current intrinsic value. The discount rate, r, equals the 4.90 percent risk free rate plus the 7.5 percent risk premium; r = 12.4 percent. You could compute intrinsic value as

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{\$1.60}{(1+.124)^1} + \ldots + \frac{\$1.94}{(1+.124)^4} + \frac{\$2.07}{(1+.124)^5} + \ldots + \frac{\$37.39}{(1+.124)^{50}} \quad to\ \infty$$

Formula 8.8 enables a simpler and more efficient calculation that obtains exactly the same answer as summing the infinite terms above:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{\$1.60}{0.1240 - 0.0664}$$

$$= \$27.78$$

Finally (iii), invoke the decision rule. The stock's intrinsic value of \$27.78 exceeds the actual stock price of \$20. The stock is worth more than it costs, so put it on the buy list! If the actual share price instantly moved up to the intrinsic value, the investor would realize a 38.9% rate of return (that is, 0.3890 = 27.78/20 – 1).

Investment professionals often use intrinsic value estimates to rank stocks from most promising to least promising. Stocks that appear significantly undervalued are most promising. Professionals do not expect the share price to move instantly up to intrinsic value. Instead, the expectation is that over a longer horizon, say one year, the share price converges to intrinsic value.

Next year's intrinsic value, however, differs from today's intrinsic value. Glean insight about the relation between next year's and today's intrinsic values by constructing a ratio with formula 8.8.

$$\frac{\begin{pmatrix}\text{intrinsic}\\ \text{value}\end{pmatrix}_1}{\begin{pmatrix}\text{intrinsic}\\ \text{value}\end{pmatrix}_0} = \frac{\dfrac{\text{dividend}_1\,(1+g)}{r-g}}{\dfrac{\text{dividend}_1}{r-g}}$$

$$= (1+g)$$

Next year's intrinsic value is *g* percent bigger than this year's. The constant growth dividend valuation model implicitly assumes that intrinsic value, just like the dividend, grows smoothly at rate *g*. The example below uses this insight.

EXAMPLE 10 Find ROR when today's price converges to next year's intrinsic value

The company just paid its annual dividend of $3.30 per share. You believe that the dividend grows smoothly at a 9.8 percent annual rate. Today's price-to-earnings ratio is 12.5, and the company's payout ratio always is 80%. You assess the stock's intrinsic value by discounting future dividends with a 16 percent annual rate. You buy the stock at today's price with the expectation that the price next year converges to next year's intrinsic value. What is the expected annual rate of return on this stock investment?

SOLUTION

The periodic rate of return, according to formula 4.2, equals the percentage change in wealth. Beginning wealth equals today's stock price because this is the amount you pay for the investment. Ending wealth equals next year's stock price plus the dividend. Thus, the shareholders' rate of return is:

$$ROR_t = \frac{\text{price}_t + \text{dividend}_t - \text{price}_{t-1}}{\text{price}_{t-1}}$$

Find the rate of return throughout the next year (ROR_1) by substituting values for the three right-hand-side variables.

Next year's dividend equals today's dividend times *(1 + g):*

$$\text{dividend}_1 = \$3.30\,(1 + .098)$$

$$= \$3.62$$

You expect to receive a dividend next year of $3.62, at which time you might sell the stock.

Today's price stems from the definition for the price to earnings ratio:

$$\text{price-to-earnings ratio} = \frac{\text{price}}{\text{earnings}}$$

$$= 12.5$$

Earnings times the payout ratio equals dividends. Because today's payout ratio and dividend equal 0.80 and $3.30, respectively, find today's share price as:

$$12.5 = \frac{price_0}{\$3.30/0.80}$$

$$price_0 = \$51.56$$

The investment today costs $51.56.

Finding next year's price requires finding next year's intrinsic value. First, however, use formula 8.8 to find this year's intrinsic value:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{\$3.30\,(1+.0980)}{0.16\ -\ 0.0980}$$

$$= \$58.44$$

Next year's intrinsic value simply equals this year's intrinsic value times *(1 + g).* Thus, next year's price is $64.17 (that is, $64.17 = $58.55(1.098)).

Substitute and find

$$ROR_1 = \frac{\$64.17\ +\ 3.62\ -\ 51.56}{51.56}$$

$$= 31.48\%$$

The stock investment promises a hefty return!

Trading stocks a half-century ago was more costly since information was less widely available and access to trade execution had few gateways. Examine the snippet below from table 2.1 showing SCHW sorted 51st by *Total assets* in the list of 11,000 U.S. companies circa beginning of year 2014.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
SCHW	$143,642	14	1071	5539	33719
MSFT	$142,431	99	21863	77849	287691
PG	$139,263	121	11312	84167	211132
JNJ	$132,683	128	13831	71312	258341
FITB	$130,443	19	1836	6864	17987
IBM	$126,223	431	16483	99751	197772

SNIPPET from table 2.1 in chapter 2: SCHW

Charles Schwab revolutionized the equity markets through services and pricing policies pioneered by his company. The *Total assets* at $143.6 billion are larger than IBM in the bottom row. They within a percent of the Microsoft *Total assets* in the second row. Yet the *market cap* of SCHW is little bigger than 1/10th the size of MSFT. The balance sheet assets of SCHW differ greatly from those of MSFT and IBM. For the latter the assets embody return streams squeezed from the assets by specialized employees, large institutional structures, and durable assets and capital. SCHW largely shows *financial assets* on its balance sheet and has relatively less institutional and fixed capital carry costs. The competitive relatively elastic supply of *financial capital* is assurance that security returns tend toward covering carry costs plus capital costs plus barriers to entry economic profits. The SCHW residual cash flows to equity embody less return from assets than the durable assets. Hence, smaller market cap. The Charles Schwab Co. empowered households as major players in the equity markets by embracing technology with a smart gateway and a dedicated small army of employees to make it easier and cheaper to trade stocks. Though less profit per stock trade for SCHW exists because of efficient pricing, Chuck got his gain by opening new markets and growing. $1 invested in 1981 in SCHW is worth $120 in 2015! Thank you, Chuck!

EXERCISES 8.2B

Numerical quickies

1. Exactly 8 years ago the company paid a 32 cent annual dividend. Today's annual dividend is $1.22 . Find the average annual dividend growth rate throughout the past 8 years. ©ST5 .

2. Yesterday (year-end 2525) the company paid its annual dividend of $1.45. The annual dividend history is:

year	dividend
2521	$0.87
2522	$1.07
2523	$1.16
2524	$1.20

Assume that dividends grow smoothly in accordance with the constant exponential growth model, even though each year there may be random errors that on average equal zero. Compute the best estimate of the dividend growth rate and find the expected values for dividends in years 2524 and 2525. ©ST18 .

3. Yesterday (year-end 2525) the company paid its annual dividend of $1.60. You believe that the stock merits a buy recommendation if it returns 16% per year. Your estimate of intrinsic value assumes that dividends grow smoothly in accordance with the constant exponential growth model. The annual dividend history is:

year	dividend
2521	$0.99
2522	$1.28
2523	$1.26
2524	$1.63

Find the best estimate of the dividend growth rate and intrinsic value. ©ST13 .

4. The Company dividend appears to grow smoothly at a constant exponential rate of 5.5%. Analysts forecast that next year's dividend should equal $3.80. For you to receive a 14% average annual rate of return, how much should you offer for the stock? ©ST19 .

5. The Company is expected to announce their annual dividend tomorrow. One year ago they paid a dividend of $2.40, and 4 years ago they paid $2.12. You believe that

future dividends will grow by the same rate as past ones. You are aware that riskless government securities are yielding 3.5%, and you make an offer to purchase the stock so that you earn 7.2% above the risk-free rate. How much is your offer price? ©ST2am .

6. The company yesterday paid their annual dividend of $2.00 and the share price was $35.10. The company growth rate is 8.6%. Suppose the stock is always priced in accordance with the constant growth dividend valuation model. Find the stock's annual total rate of return. ©ST9 .

Numerical challengers

7. The Company is expected to announce their annual dividend tomorrow. One year ago they paid a dividend of $1.60 and 8 years ago they paid $1.00. You believe that future dividends will grow by the same rate as past ones. You are aware that riskless government securities are yielding 3.7%, and you make an offer to purchase the stock so that you earn 10.3% above the riskless rate. Due to market conditions, you must purchase the stock for $7.50 above your offer price. Find your expected total rate of return at this higher purchase price. ©ST2bm .

8. The company just paid its annual dividend of $4.25. You believe the dividend will grow perpetually at 7.8% per annum. Today's price-to-earnings ratio is 10.9 and the payout ratio always equals 65%. You assess intrinsic value with a 16.6% discount rate. Find the one-year rate of return from buying the stock today and holding it one year, given that next year's share price converges to next year's intrinsic value. ©ST8 .

2.C. Total return partitions into dividend and capital gain yields

Stocks compensate investors two ways. Stockholders receive dividends when the company pays them. Stockholders receive capital gains when market forces push the share price higher. Both are important components of the total rate of return.

Discussion in the time value chapters establishes that the discount rate, *r*, represents the total rate of return. Rearrange formula 8.8 for a specification of *r* from the constant growth dividend valuation model:

FORMULA 8.9 Components for stocks of the total rate of return

$$r = \frac{dividend_1}{\begin{pmatrix} stock \\ value \end{pmatrix}_0} + g$$

$$= \begin{pmatrix} expected \\ dividend \\ yield \end{pmatrix} + \begin{pmatrix} capital \\ gains \\ yield \end{pmatrix}$$

The total return from a stock investment has two sources. A current income component provides immediate cash flow in the form of dividends, a changing price component causes capital gains or losses. Table 8.4 contrasts characteristics for these two components.

Expected dividend yield ($dividend_1 / price_0$)	Capital gains yield (g)
realized cash flow	accrued cash flow
immediately taxable	taxes are deferred
relatively predictable & more certain	very unpredictable & more uncertain
relatively small, averaging 1 to 4%	relatively large, averaging 8 to 12%

TABLE 8.4 Component characteristics for the total rate of return

The table shows the dividend is a realized and taxable cash flow that is fairly predictable and relatively small. The expected capital gains, conversely, is an accrual that is not taxable until the security is sold. Predicting capital gains is relatively difficult, but they comprise the largest component of total stock returns. These characteristics are important when selecting stocks. Two alternative stocks, for example, may provide the same expected total rate of return. The stocks may not be equally appealing, however, if the partitioning of their total returns into dividends and capital gains varies.

The actual share price set through market trading arguably equals the best estimate of a stock’s true value. Linking the actual stock price to a good estimate of the growth rate enables insight about the promised total rate of return. The example below relies on formula 8.9 to answer a powerful question.

EXAMPLE 11 Find the total rate of return

The company just increased its annual dividend by 32 cents relative to last year, so that today’s dividend per share is $4.06. Dividends for this company grow smoothly at a constant rate. The payout ratio is constant at 65 percent. The stock's price-to-earnings ratio today equals 17.3, and you believe the market properly values the stock. What is the expected total rate of return that the constant growth dividend valuation model implies?

SOLUTION

The percentage change in dividend equals the growth rate:

$$g = \frac{dividend_0 - dividend_{-1}}{dividend_{-1}}$$

$$= \frac{\$0.32}{\$4.06 - \$0.32}$$

$$= 8.56\%$$

Use the definition for the price-to-earnings ratio to find the current price (recall that earnings equals dividend ÷ payout ratio):

$$17.3 = \frac{price_0}{\$4.06/0.65}$$

$$price_0 = \$108.06$$

Formula 8.9 specifies the total rate of return:

$$r = \frac{\$4.06\,(1+.0856)}{\$108.06} \quad + \quad 0.0856$$

$$= \quad 4.07\% \quad + \quad 8.56\%$$

$$= \quad 12.63\%$$

The bottom line inference is as simple as 1-2-3: *If* (1) you are confident about the growth rate; (2) persistent and smooth growth is reasonably accurate; and (3) the actual stock price is the best guess about true value, *then* the expected rate of return from a stock investment equals, as formula 8.9 shows, the sum of the expected dividend yield plus the growth rate.

Dividend yields historically are an important component of the total rate of return. Dividends in the preceding example are the source of about one-third of the total rate of return. That historically is a realistic partition for the total rate of return.

Dividend yields rise in fall in relatively long-run disimilar cycles. Dividend yields for S&P 500 companies sometimes average less than 2 percent even though total returns during the same horizon averaged 15 percent. Growth stocks sometimes gain glamour with investors then dim. Perhaps, this is the crowd behavior driver of stock returns.

Long-run behavior created U.S. stock market capitalization in 2015 of $35,127 trillion. These companies sell stocks in primary market transactions. Many raise money from credit market borrowing, too. Secondary markets settle on marginal stock prices and we measure company market caps. The biggest of these companies appear in table 2.1. Find in table 8.5 the distribution of market cap throughout the U.S. corporate sector for a sample of 11,000 U.S. companies that underlie tables 2.1 and 8.5. Each company represents a buy side opportunity.

Type of securities on the sell-side	Snapshot of the balance 9/30/2014
	$ billions
Total market capitalization of U.S. corporate equities	*$35,127*
Market cap of company equities by industry, North American Industry Classification code	
Agriculture, forestry, fishing and hunting (31 companies, NAIC 11)	$91
Mining, quarrying, and oil and gas extraction (1,479 companies, NAIC 21)	$2,194
Utilities (302 companies, NAIC 22)	$1,035
Construction (108 companies, NAIC 23)	$177
Manufacturing – Food, beverages, textiles, and leather (241 companies, NAIC 31)	$1,795
Manufacturing – Wood products, chemicals and plastics, paper & printing (1,040 companies, NAIC 32)	$5,502
Manufacturing – Primary metal, fabricating, computer and electronic, transportation equipment (1,430 companies, NAIC 33)	$5,559
Wholesale trade (193 companies, NAIC 42)	$446
Retail trade (231 companies, NAIC 44-45)	$1,389
Transportation and warehousing (2,295 companies, NAIC 48-49)	$3,332
Information (791 companies, NAIC 51)	$4,504
Finance and insurance (2,896 companies, NAIC 52)	$5,436
Real estate and rental and leasing (391 companies, NAIC 53)	$1,096
Professional, scientific and technical services (294 companies, NAIC 54)	$865
Administrative and support, waste management and remediation services (131 companies, NAIC 56)	$359
Educational services (39 companies, NAIC 61)	$25
Health care and social assistance (102 companies, NAIC 62)	$183
Arts, entertainment and recreation (53 companies, NAIC 71)	$61
Accommodation and food services (113 companies, NAIC 72)	$622
Other services (except public administration) (17 companies, NAIC 81)	$18

TABLE 8.5 Distribution of U.S. corporate market capitalization by industry, 2014

After a sustained flat-to-falling stock market maybe investors will flock back toward fundamentally strong dividend yields. It's hard to predict the direction they'll turn next!

EXERCISES 8.2C

Numerical quickies

1. A share of company stock just paid its annual dividend of $1.45. Exactly 4 years ago the dividend was $0.90. Your analyst tells you the stock's expected dividend yield is 5%. You believe the constant growth dividend valuation model applies perfectly to this properly valued stock. Find (i) the expected total rate of return and (ii) the stock's current intrinsic value. ©ST16

2. You pick up the *Wall Street Journal* and see that riskless government securities are offering 6.35%. You read that the Company just increased their annual dividend by $0.12 cents so that today it is paying a $1.94 dividend per share. You also read that its share price is $36.20 . You believe the constant growth dividend valuation model applies perfectly to this properly valued stock. What is the implied risk premium that is earned from owning the stock; that is, by how much does the expected return on the stock exceed the riskless interest rate? ©ST17

Numerical Challengers

3. The company just increased its annual dividend by $0.71 relative to last year, so that today's dividend per share is $8.02. Dividends for this company grow smoothly at a constant rate. The payout ratio is constant at 25%. The stock's price-to-earnings ratio today equals 15.0, and you believe the market properly values the stock. Partition the expected total rate of return that the constant growth dividend valuation model implies into the leading dividend yield and capital gains yield. ©ST14b .

4. You believe that if today you buy a share of Company stock and sell it in one year for $33.72 your total rate of return should equal 17.1%. You expect share price movements will reflect a capital gains yield of 10.9%. Just yesterday the company paid its annual dividend. According to the constant dividend growth model, what dividend should you expect next year? ©ST4a .

3. The fundamental search for true intrinsic value

Assessing a stock's intrinsic value provides important information for investment decision-making. Finding an asset's true intrinsic value and subsequently investing only in undervalued stocks does not, however, guarantee superior investment performance. Imagine that you discover a stock selling for $70 really is worth $100. Imagine, too, that you are absolutely right. Are you certain that if you buy the stock you will realize the $30 capital gain? Absolutely not! You only make the excess return if the market pushes the price up to its proper level. If the market pushes the price even lower then you lose. Perhaps the market is wrong and you are right. But we are all price-takers in financial markets and the best we can do is make educated investment decisions and trust our forecast of expected outcomes.

Stock prices in the long run and on-average must nonetheless follow intrinsic values. Serious investors search for true intrinsic values. The two subsections below investigate important elements of fundamental analysis. The first subsection examines in more detail the role of growth. The second subsection investigates multiplier analysis.

3.A. *Intrinsic value and the sustainable growth rate*

Growth is an extremely important component of total return. Growing sales, net income, dividends, and cash flow support rising stock prices. Yet how much growth can a company sustain? For this answer, recall the lessons from financial accounting about sustainable growth. A company growing at the sustainable growth rate maintains steady asset turnover, net profit margin, payout policy, and debt ratio. Formula 8.10 reprints the sustainable growth rate equation presented previously as formula 3.4. These three formulations are algebraically equivalent:

FORMULA 8.10a, 8.10b, and 8.10c The sustainable growth rate

$$g^{sustainable} = \frac{R_t(1+D_t/SE_t)}{A_t - R_t(1+D_t/SE_t)}$$

$$= \frac{(retention\,ratio)(ROE)}{1-(retention\,ratio)(ROE)} \qquad when\ ROE = \frac{Net\ income_t}{Stockholders\ equity_t}$$

$$= (retention\,ratio)(ROE) \qquad when\ ROE = \frac{Net\ income_t}{Stockholders\ equity_{t-1}}$$

The variables *R*, *A, D,* and *SE* denote *New retained earnings, Total assets*, *Total debt,* and *Stockholders' equity,* respectively.

The top line (formula 8.10a) uses from the income statement *New retained earnings* (R_t) and from the contemporaneous balance sheet *Total assets* (A_t), *Total debt* (D_t), and *Stockholders equity* (SE_t). The middle and bottom lines (8.10b and 8.10c, respectively) use the retention ratio (that is, *1 − dividends/Net income*) and return-on-equity (*ROE*). The *ROE* is the ambiguous ratio of a flow and a balance with different definitions in-use. The two *ROE* definitions above differ because *Net income* is divided by *Stockholders equity* at either the end or beginning of period.

The sustainable growth rate is an important consideration when assessing the likelihood of sustainable support for today's stock price.

EXAMPLE 12 Sustainable growth and the one-year ROR

The company just paid a dividend of $1.60 per share. The company has a 14 percent return on equity (= $net\ income_t \div stockholders\ equity_t$), a 30 percent payout ratio (= *dividends ÷ net income*), and today's price-to-earnings ratio is 18.1. You believe the company operates at their sustainable growth rate. Find (i) the implied total rate of return if the stock is properly valued; (ii) today's intrinsic value when assessed with a 16 percent discount rate; and (iii) the annual rate of return from purchasing the stock at today's price and selling it next year after the price has converged to next year's intrinsic value.

SOLUTION

Computing the implied total rate of return with formula 8.9 requires finding numbers for *g* and *dividend,* and using the actual share price as *stock value*. Compute $g^{sustainable}$ with formula 8.10b since this *ROE* definition corresponds to the problem set-up. The *retention ratio* equals (1 – *payout ratio*). Thus:

$$g^{\,sustainable} = \frac{(1 - 0.30)(0.14)}{1 - (1 - 0.30)(0.14)}$$

$$= 10.86\%$$

Find today's share price from the price-to-earnings ratio definition:

$$18.1 = \frac{price_0}{\$1.60/0.30}$$

$$price_0 = \$96.53$$

Substitute into formula 8.9 and find (i) the implied total rate of return:

$$r = \frac{\$1.60\,(1 + .1086)}{\$96.53} + 0.1086$$

$$= 12.70\%$$

Notice that the 12.7 percent implied total rate of return is less than the 16 percent discount rate with which you intend to assess intrinsic value.

Find (ii) intrinsic value with formula 8.8:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{\$1.60\,(1 + .1086)}{0.16 - 0.1086}$$

$$= \$34.54$$

The intrinsic value of \$34.54 is significantly less than the actual stock price of \$96.53 and suggests rather extreme overvaluation.

Finding (iii) the one year *ROR* from purchasing the stock at today's price requires numbers for next year's dividend and intrinsic value. The dividend and intrinsic value grow at $g^{sustainable}$. Thus:

$$ROR_1 = \frac{price_1 + dividend_1 - price_0}{price_0}$$

$$= \frac{\$34.54(1.1086) + \$1.60(1.1086) - \$96.53}{\$96.53}$$

$$= -58.49\%$$

The investment likely represents a losing proposition. Sustainable growth does not seem able to sustain the current stock price. Either the lofty price results from speculative factors or, alternatively, an error in the fundamental analysis wrongly biases downward the intrinsic value estimate.

EXERCISES 8.3A

Numerical quickies

1. The Company just paid a dividend of $2.30 per share. The Company offers a 17.10% return on equity (= $\textit{Net income}_t \div \textit{Stockholders equity}_t$), a 60% payout ratio (= dividends ÷ net income), and equity investors assess intrinsic value with a 8.8% rate of return. The Company always operates at their sustainable growth rate and successfully holds constant all relevant financial ratios. Find the share's intrinsic value. ©SV2a .

2. The Company's total assets at year-end 2525 equal $6,200 and are financed by debt of $2,500 and stockholder's equity of $3,700 (160 shares outstanding). Their sales for year 2525 were $6,820 and yielded a net profit margin (= net income ÷ sales) of 2.70%; the payout ratio (= dividends ÷ net income) always is 60%. The price-to-earnings ratio at year-end 2525 is 6.28. For the foreseeable future the company intends to operate at their sustainable growth rate. Assess the share's intrinsic value by using a 10.9% discount rate. Contrast intrinsic value with share price. ©SV1a .

Numerical challengers

3. The Company just announced earnings per share of $3.50, which means that their price to earnings ratio is 16.79. The Company has an asset turnover ratio (= $\textit{Sales}_t \div \textit{Total assets}_t$) of 2.09, a net profit margin (= *Net income* ÷ *Sales*) of 4.6%, a debt ratio (= *Total debt* ÷ *Total assets*) of 40%, and a payout ratio (= *Dividends* ÷ *Net income*) of 45%. The Company always operates at their sustainable growth rate and successfully holds constant all relevant financial ratios. You would like to invest in the stock such that you'll get a 13.8% total rate of return. Contrast intrinsic value with share price. ©SV3a .

4. Company *Total assets* at year-end 2525 equal $3,900 and are financed by *Total debt* of $1,300 and *Stockholders' equity* of $2,600 (200 shares outstanding). Their *Sales* for year 2525 equal $8,190 and yielded a net profit margin (= *Net income* ÷ *Sales*) of 1.70%; the payout ratio (= *Dividends* ÷ *Net income*) always is 55%. The price-to-earnings ratio at year-end 2525 is 7.5. For the foreseeable future, the company intends to operate at their sustainable growth rate. You assess the share's intrinsic value by using a 11.7%

discount rate. Suppose you buy the share today at its market price of 12/31/2525. You hold the stock until 12/31/2526 at which time you receive next year's dividend. Also, suppose the market share price has converged to its intrinsic value of 12/31/2526. What is the one-year rate-of-return from investing in the share? ©SV4dm .

3.B. Price multiples and fundamental analysis

The most common fundamental analysis does not explicitly discount expected cash flow streams. Instead the analysis compares fundamental ratios for a prospective investment with those from a peer group. Typical fundamental ratios include the price-to-earnings, price-to-book, price-to-cash flow, and price-to-sales. Other ratios are possible but all contain stock price in the numerator. The variable in the denominator always is per share, too. Each fundamental ratio measures the market value per dollar of the variable in the denominator.

The data in table 8.6 illustrates a multiplier analysis for six dominant companies in the air courier industry. Suppose the prospective investment is FedEx. The share price of FedEx on the New York Stock Exchange is $44.29. Column 1 clearly shows that among this peer group the share price for FedEx is highest. The relevant question, however, is whether the price undervalues the share's intrinsic value.

Company (ticker)	*stock price* - 1 -	*price-to-earnings* - 2 -	*price-to-book* - 3 -	*price-to-cash flow* - 3 -	*price-to-sales* - 5 -
Air T, Inc. (AIRT)	$ 4.00	10.0	1.18	5.5	0.17
Airborne, Inc. (ABF)	12.65	45.2	0.69	2.5	0.19
AirNet Systems, Inc. (ANS)	4.25	7.1	0.60	2.3	0.34
Atlas Air, Inc. (CGO)	35.47	15.6	2.60	7.4	1.71
FedEx Corporation (FDX)	44.29	17.8	2.45	6.7	0.66
TNT Post Group N.V. (TP)	25.94	51.9	5.78	17.3	1.42

TABLE 8.6 Multiples for a peer group of air courier companies.
Historical snapshot. Compiled by author.

Multiplier analyses glean valuation insights by relating the share price to a key measure of entrepreneurial activity. The most popular fundamental ratio in the financial press is the P/E, that is price divided by earnings per share. For FedEx the P/E is 17.8. This indicates that the market assigns a value of $17.80 to every dollar of FedEx earnings. For AirNet Systems the market values a dollar of earnings at only $7.10. A common but often incorrect conclusion is that the likelihood of overvaluation increases as the fundamental ratio gets higher. Naive signals from the P/E ratio suggest that FedEx is overvalued relative to AirNet. But Airborne and TNT appear even more overvalued!

The constant growth dividend valuation model allows important insights about multiplier analyses. Formula 8.8 expresses intrinsic value as the discounted sum of expected dividends. With a constant payout ratio, however, substitution and rearrangement shows:

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 = \frac{dividend_1}{r - g}$$

$$= \frac{payout \times earnings_1}{r - g}$$

Divide both sides by $earnings_1$ to obtain:

FORMULA 8.11 The intrinsic P/E ratio, concise version

$$\begin{pmatrix} stock \\ value \end{pmatrix}_0 \Big/ earnings_1 = \frac{payout}{r - g}$$

When numbers are given for all variables except *stock value*, then *stock value* represents intrinsic value. The right-hand-side of the formula computes the “intrinsic P/E ratio.” Alternatively, when *stock value* and *earnings* are set to the actual numbers then the ratio is the actual price-to-leading-earnings ratio (the earnings next period are known as “leading earnings”).

Intrinsic value of earnings equals present value of earnings. In the constant growth special case the intrinsic value equals payout ratio divided by $(r - g)$. Differences among companies in *r, g,* and payout policies justify differences in P/E ratios. Fundamental ratios should differ when present values of expected cash flow streams differ.

Consider the effect on the intrinsic P/E ratio if companies have different growth rates. Inspect formula 8.11 and notice that as *g* gets bigger the intrinsic P/E ratio gets larger. We expect, all else equal, that ever-larger growth rates correspond with ever-larger P/E ratios. For the five years ending with table 8.6, in fact, annual average growth in operating cash flow is 1.4 percent for AirNet Systems and 9.0 percent for FedEx. It is even higher for Airborne. Perhaps the P/E ratio is larger for FedEx than for AirNet due to differences in growth. Perhaps valuation errors have nothing to do with it.

Differences in discount rates also impact the intrinsic P/E ratio. Inspect formula 8.11 and notice that as *r* gets bigger the intrinsic P/E ratio gets smaller. Larger *r* correspond to higher risk premia. That is, two identical cash flow streams have different present values if their risks differ. We expect, all else equal, that riskier companies associate with small P/E ratios irrespective of valuation effects. The market capitalization of AirNet Systems is $46 million and its stock price is extremely volatile. FedEx market capitalization exceeds $12 billion and its stock price is less volatile. Perhaps the P/E ratio is larger for FedEx than for AirNet due to differences in risk, and valuation effects have nothing to do with it.

Consider finally the effect on the intrinsic P/E ratio if companies have different payout policies. This consideration requires recognition that the payout ratio has offsetting effects. A direct effect is that higher payout ratios associate with ever-larger P/E ratios because the dividend stream is a larger proportion of earnings. This effect appears in the numerator of formula 8.11. An indirect effect appears in the denominator. Higher payout ratios associate with smaller growth rates (see formula 8.10c). Smaller growth rates in turn associate with ever-smaller P/E ratios. The effect of the payout policy on the intrinsic P/E ratio is ambiguous due to offsetting effects.

Additional insight about the effect of payout policy on intrinsic value results by generalizing formula 8.11. Formula 8.10c shows:

$$g = (1 - \text{payout}) \times \text{ROE}$$

Represent the *ROE* as the total rate of return *r* multiplied by a constant:

$$\text{ROE} = \alpha r$$

The number α simply equals a constant of proportionality that depends on a company's investment and financing opportunities. *ROE* measures the book rate of return that the company earns on its investments. Discount rate *r* measures the rate of return that shareholders earn from capital. Substitution into the intrinsic P/E formula and rearrangement shows:

FORMULA 8.12 The intrinsic P/E ratio, general version

$$\left.\begin{pmatrix} stock \\ value \end{pmatrix}_0 \middle/ earnings_1\right. = \frac{1}{r}\left[\frac{payout}{1 - \alpha\,(1 - payout)}\right]$$

The intrinsic P/E ratio equals the reciprocal of the discount rate multiplied by the term in square brackets. The term in square brackets captures the effect of payout policy on intrinsic value. The effect of the payout policy depends on the value of α.

The constant α equals 1 when *ROE* equals *r*. For this special case the intrinsic P/E ratio simplifies as follows:

$$\left.\begin{pmatrix} stock \\ value \end{pmatrix}_0 \middle/ earnings_1\right. = \frac{1}{r}$$

When the rate of return investors receive on their stock (*r*) exactly equals the rate of return the company earns from its investments (*ROE*), company payout policy is irrelevant and the intrinsic P/E ratio equals the reciprocal of the discount rate. This special case typifies mature competitive industries in which economic profit equals zero.

The constant α exceeds 1 when *ROE* exceeds *r*. For this special case an ever-larger payout ratio makes an ever-smaller intrinsic P/E ratio. The stock market penalizes high payout policies when company investment opportunities surpass rates of return that shareholders earn from financial investments. Because company opportunities are exceptional, shareholders prefer that the company retain earnings and grow.

Alternatively, the constant α is less than 1 when *ROE* is smaller than *r*. For this special case an ever-larger payout ratio makes an ever-larger intrinsic P/E ratio. The stock market rewards high payout policies because company investment opportunities are sub par. Shareholders prefer that the company payout earnings thereby allowing shareholders the opportunity to reinvest the money in alternative uses.

The upshot of multiplier analysis is that many factors cause variation in fundamental ratios besides valuation errors. Using a fundamental ratio to infer misvaluation requires extreme care constructing the peer group. Undeniably if one company is a clone of the other in every imaginable way, that everything about the two is identical except the share prices, then a valuation inference is valid. As differences arise between company characteristics, however, the inference weakens. And every company, like every household, is unique. Fundamental differences in entrepreneurial factors cause differences in intrinsic values.

EXERCISES 8.3B

Conceptual

1. Suppose the company stock price is $56, earnings per share is $1.56, operating cash flow per share is $3.19, and book value per share is $24.78. For a carefully constructed peer group you find the following average multiples: the price-to-earnings ratio is 30, the price-to-cash flow ratio is 15; the price-to-book is 3. Compare the company and peer group multiples and, assuming the peers are virtual clones of the company, make inferences about the company share price. ©ST10

ANSWERS TO CHAPTER 8 EXERCISES

EXERCISES 8.1

1. The short-run is 1 day and long-run is 20 days. Use formula 8.3 and compute the cross-over price = (20 x $26 – $21.25) / (20 – 1), which is $26.25.

2. The short-run is 2 days and long-run is 20 days. Use formula 8.2 and compute the cross-over price = { 2 x (20 x $25.25 – $20.50) – 20 x (2 x $23 – $23.50) } / (20 – 2), which is $28.83.

3. The short-run is 2 days and long-run is 5 days. Use formula 8.2 and compute the cross-over price = { 2 x (5 x $39.75 – $32.25) – 5 x (2 x $36.25 – $37.00) } / (5 – 2), which is $51.83.

EXERCISES 8.2

1. The present value of the perpetual stream is $6.80 / 0.103, which is $66.01. That present value exists one period before the first dividend, that is 7 years from now. Discount $66.01 / 1.103^7 and find the intrinsic value today equals $33.24.

2. You don't get yesterday's dividend, but in one year will receive $2.30 (= 1.044 x $2.20). Sell the stock for $24 and get a total of $26.30. Discount with 12.2% and find the intrinsic value is $23.44 (= $26.30 / 1.122).

3. This solution relies on standard time value relationships. In one year you receive a total of $92.60 (= $89 + $3.60). Present value today is $84. Use the lump-sum formula to find the *ROR* is 10.2% (= ($92.60 / $84)^{1} – 1).

4. In ten years receive a total of $37.70 (= $34 + $3.70). Present value today is $13.64. Use the lump-sum formula to find the *ROR* is 10.7% (= ($37.70 / $13.64)$^{1/10}$ – 1).

5. The present value at time 4 of the perpetuity is \$5.60 / *ROR*. Discount it back 4 periods and use formula 8.4 and find the *ROR*:
$\$40.43 = \$0 \times PVIFA_{ROR, 4} + (1 + ROR)^{-4} \times (\$5.60 / ROR)$.
Solve this on the financial calculator by setting CF_0 = \$-40.43; CF_1 = 0 for 4 periods; CF_2 = \$5.60 for 100 (the value of the perpetuity for the first 100 years is basically the same as for the first million years). Compute that *IRR* equals 9.56%.

EXERCISES 8.2A

1. Use formula 8.5 and find the *ROR* is 7.69% (= \$6 / \$78).

2. Add 175 BP to 4.5% and find that the target *ROR* for the preferred stock is 6.25% (= .0450 + .0175). Find the intrinsic value with formula 8.5:
intrinsic value = \$3.75 / 0.0625; = \$60.00.

3. Find the actual *ROR* for the preferred stock with formula 8.5:
\$101.50 = \$6.25 / *ROR*; or *ROR* = 6.16%.
The actual risk premium that the preferred stock offers investors equals the difference between 6.16% and the CD rate of 4.65%. The risk premium is 141 basis points (= 0.0616 – 0.0465). Note that the risk premium is not big enough to satisfy your preferred risk premium of 6.65% (= .0465 + .0200). Find the intrinsic value with
intrinsic value = \$6.25 / 0.0665; = \$93.98.
The intrinsic value (= \$93.98) is less than the actual price (= \$101.50) and according to rule 8.2 this is a sell.

4. You pay \$58.10 at time 0 and receive \$5 at time1. Then at time 2 you receive \$5 plus the sell price. The sell price satisfies the perpetuity formula *price* = *PMT/r* , or *price* = \$5 / .065, or *price* = \$76.92. Total cash flow at time 2 is \$81.92. The annual rate of return satisfies the equality
$\$58.10 = \$5/(1+r) + 81.92/(1+r)^2$.
Use the financial calculator and find that r = 23.1%

5. You pay \$52.25 at time 0 and receive \$6 at times 1-3. Then at time 4 you receive \$6 plus the sell price. The sell price satisfies the perpetuity formula *price* = *PMT/r* , or *price* = \$6 / 0.07, or *price* = \$85.71. The annual rate of return equals 22.9% and satisfies constant annuity formula 5.1 wherein *CF* = \$6, *FV* = \$85.71, *PV* = \$52.25, and *N* = 4:
$\$52.25 = [\$6 \times (1 - (1+r)^{-4}) \div r] + (\$85.71 \times (1+r)^{-4})\}$

EXERCISES 8.2B

1. $g = (\$1.22 / \$0.32)^{1/8} - 1$; or *g* = 18.2%.

2. This problem involves execution of the *Calculator clue* in Example 8 for the 5 data points in this problem. Enter the data, set the calculator to EXPonential, solve that *b = 1.1203* which implies that *g* = 12.03%. Enter that X' = 4 and find that the expected div_{2524} = \$1.27. Likewise, div_{2525} = \$1.42.

3. Execute calculator clue from Example 8 and solve that *b = 1.1277* which implies that *g* = 12.77%. Enter that X' = 6 and find that the expected div_{2526} = \$1.91. Use formula 8.8 to find that intrinsic value equals \$59.06 (= \$1.91 / (0.16 – 0.1277). The stock is a buy if the actual price is less than \$59.

4. Use formula 8.8 to find that intrinsic value equals \$44.71 (= \$3.80 / (0.14 – 0.055). The stock is a buy if actual price is less than \$44.71.

5. Find that g = 4.22% [=($2.40 / $2.12) $^{1/3}$ – 1. Then find that r = 10.7% (= 0.035 + 0.072). Then use formula 8.8 to find that intrinsic value equals $40.24 [= $2.40(1 + 0.0422)^2 / (0.1070 – 0.0422); = $2.61 / 0.0648].

6. Use formula 8.8 to find unknown variable r equals 14.3%:
$35.10 = $2.00 / (r – 0.086); or r = $2.00 / $35.10 + 0.086.

7. Find that g = 6.94% [=($1.60 / $1.00) $^{1/7}$ – 1]. Then find that r = 14.0% (= 0.037 + 0.103). Then use formula 8.8 to find that intrinsic value equals $25.94 [= $1.60(1 + 0.0694)^2 / (0.1400 – 0.0694); = $1.83 / 0.0706]. You offer to purchase for $25.94 and seller counteroffers at $33.44.

Again use formula 8.8 but this time find that unknown variable r equals 12.4%:
$33.44 = $1.83 / (r – 0.0694); or r = $1.83 / $33.44 + 0.0694.

8. First find today's price by using the P/E ratio and dividend:
P_0 / E_0 = 10.9; E_0 x *payout* = div_0; E_0 = $4.25 / 0.65; P_0 = 10.9 x ($4.25 / 0.65); or P_0 = $71.27.
Now use formula 8.8 to find intrinsic value equals $52.06 [V_0 = $4.25(1 + 0.078) / (0.1660 – 0.0780); = $4.58 / 0.0880]. Note that next year's intrinsic value is $56.12 [= $52.06 x (1 + 0.078)].

Buy the stock today for $71.27 and in one year receive a dividend of $4.58 and sell the stock for $56.12. Find that the *ROR* equals -14.8% [= ($56.12 + $4.58 – $71.27) / $71.27].

EXERCISES 8.2C

1. Find the dividend growth rate as g = ($1.45 ÷ $0.90)$^{1/4}$; g = 12.66%. The total rate of return equals, according to formula 8.9, the sum of the growth rate (12.66%) and the expected dividend yield (5%). Thus, r equals 17.66 percent. All numbers required by formula 8.8 to find intrinsic value are known: $V_0 = div_0(1+g)/(r-g)$; = *$1.45(1.1266)/(.1766 – .1266);* V_0 = *$32.67.*

2. First observe that $g = (div_1 - div_0)/div_0$; *g = .12/(1.94-.12); g = 6.59%.* Now use formula 8.9 to find that *r = div1/V0 + g; r = $1.94(1.0659)/$36.20 + 6.59%;* or *r = 12.31%* . The share offers a risk premium relative to the government security equal to 5.96% (= 12.31% - 6.35%).

3. Find g by dividing the dividend increase by last period's dividend:
g = $0.71 / ($8.02 - $0.71); or g = 9.71%.
Find today's price by using the P/E ratio and dividend:
P_0 / E_0 = 15; E_0 x *payout* = div_0; E_0 = $8.02 / 0.25; P_0 = 15 x ($8.02 / 0.25); or P_0 = $481.20.

Now use formula 8.8 (or 8.9) wherein intrinsic value equals $481.20 and solve for the unknown total rate of return r:
$481.20 = $8.02(1 + 0.0971) / (r – 0.0971); or r = $8.80 / $481.20 + 0.0971; or r = 1.83% + 9.71%; = 11.54%.

The 11.54% total rate of return includes a 1.83% leading dividend yield plus a 9.71% capital gains yield.

4. The total rate of return r of 17.1% equals the capital gains yield of 10.9% plus the leading dividend yield, implying the dividend yield is 6.2% (= 0.171 – 0.109). Today's price is 10.9% smaller than next period's price; today's price is $30.41 (= $33.72 / 1.109). Thus, 6.2% = div_1 / $30.41, or div_1 = $1.89.

EXERCISES 8.3A

1. The *Retention ratio* is 0.40 (= 1 – 0.60). Use formula 8.10b to find the sustainable growth rate is 7.34% [= 0.40 x 0.1710 / (1 – (0.40 x 0.1710))]. Use formula 8.8 to find intrinsic value is $169.36 [= $2.30(1.0734) / (0.0880 – 0.0734)].

2. Compute that *New Retained earnings*$_{2525}$ equals $73.66 (= $6,820 x 0.0270 x (1 – 0.60)]. Use formula 8.10a to find the sustainable growth rate is 2.03% [= $73.66 (1 + $2,500/$3,700) / ($6,200 – {$73.66 (1 + $2,500/$3,700)})]. Compute that div_{2525} equals $0.69 (= $6,820 x 0.0270 x 0.60 / 160) and div_{2526} is $0.70 (= $0.69 x 1.0203). Use formula 8.8 to find that intrinsic value equals $7.94 per share [= $0.70 / (0.109 – 0.0203)].

Find price P_{2525} by using the P/E ratio and dividend:
P_{2525} / E_{2525} = 6.28; E_{2525} = $0.69 / 0.60; P_{2525} = 6.28 x ($0.69 / 0.60); or P_{2525} = $7.23. The stock is undervalued by 9.9% [= ($7.94 – $7.23) / $7.23].

3. Use the DuPont decomposition of *ROE* from formula 2.8 to compute that *ROE* equals 16.02% [= 0.046 x 2.09 x $(1 – 0.40)^{-1}$]. Use formula 8.10b with retention ratio of 55% (= 1 – 0.45) to find the sustainable growth rate is 9.66% [= 0.55 x 0.1602 / (1 – (0.55 x 0.1602))]. Use formula 8.8 to find intrinsic value is $41.72 [= ($3.50 x 0.45) (1.0966) / (0.1380 – 0.0966)].

Find price P_{2525} by using the P/E ratio and dividend:
P_{2525} / E_{2525} = 16.79; E_{2525} = $3.50; P_{2525} = 16.79 x $3.50; or P_{2525} = $58.76. The stock is overvalued by 28.9% [= ($41.72 – $58.76) / $58.76].

4. Compute that *New Retained earnings*$_{2525}$ equals $62.65 (= $8,190 x 0.0170 x (1 – 0.55)]. Use formula 8.10a to find the sustainable growth rate is 2.47% [= $62.65 (1 + $1,300/$2,600) / ($3,900 – {$62.65 (1 + $1,300/$2,600)})]. Compute that div_{2525} equals $0.38 (= $8,190 x 0.0170 x 0.55 / 200) and div_{2526} is $0.39 (= 0.38 x 1.0247). Use formula 8.8 to find that intrinsic value V_{2525} equals $4.25 per share [= $0.39 / (0.117 – 0.0247)]. Intrinsic value V_{2526} equals $4.36 (= $4.25 x 1.0247).

Find price P_{2525} by using the P/E ratio and dividend:
P_{2525} / E_{2525} = 7.5; E_{2525} = $0.38 / 0.55; P_{2525} = 7.5 x ($0.38 / 0.55); or P_{2525} = $5.22. The stock is overvalued by 18.6% [= ($4.25 – $5.22) / $5.22].

Buy the stock at $5.22 and one year later receive a dividend of $0.39 and sell the stock for$4.36. The *ROR* is -9.0%.

EXERCISES 8.3B

1. Tabulate the price multiples.

	Industry	Company	Inference about company
P / E	30	35.9 (= $56 / $1.56)	overvalued
P / CF	15	17.6 (= $56 / $3.19)	overvalued
P / B	3	2.3 (= $56 / $24.78)	undervalued

The multiples give mixed signals. This happens often. Statistical studies suggest that the price-to-book ratio contains significant information about subsequent stock returns. Those studies find that P/E and P/CF are unrelated to stock returns. The analysis suggests, albeit unconvincingly, that the company stock may be undervalued. Almost certainly, however, differences in multiples suggest that the stock market does not believe the company and peer group are clones.

PART 2: ECONOMIC PROFIT#2; THE COST OF CAPITAL AND RISK-ADJUSTED RATE OF RETURN

Till now the book's focus has been how flows and balances within different settings pertain to fundamental time value relationships. That is because Part 1 builds the structure for the first source of economic profit: *net present value*. Part 2 is about a different source of economic profit: the *risk-adjusted rate of return*. Getting to those lessons requires learning first about transformation value.

> *Transformation value* is the economic value-added by combining different inputs to produce a unique output. A special case of transformation value is the diversification benefit from combining security cash flow streams.

Assigning value to diversification benefits is one of the more important objectives of financial science. Making those measurements requires study of the risk-return relation. In textbook Part 1 the lessons largely employ the discount rate as though it were an exogenous number pulled out of the air; like "What is the *NPV* if the discount rate is 5%, you invest \$100 and get back \$110?" Lessons in Part 2 on the second source of value, however, seek insights about the question "what is the financial rate of return consistent with market equilibrium?" Our objective is to endogenously determine discount rates. Part 2 examines the structural forces driving financial economy market equilibrium.

Chapter 9 commences Part 2 of *Lessons* and describes the market backdrop for determining financial equilibrium. This chapter is totally descriptive and contains no problems or computations. Discussions explain that primary market supply and demand for financial securities is a determinant of equilibrium security prices. *Buy-side* institutional investors, households, and some companies demand financial market securities for inclusion in portfolios. The *sell-side* brings securities to the marketplace in order to raise financing for companies, institutions, government, and households. Chapter 9 next explains the rationale for financial market equilibrium by distinguishing between expected and required rates of return. These two rates are as different as supply price from demand price. $ROR^{expected}$ is the internal rate of return that equates the actual security price to the discounted sum of expected cash flows. $ROR^{required}$ is the minimum discount rate that an investor willingly accepts for computing intrinsic value.

Chapter 10 presents lessons on risk, return, and diversification benefits. The chapter opens with discussion on dominance and the risk-for-return trade-off. Explanations and examples describe easy methods for graphing the parabola representing a two-security risk-return profile. This graphic technique vividly illustrates that portfolio risk often diminishes when one sells a low risk and buys a high risk investment. Correlation influences diversification benefits and these *DB* represent a unique source of economic value. Chapter 10 closes with investment advice that the preceding analyses suggest.

Chapter 11 concludes Part 2 on the second source of economic value and explains why $ROR^{required}$ equals the risk-free interest rate plus a risk premium. Further discussion explains why the efficient market hypothesis implies that in the long-run and on-average expected returns vibrate around required returns. A common structural market model for measuring economic profit as $ROR^{risk\text{-}adjusted}$ is studied. The chapter concludes with definition, specification and problem solving with measurements of the company weighted average cost of capital.

CHAPTER 9: BUY-SIDE DEMAND, SELL-SIDE SUPPLY, AND THE RATIONALE FOR FINANCIAL MARKET EQUILIBRIUM

Lessons about financial market equilibrium include some of the most important concepts in finance. The first lesson occurs in Section 1 with an overview on forces of supply and demand that drive security prices. Section 2 examines significant market participants that demand financial securities. Then Section 3 switches sides and examines security supply. Lessons from Section 4 about the "Efficient market hypothesis (EMH)" provide the rationale for determining how forces in financial markets determine equilibrium rates of return.

1. SUPPLY AND DEMAND IN THE FINANCIAL MARKETS

Financial security prices, just like other prices, respond to the forces of supply and demand. An increase in the demand for securities leads to a price increase, all else equal. Likewise, an increase in supply causes security prices to fall. Security price movements translate into rates of return. The supply and demand for financial securities is an important and often overlooked determinant of equilibrium rates of return.

A loose definition in the financial markets for the supply-side is the "sell-side." Companies want to sell stocks and bonds because they seek financing sources. People that work in the "sell-side" help companies, and governments too, issue securities and raise cash. Other people work on the demand-side, known loosely as the "buy-side." Institutional investors demand securities to put into huge portfolios, or brokers drum-up business by looking for individuals that want to buy stocks. Table 9.1 presents a historical snapshot for the distribution of workers on both sides of the street. At that time 50,000 members of the CFA Institute (http://www.cfainstitute.org/Pages/index.aspx) according to the table mostly were workers are on the buy-side. The sell-side was much smaller, at least-by number of employee-members. It is as though securities issued by business and government funnel into the financial markets through a small high velocity opening, after which many, many buyers use the securities to pursue a near infinity of objectives. CFA membership has grown to 127,000 people in 2015.

Buy-side total employment: 67% *45% Institutional investors* These CFA members typically are in-house analysts or investment managers who research, select and manage investments for their own institutions (mutual or pension funds, banks, etc.) 24% Investment companies and mutual funds 13% Banks 5% Insurance companies 3% Pension funds *16% Investment advisors and counselors* These CFA members serve either institutional investors or high net-worth individuals or both. They analyze and recommend appropriate investments and manage client portfolios. *6% Investment consultants* These CFA members help clients identify investment goals and find the right advisors or counselors to manage their investments.
Sell-side total employment: 18% These CFA members typically are analysts that research and rate financial securities. They mostly work at investment banks and brokerage firms. Investment banks advise companies and underwrite stock and bond issues. Broker-dealers act as intermediaries between buyers and sellers.
Other: 15% These CFA members typically work with government regulatory agencies, or in academics, or in specialty finance areas.

TABLE 9.1 Distribution by employment of 50,000 members of the CFA Institute.
Historical snapshot, 2003.

Supply and demand for financial securities determine equilibrium rates of return. Figure 9.1 illustrates supply and demand schedules for financial securities in the primary market. The downward sloping demand schedule D_0 depicts normal buy-side demand for securities. When security prices are high the quantity demanded is low and vice versa. The equilibrium price for financial securities, p_0, occurs where the demand and supply schedules intersect.

The supply schedule for primary market securities is extremely elastic. This means it is very flat, maybe even horizontal. A small change in equilibrium price associates with an extreme change in quantity of securities supplied. Consider, for example, an upward shift in the demand schedule. The demand schedule shifts whenever there is a change in the behavioral characteristics, expectations, or incentives of major buy-side players. For example, perhaps changing household demographics increase security demand, or perhaps Congress enacts legislation increasing intermediary security demand, or perhaps the latest news reports hearten consensus expectations and investor optimism. If the supply schedule were inelastic, that is, if the supply curve were vertical through the point (q_0,p_0), then the demand shift from D_0 to D_1 would push the equilibrium price to p_n. With elastic security supply, however, the equilibrium quantity and price become (q_1,p_1). Quantity supplied in the primary market changes significantly in response to demand shifts, but equilibrium prices (all else equal) are quite robust.

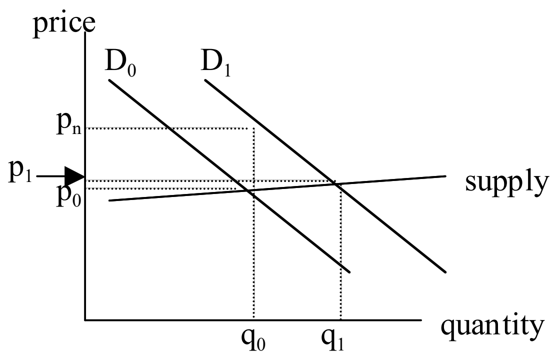


FIGURE 9.1 Supply and demand schedules for primary market financial securities

The elastic supply of securities is a consequence of opportunistic company behavior. The company cash flow cycle from chapter 1 depicts a company issuing securities in financial markets in order to transform real goods and services from stakeholder markets into products that customers want. When companies perceive an economically profitable opportunity to make a product, the company finds the financing required for pursuing the project. A lesson from the capital budgeting chapter is that companies pursue positive net present value projects. The quantity of securities supplied rises whenever entrepreneurs perceive the existence of positive NPV opportunities. Conversely, the supply of primary market securities dries up when opportunities evaporate.

2. MAJOR PLAYERS FOR THE BUY-SIDE

Table 9.2 lists the balance sheet entry of financial assets in 2014 owned by major players on the buy-side. The top left cell shows that the *household sector balance sheets* directly own $47.5 *trillion* of financial assets. This makes households the largest investor group on the buy-side. All the other major players are “institutional buyers” of financial securities. Collectively, the institution balance sheets are bigger than the households. Remember, however, that the financing source for most institutions is households. The institutions are financial intermediaries. Many intermediaries collect funds from households or other depositories for the primary purpose of buying securities in financial markets. Households invest money in financial markets two ways: directly (column 1) and indirectly through intermediaries (columns 2,3,4,5). Find below a brief discussion about the effect that each of the major players has on the buy-side demand for financial securities.

	Household sector - 1 -	Pension & retirement funds - 2 -	Mutual funds - 3 -	Insurance companies - 4 -	Banks & Private Depository Institutions - 5 -
total financial assets	$47,464	$17,435	$16,946	$7,722	$16,764
credit market instruments	$ 3,365	$2,379	$6,551	$4,504	$12,384
corporate equities	$12,884	$5,160	$8,849	$2,156	$ 105
mutual fund shares	$ 7,620	$3,652	…	$ 203	$ 62
pension entitlements	$20,499	…	…	…	…
other financial assets	$3,096	$6,244	$1,546	$ 859	$4,213

TABLE 9.2 Financial assets for the buy-side, by type of investor and asset type. All dollars in billions, 2014.

Notes: Data are from the Board of Governors of the Federal Reserve System, "Flow of Funds Accounts for the United States", 2014Q3. (1) Column 1 equals table L100 Household and nonprofit organizations *Total financial assets* minus *Deposits* minus *Equity in noncorporate business*. Because of the residual nature of the household sector, assets of entities for which there is no data source, such as personal trusts, domestic hedge funds, private equity funds, and nonprofit organizations are included in the *household sector*. (2) Column 2 equals table L116 for private and public pension funds. (3) Column 3 equals the sum from tables L120 Money market fund, L121 Mutual funds, and L122 Closed-end and exchange-traded funds. (4) Column 4 equals the sum of tables L114 and L115 and includes, property, casualty, and life insurance companies. (5) Column 5 equals table L109 private depository institutions and includes commercial banking, savings institutions, credit unions, and others.

2.A. Households

The single largest type of financial security owned by the household sector is "corporate equities". Households, as table 9.2 shows, directly own $12.9 trillion of stocks. Households also collect interest through direct ownership of $3.4 trillion of credit market instruments. The category "mutual fund shares" ($7.6 trillion, includes money market shares) represents indirect ownership of equities and credit market instruments. Furthermore, the $20.5 trillion in pension entitlements is to a large extent invested in stocks, bonds, and other real or financial assets.

Household demand for financial securities depends on many factors. Clearly, household demand for stocks and other financial securities depends to a large extent on household income. Several factors correlate with household income. These factors consequently affect buy-side demand for financial securities.

Educational accomplishment correlates with income (thank goodness!). Table 9.3 lists average earnings of U.S. workers by level of educational attainment.

	Employment				Change		Total job openings due to growth and replacement needs, 2012—22		Median annual wage, 2012
	Number		Percent distribution		2012—22				
	2012	2022	2012	2022	Number	Percent	Number	Percent distribution	
Typical entry-level education									
Total, all occupations	145,355.8	160,983.7	100.0	100.0	15,628.0	10.8	50,557.3	100.0	$34,750
Doctoral or professional degree	4,002.4	4,640.8	2.8	2.9	638.4	16.0	1,426.8	2.8	$96,420
Master's degree	2,432.2	2,880.7	1.7	1.8	448.5	18.4	950.8	1.9	$63,400
Bachelor's degree	26,033.0	29,176.7	17.9	18.1	3,143.6	12.1	8,618.7	17.0	$67,140
Associate's degree	5,954.9	7,000.9	4.1	4.3	1,046.0	17.6	2,269.5	4.5	$57,590
Postsecondary non-degree award	8,554.2	9,891.2	5.9	6.1	1,337.1	15.6	3,067.2	6.1	$34,760
Some college, no degree	1,987.2	2,212.2	1.4	1.4	225.0	11.3	642.6	1.3	$28,730
High school diploma or equivalent	58,264.4	62,895.2	40.1	39.1	4,630.8	7.9	17,667.4	34.9	$35,170
Less than high school	38,127.6	42,286.0	26.2	26.3	4,158.4	10.9	15,914.3	31.5	$20,110

TABLE 9.3 Employment and median annual wage in the U.S.A. by degree earned.
Source: ***http://www.bls.gov/emp/ep_table_education_by_train.htm .***

Scanning down any column shows that income increases with education. Scanning along rows not shown indicates that income generally increases with age. The value of a degree, irrespective of age, is significant. Education increases income potential – and income potential raises the buy-side demand for financial assets!

The demand for financial securities by households probably responds to changes in underlying societal factors. Some authors argue, for example, that changing population demographics exert forces on security prices. The argument is that as a huge mass of aging "baby-boomers" moves through mid-life when careers and income reach their maximum, the demand for securities is a little higher than it might otherwise have been. Conversely, when a proportionately large number of households eventually downsize and draw-down retirement savings there may be a relative decline in security demand. The statistical evidence that demographic movements drive security prices is inconclusive. Undeniably, however, prices respond to the forces of supply and demand so therefore the arguments merit attention.

Household attitudes toward risk also merit discussion. Sometimes societal events, such as fear of war or recession, may cause households to reduce the overall financial risk of assets owned. Usually risk-exposure is less from owning credit market instruments than equities. Conceivably, then, widespread shifts in risk attitudes may drive buy-side demand and cause one type of asset to be substituted for another. Equally important is that household risk attitudes generally depend on the individual's life-cycle stage. Table 9.4 describes how household attitudes toward risk and return generally change with age.

	Explanation	Range	Security Groups With These Characteristics	Household life-cycle profile
Risk Tolerance	How much of a loss can you stomach over a one-year period without abandoning your investment plan?	Low: 0% to 5% loss Moderate: 6% to 15% loss High: 16% to 25% loss	Low: Money market funds, certificates of deposit ("CDs") Moderate: Intermediate and long-term bonds, conservative high dividend-paying stocks High: Growth stocks	*Life cycle stage and risk tolerance* Early career: high Mid-career: high Late-career: moderate Early retirement: moderate Late retirement: low
Return Needs	What form of portfolio return do you need to emphasize: income, growth or both?	Income: Steady source of annual income Growth/Income: Some steady annual income, but some growth is also needed Growth: Growth to assure real (after inflation) increase in portfolio value	Income: Bonds Growth/Income: Dividend-paying stocks Growth: Growth Stocks	*Life cycle stage and return needs* Early career: growth Mid-career: growth Late-career: growth Early retirement: growth/income Late retirement: income
Time Horizon	How soon do you need to take the money out of your investment portfolio?	Short horizon: 1 to 5 years Long horizon: Over 5 years	Short: Money market funds, CDs, short-term bonds; intermediate-term bonds (less than 5 years) Long: Growth stocks, aggressive growth stocks	*Life cycle stage and time horizon* Early career: long Mid-career: long Late-career: long Early retirement: short/long Late retirement: short/long
Tax Exposure	Based on your annual income, at what tax bracket will additional income from portfolio earnings and gains be taxed?	Lower tax exposure: Annual income is such that marginal tax bracket is among lower rates Higher tax exposure: Annual income is such that marginal tax bracket is among higher rates	Higher tax exposure securities (stressed by lower tax-exposure investors): Fixed income securities, high dividend paying stocks Lower tax exposure securities (stressed by high tax-exposure investors): Municipal bonds, non-dividend paying growth stocks	*Life cycle stage and tax exposure* Early career: lower Mid-career: higher Late-career: higher Early retirement: lower Late retirement: lower

TABLE 9.4 Considerations relevant to households for buy-side decision-making.

A common generalization is that the adult life cycle comprises 5 stages. The early career stage typically begins at commencement from secondary or tertiary or vocational education. The individual finds a job (and usually several job-changes), locates a mate, and refines professional interests. The mid-career stage typically involves home ownership, children, community volunteerism, and career/income advancement. Late career stage probably attains maximum income and, with children going to college, also attains maximum expenditures. Early and late retirement generally are varying stages of asset deccumulation and changing interests.

Table 9.4 shows that the typical household in the early career stage of the life cycle seeks high growth, high risk securities for long-term investments. As time passes, the household shifts toward low risk credit market instruments of shorter investment horizon. Surely the buy-side demand by an individual household changes as the life cycle evolves. Also interesting to consider is the extent to which large-scale demographic changes in the population interact with life cycle profiles to affect aggregate demand for financial securities. Household behavior drives the financial economy.

2.B. Pension funds

Pension funds are among the largest institutional investors in the U.S.A. and, consequently, their behavior is important for understanding equilibrium rates of return. Each pay period workers contribute wages to retirement accounts. The managers of the account buy financial securities so that the account accumulates wealth and, eventually, provides financial security for workers after they retire. The companies that manage the retirement accounts are called “pension funds.”

Table 9.5 shows phenomenal growth during the past two decades in retirement assets. Households embrace pension funds as a primary savings vehicle. There are obviously many fewer pension fund companies than households. This huge concentration of wealth means that pension fund managers exert more influence than any other investor group on buy-side demand for financial assets.

	1980 - 1 -	1985 - 2 -	1990 - 3 -	1995 - 4 -	2000 - 5 -	2005 - 6 -	2010 - 7 -	2014 - 8 -
Total assets, all pension funds	$882	$1,885	$3,089	$5,269	$9,043	$11,379	$14,551	$17,435
credit market instruments	298	581	912	1,162	1,582	1,393	1,833	2,379
corporate equities	276	636	877	2,080	3,936	4,158	3,920	5,160
mutual fund shares	7	11	40	327	838	1,585	2,699	3,652
other assets	301	657	1,260	1,700	2,687	4,243	6,099	6,244

TABLE 9.5 Financial assets of private and public pension funds
Dollars in billions.

There are two primary types of pension plans that an employer may offer employees: a *defined benefit plan* or a *defined contribution plan*. The plans differ by the type of promise that the employer makes to employees.

B1. Defined benefit retirement plans

A defined benefit plan is the traditional type of pension plan. Many of the oldest and largest companies, such as IBM and ATT, as well as most government employers (city, county, state, and federal agencies), enroll workers in defined benefit plans. Each pay period the worker makes contributions (often mandatory) to the plan, and perhaps the employer matches the contribution. The defined benefit plan promises to pay specific sums of money to the workers when they retire. Once an employee is eligible for retirement (eligibility often occurs when age plus years-of-service equals a specific number), then a formula similar to this determines the retirement benefit:

$$\text{monthly pension} = 2.0125\% \times (\text{years of service}) \times (\text{highest annual salary}) \div 12$$

All plans use their own formula. With the preceding formula, for example, a worker retiring after 30 years of service and an annual salary of $50,000 receives a monthly pension of $2,516. The benefit continues for life. Typically, however, pension plans allow for participants to cash-out upon quitting or retiring as long as the worker satisfies vesting requirements. For example, consider a schoolteacher that contributes to a defined benefit plan with a 10-year vesting period. Once employment passes the 10-year mark, the schoolteacher qualifies for the pension benefit, even though he/she may quit the job long before retirement. Upon quitting, the individual may elect to immediately receive a one-time lump sum payment, thereby cashing-out the plan, or he/she may leave the money in the pension fund and eventually receive a monthly pension throughout retirement. Some retirees may receive pensions from several different plans, depending on their work history.

Traditional defined benefit plans have become less common during the past few decades. The largest difficulty is the liability that the company incurs because of its legally binding promise to deliver future retirement benefits. It is costly for the company to maintain and accumulate the assets necessary for delivering its pension promises. Even keeping track of vesting records for ex-employees that quit long ago is a distraction unrelated to the employer's main line of business. Accounting policies for monitoring the present value of accumulated benefit obligations are complex, even affecting the company's earnings per share. Another difficulty with defined benefit plans is that the ownership of the assets accumulating for payment of retirement benefits is unclear. Do the pension assets belong to the employer on whose balance sheet the assets (and liability) appear? Or do the pension assets belong to the employee, the principal party that sacrificed some current wages for a future promise? What happens in event of merger or bankruptcy? Court judgments about these questions are all over the place. Defined benefit plans largely mimic ideas enacted with the Social Security Act of 1935 as part of Franklin Roosevelt's "New Deal." Defined benefit plans were a grand idea, but they are disappearing - they are too costly for businesses to maintain. Someday, they may be too costly for government, too.

B2. Defined contribution retirement plans

Defined contribution plans make absolutely no promises about retirement benefits. Instead, defined contribution plans promise the amount that the employer contributes each pay period to the employee's pension fund. Typically, the employee contributes a portion of wages to the pension plan. The employer matches all or part of the contribution. With a one-to-one match, for example, the employee may contribute 5% and the employer contributes 5%. This represents an instantaneous doubling of employee wealth – a 100% rate of return without any risk! Employees definitely should contribute up to the limit that the employer matches.

Among the several types of defined contribution plans, the 401(k) plan is most popular. Table 9.6 shows the rapid increase in number of qualified plans that employers sponsor.

	1985 - 1 -	1990 - 2 -	1995 - 3 -	2000 - 4 -	2005 - 5 -	2010 - 6 -	2013 - 7 -
Number of 401(k) plans	29,869	97,614	200,813	320,000	…	…	513,000
active participants (thousands) in 401(k) plans	10,339	19,548	28,061	42,000	…	…	73,668
assets (billions) in 401(k) plans	$144	$385	$864	$1,800	$2,399	$3,148	$4,190
assets (billions) in all private defined contribution plans	$424	$713	$1,706	$2,867	$3,575	$4,543	$5,860

TABLE 9.6 Summary of 401(k) defined contribution plans

The 401(k) plan, like all defined contribution plans, shifts responsibility of financial security away from the employer and toward the employee. The retirement benefits that the employee receives depend on the performance of the investments. With good investments it will be a plentiful retirement, but if things go badly the pension may be inadequate. The employer makes no promises and bears no burden about the size of retirement benefits with a defined contribution plan.

In most cases, the employer hires an external company to manage the defined contribution plan. The management company, that is the pension fund, communicates directly with the employee about investing the contributions. Many pension funds provide employees with several investment choices. The largest pension fund managing retirement accounts for most professors in the U.S.A., for example, is the TIAA-CREF Company. This pension fund allows professors to allocate contributions into many different asset classes: a diversified stock fund, a global equities fund, a growth stock fund, an equity index fund, a bond fund, a real estate fund, a money market fund, etc.

In some pension plans the employer severely restricts employer choices. Sometimes, as with the infamous debacle of the Enron bankruptcy in 2001, the employer forces the employee to hold company stock. This episode motivated the U.S. Congress to amend laws that limit employer abuses of 401(k) plans. Despite the few bad instances, defined contribution plans are growing quickly. Evidence suggests that they may encourage higher savings rates, and certainly they empower employees for taking charge of retirement dreams.

2.C. Mutual funds

Mutual funds also are among the largest institutional buyers of financial securities in the U.S.A. Mutual funds collect money from many investors. The fund managers carefully analyze possibilities and use the money to buy assets. Quite often the managers select assets subject to guidelines that the mutual fund *prospectus* describes. The prospectus is an official document describing the mutual fund to prospective investors. The Securities Exchange Commission requires that the prospectus contains specific information such as mutual fund objectives and policies, risks that the fund faces, fees that investors pay, and investor services that the fund offers. The mutual fund balance sheet in table 9.7 provides insight on how a mutual fund operates.

ASSETS	($millions)		LIABILITIES
2.4 million shares American Express	84.9	223.2	Debt & misc.
3.9 million shares Amgen, Inc.	187.2	7,743.2	Mutual fund shares (176.3 million)
0.6 million shares Anheuser-Busch	31.3		
1.6 million shares Avon Products	84.9		
0.1 million shares Black & Decker	5.9		
15.9 million shares Cisco Systems	208.9		
0.1 million shares Electronic Arts, Inc.	3.1		
3.1 million shares Fannie Mae	199.5		
0.1 million shares Genentech	3.8		
14.8 million shares General Electric	360.1		
15.0 million shares Intel	233.7		
2.4 million shares IBM	184.2		
5.4 million shares Johnson & Johnson	291.6		
8.4 million shares Microsoft	435.9		
4.4 million shares PepsiCo, Inc.	186.8		
11.2 million shares Pfizer	343.2		
0.7 million shares Starbucks Corp.	14.2		
0.7 million shares United Parcel Serv.	44.3		
other equities & misc. assets	5,062.9		
Total assets	$7,966.4	$7,966.4	

TABLE 9.7 Balance sheet for CREF Growth Mutual fund, historical snapshot.
Dollars in millions

Notice that "shares" appear on both sides of the above balance sheet. On the right-hand-side, the mutual fund issues its own shares to investors (mostly households) in order to raise money. The money probably flows most immediately into the checking account. The fund managers then analyze prospective uses of the cash and, eventually, may purchase common stocks for companies such as IBM. Paper-on-the-left and paper-on-the-right – such is a financial intermediary!

The financial securities that mutual funds purchase almost always may be bought directly by households. Relative to direct ownership, however, households realize several advantages by owning mutual funds.

(a) Diversification benefits accrue from ownership of mutual funds because each fund typically own dozens or more different security issues. Owning one share of a mutual fund represents indirect ownership of many different securities.

(b) Investing in a mutual fund typically is easier and involves fewer transaction costs or commissions than investing directly in stocks and bonds.

(c) Mutual funds hire talented professional investment managers. Most individual households cannot allocate as much time as a full-time fund manager collecting information and monitoring securities.

(d) The astounding variety of mutual funds presents investors with access to a convenient mechanism for pursuing personal investment objectives. Even though two funds may hold exactly the same set of securities, fund characteristics may differ dramatically when each fund allocates among component stocks differently. The analogy is that there are many different ways to combine flour, sugar, and eggs – each combination tastes really different, too.

There are about 8,000 mutual funds in the U.S.A. pursuing many different investment objectives. Table 9.8 describes several common categories for mutual funds.

The categories are not mutually exclusive and perhaps one fund may qualify for several different categories.

Notable risk and return characteristics - 1 -	Types of financial securities that the mutual fund owns - 2 -
1. Money market mutual funds	
Risk of losing principal is nil. Rates of return are relatively low and follow movements in short-term interest rates.	These funds own short-term credit market securities issued by U.S. corporations and federal, state and local governments and their agencies. The interest income that the fund receives generally is taxable income to investors.
2. Bond mutual funds	
Risk of losing principal depends on two separate traits: quality and term. The lowest quality are junk bond funds which may have high risk. High quality bond funds have lower risk. Long-term bond funds have higher risk of sharp price declines; short-term bonds funds have less price risk. Bond fund rates of return: (1) have a long-run average that is larger than money market but smaller than equity funds; (2) move inversely with interest rates. Falling interest rates push up the returns for existing bond fund investors, especially long-term bonds.	*2a. Taxable bond funds* These funds own short or long-term credit market securities issued by U.S. corporations and federal, state and local governments and their agencies. The interest income that the fund receives generally is taxable income to investors. *2b. Tax-exempt bond funds* These funds own credit market securities called "municipal bonds." Investors do not pay federal taxes on interest from municipal bonds (state tax liability depends on details). The tax-exempt interest rate, all else equal, is less than the taxable interest rate. Municipal bonds presumably finance public goods such as schools, hospitals, and transportation projects. There are two types of municipal bonds. Repayment of "revenue bonds" depends on cash flows that the project generates. Repayment of "general obligation bonds" depends on the creditworthiness of the government organization that sponsors the issue.
3. Equity mutual funds	

Risks generally are higher for equity mutual funds than for bond mutual funds. Equity mutual fund risk reflects component security risks although the fund realizes diversification benefits. Foreign equity investment introduces additional risks. Long-run average rates of return are higher for equity mutual funds than any other category.	*3a. Index funds* The objective of equity index mutual funds is to closely match the movement in an underlying stock index, such as the S&P500, or Dow Jones Industrial Average, etc. The funds do not promise to pick great stocks, they simply promise to track the target index. *3b. Growth funds* The objective for managers of equity growth funds is identification of companies with strong sales, asset, and/or profit growth. *3c. Value funds* The objective for managers of equity value funds is identification of stocks that appear undervalued relative to peers. *3d. Income funds* The objective for managers of equity income funds is identification of companies that pay relatively large dividends. *3e. Sector funds* The prospectus sometimes restricts an equity mutual fund to buy stocks for companies satisfying a criterion. The criterion usually describes a specific market sector. Common criteria include (1) line of business, for example, biomedical mutual fund versus telecommunications fund; (2) company size, for example, small versus large market capitalization company; (3) geographical, for example, U.S.A. companies versus European companies versus Latin American companies; (4) economic backdrop, for example "emerging economies" versus "developed nation status".
4. Balanced mutual funds	
Risk of loss usually is less for a balanced fund than for a pure equity fund. Returns tend to be less volatile, too.	A balanced mutual fund diversifies broadly across many types of both bonds and equities.
5. Real estate mutual funds	
Risk of loss usually is less for a real estate fund than for a pure equity fund. Returns tend to be less volatile, too.	The objective for managers of real estate mutual funds is identification of prime commercial properties that promise relatively high rental incomes and opportunities for price appreciation. These "real estate investment trusts (REITs)" are subject to different regulations because the asset side of the balance sheet contains bricks-and-mortar as well as financial securities.

TABLE 9.8 Summary of mutual fund categories.

The incredible variety of objectives illustrates how mutual fund managers exert buy-side demand pressure in many sectors of financial markets. Mutual funds also are important players on the sell-side. Later in this section appears discussion about mutual fund sell-side characteristics.

2.D. Insurance companies

Table 9.2 shows the value of financial assets owned by insurance companies equals $7.7 trillion during a balance sheet snapshot from 2014. Insurance companies accumulate funds by selling products that promise customers financial benefits when specific events occur. Quite often (but not always) the events are catastrophic: car insurance pays benefits when accidents happen, fire insurance pays benefits when buildings burn, life insurance pays benefits when, well, you get the idea. The availability of insurance products dates to ancient times when marine insurance provided coverage for ships transporting goods across treacherous seas. Then as now, customers buy insurance and usually hope they never file a claim. But when the catastrophic event occurs the insurance allows the customer to rebuild and carry-on. Insurance companies survive through the centuries because customers face risks – insurance helps customers manage risk.

The price that customers pay for an insurance policy is the "premium." Insurance companies exert buy-side demand for financial securities because accumulated annual premiums usually exceed annual benefits paid to claimants. The insurance company invests the excess in order to accumulate the wealth required for paying future claims. Table 9.9 presents information about the size of different insurance markets.

	1990 - 1 -	1995 - 2 -	2000 - 3 -	2005 - 4 -	2009 - 5 -
Life insurance premiums	$ 77	$103	$131	$142	$125
Annuity premiums	129	158	307	277	232
Health insurance premiums	58	90	106	118	166
Automobile insurance premiums	95	119	139	187	178
Other property and casualty premiums	123	141	166	241	245

TABLE 9.9 Annual insurance premiums by line ($billions)

Automobile premiums grow over time in response to inflation and demographic effects, but this not a high growth segment. Other property and casualty insurers face severe challenges due to dramatic changes in the American landscape. Prior to 1992 many insurers placed the probability of a $10 billion catastrophe near zero percent. Premiums were priced on that assumption. Hurricane Andrew blew through the southeast in 1992 and insurance claims totaled $15.5 billion. California's Northridge earthquake of 1994 caused claims of $12.5 billion. During year 2000 the homeowners insurance industry paid benefits totaling $36 billion; they paid-out $1.11 for every one-dollar received in premiums. When the insurance industry must pay tens of billions of dollars in claims, they must sell securities that already they own. Arguably security prices respond to such shifts in supply and demand.

The fastest growing segment of the insurance market was for annuities but that seems to have peaked. Life insurance annuities represent savings vehicles. These financial products include "fixed annuities" and the more popular "variable annuities." The customer pays premiums and receives a policy that promises to pay benefits once a specific age or condition is met. The benefit is either a fixed amount, in which case the annuity is analogous to a defined benefit retirement plan. Or the benefit is variable and depends on performance of underlying financial assets; the variable annuity is analogous to a defined contribution retirement plan. The annuity represents an insurance product for which a benefit is not triggered by a catastrophic event.

The McCarran-Ferguson Act of 1945 stipulates that insurance commissions in each state regulate insurance companies operating in that state. Insurance companies, however, are lobbying for removal of barriers to interstate commerce. Just as insurance companies found profits by offering traditional retirement products, so too they seek profits by integrating interstate insurance products. Policy-makers must decide whether the increasing mobilization of households and businesses and integration of financial markets justifies changing the realm of insurance regulation. The effects on buy-side demand for financial assets by insurance companies likely would change, too.

2.E. Commercial banking, savings institutions, credit unions, and other depository institutions

Commercial banks and other depository institutions together own huge stocks of financial assets. The historical influence of the bank sector on the buy-side demand for financial securities shows today through the government regulations defining the banking landscape. Glean insight about the depository institutions through inspection of selected line items from the sector balance sheet in table 9.10.

FINANCIAL ASSETS USD billions			**SELECTED LIABILITIES**
Vault Cash	65	1,942	Checkable deposits
Receivables at Federal Reserve	2,538	8,346	Small time and savings deposits
U.S. government securities	2,831	1,916	Large time deposits
Municipal securities	448	736	Credit market instruments
Corporate and foreign bonds	735		
Mortgages	4,491		
Consumer credit	1,599		
Other loans	2,531		
Corporate equities	105		
Mutual fund shares	62		
Other financial assets	1,359		
Total financial assets	$16,764		

TABLE 9.10 Financial assets and selected liabilities for U.S. private depository institutions, 2014

Notice that for banks a large liability is deposits. A customer or business that deposits money at the bank is a source of financing for the bank. The bank receives the deposit and the increasing liability represents a source of funds. Probably next the bank puts the money into the vault and the increasing asset represents a use of funds. The bank eventually puts the money to other uses in order to generate profit. The biggest use of funds is lending for mortgages and other loans. Borrowers repay loans with interest and, for the bank, interest is income. Loans are the banking sector’s largest income-producing asset.

Buy-side demand by banks for marketable financial securities largely is limited to U.S. government securities, corporate bonds, and municipal securities. Banks due to historical reasons do not (yet) own many equities.

History of the banking system is fascinating. The importance of banks since the middle ages discourages politicians from taking a hands-off approach. Businesses must borrow money to make money, banks choose which businesses (and households) receive loans, and politicians enable legislation directing banks to lend toward objectives satisfying the common good.

Congress chartered "The First Bank of the United States" in 1791 to engage in general commercial banking and perform treasury functions for the federal government. The bank was privately owned and rife with controversy. It was disbanded in 1811 and replaced by the "Second Bank of the United States." President Andrew Jackson in 1836 closed that one. Between 1838 and World War 1 thousands of banks in the U.S.A. issued their own bank notes. The private bank notes were commonly accepted as a medium of exchange. Hence, they were currencies. The first truly national currency issued by the federal government was in 1863, but private bank notes persisted. Throughout the 1800s many different currencies were commonplace in the U.S.A. (8,000 different currencies in 1860!). Travelers going from Philadelphia to Chicago might exchange currency issued by a Pennsylvania bank into currency issued by an Illinois bank. Exchange rates among bank notes varied in much the same way that international exchange rates vary today. Bank notes during "panics" often became worthless because issuing banks went bankrupt.

Enabling of the Federal Reserve System in 1913 assured movement toward a pervasive national currency valid for all debts, public and private. By that time, however, a dual banking system was in place that continues through today. Banks today organize with either "state" or "national" charters. The type of charter historically meant different regulatory requirements. Today, however, differences largely have vanished except that the government chartering the bank empowers a regulatory agency monitoring the bank.

Widespread losses by banks from securities investments during the Great Depression of 1929-1934 and subsequent bank failures caused Congress to pass laws tightly restricting commercial bank activities. The laws effectively placed "firewalls" between companies that differed by line of business or even by geography. Interstate banking was prohibited, banks could not dally with securities, insurers could not dally with banking, etc. Congress also established the Federal Depository Insurance Corporation (FDIC) for the purpose of guaranteeing deposits, thereby eliminating bank panics. Today the FDIC insures each account for up to $100,000.

Commercial banks tended to focus on business lending so Congress passed laws facilitating mortgage lending by Savings Institutions and Credit Unions. Millions of households realized the dream of homeownership due to these beneficial policies. Continued economic growth and increasing integration of financial markets gave rise, however, to an environment in which micro-managing regulations caused more harm than good. Finally, in 1999 Congress passed the Financial Services Modernization Act (Gramm-Leach-Bliley Act). This law allows banks, securities firms, and insurance companies, to affiliate under a parent financial holding company. The law also stipulates regulation that depends on the company's functional activities: the Federal Reserve regulates banking activities, The Securities and Exchange Commission regulates companies dealing with securities, and state insurance commissions regulate insurance activities within that state. The Financial Services Modernization Act uninstalls dysfunctional firewalls installed during the 1930s.

The bank sector is consolidating from mergers and the number of companies is diminishing even while the number of branches are increasing. There are over 8,000 commercial banking companies in the U.S.A. In many other developed nations the numbers of banking companies are in the dozens or low hundreds, not thousands. The large number in the U.S.A. is a historical artifact of the segmentation enforced by Congressional firewalls. Meanwhile, the overall asset base for the bank sector is increasing. Credit unions are benefiting from rules relaxing the common-bond requirement for membership. The effect of bank sector consolidation on the buy-side demand for financial securities is uncertain yet important.

2.F. Nonprofit institutions

Henry Ford and his son Edsel established the Ford Foundation in 1936. Total assets for the Ford Foundation in 2015 are $11 billion. Their balance sheet lists mostly financial assets. Andrew Carnegie in 1911 founded the Carnegie Foundation and today their financial assets total $3 billion. Harvard University, established in 1636, owns an endowment fund with financial assets worth $31 billion in 2015, down a little from the previous year. The Bill and Melinda Gates Foundation, established in 1994 is the world's wealthiest nonprofit balance sheet with financial assets surpassing $42 billion in 2015. The financial assets owned by these and other nonprofit institutions provide financing that enable the institutions to pursue their missions. The mission of the Ford and Carnegie foundations is betterment of the human existence; the mission of the Harvard endowment is to foster learning by students and faculty. Foundation and endowment funds exert buy-side demand for financial assets because they invest in securities.

Endowments and foundations accumulate wealth in order to award grants or spend money in pursuit of their mission. University endowments, for example, own credit market securities that provide interest income. The endowment uses the income to pay for student scholarships or faculty salaries. Endowments and foundations also hire investment managers that analyze equity securities with the intention of "buying-low, selling-high" and providing profit for the institutional mission. In 2012 approximately 35 percent of U.S. nonprofits registering with the IRS reported $4.84 trillion in assets. Crude extrapolation puts the total nonprofit buy-side sector at perhaps $10 trillion in total assets. These balance sheets show up in column 1 of table 9.2. The *household sector* balance sheets include personal trusts, domestic hedge funds, private equity funds, nonprofits, and others. Ownership of financial assets represents buy-side demand for securities, however, just ask anyone from the list in table 9.11 employed at the largest university endowment funds in the U.S.A.

Institution	Value in 2014 ($billions)
Harvard University	$35.88
University of Texas System (system-wide)	$25.43
Yale University	$23.90
Stanford University	$21.45
Princeton University	$21.00
Massachusetts Institute of Technology	$12.43
Texas A&M University System (system-wide)	$11.10
Northwestern University	$9.78
University of Michigan	$9.73
University of Pennsylvania	$9.58
Columbia University	$9.22
University of Notre Dame	$8.04
University of Chicago	$7.55
University of California (system-wide regents portions only)	$7.38
Duke University	$7.04
Emory University	$6.68
Washington University in St. Louis	$6.64
University of Virginia	$5.95
Cornell University	$5.89

Rice University	$5.53
University of Southern California	$4.59
Dartmouth College	$4.47
Vanderbilt University	$4.09
Ohio State University	$3.55
University of Alabama System	$3.20

TABLE 9.11 The largest university endowment funds, 2014, U.S.A.

3. CHARACTERISTICS ON THE SELL-SIDE

The “sell-side” of financial markets loosely refers to security suppliers. Where do financial securities come from, what are they like, how are they made? The lessons below examine these types of questions.

Table 9.12 lists financial securities on the sell-side by type of instrument. The table lists the balance of credit market securities during a snapshot in 2014 as worth $57.9 *trillion* dollars. The market value of all equities was about half that, at $35.1 trillion. $6.6 trillion of mutual fund assets participate on both buy and sell sides because their balance sheets list shares both on the asset and liability sides. Lessons on credit and equity market securities appear in chapters 7 on bonds and 8 on stocks. The remainder of this section discusses the sell-side supply of credit market securities.

Type of securities on the sell-side	Snapshot of the balance 9/30/2014
	$ billions
Total credit market debt	*$57,982*
Open market paper	$996
U.S. treasury and agency securities	$20,590
Municipal securities	$3,631
Corporate and foreign bonds	$11,441
Mortgages	$13,360
Consumer credit	$3,246
Other credit market debt	$3,030
Total market capitalization of U.S. corporate equities	*$35,127*
Mutual funds & exchange traded funds	*$12,266*
Money market mutual funds	$2,565
Open-end mutual funds	$9,410
Closed-end & exchange traded funds	$291

TABLE 9.12 Sell-side securities by type of issuer and instrument

3.A. Credit market securities

The common characteristic for credit market securities is that the cash flow stream is relatively well specified. The issuer promises to repay principal plus interest to the lender and, in the event of default, the lender often has claims on the issuer's assets that are senior to any equity claimants. Still, many other characteristics of credit market securities vary widely, as the discussion below reveals.

A1. Open market paper

All "open market" securities mature in less than 270 days, and most mature in less than 60 days. Recall that according to one of the categorization schemes for financial markets in chapter 1 (table 1.2), the "money market" contains all securities with original maturity of 1-year or less. Securities with longer maturities are in the "capital markets." All open market paper is in the money market. Open market paper ($996 billion, 2014) includes two general security classes: commercial paper and the less used bankers' acceptances. Bankers acceptances' represent contracts in which the bank acts as an intermediary between two companies. The bank issues an "acceptance" to one company in exchange for cash. The acceptance stipulates that another company receives the cash after satisfying some condition, such as delivery of goods or services to the first company. The relatively small market size of bankers' acceptances means that "open market paper" and "commercial paper" are nearly synonyms.

Companies that issue open market securities realize an increase in a liability on the balance sheet. For issuing companies, open market securities are a financing source. The companies borrow cash for this very short-term because their intended use of the cash also is short-term. Generally speaking, companies obtain short-term financing for short-term uses, long-term financing for long-term uses. Figure 9.2 shows the types of companies that issue open market paper.

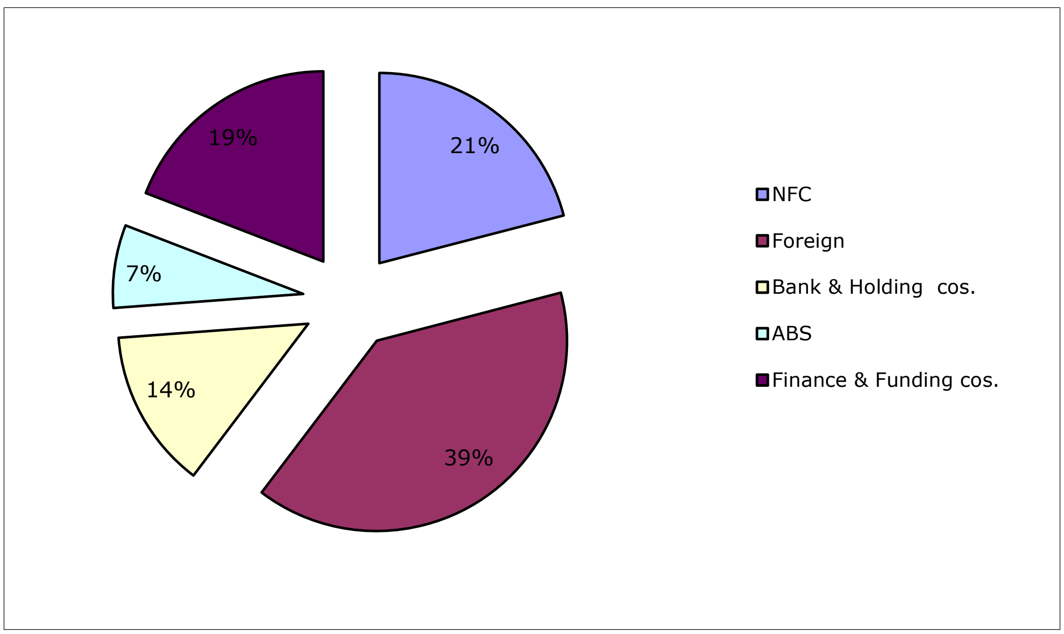


FIGURE 9.2 Issuers of outstanding open market securities, 2014.

Notes: "NFC" is nonfinancial corporate U.S. companies. "Foreign" represents non-U.S. issuers. "Banks" represents commercial bank issuers. "ABS" are asset-backed commercial paper. Data are from the Board of Governors of the Federal Reserve System, "Flow of Funds Accounts for the United States", table L208.

Nonfinancial corporate businesses in the U.S.A. ("NFC") had $208 billion of commercial paper outstanding in 2014 – that represents 21% of total open market securities. Mostly the companies borrow this money to provide customers with credit. The NFC consolidated balance sheet shows, to some extent, *Receivables* on the asset side that are financed by commercial paper on the liability side.

Many companies do not issue their own commercial paper. Instead, they out-source customer financing. "Finance & funding companies", for example, raise money by issuing commercial paper in financial markets (19% of the total open market paper according to figure 9.2). The companies subsequently use the money to offer short-term consumer loans for purchases of durable goods such as refrigerators, furniture, automobiles, etc. The loan may be offered directly to consumers and subsequently the consumer finds the right retailer. Alternatively, a retailer may contract directly for a finance company to offer its customers credit. For example, perhaps a retailer makes a credit sale (meaning there is no immediate cash revenue) thereby causing a decrease in *Inventory*. Instead of increasing *Receivables*, however, the retailer contracts with a finance company for immediate revenue (perhaps at a discounted sale price). The finance company subsequently receives payment from the customer (with interest, of course). The source of money that the finance company uses to pay the retailer is from issuance of commercial paper.

"ABS" in figure 9.2 stands for short-term asset-backed securities and in 2014 they represent only 7% of all NFC commercial paper, down from 52% in 2001. The balance sheet for the company creating the ABS provides key insight about this important security. The company issues (that is, sells) the asset backed security, the sale represents a source of financing, and there is an increase on the liability side for the line item "ABS". Investors purchasing the ABS includes institutional players on the buy-side (pension funds, etc.). Companies creating open market ABS use the money to purchase *Receivables* from hundreds of different companies. The balance sheet's asset side lists all the different *Receivables* on which the ABS have a claim. Ownership of an asset-backed security represents indirect ownership of revenues from a large pool of financial assets.

Commercial banks issue a lion's share of open market paper. Mostly, these are certificates of deposit ("CD's"). CD's represent a liability to the bank. The bank sells the CD to an investor. Investors purchasing the CD include major players on the buy-side (household, pension funds, etc.) The bank stipulates a specific rate of interest that the investment earns. The investor commits the funds for a specific time period. CD's vary in size: relatively small ones appeal primarily to households; large ones over $1 million appeal primarily to institutional investors. The large ones include "negotiable certificates of deposits" (NCD's) for which a rather active secondary market exists. NCD investors may hold the security until the bank repays the principal, thereby retiring the security. Or they may find another investor that wants to buy it.

Repurchase agreements ("repos") are another common type of short-term credit market security. There is, in fact, a very active market for overnight repos. The bank (or any other company) issuing a repo receives cash and delivers to the buyer a portfolio of securities, usually U.S. government securities (or less common is delivery of only the repo security itself). The issuer promises to buy-back the securities (or repo) at a later date for a somewhat higher price. Why do banks go to so much trouble to invest money for, say, only 1 day. Consider this: a bank with $10 million of idle cash prefers to put the money to use even if only for a day – a day's interest on $10 million at a 5% annual interest rate is $1,370. And there are a lot of days in the year.

A2. U.S. government securities

The U.S. government is the largest sell-side supplier of credit market securities in the world. The government sells securities in order to pay for the infrastructure, national defense, and social programs that are vital components of the U.S. political economy. The outstanding balance of U.S. government securities at year-end 2001, $8.3 *trillion*, includes three primary security classes: Treasury securities (41%), government sponsored enterprise securities (25%), and federally related mortgage pools (34%).

Treasury securities

The U.S. Treasury sells securities to finance the national debt. The full faith and credit of the U.S. government guarantees the timely payment of principal and interest on Treasury securities. The interest income from Treasury securities generally is taxable at the federal level but exempt from state and local income taxes. About 5% of Treasury securities outstanding are traditional U.S. Savings Bonds. These non-marketable securities appeal to households and are sold today in denominations ranging from $50 to $10,000. Throughout history different types of savings bonds have been called “liberty bonds”, “patriot bonds”, etc. Most savings bonds that the Treasury sells today are classified as either Series EE, Series HH, or Series I. Series EE and I savings bonds accrue interest monthly at a variable rate and the interest is compounded semiannually. The Series I interest rate tracks the inflation rate, the Series EE interest rate tracks a 5-year interest rate. Investors receive all income when they redeem the savings bond. Series HH savings bonds have a fixed interest rate, interest is paid-out semiannually and, upon redemption, the investor receives the face value. All owners of U.S. Savings Bonds register with the Treasury. In event of loss, the bonds are replaceable. The savings bonds earn interest for up to 30 years, they may be redeemed early, but they may not be sold. Households may buy U.S. Savings Bonds at over 40,000 financial institutions nationwide, or online with a credit card at www.savingsbonds.gov

About 95% of U.S. Treasury securities are marketable, and active secondary markets make these the most widely traded securities in the world. In 2013 about $5.9 trillion of the total $11.6 trillion outstanding Treasury securities were owned domestically in the U.S.A. The other $5.7 trillion are foreign owned, primarily in China and Japan. Figure 9.3 shows the four classes of marketable Treasury securities.

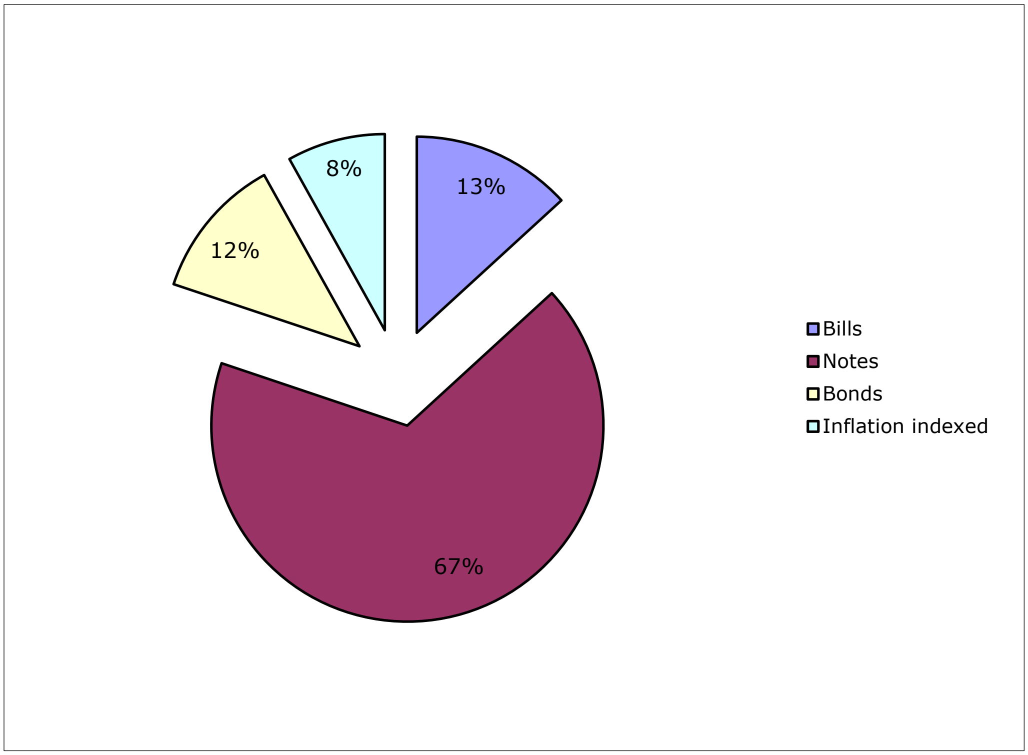


FIGURE 9.3 Types of sell-side Treasury securities

Notes: Total outstanding marketable sell-side Treasury securities 9/30/2013 equals $11,577 billion.
http://www.fiscal.treasury.gov/fsreports/rpt/finrep/finrep13/note_finstmts/fr_notes_fin_stmts_note12.htm

Perhaps U.S. Treasury bills are the world's most risk-free security. T-bills have maturity of one-year or less (and the majority are less than 26 weeks), meaning that they trade in the money market. This also means that T-bill prices are less sensitive to interest rate changes than are the longer-term notes and bonds. T-bills do not pay interest, but instead sell at auction for a price that is less than face value. When redeemed at maturity for face value, the price appreciation represents the investor's profit. T-bill price and return mechanics certainly abide by time value principles. An historical formula, however, relates quoted price and T-bill rate:

$$\begin{pmatrix} T-bill \\ price \\ per\ \$1,000 \end{pmatrix} = \$1,000\left(1 - \frac{(days\text{-}to\text{-}maturity)(T\text{-}bill\ rate)}{360}\right)$$

Look at a recent *Wall Street Journal* and you may see, for example, a listing for a T-bill with 113 days-to-maturity and rate of 1.05%. Substitute these values into the formula to compute the T-bill price of $996.70. The investor pays $996.70 for the T-bill and 113 days later receives $1,000. The profit, that is the "interest", is a modest $3.30. Low risk T-bills certainly provide low returns!

Treasury notes comprise 49% of all Treasury securities. T-notes have original maturities between one and ten years, but today's most common issues are for 2-years, 5-years, and 10-years. Treasury bonds have original maturities between 10 and 30 years. No T-bonds have been issued since mid-2002. T-notes and T-bonds pay interest semiannually. Interest rates and prices are determined at auction.

During the 1990s the Treasury began selling notes and bonds that provide a return indexed to the inflation rate. The coupon rate is constant, but the principal value adjusts by the percentage change in consumer price index since date of issue. The amount of each semiannual interest payment equals the semiannual coupon rate times the inflation-adjusted principal value. These securities comprise only 8% of Treasury issues outstanding. The importance of inflation-indexed Treasury's on future sell-side security supply probably will increase when inflation accelerates.

Government sponsored enterprise securities

The U.S. Congress responds to political economic pressure and establishes enterprises that provide financing in pursuit of a public objective. Government sponsored enterprises ("GSE's") are not included in the Federal budget because they are private companies. Several GSE's issue equity that trades on stock exchanges just as if they were a regular corporation. Debt that a GSE issues is not backed by the full faith and credit of the U.S. government. GSE debt sometimes is referred to as "agency debt." The default risk for agency debt is somewhat greater than Treasury securities but less than corporate bond risk. Many buy-side investors believe that the U.S. government never would allow a GSE to fail. Table 9.13 lists the main GSE's.

Enterprise & objective	Snapshot of the balance 9/30/2014
	$ billions
Total outstanding agency and GSE securities	*$7,834*
Mortgage-backed security pools by Government Sponsored Enterprise "Fannie Mae" and "Freddie Mac" provide liquidity to the market for residential mortgages	$1,624
Other related GSE liabilities	$5,230
Farm Credit System Institutions of the FCS provide privately financed credit to agricultural and rural communities.	$191
Federal Home Loan Bank Board The "FHLB" assists banks, insurance companies, savings institutions, and credit unions in providing financing for housing and community development.	$789

TABLE 9.13 Government sponsored enterprises

The Sallie Mae story is particularly relevant to students and illustrative of public finance history. Sallie Mae was created as a shareholder-owned government sponsored enterprise by the Education Amendments of 1972. The public policy objective of this GSE was to expand the funds available for student loans thereby enabling education investments. Sallie Mae pursued this objective for decades by purchasing student loans from eligible financial institutions. This gave incentive to private institutions to make student loans. Sallie Mae also made loans directly to students. By year 2001 the Sallie Mae balance sheet had about $84 billion of total financial assets. A student may fill out a form that authorizes Sallie Mae to purchase the loans from all the different institutions to which the student owes money. Sallie Mae then bundles it all into one payment and account for you, maybe graduating repayment schedules, etc. The source of funds for

Sallie Mae to buy the loans was for decades by issuance of agency securities (buy-side investors purchase the securities). Since 2004 Sallie Mae has been uncoupled from the government, privatized and gone public, one could say. In 2015 the SLM company has nearly $12 billion of *Total assets* and a common stock that trades on the Nasdaq with a market capitalization of about $4 billion. The political consensus was that this GSE was hurting the public policy objective more than it was helping, that the private sector could do a better job, and that the government should cease involvement.

The Fannie Mae story may be illustrative of the public finance future. More than $5.2 trillion of *Total assets* is visible in the snippet below from table 2.1 in chapter 2.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
FNMA	3,270,108	7	83,963	122,606	17,332
JPM	2,415,689	251	17,923	105,790	219,657
DB	2,220,348	98	918	58,812	49,172
BAC	2,102,273	242	11,431	101,697	164,914
FMCC	1,966,061	5	48,668	75,311	9,425
C	1,880,382	251	13,673	92,543	157,854

SNIPPET from table 2.1 in chapter 2: FNMA and FMCC

FNMA in row 1 with $3.27 trillion of mortgage financial securities on the asset side of its balance sheet is Fannie Mae, a government sponsored enterprise that played a big role in the great recession of 2008-2012. FMCC in row 5 with $1.97 trillion in mortgages is Freddie Mac. Together Freddie and Fannie bought $5.2 trillion of mortgages from depository institutions that had made loans leading up to 2008. Companies above like Bank of America (BAC) or Deutsche-Bank (DB), or J.P. Morgan Chase & Co. (JPM), all made loans, lots of loans. In 2014 the CitiGroup (C, row 6) with market cap of $158 billion agreed to pay $7.5 billion to the U.S. Treasury for knowingly lending and reselling shoddy mortgages. FNMA and FMCC have combined market cap of about $28 billion, relatively tiny given the huge assets. The mortgage assets that these ex-GSEs own reflect a financial claim on the real domicile of tens of millions of households. During the recession the repayment of principal and interest that was promised to the sources of *Total liabilities* on the FNMA balance sheet did not materialize. FNMA and FMCC cash surpluses dove deeply negative. The U.S. Treasury intervened to avoid default by many balance sheets. The Department of the Treasury threw a Congressional tarp over damaged balance sheets, from the very smallest ones to the very largest, and restructured cash flow contracts with trillion dollar infusions from the Troubled Assets Relief Program. Those infusions made the Fannie and Freddy numbers above pretty big!

4. RATIONALE FOR FINANCIAL MARKET EQUILIBRIUM

Security supply in the primary financial market directly mirrors opportunities in real asset markets. Real asset markets, and hence the elastic supply of securities, are more stable than ever-changing buy-side security demand. Relatively frequent shifts in normal demand provide more information than relatively infrequent shifts in elastic supply about equilibrium prices. Inferences on financial market equilibrium focus primarily on investor behavior.

4.A. Expected returns equilibrate with required returns

An investor’s expected rate of return from a stock investment equates security price to discounted returns.

DEFINITION 9.1 Expected rate of return ($ROR^{expected}$)
The "expected rate of return" is the discount rate that equates the actual security price to the discounted sum of expected cash flows. $ROR^{expected}$ is the internal rate of return for that cash flow stream.

Company profitability, sales trends, market share, growth opportunities, price multiples, these and other information sources shape expected rates of return.

A trading rule from the stock valuation chapter is that an unconstrained investor buys a stock when the security's intrinsic value exceeds its price. The investor implicitly figures expected cash flows from the stock. Maybe they are dividends growing at a smooth constant rate. Maybe they are not. Regardless, the investor implicitly establishes the minimal acceptable rate of return at which the stock would be considered a "buy." Define the "required rate of return" as follows.

DEFINITION 9.2 Required rate of return ($ROR^{required}$)
The "required rate of return" is the minimum discount rate that an investor willingly accepts for computing intrinsic value.

The investor implicitly uses $ROR^{required}$ to discount expected cash flows and arrives at the stock's intrinsic value. If the stock price is less than intrinsic value then the investor perceives the stock as a buy. Restatement of this rule follows:

RULE 9.1 Restatement of the intrinsic value trading strategy
The signal a fundamental analysis generates is that:

$$if\ \ ROR_t^{expected} \begin{Bmatrix} > \\ < \end{Bmatrix} ROR_t^{required}\ \ then\ \begin{pmatrix} intrinsic \\ value \end{pmatrix}_t \begin{Bmatrix} > \\ < \end{Bmatrix} \begin{pmatrix} actual \\ price \end{pmatrix}_t\ so\ \begin{Bmatrix} buy \\ sell \end{Bmatrix}$$

When $ROR^{expected}$ exceeds $ROR^{required}$ the asset is undervalued and the asset returns more than the investor requires. Always buy undervalued assets. When $ROR^{expected}$ is less than $ROR^{required}$ the asset is overvalued. Financial market equilibrium occurs when expected and required rates of return are equal.

FORMULA 9.1 Financial market equilibrium condition
The financial market for a specific security, say security A, is at equilibrium when that security's expected and required rates of return are equal:

$$ROR_A^{expected} = ROR_A^{required}\ .$$

The section below presents a lesson on long-run relations between risk and return.

4.B. Historical record of financial market rates of return

Historical data provide insight about long-run risk and rates of return. Figure 9.4 illustrates movement in financial rates of return by showing the value at year-end 2010 of a $1 investment made at year-end 1925.

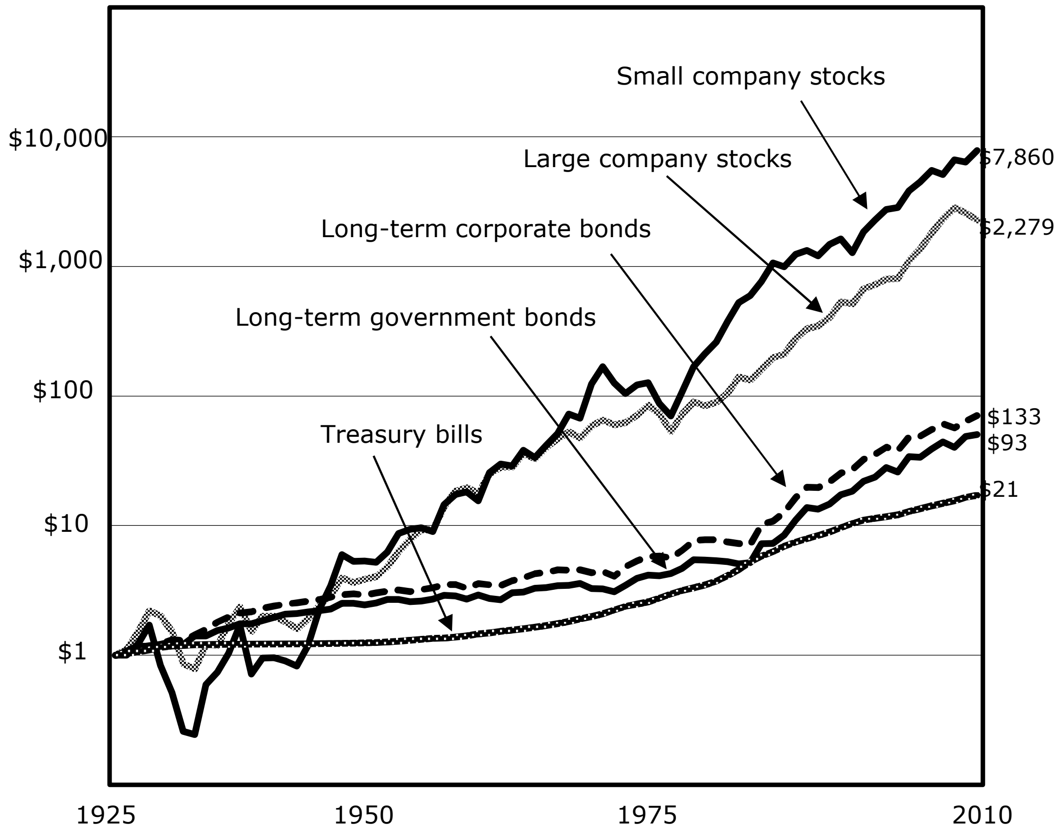


FIGURE 9.4 Growth of a $1 investment at year-end 1925 in different asset classes

A one-dollar investment in Treasury bills would have grown to become worth $21 (all figures assume automatic reinvestment of the total return). The 1,600% cumulative rate of return from T-bills is pretty big because the 85-year sample period is pretty long. Investment in long-term government bonds results in ending wealth of $93, about thrice the accumulation from T-bills. Long-term corporate bonds grow even more, to $133, and a $1 investment in large company stocks grows to an incredible $2,279. Finally, the average small company stock investment of $1 from 1925 attains an ending wealth in 2010 of $7,860.

Table 9.14 summarizes statistics about annual rates of return that underlie the preceding figures. Columns 1 and 2 list average annual ROR computed by the arithmetic and geometric approaches, respectfully. The general discussion about these measurements (see chapter 4, formulas 4.3 and 4.4) establishes that the geometric average annual ROR properly links beginning and ending wealth in accordance with the lump-sum time value relation (formula 4.6). The sensitivity of wealth accumulation over long horizons to the rate of return is apparent through comparison of $ROR^{geometric}$ in the bottom two rows. The annual average is about 160 basis points bigger for small company stocks than for large company stocks (11.1 versus 9.5 percent), yet the total accumulations differ by more than three-fold ($7,860 versus $2,279). Relevant to the current topic, however, is the tendency for all numbers to get bigger as one scans down the table.

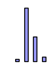
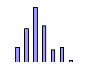
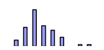
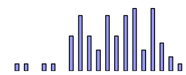
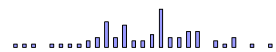
	Arithmetic mean - 1 -	Geometric mean - 2 -	Standard deviation - 3 -	Frequency distribution - 4 -
U.S. Treasury bills	3.9%	3.8%	3.2%	
Long-term government bonds	5.7	5.3	9.4	
Long-term corporate bonds	6.1	5.8	8.6	
Large company stocks	12.7	10.7	20.2	
Small company stocks	17.3	12.5	33.2	
				- 0% +

TABLE 9.14 Summary statistics for annual rates of return

Column 3 lists standard deviations of annual rates of return for each asset class. The standard deviation ("σ") measures dispersion or volatility. Much discussion appears later about σ but, for now, suffice to say that σ relates directly to risk. Column 4 illustrates the frequency distribution of annual rates of return. The distribution for Treasury bills clusters tightly toward the middle. The dispersion of T-bill returns is low – its σ is relatively low, too. Scanning down the table shows increasing dispersion and increasing σ. Table 9.14 reveals an almost perfect alignment: average rates of return rise with risk!

Table 9.15 reveals insight about risk that always is important to remember. Columns 1 and 2 of panel A list the minimum and maximum annual rates of return for each asset class, and column 3 lists the number of times the annual ROR is positive. The sample period for this table contains 76 consecutive annual ROR. The minimum and maximum for T-bills equal –0.02% and 14.7%, whereas for small company stocks the extrema equal –58.0% and 142.9%, respectively. High risk investments naturally have a higher likelihood of extreme outcomes.

	Minimum value (year) - 1 -	Maximum value (year) - 2 -	#windows with ROR > 0 - 3 -
Panel A: 1-year investment horizon (76 non-overlapping 1-year windows)			
U.S. Treasury bills	-0.02 (1938)	14.7 (1981)	75
Long-term government bonds	-9.2 (1967)	40.4 (1982)	55
Long-term corporate bonds	-8.1 (1969)	42.6 (1982)	59
Large company stocks	-43.3 (1931)	54.0 (1933)	54
Small company stocks	-58.0 (1937)	142.9 (1933)	53
Panel B: 5-year investment horizons (72 overlapping 5-year windows)			
U.S. Treasury bills	0.1 (1938-42)	11.1 (1979-83)	72
Long-term government bonds	-2.14 (1965-69)	21.6 (1982-86)	66
Long-term corporate bonds	-2.2 (1965-69)	22.5 (1982-86)	69
Large company stocks	-12.5 (1928-32)	28.6 (1995-99)	65
Small company stocks	-27.5 (1928-32)	45.9 (1941-45)	63
Panel C: 10-year investment horizons (67 overlapping 10-year windows)			
U.S. Treasury bills	0.1 (1933-42)	9.2 (1978-87)	67
Long-term government bonds	-0.1 (1950-59)	15.6 (1982-91)	66
Long-term corporate bonds	1.0 (1947-56)	16.3 (1982-91)	67
Large company stocks	-0.9 (1929-38)	20.1 (1949-58)	65
Small company stocks	-5.7 (1929-38)	30.4 (1975-84)	65
Panel D: 15-year investment horizons (62 overlapping 15-year windows)			
U.S. Treasury bills	0.2 (1933-47)	8.3 (1977-91)	62
Long-term government bonds	0.4 (1955-69)	13.5 (1981-95)	62
Long-term corporate bonds	1.0 (1955-69)	13.7 (1982-96)	62
Large company stocks	0.6 (1929-43)	18.9 (1985-99)	62
Small company stocks	-1.3 (1927-41)	23.3 (1975-89)	59
Panel E: 20-year investment horizons (57 overlapping 20-year windows)			
U.S. Treasury bills	0.4 (1931-50)	7.7 (1972-91)	57
Long-term government bonds	0.7 (1950-69)	12.1 (1982-01)	57
Long-term corporate bonds	1.3 (1950-69)	12.1 (1982-01)	57
Large company stocks	3.1 (1929-48)	17.9 (1980-99)	57
Small company stocks	5.7 (1929-48)	21.1 (1942-61)	57

TABLE 9.15 Maximum and minimum values of returns for 1-, 5-, 10-, 15-, and 20-year holding periods (compound annual rates of return in percent)

Panel B cumulates five consecutive annual returns together and recomputes statistics for the entire five-year horizon. The sample period contains 72 overlapping 5-year windows. In all 72 five-year windows T-bills provide positive cumulative returns.

Small company stocks have positive cumulative returns for 63 of 72 overlapping 5-year windows. Small company stocks lose money during nine 5-year windows (that is, the odds of losing with a 5-year investment horizon was one-out-of-eight). Indeed, $100 invested in small company stocks at year-end 1927 was worth only $77.50 by year-end 1932.

Panel D cumulates fifteen consecutive annual returns together and recomputes statistics for the entire fifteen-year horizon. The sample period contains 62 overlapping fifteen-year windows. Large company stocks provide positive cumulative returns in all 62 windows (although from 1929-1943 the cumulative return is less than 1 percent!). For small company stocks, however, there are three 15-year windows for which cumulative returns are negative. In those cases high risk did not get high return. For the long 20-year investment horizon that panel E tabulates, small company stocks provide positive returns for 57-out-of-57 overlapping 20-year windows.

An important insight is that high risk investments do not always earn high returns. Quite often, even for decades-long investment horizons, high risk investments may receive paltry or negative returns. Yet in efficient financial markets logic dictates a positive relation between risk and equilibrium rates of return. Formula 9.2 summarizes this dictum:

FORMULA 9.2 $ROR^{required}$ and the risk premium

The required rate of return for security A, denoted $ROR_A^{required}$, compensates investors for bearing risk by offering, in the long-run and on-average, a positive risk premium:

$$ROR_A^{required} = ROR^{risk-free} + (security\,risk\,premium)_A \ ,$$

where $ROR^{risk\text{-}free}$ denotes the nominal risk-free interest rate.

Investors that completely forgo risk may invest in T-bills, for example, and earn the risk-free rate $ROR^{risk\text{-}free}$. Yet to pursue a higher risk investment, investors require the promise of higher returns. Investors require a risk premium. Without the promise of a positive risk premium investors shy away from risky investments. High risk earns high return in the long-run but, always remember, one never can be sure exactly how long is the long run.

4.C. The efficient markets hypothesis implies expected returns vibrate around required returns

Analysts and serious investors spend time and money gathering information and developing forecasts for expected rates of return. One of the most important hypotheses in financial science explores the importance of information for forecasting equilibrium rates of return. Ideas inherent with the “efficient market hypothesis” were formalized by Eugene Fama (University of Chicago) and popularized by Burton Malkiel (Princeton University) during the early 1970s. This definition contains the crux of the hypothesis:

DEFINITION 9.3 Efficient market hypothesis (“EMH”)

A market is efficient with respect to an information event if it is impossible to devise a trading strategy that uses the information to consistently earn economic profit.

In an informationally efficient market satisfying the preceding definition all security prices quickly adjust to new information. This means that by the time the marginal investor learns information that may move a stock's price, other market participants already acted on it, the stock price already moved to its new and fair position, and already the stock price reflects consensus expectations about the future. The information has no incremental value to the marginal investor because the stock price embodies all available information.

A simplistic analogy for the EMH is this: there are no five-dollar-bills lying on the sidewalk of the busiest street in town. This analogous hypothesis obviously is true (at least 99.99999999% of the time!). If there were a five-dollar-bill on the sidewalk, someone else already would have picked it up. You cannot walk down a busy street and find five-dollar bills just like you cannot listen to the news and use what you hear to consistently earn economic profit.

An important implication of the EMH is that equilibrium rates of return, in the long run and on average, are not driven by information about company profitability, sales trends, market share, growth opportunities, price-multiples, or any of the other information events that possibly shape expected rates of return. Unanticipated information events cause price adjustments after the fact. Before the fact, however, investors anticipate information thereby causing convergence of $ROR^{expected}$ with $ROR^{required}$. Ex ante equilibrium rates of return vibrate around required returns. Before continuing our investigation of the relation between risk and return, consider important nuances of the efficient market hypothesis.

C1. Nuances of the efficient market hypothesis

This subsection examines the EMH and overviews tests of market efficiency. The hypothesis pertains to all well-developed financial markets. Typically, however, the stock market is the stage on which most discussions of the EMH center. First, consider components of EMH definition 9.3 that are notable.

1. The financial market may be efficient with respect to one information event but not others. The question "is the stock market efficient?" is underspecified and does not have a definite answer. Instead, a definite information event must be specified for which a definite yes/no answer exists. For example, a valid question is this: "Is the stock market efficient with respect to announcement of a stock split?"

2. The EMH argues that stock prices quickly respond to new information. The price adjustment process, however, takes time. The length of time varies from seconds, to minutes, and maybe longer for surprising information that is difficult to assess. Lengthy adjustment periods may reflect information complexity rather than market inefficiency. Sometimes, for example, new information about a single event reaches the market in bits and pieces. Consequently, the price response may diffuse across time. The important issue is whether security prices adjust to unanticipated changes in consensus expectations.

3. There is no violation of the EMH simply because a trading strategy uses information and once earns economic profit. Instead, the EMH argues that a trading strategy cannot consistently earn economic profit.

4. If the EMH is true then all security prices are "fair." This means that consensus expectations push prices to the point that none are undervalued nor overvalued. Security investment earns a return that depends exclusively on risk and unanticipated changes in consensus expectations.

5. The EMH uses economic profit as its benchmark for comparison. Perhaps a trading strategy consistently earns positive accounting profit, but this does not violate the

EMH. Instead, the EMH argues that a trading strategy cannot consistently earn returns that exceed fair compensation for risk exposure.

Figure 9.5 shows possible price adjustments to an information event. Perhaps the event is announcement of a stock split, or a merger, corporate name change, labor strike, or even announcement of federal legislation changing the regulatory environment. Time is on the horizontal axis. For convenience, the three panels break the entire time horizon into three subperiods. The middle panel illustrates the subperiod with the information event. The top and bottom panels are the pre-event and post-event time-windows, respectively.

The vertical axis always shows the *cumulative abnormal return* ("CAR"). The CAR equals the cumulative economic profit that the stock earns. Compute the CAR in two steps. The first step computes each period's abnormal return. The periodic abnormal return equals the stock's actual ROR minus the ROR earned by a security (or securities) of equivalent risk that is unaffected by the event. This latter ROR is the "control sample ROR" and its subtraction from the actual ROR adjusts for fair returns that normally accrue. The periodic abnormal return represents the economic profit the security earns per period and its frequency may be daily or monthly. The second step sums the periodic abnormal returns for all periods within a time-window. The sum equals the CAR and represents total economic profit throughout the window.

Panel A illustrates how the CAR moves during the pre-event window. The CAR vibrates around zero before anticipation of the event begins. Securities at equilibrium earn zero economic profit, thereby implying a CAR of zero. At some point the market starts anticipating the event. Perhaps rumors trigger the anticipation, or perhaps the company begins fitting the profile of companies that tend to experience the event. Regardless, consensus expectations about the company change and, according to the EMH, the stock price responds by reflecting the expected event. If the market anticipates that the event will be good news, such as higher company profitability, the price rises abnormally and the CAR gets bigger. Anticipation of bad news pushes the CAR negative. Figure 9.5 illustrates that total economic profit throughout the pre-event window sums to CAR_g and CAR_b in anticipation of good news and bad news, respectively. With anticipation of neutral news, or if the market never anticipates the event, the CAR continues vibration around zero.

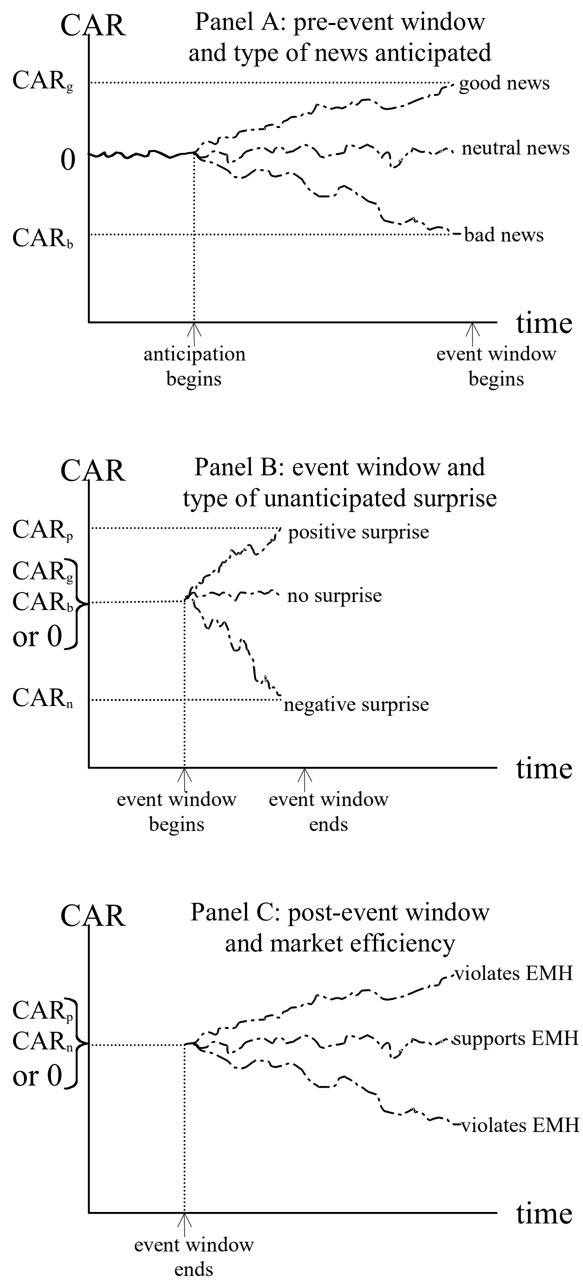


FIGURE 9.5 Price adjustment process to an information event

Panel B illustrates how the CAR moves during the event window. An event window begins when finally the anticipation is over and facts arrive. The CAR when the event window begins has whatever value is attained in Panel A (either CAR_g or CAR_b or 0). Movement of the CAR during the event window depends on the accuracy of the anticipation. When actual information is better than expected the CAR moves up toward CAR_p. The positive surprise drives up the stock price as the market digests the new information. Conversely, for negative surprises the stock price moves down and the CAR drops to CAR_n. With accurate anticipation the CAR does not move because there is no surprise.

Sometimes it may seem puzzling when a company's stock price hardly responds to an announcement of significant news. The explanation is that the market accurately anticipated the news. Likewise, sometimes a company announces bad news but the stock price rises. Why? The answer is that the market anticipated worse news. When finally the company announces that things are bad, but the market perceives it's not as bad as expected, the positive surprise sends the stock price upwards.

The event window ends when consensus expectations fully digest effects of the event on company cash flows. The length of the event window may be seconds, minutes, days, or more. Regardless, once the flow of new information ceases then the event window closes.

Panel C illustrates how the CAR moves during the post-event window. The EMH claims that the stock price quickly reflects all available information. As long as the post-event window is absent of new information then, according to the EMH, actual stock returns contain zero economic profits. The CAR should vibrate around whatever value is attained in Panel B (either CAR_p or CAR_n or 0). The EMH claims that during the post-event window the CAR drifts sideways, neither rising nor falling significantly. Significant drift, up or down, violates the EMH.

Hundreds of tests on market efficiency examine many different types of information events. Information events belong to either of three traditional categories. "Weak" information events pertain exclusively to stock market data such as historical prices, trading volume, price multiples, calendar effects, etc. "Semi-strong" information events pertain to all publicly available data such as economic or political announcements, corporate news, etc. "Strong" information events pertain to all data, public and private, such as insider or managerial insights. Irrespective of category, an efficient market exists when a trading strategy cannot use an information event to consistently earn economic profit. The super-majority find tendencies that figure 9.5 illustrates as supportive of the EMH.

A handful of studies find tendencies violating the EMH. Important reasons why a CAR may show significant non-zero drift during the post-event window include:

(1) The information flow is wrongly identified. Perhaps the post-event window contains new information that drives the CAR. Perhaps even the event window is mis-specified. Occurrence of this type of error means that the EMH may be rejected even though it really is true.

(2) The control sample ROR may be computed wrongly. The control sample ROR purports to measure the stock's rate of return that would have occurred if the information event did not exist. But in actuality the event exists and divining what might have happened if it didn't exist is difficult. Reliance on CARs necessarily results in a joint test of two hypotheses: (a) market efficiency is true and (b) control sample design is correct. It is impossible to know which reason causes the significant drift in CARs. Medical studies face an analogous problem asking what happens to patients if they take a medicine versus what happens if they don't – in actuality some patients take it and the control sample pretends to mimic those same patients as if they hadn't. Stock market studies, like medical ones, generate controversial findings.

(3) The EMH is false for this particular type of information event. This implies existence of a trading strategy that consistently earns economic profit. As large numbers of investors utilize the strategy its effectiveness probably would disappear.

With efficient markets no trading strategy consistently earns positive economic profits because no investor consistently finds new information offering an advantage for predicting $ROR^{expected}$. The implication is that expected rates of return vibrate around required rates of return like a rubber band conforms to a deck of playing cards. New information may momentarily stretch $ROR^{expected}$ away from $ROR^{required}$, but very quickly it snaps back. Rule 9.2 summarizes this lesson.

RULE 9.2 Risk determines $ROR^{equilibrium}$
Equilibrium rates of return track required returns in efficient financial markets. Required returns depend exclusively on risk. Risk, and risk alone, determines the level of equilibrium rates of return.

Equilibrium rates of return track required returns in efficient financial markets, and required returns depend exclusively on risk.

CHAPTER 10: MEASURING RISK, RETURN, AND DIVERSIFICATION BENEFITS

Lessons from the previous chapter explain that expected rates of return depend on time value principles that Part 1 of this book discusses. Investors digest information and perceive a cash flow stream, they observe the actual security price, and $ROR^{expected}$ is the discount rate that links price with present value of expected cash flows.

Required rates of return depend on a fundamentally different process. Investor perceptions of tradeoffs and opportunity costs drive determination of $ROR^{required}$. The required rate of return, restated below, largely depends on risk:

FORMULA 10.1 Required rate of return ($ROR^{required}$)

The "required rate of return" is the minimum discount rate that an investor willingly accepts for computing intrinsic value. The required rate of return for security *A* equals the nominal risk-free interest rate ($ROR^{risk\text{-}free}$) plus that security's risk premium:

$$ROR_A^{required} = ROR^{risk-free} + (security\ risk\ premium)_A .$$

Investors may purchase riskless government securities and earn rate $ROR^{risk\text{-}free}$. For bearing risk, however, investors must perceive and receive, in the long-run and on-average, a positive risk premium. The risk premium induces investors to bear *ex ante* risk –the risk premium compensates investors *ex post* for bearing risk.

The previous chapter also explains that in efficient financial markets $ROR^{expected}$ vibrate around $ROR^{required}$. Equilibrating forces push expected returns toward required returns. Consequently, equilibrium rates of return track required returns. Explanations of market equilibrium focus on explanations of the relation between risk and return. This chapter presents several lessons on risk and return. Section 1 explains how risk and return relate to the dominance concept. Section 2 examines primary risk sources. Sections 3 & 4 introduce measures for risk, return and diversification benefits.

1. Overview of risk, return, and the dominance concept

Financial science generally assumes that investors prefer more return instead of less return, and less risk instead of more risk. The dominance concept illustrates implications of this assumption about investor risk preferences.

DEFINITION 10.1 Dominance
One strategy dominates the alternative if in every dimension the strategy is at least as good or better than the alternative. The absence of dominance implies the existence of a tradeoff.

Every comparison of two alternatives shows either the existence of dominance or a tradeoff. When dominance exists, investors prefer the dominant strategy. When a tradeoff exists, one strategy is better than the alternative in some ways, worse in other ways, and the one that is best depends on other factors.

A dominant investment strategy is one with expected return equal or exceeding an alternative's and with risk less than the alternative's. Figure 10.1 illustrates for financial investments the concept of dominance versus tradeoff. The vertical axis measures expected rate of return and horizontal axis measures risk. Suppose an investor contemplates buying either security A or security B. Consider the risk-return characteristics at point A. Now consider how characteristics change by moving to point B. The top panel illustrates that with movement from point A toward B the return increases – and that is good. Risk decreases – that is good, too. It's a win-win move. Security B dominates A. To the extent that investors only care about risk and return, and that our graph properly portrays them, B is preferable to A in every dimension. When an investor faces a constraint that only one security may be bought, A or B, then the decision is a no-brainer. Investors prefer dominant strategies, B dominates A, so buy B.

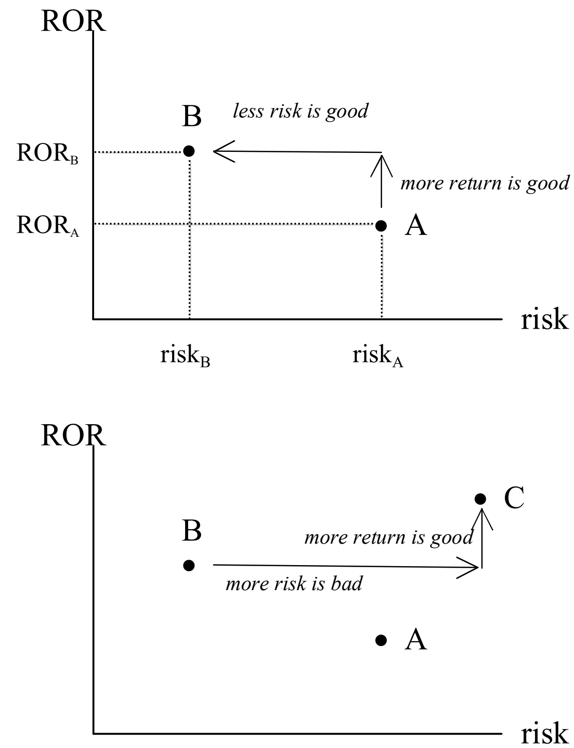


FIGURE 10.1 Risk, return, and dominance

Now consider comparison of securities B and C shown in the lower panel. Security C has more risk – that is bad. But security C also has more return – and that is good. Securities B and C coexist as tradeoffs. The one that is best does not depend on the security characteristics. Instead, the one that is best depends on risk preferences of the investor. To some investors the extra return may be worth the extra risk, whereas to other investors the tradeoff may not be worthy. We cannot say whether C is a better choice than B by looking simply at the security risk-return characteristics. Information about the investor risk preferences is crucial. Similar analysis shows no dominance between A and C either. A and C coexist as tradeoffs.

When one security dominates the alternative, the security risk-return characteristics determine which is best. When there is not dominance, that is they coexist as tradeoffs, then the securities do not determine which is best – investor risk preferences do. For this reason many advertisements in the financial press advocate the importance of tailoring investment decisions to the investor's personal situation. Individuals in early life-cycle stages (see table 9.4) generally have longer investment horizons and tolerate high risk strategies. Retirees, conversely, generally have shorter horizons and possess low risk tolerances.

Investments expose investors to risk that emanates from many sources. At the most basic level, however, all sources of risk belong to one of two classes: idiosyncratic risk or systematic risk.

DEFINITION 10.2 Two primal risk classes: idiosyncratic and systematic

Idiosyncratic risk is unique to a specific security and may be reduced or eliminated through prudent diversification strategies.

Systematic risk is unaffected by diversification strategies and is common to all securities although security sensitivities to systematic risk may vary.

The definitions emphasize the importance of diversification attributes for classifying risk. Financial theory shows that diversification eliminates risk when the distribution of risk across securities and markets has certain properties. In reality, however, there are many sources of risk with varying and "in-between" properties. Pigeonholing risk sources into one class or the other is difficult. The classification scheme, nonetheless, is useful. There exist, at the limit, two types of risk: risk that can be managed or eliminated (idiosyncratic) and risk that can't (systematic).

EXERCISES 10.1

Conceptual

1. Your analysis of common stocks for companies X and Y lead you to believe their expected rates of return and standard deviation of returns are as listed below. Is there dominance or a tradeoff? ©ER6

	E(ROR)	σ
security X	15.9%	15.7%
security Y	16.4%	13.0%

2. Suppose that you are able to perfectly measure risk and expected return, and the bigger the number the bigger the risk or return. Measurements of (*risk, return*) for three possible asset investments, call them *X*, *Y*, and *Z*, are as follows: *X*: (20,9); *Y*: (4,6); *Z*: (11,7). Compare the three with regards to dominance or tradeoff. ©ER11

3. Suppose that you are able to perfectly measure expected return. Also, suppose that there exist two different kinds of risk that you can measure, call them $Risk_1$ and $Risk_2$. The amount of $Risk_1$ an investment possesses is totally unrelated to the amount of $Risk_2$ that it possesses. Three possible asset investments, call them *A*, *B*, and *C*, have measurements for ($Risk_1$, $Risk_2$, *return*) as follows: *A*: (10,40,8); *B*: (10,20,9); *C*: (15,50,10). Compare the three with regards to dominance or tradeoff. ©ER10

2. Sources of idiosyncratic risk

When a company issues a credit market security they promise to repay principal plus interest to the investor. But there is no guarantee. When a company issues a common stock the hopeful investor expects to collect future dividends or capital gains. But there is no guarantee. These securities have risk, yet where does it come from? What are the sources of risk?

Traditional sources of idiosyncratic security risk have been known for a long time. Subsections below discuss several prominent risk sources. The sources are not mutually exclusive and, perhaps, not every security exposes its investor to each type of risk.

Liquidity risk

The lesson on liquidity ratios in chapter 2 (section 3.A) teaches that a *liquid* asset is one that easily and quickly converts to cash at a fair value. Not all financial securities convert to cash with the same liquidity. Some securities trade so frequently that changes in market value are observable second-by-second. They are very liquid. Other securities may not trade for months at a time (or even longer!); these are illiquid. Imagine an investor comparing two securities. Suppose the two promise identical cash flow streams but one is frequently traded whereas the other is not. The investor knows that the liquid security is easier to buy or sell at the prevailing market value. Trading the illiquid security is riskier because willing traders aren't as prevalent. All else equal, demand and equilibrium price are greater for a liquid security than for an illiquid one. In other words, liquidity is a source of idiosyncratic risk:

DEFINITION 10.3 Liquidity risk premium

Liquidity risk refers to the likelihood that a security easily and quickly converts to cash at the prevailing market value. The *liquidity risk premium*, nil for very liquid securities, gets bigger as liquidity diminishes:

$$\text{When security liquidity}\begin{Bmatrix}\text{increases}\\ \text{decreases}\end{Bmatrix}\text{then the liquidity risk premium}\begin{Bmatrix}\text{decreases}\\ \text{increases}\end{Bmatrix}.$$

Common stocks trade with different frequencies, thereby suggesting differing liquidities. Table 10.1 tabulates facts about equity liquidity for the New York Stock Exchange and for NASDAQ.

% of stocks with less than number at right	Average value of trades per day ($1,000s) - 1 -	Average number of shares traded per day (1,000s) - 2 -	Share turnover period (days) - 3 -
NYSE (2,555 stocks)			
100% (maximum)	$534,188	36,632	193,000
80%	$13,038	561	810
60%	$3,186	173	423
40%	$791	62	273
20%	$200	18	170
0% (minimum)	$0.18	0.01 shares	0.8 days
NASDAQ (3,650 stocks)			
100% (maximum)	$1,539,746	56,884	infinite
80%	$1,514	194	1327
60%	$254	52	640
40%	$65	16	326
20%	$16	4	145
0% (minimum)	$0.00	0.00 shares	0.6 days

TABLE 10.1 Quintile breakpoints on equity liquidity variables.
A historical snapshot from 22 consecutive trading days. The "share turnover period" equals total common shares outstanding ÷ (monthly share volume/22). Compiled by author.

Column 1 of the upper panel shows that among 2,555 stocks on the NYSE the average value of trades per day ranges from a maximum of $535,188,000 (that's half-billion dollars per day!) to a minimum of $180. This suggests a huge difference in liquidity! Stock for the company at the upper percentile (IBM) is more liquid than stock for the company at the lowest percentile (Companhia de Bebidas das Americas). Inspection of this column shows that daily dollar volume for 20% of all companies on the NYSE averages less than $200,000.

The lower panel lists analogous facts for 3,650 NASDAQ stocks. The most traded NASDAQ stock (Microsoft) averages daily volume of $1.54 billion (that's almost 3 times more than for IBM). Generally, however, dollar volume is less for stocks on NASDAQ than NYSE. Sixty percent of NASDAQ stocks, for example, average dollar volume less than $254,000 – this would put them at or below the 21st percentile on the NYSE.

Columns 2 and 3 examine other measures of equity liquidity. Column 2 lists number of shares instead of dollar value. The maximum number of shares traded per day is 36.6 million (for Lucent Technologies) on the NYSE and 56.9 million (for Cisco Systems) on NASDAQ. Twenty percent of stocks on the NYSE trade fewer than 18,000 shares per day; on NASDAQ twenty percent trade fewer than 4,000 shares.

Column 3 offers another comparison of equity liquidity by examining the "share turnover period." The share turnover period is number of days that, at average daily share volume, the entire balance of outstanding company stock would be traded. With IBM, for example, there are 1,690 million shares outstanding and average daily volume (throughout December 2002) is 6.89 million shares. At this rate it takes 245 days to completely turnover all IBM shares (245 = 1,690,088,000 ÷ 6,892,745). The IBM share turnover period, 245 days, is at the 34th percentile of all NYSE stocks. The table shows that 60% of all NYSE stocks have turnover periods less than 423 days. The 54th percentile is 365 days, implying that it takes longer than one year for 46% of NYSE stocks to completely turnover all outstanding shares.

Column 3 for NASDAQ presents a similar picture. A turnover ratio of 365 days is at the 41st percentile and indicates it takes longer than one year for 59% of NASDAQ stocks to completely turnover all outstanding shares. Even though columns 1 and 2 suggest NYSE stocks are more liquid than NASDAQ stocks, column 3 shows lesser differences among share turnover periods.

Table 10.2 shows other similarities between NYSE and NASDAQ equity liquidity. Column 1 for the NYSE ranks in descending order the 2,555 stocks by average value of trades per day. The top 20% (511 stocks) account for 86.7% of all total daily trading volume (by dollars). The bottom 60% (1,533 stocks) account for only 3.4% of all trading. There is a huge concentration of trading activity. This is reminiscent of tables from chapter 1 that show a huge concentration of business activity in the upper crust of large companies.

Column 2 for the NYSE ranks in descending order the 2,555 stocks by average daily share volume. The top 20% (511 stocks) account for 83.1% of total shares traded. Columns 1 and 2 convey similar stories. NASDAQ trading concentrates even more.

The median shareprices in parentheses of columns 2 and 3 convey insights about the relation between stock price and liquidity. The most liquid stocks are in the "biggest quintile" when sorted by average daily share volume. Conversely, when sorted by the share turnover period the most liquid stocks are in the "smallest quintile." Observe that for NYSE stocks the most liquid stocks have high stock prices. As liquidity diminishes so too does the shareprice. The relation is different for the NYSE than for NASDAQ. For NASDAQ stocks there is a significant quadratic relation between liquidity and shareprice. A stock with a relatively high price is likely to be extreme – it's either extremely liquid or extremely illiquid. Medium liquidity NASDAQ stocks tend to have relatively low stock prices.

	Percentage of total dollar volume for all stocks in quintile (and median stock price) when sorted by "Average value of trades per day" - 1 -	Percentage of total shares traded for all stocks in quintile (and median stock price) when sorted by "Average daily share volume" - 2 -	Percentage of total shares traded for all stocks in quintile (and median stock price) when sorted by "Share turnover period" - 3 -
NYSE 2,555 stocks 1.43 billion average daily volume (shares) $34.5 billion average daily volume (dollars) $16.44 median share price			
biggest quintile	86.7%	83.1% ($23.81)	1.0% ($13.64)
upper middle	10.0%	11.4% ($22.47)	9.1% ($13.40)
middle	2.6%	3.9% ($16.24)	19.8% ($19.93)
lower middle	0.7%	1.3% ($13.91)	27.6% ($21.51)
smallest quintile	0.1%	0.3% ($13.43)	42.4% ($20.45)
NASDAQ 3,650 stocks 1.35 billion average daily volume (shares) $19.4 billion average daily volume (dollars) $6.51 median share price			
biggest quintile	96.8%	91.9% ($10.24)	0.2% ($8.00)
upper middle	2.5%	5.9% ($5.31)	0.9% ($4.91)
middle	0.5%	1.7% ($4.05)	2.7% ($3.94)
lower middle	0.1%	0.5% ($5.48)	11.3% ($5.72)
smallest quintile	0.0%	0.1% ($9.47)	84.8% ($11.55)

TABLE 10.2 Percentage of total trading volume (and median stock price) in each quintile according to various liquidity measures.
Historical snapshot compiled by author.

Liquidity risk also is pertinent to credit market securities. The common consensus is that liquidity risk is greater for credit market securities than for equities, with the notable exception of U.S. Treasury securities which are the most liquid of all. Certainly the market for corporate notes and bonds is less complete than the market for equities. This occurs even though, as table 9.12 shows, the value of credit market securities in the U.S. exceeds the value of equities. Perhaps greater liquidity risk for credit market securities is attributable to two facts. First, there is relatively less company information available for many issuers of credit market securities. Many credit market issuers do not issue publicly traded stock and are not subject to the same disclosure requirements as public companies. Investor interest wanes and liquidity may lessen when information about the issuer becomes scant. Second, a large publicly traded company may have more than a dozen different credit market securities outstanding but only one common stock! The number of securities in the credit market, in other words, is orders of magnitude larger than the number of equity securities and this may translate into greater liquidity risk premia for credit market securities.

Term risk

Term is the time period for which a financial contract is valid. Chapter 7 illustrates that market financing rates normally are lower for short-term securities. *Term risk* is a catchall for phenomena that cause two securities with different term to have different required returns even though they are alike in every other way (e.g., identical liquidity, default risk, etc.). *Term* is a source of idiosyncratic risk.

DEFINITION 10.4 Term risk premium

Term risk relates to the timing of cash flows that a security implicitly or explicitly promises. The *term risk premium*, nil for very short-term securities, normally gets bigger as term increases:

$$When\ the\ timing\ of\ security\ cash\ flows\ tilt\ toward\ the\ \begin{Bmatrix} near-term \\ remote\ future \end{Bmatrix}$$

$$then\ normally\ the\ term\ risk\ premium \begin{Bmatrix} decreases \\ increases \end{Bmatrix}.$$

Term risk is a dynamic concept that reflects complex phenomena occurring as cash flows distribute across time. Term risk affects all types of investments. Credit market securities have differing maturities – the term risk premium increases with maturity (all else equal). Real estate and other capital budgeting projects have different payback periods – the term risk premium increases with payback period (all else equal). Equities trade for many different kinds of companies – startups sometimes don't return dividends for decades.

Financial research suggests that when two securities are alike in every way except term then the following factors may influence the term risk premium.

1. *Price risk:* Discounted values of long-term cash flow streams are more sensitive than short-term cash flow streams to changes in the discount rate (see chapter 7). Consequently, near-term price movement is relatively volatile for a long-term security. The price in the near-term becomes very relevant if liquidation becomes necessary. Perhaps investors respond to greater uncertainty about near-term prices for long-term securities by increasing the term risk premium.

2. *Inflation and expectations risk*: Required rates of return compensate investors for opportunity costs such as (a) the decline in the purchasing power of money caused by inflation, as well as (b) interest rates on competing investments. People develop expectations that sometimes suggest levels of near-term inflation and interest rates may not persist. For example, they may think that rates may be steadily rising for awhile and then falling, or vice versa. Term risk premia for short and long-term securities differ (all else equal) due to embodiment of expectations over the relevant horizon. Find the approximate inflation component of the term risk premium as:

$$\begin{pmatrix} \textit{inflation premium} \\ \textit{for } N-\textit{period} \\ \textit{horizon} \end{pmatrix} = \sum_{t=1}^{N} \frac{(\textit{inflation rate})_t}{N}$$

Suppose, for example, that over the next one year expected inflation is 0% but that during the second year expected inflation is 10%, during the third year 20% and during the 4th year 0%. The inflation premium equals 0% on a one-year bond, 5% on a 2-year bond, 10% on a 3-year bond, and 7½% on a 4-year bond.

3. *Reinvestment option*: A short-term security returns invested capital to the investor who then has an option for reinvesting. At that time the investor has an option to reinvest the capital in another security, or use the money to party, etc. A longer term security does not provide that option because cash flow tilts toward the remote future. Perhaps the reinvestment option is valuable and investors require more return per year as compensation for its sacrifice.

4. *Nonlinear risk*: Investors normally respond to an increasing term by demanding more return per year. This revealed behavior suggests that perhaps risk is nonlinear. Research with stochastic financial models suggests that investors may behave as though two years of risk "compounds" and requires more than twice the compensation of one year's risk. That is, while getting 10% for one year may be adequate, the addition to term of an extra year requires more than an additional year of 10% returns.

The timing of cash flows differs across securities. Some securities promise returns in the near-term. Others offer returns in the remote-term. Investors overwhelmingly reveal a preference for near-term returns. To the extent that equilibrium rates of return rise with risk, the implication is that risk normally rises with term. Timing and duration of the expected cash flow stream determine the term risk premium.

Default risk

Default occurs when a borrower is unable to make payments promised to creditors. When default occurs then bankruptcy or reorganization become possibilities. When a borrower defaults on one loan then all the securities they have issued become at-risk. The likelihood of default is a source of idiosyncratic risk.

DEFINITION 10.5 Default risk premium

Default risk refers to the likelihood that an issuer may be unable to fulfill financial obligations. The *default risk premium*, nil for short-term U.S. government credit market securities, gets bigger as the likelihood of default increases:

$$\textit{When the likelihood of default by the issuer} \begin{Bmatrix} \textit{increases} \\ \textit{decreases} \end{Bmatrix}$$

$$\textit{then the default risk premium} \begin{Bmatrix} \textit{increases} \\ \textit{decreases} \end{Bmatrix}.$$

Issuers of financial securities hire rating agencies to assess security default risk. The agencies assign ratings to many different types of securities issued by many different types of organizations. With corporate bonds, for example, the ratings range from triple-A for highest quality bonds (lowest default risk) to C-grade for lowest quality (highest default risk). Default risk, however, also affects equities because shareholders lose when the company defaults on its bonds.

Company default risk depends on many factors. Glean important lessons about default risk by re-examining the stylized income statement from chapter 2 (section 3c, Breakeven ratios):

Sales revenue	*p Q*
- Total fixed costs	*F*
- Total variable costs	*v Q*
= Earnings before interest & taxes	*EBIT*
- Interest	*I*
- Taxes	*T*
- Preferred dividends	*PD*
= Earnings available for common	*EAC*

The variable *Q* represents the quantity of product that the company sells, *p* is the unit sales price of the product (*p* times *Q* equals total *Sales revenue*), and *v* is the variable cost per unit. Default risk relates to the inability of the company to fulfill financial obligations. Recall that the cash flow diagram from chapter 1 (figure 1.3) illustrates the company transferring wealth to two groups of economic entities: Stakeholders in real asset markets and capitalists in financial markets. Discussions below refer to the preceding stylized income statement to explain how financial obligations to stakeholders and capitalists relate to default risk.

Operating leverage

Senior financial obligations include operating expenses that the company owes employees and other stakeholders in real asset markets. The preceding income statement categorizes operating costs as either “fixed” or “variable.” The “operating breakeven point” occurs when company *Sales revenue* equals fixed plus variable operating costs. That is, when *EBIT* equals zero the company is at the operating breakeven point (see formula 2.9). Companies prefer to operate as far beyond breakeven as possible because, as everybody knows, sales fluctuate through time. Fluctuations in sales and costs influence the default risk premium. The degree of operating leverage measures the sensitivity of operating income to sales fluctuations:

FORMULA 10.2 Degree of operating leverage (DOL)

The "degree of operating leverage" measures the percentage change in *EBIT* that results from a 1% sales increase:

$$\begin{pmatrix} \text{degree of} \\ \text{operating} \\ \text{leverage} \end{pmatrix} = \frac{\%\Delta\, EBIT}{\%\Delta\, Sales\ revenue}$$

$$= \frac{(p - v)Q}{(p - v)Q - F}$$

$$= \frac{Sales\ revenue - Total\ variable\ costs}{EBIT}$$

where *p* is the unit price of the product, *v* is the variable cost per unit, *F* is *Total fixed costs*, and *Q* represents the quantity of product that the company sells. This model assumes the ratio of *Total variable costs* to *Sales revenue* is constant.

The numerator of the *DOL* formula usually is bigger than the denominator. Consequently, the *DOL* usually is bigger than one. The *DOL* is huge when *Sales revenue* barely surpasses *Total variable* plus *fixed costs*. As *Sales* grow far beyond *fixed costs* the *DOL* diminishes toward unity.

EXAMPLE 1 Find and interpret the degree of operating leverage

The most recent annual report lists company *Sales revenue* at \$175,000. Cost analysis suggests that annual *Total fixed costs* and *Total variable costs* equal \$42,000 and \$108,000, respectively. Find and interpret the company's degree of operating leverage.

SOLUTION

For this problem plug the numbers into formula 10.2:

$$\begin{pmatrix} \text{degree of} \\ \text{operating} \\ \text{leverage} \end{pmatrix}_{@Sales=\$175,000} = \frac{175{,}000 - 108{,}000}{175{,}000 - 42{,}000 - 108{,}000}$$

$$= 2.68$$

The DOL of 2.68 means *EBIT* increases (or decreases) about 2 ⅔% for every 1% increase (or decrease) in *Sales revenue*.

The most recent annual report shows that sales of \$175,000 generate *Earnings before interest and taxes* of \$25,000 (= \$175,000 - \$42,000 - \$108,000). Suppose, for example, that sales were up 1%, that is, \$1,750. The additional sales incur additional variable costs of \$1,080 (variable costs equal 61.71% of sales; \$1,080 = 0.6171 *x*

$1,750). The additional *EBIT* equals $670 (=$1,750 - $1,080), which is 2.68% of $25,000. Conversely, if sales were down 1% then *EBIT* falls 2.68%. The *DOL* allows quick approximation of the sensitivity of *EBIT* to sales fluctuations. If sales were to rise (or decline) 8%, for example, *EBIT* rises (or falls) approximately 21% (≈ 2.68 x 8%).

DOL formulas depend on the amount of sales. As an alternative scenario for the preceding example consider if company sales were $150,000. Then the *DOL* equals 3.72 {= [$150,000 - (0.6171 *x* $150,000)] ÷ [$150,000 - $42,000 – (0.6171 *x* $150,000)] }. When company *Sales revenue* nears the operating breakeven point then the relatively large *DOL* means a 1% fluctuation in sales has a huge effect on *EBIT*. Conversely, if company sales were $200,000 then the *DOL* equals 2.21 {= [$200,000 - (0.6171 *x* $200,000)] ÷ [$200,000 - $42,000 – (0.6171 *x* $200,000)] }.

The degree of operating leverage directly measures the robustness of operating income to sales fluctuations. Companies operating near breakeven have high operating leverage and face relatively high default risk. Companies fortunate to operate well-beyond breakeven have a relatively small degree of operating leverage and, all else equal, face relatively low default risk.

Financial leverage

The bottom-half of the income statement documents cash flows from company to capitalists in financial markets. The company sends interest to creditors and dividends to shareholders. Payments to creditors are financial obligations that relate directly to default risk. Shareholders are residual claimants on company cash flows and cannot force default. Shareholders nonetheless have a financial stake that depends on whether the company defaults.

The lesson on debt management ratios in chapter 2 (section 3.A) teaches that a company's excess borrowing capacity relates to its ability to obtain further credit. It's bad news, as hopefully you know, when one borrows up to the credit limit because default and bankruptcy may be one misfortune away. Analysis of the preceding stylized income statement allows further insights on how default risk depends on debt.

The *Earnings available for common* (*EAC*) equals *EBIT*, minus *Interest (I)*, minus *Taxes*, minus *Preferred dividends (PD)*. When *Taxes* are proportional to taxable income we may write:

$$EAC = (EBIT - I) (1 - \text{tax rate}) - PD .$$

Manipulation of the preceding equation yields the following formula.

FORMULA 10.3 Degree of financial leverage (*DFL*)

The "degree of financial leverage" measures the percentage change in *Earnings available for common* that results from a 1% increase in *Earnings before interest and taxes*:

$$\begin{pmatrix} \text{degree of} \\ \text{financial} \\ \text{leverage} \end{pmatrix} = \frac{\%\Delta\, \text{Earnings available for common}}{\%\Delta\, EBIT}$$

$$= \frac{EBIT}{EBIT - I - PD/(1 - \text{tax rate})}$$

where *I* and *PD* represent *Interest* expense and *Preferred dividends,* respectively.

The numerator of the *DFL* formula usually is bigger than the denominator. Consequently, the *DFL* usually is bigger than one. The *DFL* grows huge as *Interest* consumes more and more of *EBIT*. Glean insight on the *DFL* with the following example.

EXAMPLE 2 Find and interpret the degree of financial leverage

Analysis of the most recent annual report reveals the following entries:

Sales revenue	*$175,000*
- Total fixed costs	*42,000*
- Total variable costs	*108,000*
= Earnings before interest & taxes	*25,000*
- Interest	*8,500*
- Taxes (@30% tax rate)	*4,950*
- Preferred dividends	*2,000*
= Earnings available for common	*$ 9,550*

Find and interpret the company's degree of financial leverage.

SOLUTION

Plug relevant numbers into the *DFL* formula:

$$\begin{pmatrix} \text{degree of} \\ \text{financial} \\ \text{leverage} \end{pmatrix}_{@EBIT=\$25,000} = \frac{25{,}000}{25{,}000 - 8{,}500 - 2{,}000/(1 - 0.30)}$$

$$= 1.83$$

The DFL of 1.83 means *EAC* increases (or decreases) 1.83% for every 1% increase (or decrease) in *EBIT*. Suppose, for example, that *EBIT* were up 1%, that is, $250. The additional taxable income incurs additional taxes of $75 (= 0.30 *x* $250). The *Interest* and *Preferred dividends* remain constant yet the *Earnings available for common* increases $175 (= $250 - $75). This represents an increase in *EAC* of 1.83% (=$175 ÷ $9,550). Analogously, if *EBIT* were to fall 8% then *EAC*, all else equal, would fall almost 15% (≈ 8% x 1.83).

A key insight is that the *DFL* increases as the company increases reliance on debt financing. Examine formula 10.3 and notice that the *DFL* gets bigger as *Interest* consumes a larger share of *EBIT.* Rising reliance on debt incurs higher interest expenses and rising default risk. A relatively huge *DFL* is an indicator of relatively large default risk.

Consider, for example, that interest were $15,000 instead of the $8,500 shown in the annual report. The *DFL* for this alternative scenario is 3.50 {= $25,000 ÷ [$25,000 - $15,000 – ($2,000/0.70)] }. The sensitivity of *EAC* to fluctuations in *EBIT* almost doubles. And as the previous subsection explains, fluctuations in *Sales* cause fluctuations in *EBIT* - a company with relatively large debt payments has relatively high default risk and volatile earnings for common.

The final insight from the stylized income statement combines the *DFL* and *DOL* to infer effects of sales fluctuations on *Earnings available for common.*

FORMULA 10.4 Degree of total leverage (*DTL*) and total breakeven point

The “degree of total leverage” measures the percentage change in *Earnings available for common (EAC)* that results from a 1% increase in *Sales revenue*:

$$\begin{pmatrix} \text{degree of} \\ \text{total} \\ \text{leverage} \end{pmatrix} = \frac{\%\Delta \, Earnings \ available \ for \ common}{\%\Delta \, Sales \ revenue}$$

$$= \begin{pmatrix} \text{degree of} \\ \text{operating} \\ \text{leverage} \end{pmatrix} \begin{pmatrix} \text{degree of} \\ \text{financial} \\ \text{leverage} \end{pmatrix} .$$

$$= \frac{Sales \ revenue - Total \, variable \ costs}{EBIT - I - PD \big/ (1 - tax \ rate)}$$

For the income statement from preceding examples the *DOL* equals 2.68 and the *DFL* equals 1.83. Multiply the two and find, according to the middle version of formula 10.4, that the degree of total leverage equals 4.91 (= 2.68 *x* 1.38). This means that *Earnings available for common* moves 4.91% in response to a 1% fluctuation in *Sales revenue.* Increase sales by $1,750 and, for example, *EAC* increases by about $469 {that is, $469 = .0491 *x* $9,550; or equivalently, $469 = [$1,750 – (0.6171 *x* $1,750)] *x* (1 –

0.30)}. The identical *DTL* of 4.91 computes from the lower version of formula 10.4:

$$\begin{pmatrix} degree\ of \\ total \\ leverage \end{pmatrix}_{@Sales=\$175,000} = \frac{\$175,000 - \$108,000}{25,000 - 8,500 - {2,000}/{(1-0.30)}}$$

$$= 4.91$$

The *DTL* depends on the level of sales. When *Sales revenue* barely surpasses the total breakeven point (see formula 2.10) the *DTL* gets huge. If sales, for example, were \$150,000 then the *DTL* equals 14.11 {that is, 14.11 = [\$150,000 – (0.6171 *x* \$150,000)] ÷ [(\$150,000 - \$42,000 -(0.6171 *x* \$150,000) - \$8,500 - \$2,000/(1 – 0.30)}. With a *DTL* of 14.11 a one percent sales decline translates into a 14% decline in *Earnings available for common*. That's a lot!

The leverage concept is universally important and has wide application. Consider a household with one wage earner making an after-tax annual salary of \$60,000 and paying annual fixed expenses of \$40,000. This wage earner contributes \$20,000 of discretionary income to the household. Now suppose a second member of the household considers a job that clears \$30,000. While at first glance one may perceive the salary of the second wage earner (\$30,000) is half-as-important as the first wage earner (\$60,000 salary), the leverage concept comes into play. Due to the second wage earner household discretionary income more than doubles, rising to \$50,000 from \$20,000! Leverage amplifies outcomes (in finance and physics, too). Analysis of the stylized income statement reveals that companies with relatively large operating and financial leverage probably have volatile earnings and substantial default risk.

Other common sources of idiosyncratic risk

The nearly infinite variety of financial securities implies that there are nearly infinite sources of idiosyncratic risk. Here are a few notable sources.

Repayment risk: Sometimes an issuer may repay the liability early, thereby yanking the security out of the investor's hands. Repayment may occur through several different mechanisms. For example, investors in asset-backed securities ("ABS", see chapter 9, section 3A) may receive loan payments that homeowners make on mortgages. Perhaps ABS investors expect to receive cash flows for twenty years. But when an unusually high percentage of homeowners refinance then the asset-back security gets repaid early. The investor does not lose money but, on the other hand, the cash flows do not occur as expected. An analogous situation occurs when companies unexpectedly repay bonds early (see the discussion on bond refunding and call protection in chapter 7). Evidence indicates that equilibrium rates of return increase with the likelihood of early yet unexpected repayment.

Exchange rate risk: Some securities trade across international boundaries. These securities therefore expose investors to risk that currency exchange rates may unexpectedly change. An investor, for example, may send U.S. dollars to a New York mutual fund that converts dollars into yen in order to purchase Japanese stocks. Even though the stock price may rise over time on the Tokyo Stock Exchange, generating a gain in yen, adverse exchange rate movement can completely offset the gain when funds reconvert into U.S. dollars. Investors likely demand a risk premium for bearing exchange rate risk.

Political risk: Other securities depend on government-legislated guarantees for repayment of principal and interest. These securities therefore expose investors to risk that political considerations may unexpectedly change the legislation guaranteeing security cash flows.

Catastrophes: There even exist securities that provide cash flows dependent on catastrophes such as earthquakes or hurricanes. Hence, one can rightly say that earthquakes and weather are sources of idiosyncratic risk. Each and every security has unique risks that are security-specific. Financial theory shows that the independence of risk sources across securities is assurance that prudent diversification strategies may manage or eliminate idiosyncratic risks.

EXERCISES 10.2

Conceptual

1. What is the extent of the range for the leverage measures, and explain the economic significance of extreme values.

3. Statistical measurements of risk and return

Explaining equilibrium in financial markets requires explaining how returns relate to risk. Fortunately, a rather sterile statistical setting offers excellent opportunity for learning fundamental characteristics about risk and return. For this imaginary laboratory setting suppose that all possible outcomes from a financial investment are known, as well as their likelihoods. Formulas below reliably measure the risk and rate of return for the investment.

3.A. Measures for individual securities

When all possible outcomes from a financial investment are known, and furthermore the probabilities for each outcome are known, the expected value of the rate of return is given by this formula:

FORMULA 10.5 *E(ROR)* given probabilities and outcomes

$$\begin{pmatrix} \text{expected} \\ \text{value of} \\ \text{the ROR for} \\ \text{security } j \end{pmatrix} \equiv E(ROR_j)$$

$$E(ROR_j) = \sum_{i=1}^{N} probability_i \times ROR_{i,j}$$

Three right-hand-side variables include: the probability that the i^{th} outcome occurs is $probability_i$, the rate of return occurring for asset *j* when outcome *i* actually happens is ROR_i, and the total number of outcomes is *N*.

Formula 10.5 coins the phrase “expected” rate of return for *E(ROR)*. This nomenclature is consistent with standard statistical practices and all finance books follow this tradition. An important realization, however, is that *E(ROR)* is <u>not</u> the same concept as $ROR^{expected}$. The expected rate of return for a financial investment, $ROR^{expected}$, represents the internal rate of return that equates, either implicitly or explicitly, the observable price to a cash flow stream consistent with the investor's information and expectations. Time value formulas and relationships determine $ROR^{expected}$. *E(ROR)* represents a simple summary statistic spit out by a simple statistical formula.

This lesson's objective is teaching how the "required" rate of return, $ROR^{required}$, relates to risk. For this purpose we rely on examining the statistical relation between *E(ROR)* and risk. Ironically by studying *E(ROR)* we learn more about $ROR^{required}$ than about $ROR^{expected}$. At equilibrium, of course, expected and required rates of return are equal.

The example below uses formula 10.5 in a very simple setting.

EXAMPLE 3 Either win-big or lose it all ©ER4

An investment costs $1,200. There is a 75 percent chance it returns $1,800. Otherwise, it loses everything. Find the expected value for the rate of return.

SOLUTION

For this problem the first step is finding the rates of return for the two different outcomes. For outcome one the investment returns $1,800 and the rate of return is:

$$ROR_1 = (\$1{,}800 - \$1{,}200) / \$1{,}200$$

$$= 50\%$$

Outcome two loses everything and ROR_2 is –100%. Now apply formula 10.5 and multiply the probability times the ROR:

$$E(ROR) = (0.75 \times 0.50) + (0.25 \times (-1.0))$$

$$= 12.5\%$$

The expected value for the rate of return is 12.5 percent.

The preceding simplistic example illustrates several attributes of *E(ROR)*.

1. The sum of probabilities over all outcomes equals 100 percent.

2. In formula 10.5 probabilities must be entered as decimal equivalents (for example, the probability of 75 percent is 0.75). *ROR* may be entered either way, as decimal equivalents or percent (for example, 50 percent on the calculator may be 0.50 or 50), but proper interpretation of the answer must be internally consistent. For example, the above *ROR* of 0.50 and –1.0 yield the answer 0.125 and the interpretation is 12.5 percent. All lessons in this book use decimal equivalents for all computations.

3. *E(ROR)* is a number that may not itself be one of the outcomes. Possible outcomes in the previous example are 75 percent and –100 percent whereas *E(ROR)* is 12.5 percent. The number represents the mean of the probability distribution function and conveys intuitive content. Knowing, for example, that the mean number of children per U.S.A. household is 2.3 conveys information even though we may be quite certain that no household has exactly 2.3 children.

Measuring risk for a financial investment is complex and fraught with difficulty. Still, the standard deviation of rates of return emerges as the simplest and most widely used measurement. The formula for σ in our sterile laboratory setting is this:

FORMULA 10.6 σ given probabilities and outcomes

The standard deviation of returns for asset *j* equals the square root of the sum of probability weighted squared deviations from *E(ROR)*. That is,

$$\begin{pmatrix} standard \\ deviation \\ for\ \ asset\ \ j \end{pmatrix} \equiv \sigma_j$$

$$\sigma_j = \left\{ \sum_{i=1}^{N} probability_i \times \left(ROR_{i,j} - E(ROR_j)\right)^2 \right\}^{1/2}$$

Several characteristics of the standard deviation merit mention.

1. σ treats upside and downside returns identically. That is, σ gets bigger as the dispersion of outcomes increases irrespective of whether the dispersion occurs on the downside (a loss) or upside (a gain). Most investors interpret risk as the chance of loss, not as the chance of gain. σ is an imperfect risk measure because it treats losses and gains the same.

2. σ is the square root of σ^2, the variance. A ranking of investments by standard deviation, smallest to largest or vice versa, is identical to a ranking by variance. The two measures differ in that σ measures "percent" whereas σ^2 measures "percent-squared." Because many people lack intuition about "percent-squareds", and also because *E(ROR)* also measures "percent", discussions about risk usually focus on σ.

3. σ conveys information about the size of confidence intervals. Statistical tables show that about 95 percent of all normally distributed outcomes lie within two standard deviations of the mean. Suppose, for example, that the small company stock returns underlying table 9.15 were normally distributed. A mean of 12.5% and standard deviation of 33.2% implies with 95% confidence that next year's stock return lies approximately between –54% and +79% (that is, the mean plus and minus 2σ.)

The following example is a final reminder that the expected rate of return links the current stock price to future cash flows.

EXAMPLE 4 Find *E(ROR)* given future stock prices ER1c

Your analysis of a small company convinces you that future movements in their stock price depend on how many automakers adopt the company's product innovations. The current price for this non-dividend paying stock is $20. The table below summarizes your belief about future outcomes.

#automakers adopting product	*probability*	*resultant intrinsic value for stock price*
1	30%	$35
2	20%	$45
3	10%	$65

If no automakers adopt the product then the company goes bankrupt and the stock is worthless. Contrast this small company's risk and return with the long-run averages in table 9.15.

SOLUTION

For this problem the first step is finding the rates of return for the four different outcomes. Assume that the stock price moves to the "resultant intrinsic value" from its beginning amount of $20. The table below summarizes resultant rates of return.

#automakers adopting product	*ROR computation*	*resultant ROR*
0	(0 – 20) / 20	-100%
1	(35 – 20) / 20	75%
2	(45 – 20) / 20	125%
3	(65 – 20) / 20	225%

The probability for bankruptcy is 60%. Apply formula 10.5 to find *E(ROR)*.

$$E(ROR) = (0.30 \times 0.75) + (0.20 \times 1.25) + (0.10 \times 2.25) + (0.40 \times (-1.0))$$

$$= 30.0\%$$

Apply formula 10.6 to find σ.

$$\sigma = \left\{ \begin{aligned} & 0.30\,(0.75 - 0.10)^2 \\ & +\ 0.20\,(1.25 - 0.10)^2 \\ & +\ 0.10\,(2.25 - 0.10)^2 \\ & +\ 0.40\,(-1.0 - 0.10)^2 \end{aligned} \right\}^{1/2}$$

$$= 114\%$$

Table 9.15 shows that the long run average rate of return for small company stocks is 12.5% with a standard deviation of 33.2%. This particular company has larger return (30%) and more risk (114%) than the average small company stock. There is a trade-off between this particular company stock and the average small company stock.

Company profitability, sales trends, market share, growth opportunities, price multiples, these and other information sources shape expected rates of return. Required rates of return, conversely, depend exclusively on risk. In order to narrow focus on the relation between risk and return our lessons become a little more statistical.

EXAMPLE 5 Compute/compare stock risk and return to find dominance or tradeoff

The business cycle obviously affects total rates of return on equities. Business economists predict the following outcomes for the economy and associated rates of return on equities for the Alpha Company and the Zed Company.

	expanding economy	*stagnant economy*	*recession*
probability	30%	50%	20%
ROR Alpha	28%	-3%	24%
ROR Zed	0%	12%	10%

a. You might invest in one of these company stocks. You cannot invest in both companies but rather must select only one. Explain your choice.
b. Your friend tells you that a security they own, Lambda, has expected return of 10% and standard deviation of returns equal to 16%. How does this one stock compare to the other two?

SOLUTION
a. For this problem,

$$E(ROR_{Alpha}) = .30(.28) + .50(-.03) + .20(.24)$$

$$= 11.70\%$$

and $$\sigma_{Alpha} = \{ .30(.28-.1170)^2 + .50(-.03-.1170)^2 + .20(.24-.1170)^2 \}^{1/2}$$

$$= 14.77\%$$

Similar computations for Zed show:

$$E(ROR_{Zed}) = 8.00\%$$

and $$\sigma_{Zed} = 5.29\%$$

Determining whether Zed or Alpha is best uses the “dominance” concept. For these companies there is not a case of dominance. That is, there exists a tradeoff. Notice that for Zed the expected return is lower, which is bad, but its risk is lower too, which is good. With a tradeoff one stock is not necessarily better than the other. The decision about which is best depends on risk preferences of the individual investor. If you are a high risk investor (wealthy or long investment horizon) choose Alpha whereas if you prefer low risk (cash poor or short investment horizon) choose Zed.

CALCULATOR CLUE 10.1 Solving this problem on the BAII Plus© is easier given efficient usage of the algebraic keys and several different calculator memories. Compute statistics for Alpha with these keystrokes:
.3 x .28 + .5 x .03 +/- + .2 x .24 = .
The display shows the mean of Alpha is 11.70% so hit STO 1 . Now obtain the standard deviation of Alpha:
.3 x (.28 - RCL 1) x^2 + .5 x (.03 +/- - RCL 1) x^2 + .2 x (.24 - RCL 1) x^2 = √ .
The display shows the standard deviation for Alpha is 14.77% so hit STO 2 . Now get the same statistics for Zed:
.3 x 0 + .5 x .12 + .2 x .10 = STO 4 .
and
.3 x (0 - RCL 4) x^2 + .5 x (.12 - RCL 4) x^2 + .2 x (.10 - RCL 4) x^2 = √ STO 5 .
Memories 1 and 2 contain the mean and standard deviation for Alpha while memories 3

and 4 contain analogous numbers for Zed.

b. Figure 10.2 summarizes risk-return characteristics of the 3 securities.

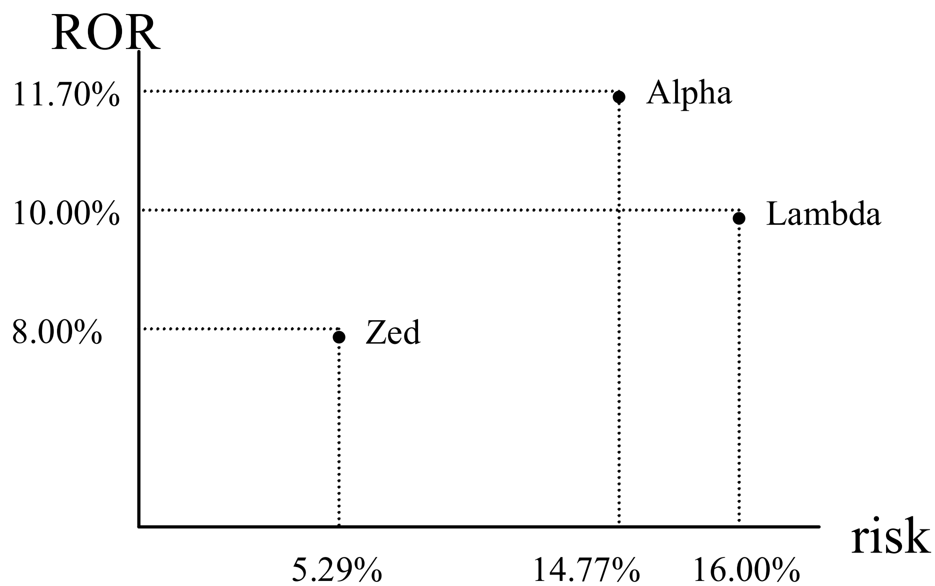


FIGURE 10.2 Risk and return for Example 5

Between Alpha and Zed there is a tradeoff, as stated previously, so one security is not necessarily better than the other. Alpha compares to Lambda differently, however. If one were to own Lambda and switch to Alpha, the expected return increases (which is good) and risk decreases (which also is good). There is not a tradeoff between Lambda and Alpha. Instead, there is a case of dominance. Alpha dominates Lambda because Alpha has higher return and less risk. When one stock dominates the other, the choice about which is best does not depend on the risk preferences of the investor. Instead, one stock is better than the other one for all rational investors (as long as the investor believes these numbers, that is). Alpha is better than Lambda Similar logic shows that between Lambda and Zed there is a tradeoff.

A summary of the preceding analysis is that an investor restricted to own only one of these stocks and none others should never choose Lambda. Feasible choices include only Alpha and Zed. The choice about which of these two is best depends on investor risk preferences. Is the extra but uncertain 3.70% return from Alpha worth the extra 9.48% risk? At this point we say only that the investor knows for sure.

Obtaining probabilities for many different outcomes is usually impractical. Instead, insights about security risk often rely on the rule that past performance is a good predictor of the future. Many believe that, like a leopard can't change its spots, high risk securities of the past tend to be high risk securities in the future. Formulas really simplify when historical outcomes are equally likely to repeat. The formula for *E(ROR)* is simply the arithmetic average:

FORMULA 10.7 *E(ROR)* with equally likely outcomes

$$E(ROR_j)=\frac{\sum_{i=1}^{N} ROR_{i,j}}{N}$$

The formula for the standard deviation simplifies like this:

FORMULA 10.8 σ with equally likely outcomes

$$\sigma_j = \left\{ \frac{\sum_{i=1}^{N} \left(ROR_{i,j}\right)^2}{N} - \left(E(ROR_j)\right)^2 \right\}^{1/2}$$

The biggest simplification of all, however, is that with equally likely outcomes all statistics compute automatically with calculators such as the Texas Instruments BAII Plus. Consider this example below:

EXAMPLE 6 With equally likely outcomes find dominance or tradeoff ©ER12

Two securities, X and Y, exhibit the following returns over the past 5 years:

	1	2	3	4	5
X:	15.15%	3.45%	-0.05%	-1.15%	14.85%
Y:	6.25%	8.45%	6.35%	10.45%	3.85%

You believe that past performance is a perfect predictor of future performance. Contrast *E(ROR)* and σ for each security if owned in isolation.

SOLUTION

For security X:

$$E(ROR_X) = (.1515 + .0345 - .0005 - .0115 + .1485) / 5$$

$$= 6.45\%$$

and $\sigma_X = \{(.1515^2 + .0345^2 + .0005^2 + .0115^2 + .1485^2)/5 - .0645^2\}^{1/2}$

$$= 7.15\%.$$

Similar computations for Y show that its expected return is 7.07% and standard deviation is 2.23%.

Summarize these findings as follows:

security	*E(ROR)*	*σ*
X	6.45%	7.15%
Y	7.07%	2.23%

Security Y dominates security X; Y is better on all counts because it has higher return and less risk. Nobody that owns only one of these securities should own X.

CALCULATOR CLUE 10.2 The BAII Plus© calculator contains a spreadsheet that enables easy estimation of statistics when all outcomes are equally likely. On the BAII Plus© type 2nd DATA (the data key is the same as the 7 key). The calculator opens a worksheet that contains two columns for holding data. The calculator refers to the two columns of data as column X and column Y. For this problem columns X and Y contain the rates of return for securities X and Y, respectively. Clear unwanted numbers already stored in the data worksheet by typing 2nd CE/C. Enter this problem's data by typing

	.1515		ENTER	↓	.0625	ENTER
↓	.0345		ENTER	↓	.0845	ENTER
↓	.0005	+/-	ENTER	↓	.0635	ENTER
↓	.0115	+/-	ENTER	↓	.1045	ENTER
↓	.1485		ENTER	↓	.0385	ENTER

The preceding keystrokes set X_1 = 15.15% and Y_1 = 6.25%, … , X_5 = 14.85% and Y_5 = 3.85%. Find the statistical estimates of the mean and standard deviation for the data in memory by typing 2nd STAT . Make sure the display shows "LIN". Hit ↓ 2 times and the display shows "X-bar = .0645". This is *E(ROR$_X$)*. Hit ↓ 2 more times and the display shows "σ_X = .0715", the standard deviation of X. Hit ↓ once and the display shows "Y-bar = .0707", and hit ↓ 2 more times to see "σ_Y = .0223". Note that the calculator also shows an alternative computation for the standard deviation, "S_X" and "S_Y". The difference between σ and S is that the former divides by N, whereas the latter divides by N-1. Our assumption for problems in this book is that the data represent the population of future outcomes so therefore reliance on σ is appropriate.

EXERCISES 10.3A

Numerical quickies

1. You invest $900 . The odds are 50% you will get back $1,300 . Otherwise, you lose everything. What is the expected rate of return on this brilliant financial investment? ©ER4

2. Your analysis of a small company convinces you that future movements in their stock price depend on how many big companies adopt the small company's product innovations. Today's price for this non-dividend-paying small company stock is $6 . The table below summarizes your beliefs about future outcomes.

#big companies adopting product	*probability*	*resultant intrinsic value for stock price*
1	40%	$11
2	20%	$17

If no big companies adopt the product then the small company goes bankrupt and the stock is worthless. Compute this small company stock's measurements for risk [= σ] and return [= *E(ROR)*]. ©ER16

3. Your analysis of common stocks for companies X and Y lead you to believe rates of return depend as follows on the future strength of the economy. Compare X and Y with regards to dominance or tradeoff. ©ER5

	weak	*moderate*	*strong*
probability	25%	40%	35%
%return X	4.0%	6.1%	25.7%
%return Y	7.3%	22.1%	10.7%

4. Each pair of rates of return for securities X and Y listed below is equally likely. Find the standard deviation and expected rates of return for securities X and Y, and also compare the two regarding dominance or tradeoff. ©ER12

X:	-3.2%	11.5%	22.3%	12.1%
Y:	18.7%	27.1%	14.2%	7.5%

3.B. Measures for portfolios of securities

A portfolio is a collection of many different types of assets. Investors sometimes hold a portfolio with stocks from many companies. Mutual funds may hold a portfolio with bonds from many different issuers. Businesses may hold a portfolio with many different types of real capital goods. Different assets in a portfolio potentially provide different rates of return. Lessons below focus on how portfolio risk and return characteristics relate to component assets.

Time value lessons from chapter 4 show that for a time series of periodic security returns the geometric average rate of return properly measures the percentage change in wealth. The example below demonstrates that measuring a portfolio's periodic rate of return from component asset periodic returns is somewhat different.

EXAMPLE 7 Find the periodic ROR for a portfolio of stocks A, B, and C

At the beginning (*t=0*), you invest $1000 equally among three different shares (A, B, and C). The prices for each of the three shares changes so that now, at *t=1*, your holdings are worth:

	A	B	C
p(0)=	$333.33	$333.33	$333.33
p(1)=	$191.00	$633.00	$200.00

Find each share's periodic rate of return and then compute the average return (arithmetic and geometric).

SOLUTION

	A	B	C	wealth
p(0)=	$333.33	$333.33	$333.33	$1,000
p(1)=	$191.00	$633.00	$200.00	$1,024
ROR=	-42.70%	89.90%	-40.00%	%ΔWealth = 2.40%

Compute periodic security returns as follows (formula 4.2):

$$ROR \;=\; (\text{ end-of-period wealth} \div \text{beginning-of-period wealth }) - 1$$

For example, security A's rate of return is -42.7% [= (191 ÷ 333.33) -1], security B's is 89.90%, etc. Next, recall formula 4.3 for an arithmetic average return:

$$\begin{aligned} \text{arithmetic average rate of return} &= \Sigma ROR_t \div N \,. \\ &= (-.4270 + .8990 - .40) \div 3 \\ &= 2.40\% \end{aligned}$$

Compute the geometric average from formula 4.4 as

$$\begin{aligned} \text{geometric average rate of return} &= \{ (1+ROR_1) \times (1+ROR_2) \times \ldots \times (1+ROR_N) \}^{1/N} - 1 \\ &= [(1+(-.427))(1+.8990)(1-.40)]^{1/3} - 1 \\ &= -13.25\% \end{aligned}$$

Add together the values of all shares; this is total portfolio wealth. Notice it begins at $1000 and rises to $1,024 . Its percentage change is 2.40% (=$1,024 ÷ 1,000 - 1). The arithmetic and geometric average rates of return computed above equal 2.40% and -13.25%, respectively. The arithmetic average *ROR* across all securities is a proper measurement for the portfolio's periodic rate of return.

For a portfolio the arithmetic average of component periodic *ROR* corresponds to the actual change in wealth. Formula 10.9 summarizes this finding.

FORMULA 10.9 $E(ROR_{portfolio})$

The periodic rate of return for a portfolio is a weighted average of component returns:

$$E(ROR_{portfolio}) = \sum_{j=1}^{M} w_j \, E(ROR_j) \;,$$

where w_j is the proportion of portfolio wealth allocated to asset *j* and *M* is how many assets the portfolio contains. $E(ROR_j)$ is the expected value of the rate of return from formula 10.5.

In the preceding example securities A, B, and C provide periodic rates of return equal to -42.70%, 89.90%, and -40.00%, respectively, and 1/3 of funds are invested in each. Applying formula 10.9 to find the portfolio periodic rate of return shows:

$$E(ROR_{portfolio}) \;=\; ⅓(-.4270) + ⅓(.8990) + ⅓(-.40)$$

= *-2.40%*.

This is the same answer that example 7 obtains. Now apply formula 10.9 to example 5 from the previous section.

EXAMPLE 8 Find portfolio *E(ROR)* and σ for Alpha and Zed ©ER2c

The business cycle obviously affects total rates of return on equities. Business economists predict the following outcomes for the economy and associated rates of return on equities for the Alpha Company and the Zed Company.

	expanding economy	*stagnant economy*	*recession*
probability	30%	50%	20%
ROR Alpha	28%	-3%	24%
ROR Zed	0%	12%	10%

Suppose you invest 25% of your funds in Alpha and the remainder in Zed. Find the portfolio's *E(ROR)* and σ.

SOLUTION

Example 5 applies formulas 10.5 and 10.6 to find the following summary statistics for Alpha and Zed:

	E(ROR)	σ
Alpha	11.70%	14.77%
Zed	8.00%	5.29%

Now apply formula 10.9 and compute the portfolio return as the weighted average of component returns:

$$E(ROR_{\frac{1}{4}Alpha + \frac{3}{4}Zed}) = .25\,(.1170) + .75\,(.0800)$$

$$= 8.93\%$$

The portfolio return, 8.93%, is an average of Alpha's 11.7% and Zed's 8.0%. The portfolio return is nearer the asset with the most weight; $E(ROR_{\frac{1}{4}Alpha + \frac{3}{4}Zed})$ is three-quarters closer to Zed.

One approach for finding portfolio risk specifies the portfolio ROR for each outcome – then compute σ with formula 10.6. With an expanding economy every $1 invested in Alpha returns 28 cents profit. A quarter-dollar investment returns only 7 cents. Zed returns 0 in an expanding economy. The portfolio returns 7 cents of profit in an expanding economy when the allocation is 25% Alpha, 75% Zed. Use similar logic to prepare the below listing portfolio rates of return for the three different outcomes.

	expanding economy	*stagnant economy*	*recession*
probability	30%	50%	20%
¼Alpha + ¾Zed	¼(.28) + ¾(0) = 7.00%	¼(-.03) + ¾(.12) = 8.25%	¼(.24) + ¾(.10) = 13.50%

Apply formula 10.6 to the entries in the bottom row:

$$\sigma_{\frac{1}{4}Alpha + \frac{3}{4}Zed} = \{\, .30(.0700-.0893)^2 + .50(.0825-.0893)^2 + .20(.1350-.0893)^2 \,\}^{1/2}$$

= 2.35%

The portfolio risk, $\sigma_{\frac{1}{4}Alpha + \frac{3}{4}Zed}$, is not an average of component risks. It in fact is less than any individual security suggesting that some risk, almost magically, simply vanishes. The portfolio offers *diversification benefits*.

CALCULATOR CLUE 10.3 Example 5 includes a *Calculator Clue* that computes and stores the mean and standard deviation for Alpha in memories 1 and 2 and analogous numbers for Zed in memories 4 and 5. Compute the average return for the portfolio that allocates 25% Alpha and 75% Zed with these keystrokes:

RCL 1 x .25 + RCL 4 x .75 = STO 7 .

The display shows the portfolio return equals 8.93%. Now obtain portfolio returns for the three different states. Store the 3 returns in memories 3, 6, and 9 for later use:

.25 x .28 + .75 x 0 = STO 3 .

.25 x .03 +/- + .75 x .12 = STO 6 .

.25 x .24 + .75 x .10 = STO 9 .

Now get the portfolio standard deviation:

.3 x (RCL 3 - RCL 7) x^2 + .5 x (RCL 6 - RCL 7) x^2 + .2 x (RCL 9 - RCL 7) x^2 = √ STO 8 .

Memories 7 and 8 contain the mean and standard deviation for a portfolio that allocates 25% Alpha, 25% Zed.

Forming a financial portfolio usually means choosing among investment assets of incredible diversity. Among the many equity choices of company stock available for purchase find six in the snippet below from table 2.1 showing ICE sorted 100th by *Total assets* in the list of 11,000 U.S. companies circa beginning of year 2014.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
TWX	$67,994	34	$3,691	$29,795	$ 62,399
LBTYK	$67,714	35	-964	14,474	34,311
SLB	$67,100	123	6732	45,266	117,804
AMGN	$66,125	20	5,081	18,676	86,085
CMA	$65,227	9	541	2,610	8,667
ICE	$64,818	4	254	1,795	25,866

SNIPPET from table 2.1 in chapter 2: ICE

When you buy a share of ICE you are buying a share of a balance sheet that includes complete ownership of The New York Stock Exchange, Euronext, and a whole lot more. ICE is the third largest exchange group globally, behind world No. 1 Hong Kong Exchanges and Clearing and the Chicago based CME Group. ICE operates a network of regulated exchanges and clearing houses for financial and commodity markets in the United Kingdom, Continental Europe, Canada, Asia, and of course, New York. Buy a share ICE and get the company whose main line of business allows others to form portfolios.

EXERCISES 10.3B

Numerical quickies

1. At the beginning of last month about 40% of your $6,250 portfolio was in stock *X;*

stock *Y* accounted for 30% and stock *Z* for the rest. Monthly rates of return equaled -12% for stock *X*, 20% for *Y*, and -24% for *Z*. Find last month's percentage change in total portfolio wealth. ©ER13

2. The expected rate of return on common stock for company X equals 5.9%. For Company Y, the expected rate of return is 23.5%. You wish to form a portfolio by allocating some of your funds in Company X and the remainder in Company Y. In order to form a portfolio whose expected return equals 15.2%, what proportion of funds should be invested in Company X? ©ER3

Challengers

3. You form a portfolio that invests 30% of total funds in stock X and 70% in stock Z. Two possible outcomes exist. The probability is 30% that the first outcome occurs, in which case the rates of return equal 10% for X and 38% for Z. The probability is 70% that the second outcome occurs, in which case the rates of return equal 45% for X and 8% for Z. Find the expected return and standard deviation of portfolio returns. ©ER9a ©ER9b

4. Your analysis of outcomes for sales and the associated rate of return on common stocks for companies X and Y are shown below. You intend to form a portfolio by allocating 35% of your funds in Company X, and the remainder in Company Y. Find the expected return and standard deviation of portfolio returns. ©ER2c

	declining	flat	rising
probability	35%	30%	35%
%return X	-1.9%	5.4%	17.7%
%return Y	-6.5%	15.4%	2.1%

5. Each pair of rates of return for securities X and Y listed below is equally likely. You wish to form a portfolio by allocating 25% of funds in Company X and the remainder in Company Y. Find the standard deviation and expected rate of return for the portfolio. ©ER14

X:	-3.9%	4.6%	21.9%	17.6%
Y:	13.4%	13.4%	12.5%	6.7%

4. Benefits from diversification

Creating portfolios possibly creates diversification benefits. The previous example computes that the "risk" of securities Alpha and Zed, that is σ_{Alpha} and σ_{Zed} respectively, equal 14.77% and 5.29%. Allocating 25% of funds in Alpha and 75% in Zed creates a portfolio with a risk of 2.35% - less than either individual security. For this situation the portfolio offers diversification benefits. Figure 10.3 illustrates risk-return characteristics for these securities.

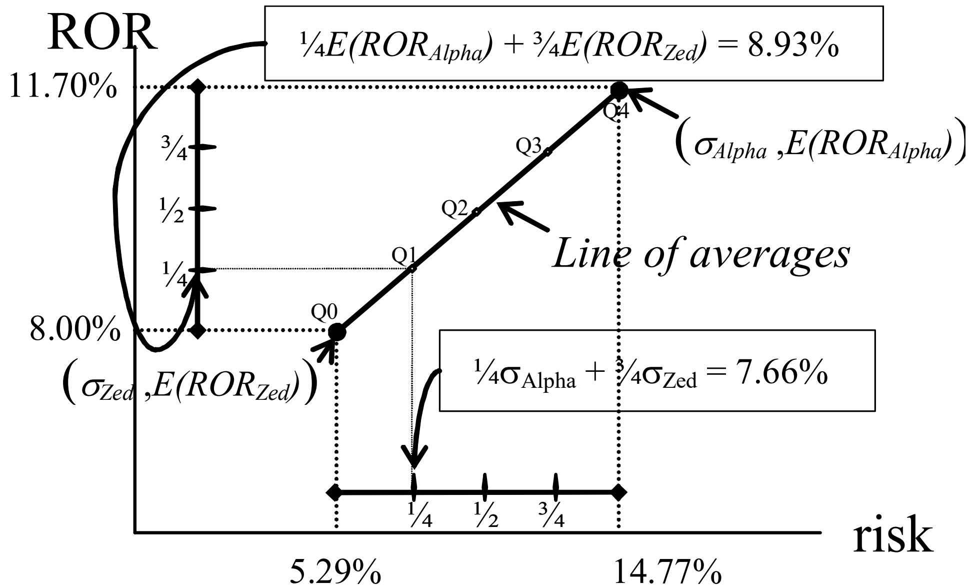


FIGURE 10.3 Risk and return for Alpha and Zed

First consider return characteristics. Figure 10.3 illustrates return on the vertical axis. Zed is a relatively low return asset (8.00%) whereas Alpha is a relatively high return asset (11.70%). Adjacent to the vertical axis is a little "step ladder." It begins at 8.00% (=$E(ROR_{Zed})$) and steps up by quarters toward 11.70%. A portfolio with ¼Alpha+¾Zed is on the first step; it is one-fourth the distance toward $E(ROR_{Alpha})$. If the allocation were half-and-half the resultant portfolio *ROR* would be on the second step, halfway between 8.00% and 11.70%. The return smoothly steps from the low return toward the high return as the allocation in Alpha steadily increases. *E(ROR)* for the portfolio is a weighted average of component returns.

The average risk of components in a portfolio is given by this formula:

FORMULA 10.10 Portfolio average risk, $\sigma_{average}$

The portfolio's average risk is a weighted average of component risks:

$$\sigma_{average} = \sum_{j=1}^{M} w_j \sigma_j \ ,$$

where w_j is the proportion of portfolio wealth allocated to asset *j* and *M* is how many assets the portfolio contains. σ_j is the standard deviation of returns for asset *j.*

Figure 10.3 illustrates risk on the horizontal axis. Zed is a relatively low risk asset (5.29%) whereas Alpha is a relatively high risk asset (14.77%). With one-quarter allocation to Alpha, three-quarters to Zed, compute average risk like this

$$\sigma_{average} = \tfrac{1}{4}\sigma_{Alpha} + \tfrac{3}{4}\sigma_{Zed}$$

$$= .25(.1477) + .75(.0529)$$

= 7.66%

Adjacent to the horizontal axis is a little "step ladder." It begins at 5.29% (=σ_{Zed}) and steps out by quarters toward 14.77%. A portfolio with ¼Alpha+¾Zed is out one step, one-fourth the distance toward σ_{Alpha} with $\sigma_{average}$ equal to 7.66%. If the allocation were half-and-half the resultant $\sigma_{average}$ for the portfolio would be out two steps, halfway between 5.29% and 14.77%.

The solid line with points Q_0 , Q_1, …, Q_4 shows combinations of $\sigma_{average}$ and $E(ROR_{portfolio})$ for different allocations and is the *line of averages* for securities Alpha and Zed. Every point on the line measures average risk and return for a unique portfolio allocation. Point Q_0 represents 100% allocation in Zed (w_{Zed}=1.0, w_{Alpha}=0). Q_1, represents return and average risk for the portfolio with ¼Alpha+¾Zed. Its coordinates are one-quarter the distance between Alpha and Zed. Q_2 represents the fifty-fifty portfolio. Its return and average risk is midway between Alpha and Zed. Q_4 at the other end of the line of averages is the all-Alpha allocation (w_{Zed}=0, w_{Alpha}=1). Allocation to Alpha increases from 0 to 100% with movement from Q_0 toward Q_4.

The portfolio with ¼Alpha+¾Zed has average risk of 7.66% but actual risk of 2.35%. The benefit from diversification is a reduction in risk of 5.31%.

DEFINITION 10.6 Portfolio diversification benefit ("*DB*")
The diversification benefit (*"DB"*) from forming a portfolio equals the difference between the average component risk and actual portfolio risk:

$$\begin{pmatrix} Diversification \\ benefit \end{pmatrix} = \sigma_{average} - \sigma_{portfolio} .$$

Lessons earlier in this chapter explain that investors prefer less risk. Hence, diversification benefits are valuable because, all else equal, investors prefer risk reduction.

CALCULATOR CLUE 10.4 After completion of the *Calculator Clue* for Example 8 the mean and standard deviation for Alpha are in memories 1 and 2, analogous numbers for Zed are in memories 4 and 5, and analogous numbers for the portfolio that allocates 25% Alpha and 75% Zed are in memories 7 and 8. Compute the average risk of the portfolio with these keystrokes:
RCL 2 x .25 + RCL 5 x .75 = STO 0 .
The display shows the average risk is 7.66%. Now obtain the diversification benefits:
RCL 0 - RCL 8 =
The display shows the diversification benefits equal 531 basis points.

4.A. Relation between diversification benefits and correlation

The preceding example computes portfolio risk by figuring the standard deviation of returns for each different outcome (e.g., stagnant economy, etc.). Formula 10.11 allows direct computation of portfolio risk:

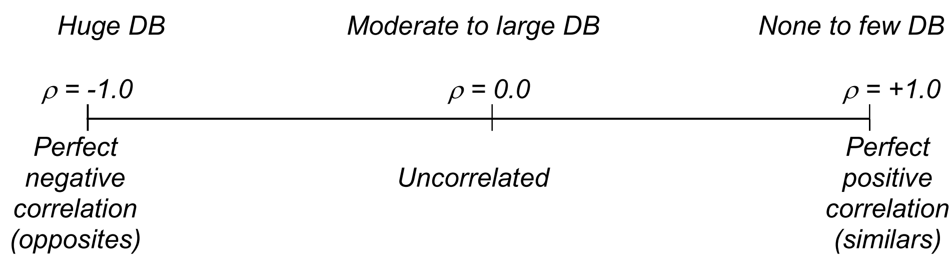
FORMULA 10.11 Actual risk for a 2-security portfolio ("$\sigma_{portfolio}$")

$$\sigma_{portfolio} = \left\{ w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + 2 w_X \sigma_X w_Y \sigma_Y \rho_{X,Y} \right\}^{1/2} ,$$

where w_X and w_Y denote the proportion of portfolio wealth allocated to asset X and Y, respectively, and $\rho_{X,Y}$ is the correlation coefficient.

The correlation coefficient is a number between -1 and +1 and measures the extent to which two variables move together.

The diversification benefit that a portfolio creates depends on the amount of correlation between component assets. Formula 10.11 shows $\sigma_{portfolio}$ relates directly with ρ. That is, relatively large ρ implies relatively large $\sigma_{portfolio}$ and small ρ implies relatively small $\sigma_{portfolio}$. Examine this graphic.



With perfect positive correlation (ρ = 1.0) the returns for two assets always move together. When one is up the other is up. When one is down the other is down. They always follow each other. Because they always follow each other there exist few diversification benefits from combining these assets into a portfolio.

With perfect negative correlation (ρ = -1.0) the returns for two assets always move oppositely. When one is up the other is down. When one is down the other is up. Because they always move oppositely a lot of offsetting occurs, thereby smoothing outcomes and reducing risk. There exist huge diversification benefits from combining these assets into a portfolio.

A correlation coefficient near zero means that returns between two assets are uncorrelated. There is no reliable relation between the two. When one is up the other sometimes is down, but sometimes it's up. They are completely unrelated. Combining these assets into a portfolio creates moderate to large diversification benefits.

The formula for computing ρ when all outcomes for assets X and Y are equally likely is this:

FORMULA 10.12 Correlation coefficient (ρ) with equally likely outcomes

$$\rho_{X,Y} = \left\{ \frac{\sum_{i=1}^{N} \left(ROR_{i,X} \times ROR_{i,Y} \right)}{N} - \left(E(ROR_X) \times E(ROR_Y) \right) \right\} \div \sigma_X \sigma_Y$$

$$\rho_{X,Y} = \left\{ covariance_{X,Y} \right\} \div \sigma_X \sigma_Y$$

The expression in curly brackets is the *covariance*. The correlation coefficient and covariance always have the same sign. Always $-1 \leq \rho \leq +1$ whereas covariance is unbounded.

The previous section explains that when all outcomes are equally likely computations simplify, largely because calculators such as the Texas Instruments BAII Plus compute statistics automatically – these calculators also compute ρ! Consider this example.

EXAMPLE 9 With equally likely outcomes find portfolio diversification benefits

Two securities, *X* and *Y*, exhibit the following returns over the past 5 years:

	1	2	3	4	5
X:	15.15%	3.45%	-0.05%	-1.15%	14.85%
Y:	6.25%	8.45%	6.35%	10.45%	3.85%

You believe that past performance is a perfect predictor of future performance. Find the diversification benefits for the portfolio that allocates 40% to *X* and 60% to *Y*.

SOLUTION

Follow example 6 in the previous section that uses formula 10.6 and 10.7 to compute that σ_X= 7.15% and σ_Y= 2.23%. Then formula 10.10 finds:

$$\sigma_{average} = 0.40(.0715) + 0.60(.0223)$$

$$= 4.20\% .$$

Computing the actual portfolio risk requires the correlation coefficient from formula 10.12:

$$\rho_{X,Y} = \left\{\frac{.1515(.0625)+.0345(.0845)+(-.0005)(.0635)+(-.0115)(.1045)+(.1485)(.0385)}{5}\right.$$

$$\left. -(.0645)(.0707) \right\} \div (.0715)(.0223)$$

$$= -0.7444$$

The large negative coefficient suggests huge diversification benefits. Substitute into formula 10.11 to find the risk for the portfolio with 40%*X* + 60%*Y.*

$$\sigma_{40\%X+60\%Y} = \left\{.40^2(.0715^2)+.60^2(.0223^2)+2(.40)(.0715)(.60)(.0223)(-.7444)\right\}^{1/2}$$

$$= 2.07\%$$

Diversification benefits equal the difference between average and actual risks:

DB = 4.20% - 2.07%

= *2.13%* , that is, 213 basis points.

CALCULATOR CLUE 10.5 Follow example 6 and enter the data for *X* and *Y* into the BAII Plus© DATA worksheet. Then find and store the statistical estimates of the standard deviations and correlation coefficient. It's a good idea to rely on stored numbers for calculations instead of re-typing. Hit 2nd STAT and make sure the display shows "LIN". Hit ↓ 4 times and the display shows "σ_X = .0715" so hit STO 4 . Hit ↓ 3 more times to see "σ_Y = .0223" so hit STO 5 . Hit ↓ 3 more times to see "*r* = –.7444". This is the correlation coefficient so hit STO 3 . Now obtain average risk with these keystrokes:
.4 x RCL 4 + .6 x RCL 5 = .
The display shows 4.20% so hit STO 9 . Obtain actual risk like this:
.4 x^2 x RCL 4 x^2 + .6 x^2 x RCL 5 x^2 + 2 x .4 x RCL 4 x .6 x RCL 5 x RCL 3 = √
The display shows 2.07% so hit STO 0 . Compute DB as
RCL 9 - RCL 0 =
The display shows the final answer of 213 basis points.

EXERCISES 10.4A

Conceptual

1. Listed below are rates of return for securities X, Y, and Z for 5 different periods. Without performing any computations, comment on the correlation between the different securities.

	period 1	2	3	4	5
X:	-3.9%	4.6%	21.9%	17.6%	14.1%
Y:	13.4%	13.1%	12.5%	12.7%	12.9%
Z:	4.3%	15.2%	3.2%	-3.7%	1.4%

Numerical quickies

2. The standard deviation of expected returns for investments X and Y equal 14.5% and 9.5% , respectively. The correlation between returns for X and Y is 0.30 . If you allocate 80% of your funds to X, and the remainder to Y, what is the portfolio's standard deviation of expected returns? ©MR3am

3. Investment risk, as measured by the standard deviation of returns, equals 21.0% for stock X and 12.1% for stock Y. The correlation between the securities is zero. You form a portfolio allocated 40% in X and 60% in Y. Find the diversification benefit, measured as percent reduction in risk, for the portfolio. ©ER8b

4. Each pair of rates of return for securities X and Y listed below is equally likely. You wish to form a portfolio by allocating 30% of funds in Company X and the remainder in Company Y. Find the diversification benefit, measured as the standard deviation reduction in basis points (BP), that the portfolio provides. ©ER15

X:	-3.2%	4.9%	22.3%	15.8%
Y:	20.9%	13.0%	8.2%	6.0%

Challengers

5. You form a portfolio that invests 60% of total funds in stock X and 40% in stock Z. Two possible outcomes exist. The probability is 15% that the first outcome occurs, in which case the rates of return equal 10% for X and 37% for Z. The probability is 85% that the second outcome occurs, in which case the rates of return equal 30% for X and 12% for Z. Find the diversification benefit, measured as the standard deviation reduction in basis points (BP), that the portfolio provides. ©ER9c

6. The standard deviation of returns equals 10.5% for stock X and 21.5% for stock Z. The correlation between the two stocks equals -0.20. You make a portfolio that allocates 75% of funds to stock X. The remainder is put in stock Z. Find the diversification benefit, measured as the standard deviation reduction in basis points (BP), that the portfolio provides. ©ER7

4.B. The minimum risk portfolio and investment advice

Several preceding examples return a familiar finding: *the portfolio risk sometimes is less than the risk of every individual component security!* This raises a very interesting question: *What allocation scheme provides the least risky portfolio and why does it matter?* This subsection looks at that lesson.

The question's quantitative solution is surprisingly simple. Formula 10.13 gives the allocation in a 2-security portfolio that provides the least risky portfolio.

FORMULA 10.13 Allocation at minimum risk portfolio ("$w^{min\ \sigma}$")

The proportional allocation to security *X* that generates the minimum risk portfolio for all possible combinations of securities *X* and *Y* is this:

$$w_X^{minimum risk} = \frac{\sigma_Y^2 - covariance_{X,Y}}{\sigma_X^2 + \sigma_Y^2 - (2 \times covariance_{X,Y})} .$$

Use formula 10.13 to find the weight for one security. Subtract from 100% the weight found above to find the weight for the other security.

Formula 10.13 implies $w_X^{min\ \sigma}$ increases as σ_Y increases. That is, as *Y* gets riskier then put more into *X* in order to form the minimum risk portfolio. Also notice that formula 10.13 depends on covariance between securities. Covariance is intuitively analogous to correlation, as the discussion for formula 10.12 suggests, except that the correlation coefficient is restricted to range between plus and minus one whereas covariance is unrestricted.

EXAMPLE 10 With equally likely outcomes find minimum risk portfolio and *DB*

Two securities, *X* and *Y*, exhibit the following returns over the past 5 years:

	1	2	3	4	5
X:	15.15%	3.45%	-0.05%	-1.15%	14.85%
Y:	6.25%	8.45%	6.35%	10.45%	3.85%

You believe that past performance is a perfect predictor of future performance. Find the risk, return and diversification benefits for the minimum risk portfolio containing securities *X* and *Y*.

SOLUTION

Follow example 9 and find that σ_X= 7.15%, σ_Y= 2.23%, and $\rho_{X,Y}$ = -0.7444. Then use formula 10.13 to compute:

$$w_X^{min\,\sigma} = \frac{.0223^2 - (.0223)(.0715)(-.7444)}{.0715^2 + .0223^2 - 2(.0223)(.0715)(-.7444)}$$

$$= 21.12\%$$

The minimum risk portfolio that combines stocks *X* and *Y* invests 21.12% in *X*. The amount to invest in Y is 100% minus 21.12%, or $w_Y^{min\,\sigma}$ = 78.88%.

For the minimum risk portfolio obtain the average return and average risk,

$$E(ROR_{21.12\%X + 78.88\%Y}) = .2112\ (.0645) + .7888\ (.0707)$$

$$= 6.94\%\ ,$$

$$\sigma_{average} = 0.2112(.0715) + 0.7888(.0223)$$

$$= 3.27\%\ .$$

Find the actual risk for this portfolio:

$$\sigma_{21.12\%X+78.88\%Y} = \left\{ \begin{aligned} &.2112^2(.0715^2) + .7888^2(.0223^2) \\ &+ 2(.2112)(.0715)(.7888)(.0223^2)(-.7444) \end{aligned} \right\}^{1/2}$$

$$= 1.19\%$$

Because of negative correlation between component securities the minimum risk portfolio offers huge diversification benefits.

$$DB = 3.27\% - 1.19\%$$

$$= 2.08\%$$

Among all possible portfolios containing securities *X* and *Y* the one with least possible risk has *E(ROR)* of 6.94%, σ of 1.19%, and the portfolio creates diversification benefits of 208 basis points.

CALCULATOR CLUE 10.6 Follow example 6 and enter the data for *X* and *Y* into the BAII Plus© DATA worksheet. Then hit 2nd STAT and make sure the display shows "LIN". Hit ↓ 2 times and the display shows "X-bar = .0645" so hit STO 1. Hit ↓ 2 times and the display shows "σ_X = .0715" so hit STO 4. Hit ↓ 1 time and the display shows "Y-bar = .0707" so hit STO 2 .Hit ↓ 2 times to see "σ_Y = .0223" so hit STO 5. Hit ↓ 3 more times to see "*r* = –.7444". While ρ is on the display obtain and store the covariance with these keystrokes:

x RCL 4 x RCL 5 = STO 6.

Now compute and store the allocation for *X*:

RCL 5 x^2 - RCL 6 = ÷ (RCL 4 x^2 + RCL 5 x^2 - 2 x RCL 6) = STO 7.

While *0.2112,* the allocation for $w_X^{min\ \sigma}$, is on the display compute and store $w_Y^{min\sigma}$:

+/- + 1 = STO 8.

Now memories 7 and 8 contain weights for *X* and *Y* at the minimum risk portfolio. Compute the portfolio return with these keystrokes

RCL 7 x RCL 1 + RCL 8 x RCL 2 = STO 3.

Memories 1, 2, and 3 now contain the E(ROR) for X, Y, and the minimum risk portfolio. The display shows the portfolio return of 6.94%. Next compute average risk.

RCL 7 x RCL 4 + RCL 8 x RCL 5 =.

The display shows average risk is 3.27% so hit STO 9. Obtain actual risk like this:

RCL 7 x^2 x RCL 4 x^2 + RCL 8 x^2 x RCL 5 x^2 + 2 x RCL 7 x RCL 8 x RCL 6 = √

The display shows 1.19%, the actual risk of $\sigma_{21.12\%X + 78.88\%Y}$ so hit STO 0. Compute DB as

RCL 9 - RCL 0 =

The display shows the final answer of 208 basis points.

The qualitative answer to the question "why does the minimum risk portfolio matter" is straightforward, too. Discover its significance by comparing these three investment strategies: Strategy 1 ("S1") invests 100% of funds in security *X*; Strategy 2 ("S2") invests 100% of funds in *Y*; Strategy 3 ("S3") invests in the minimum risk portfolio comprising securities *X* and *Y*.

Here is a summary of risk-return characteristics for these 3 strategies:

allocation	*E(ROR)*	σ
S1: 100% X	6.45%	7.15%
S2: 100% Y	7.07%	2.23%
S3: 21.12%X + 78.88%Y	6.94%	1.19%

Figure 10.4 illustrates the three strategies.

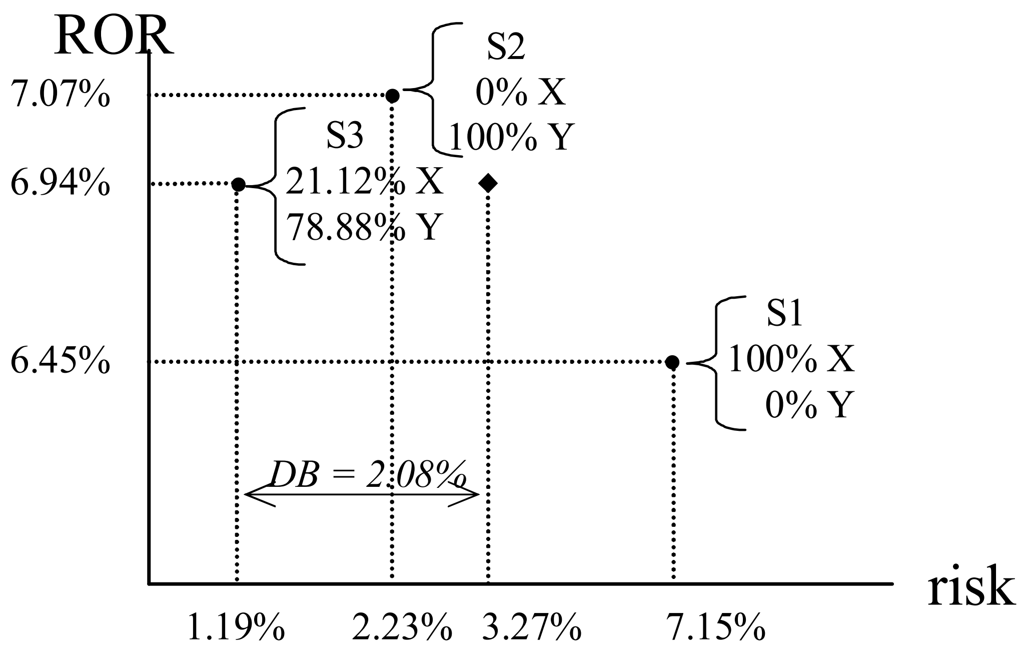


FIGURE 10.4 X, Y, and the minimum risk portfolio

First compare S1, the all-*X* allocation, with S2, the all-*Y* allocation. Security *Y* dominates security *X* because *Y* has higher return and less risk. Nobody that owns only one of these securities should own *X*. The choice between S1 and S2 is a no-brainer: choose S2 and go with *Y*.

Thus far it may appear that *X* is a loser with little redeeming value. But hold on! Strategy S3 shows that mixing a little *X* with a lot of *Y* allows a unique opportunity. S3 and S2 coexist as tradeoffs. This means both are feasible investments that appeal to investors with differing attitudes toward the risk-return tradeoff. S2 is for high-risk, high-return preferences whereas S3 is for low-risk, low-return preferences. Asset *X*, even though dominated, has purpose, a *raisoń d'ětre*. *X* is a loser by itself but when placed in a portfolio *X* offers something unique – a mechanism for trading off risk and return.

Implications of this lesson are profound and far-reaching. Imagine an aging relative that owns all-*Y* because at their point in the financial life cycle (see table 9.4) they gladly tradeoff risk for lower return. Pretend that *X* and *Y* are the only available investments and that you both believe the numbers in figure 10.4. You both know that *X*, with its high risk and low return, is a dominated dog. But you have financial training and offer this loved one the following observation: *Sell 21% of your holding in low risk Y and use the money to buy high risk X and the overall portfolio risk will fall!* And you are right!

Perhaps it seems counterintuitive that you may reduce portfolio risk by selling a low risk asset and buying a high risk asset. The key, however, is that creating portfolios possibly creates diversification benefits. These benefits are valuable; they are a special type of transformation value. From this lesson emerges a stronger intuition - *putting all your eggs in one basket is a risky strategy*.

The minimum risk portfolio is not necessarily the "best" allocation. But it is not dominated. It coexists with many other allocation schemes as part of the *feasible allocation set*:

RULE 10.1 Feasible allocation set ("*FAS*") for 2-security portfolios
The feasible allocation set for a portfolio comprising two securities includes all allocations that are not dominated. All portfolios in the *FAS* coexist as tradeoffs. The *FAS* includes all portfolios with allocation for the highest returning asset greater than or equal to its allocation in the minimum risk portfolio.

The minimum risk portfolio represents, as figure 10.5 illustrates, the beginning of the *FAS*. This figure shows the "Risk-return profile for securities *X* and *Y*," a graph of all portfolios formed by combining securities *X* and *Y* in every possible allocation.

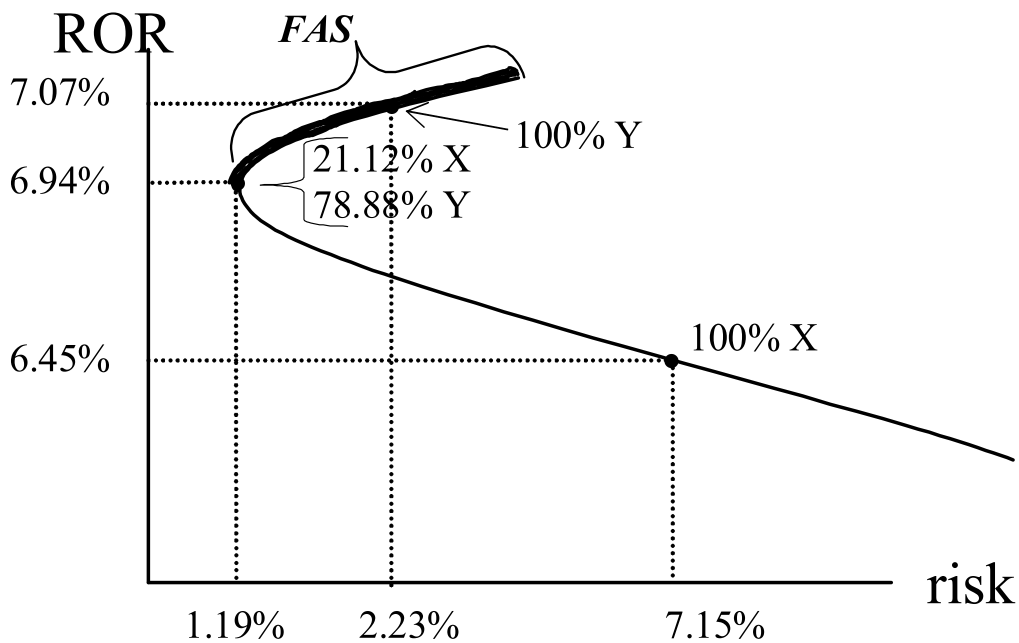


FIGURE 10.5 Risk-return profile for securities X and Y

The risk-return profile is the parabola that passes through three labeled points: the 100%*X* allocation toward the lower right, the minimum risk portfolio at the left tip of the parabola, and the 100%*Y* allocation toward the upper middle. Each unique point on the parabola represents a different portfolio with a unique allocation of funds between *X* and *Y*. Movement along the lower portion of the parabola from the all-*X* allocation toward the minimum risk portfolio occurs smoothly as w_x diminishes from 100% to 21.12%. Movement along the upper portion of the parabola from the minimum risk portfolio toward the all-*Y* allocation occurs smoothly as w_x diminishes from 21.12% to 0%.

The feasible allocation set in figure 10.5 is the bold portion of the parabola that begins at the minimum risk portfolio and extends above and to the right. The *FAS* includes the 100% allocation in security *Y*. For all portfolios in the *FAS* the allocation to *Y* exceeds 78.88% of funds. All these are good investment choices and the one that is best depends on the risk preferences of the investor.

Dominated portfolios are below and to the right of the minimum risk portfolio. These include the 100%*X* allocation. For all dominated portfolios the allocation to *X* exceeds 21.12%. These are bad investment choices for every investor, irrespective of risk preferences.

Inspection of the risk-return profile for securities *X* and *Y* leads to this advice for investors that consider creating a portfolio containing either or both of these securities. These two equivalent statements are consistent with rule 10.1:

Always allocate more than 78.88% of funds in security Y.

Always allocate less than 21.12% of funds in security X.

An important step in determining investment advice is finding the feasible allocation set; finding the minimum risk portfolio makes that simple.

EXERCISES 10.4B

Conceptual

1. Is the objective "minimizing portfolio risk" the same as "maximizing diversification benefit"?

2. Formula 10.13 computes the allocation in each of two securities that yields least risky portfolio possible. Definition 10.6 defines *Diversification benefit* as the difference between actual and average portfolio risk. Simplify the formulas and find *DB* at the minimum risk for the following special case $\rho = +1$.

Numerical quickies

3. Throughout the past, the return for Large Cap Stocks has averaged 11.2% and the standard deviation has been 32.8%. For International Stocks, the return has averaged 15.0% and the standard deviation 37.1%. The correlation between the returns for these two assets has been -0.10. What is the percentage allocation of funds in Large Cap Stocks that results in a portfolio with the lowest possible risk; the remaining funds are to be invested in the other asset. ©MR2am

Challengers

4. The standard deviation of expected returns for investments X and Y equal 14.0% and 22.0%, respectively. The correlation between returns for X and Y is -0.50. How much risk reduction, that is diversification benefit in basis points, does the minimum risk portfolio provide? ©MR4am

5. Your analysis suggests that each pair of outcomes is equally likely:

%return Alpha	0.0%	5.2%	22.2%	13.8%
%return Zed	23.7%	23.8%	9.2%	-4.8%

a. Find the combination of Alpha and Zed that yield the minimum risk portfolio. ©MR1am

b. Find σ and *E(ROR)* of the minimum risk portfolio. ©MR1dm

c. How much diversification benefit, that is risk reduction in basis points, does the minimum risk portfolio provide? ©MR1fm

4.C. The risk-return profile for 2-security portfolios

The risk-return profile shows risk and return measurements for all possible portfolios comprised of two securities. The following rule about risk-return profiles always is true.

RULE 10.2 Shape of 2-security risk-return profiles
The risk-return profile is a parabola that opens to the right and is symmetric around a horizontal ray passing through the minimum risk portfolio. Correlation between component securities determines the extent of inflection in the parabola.

Figure 10.6 shows plausible risk-return profiles for two generic securities, call them *A* and *B*. This discussion is true irrespective of whether the securities coexist as tradeoffs or one is dominant. The all-*A* portfolio (w_A=100%, w_B=0%) as well as the all-*B* portfolio (w_A=0%, w_B=100%) always lie somewhere on the parabola.

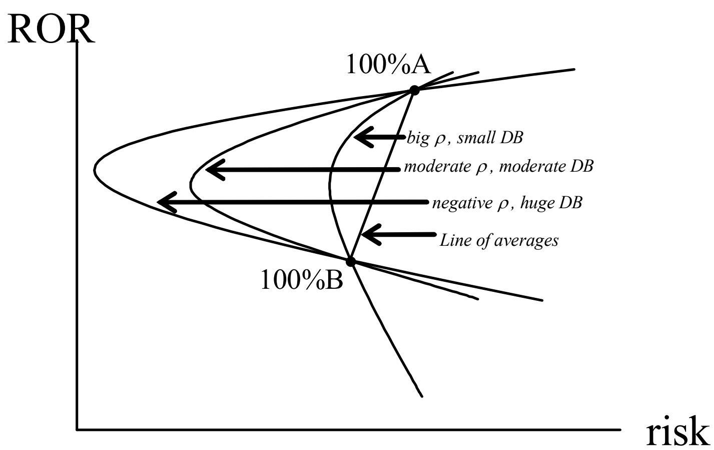


FIGURE 10.6 Risk-return profiles depend on correlation

When the correlation coefficient between returns for securities *A* and *B* is relatively large then there is not much bow or inflection in the parabola. The diversification benefit from creating a portfolio with *A* and *B* consequently is small. The limiting case occurs with perfect positive correlation when ρ = +1. For this special case the parabola actually is coincident with the line of averages connecting securities *A* and *B* – the *DB* equals zero.

When component securities are moderately correlated, uncorrelated, or even negatively correlated then the parabola has a lot of flex, like an archer's bow pulled tightly. Diversification benefits consequently are moderate to large. The limiting case occurs with perfect negative correlation. For this special case the parabola actually kisses the vertical axis – the minimum risk portfolio has σ = 0 when ρ = -1. The *DB* is huge when component securities offset each other's extreme outcomes.

A second rule for risk-return profiles pertains to the allocation at the minimum risk portfolio. Before presenting the new rule learn some lessons about the risk-return parabola. Realize that just as two points determine the location of a line, three points determine the location of a parabola. Each point on the parabola represents a unique portfolio. It's simplest and most useful to find the risk-return coordinates for these three portfolios:

(i) the all-*A* portfolio;
(ii) the all-*B* portfolio;
(iii) the minimum risk portfolio.

Previous examples show how to obtain measures of *E(ROR)* and σ for securities *A* and *B*, and formula 10.13 allows computation of those measures for the minimum risk portfolio.

Sometimes the answer from formula 10.13 is a negative number. When the proportional allocation for security *A* is a negative number, that is $w_A^{min\ \sigma}<0$, the minimum risk portfolio takes a *short* position in security *A*. Whereas a long position in an asset ($w>0$) means the asset is owned, a short position ($w<0$) means the asset is owed – it is a liability! Suppose, for example, an investor forms a $1,000 portfolio with securities *A* and *B* where $w_B=1.25$ and $w_A=-0.25$. Notice that the sum of weights equals 100% as required. The weight in *A* is -25%. The investor takes a short position by borrowing and selling $250 worth of security *A*. Next the investor allocates 125% of portfolio wealth ($1,250) and buys security *B*. The financing source for the purchase is $1,000 of initial endowment plus $250 from the short sale. The short sale increases the investor's liabilities - an increase in a liability is a source of financing.

The objective of most financial investments is making profit. One way to profit is *buy low now and sell high later*. Buy the stock today for $50, for example, and sell later for $70. In developed financial markets, however, short sales are fairly common. Short sales reverse the timing to *sell high now and buy low later*. A short sale occurs, for example, when you borrow a share from your broker and sell the stock today for $50. The broker sells stock from inventory and credits your account with the proceeds. You are obligated to eventually cover your short position. Motivation for covering the short sale may include: (1) the stock price later drops, say to $30, and you order the broker to buy it so you can take your profit. You sold high, bought low, and profited. Motivation also may occur because: (2) the stock price later rises, say to $70, and you wish to cut your losses. For this case you sold high but unfortunately bought higher. A third motivation may be: (3) the broker for whatever reason wants the stock back and has the right to force you at any time to cover the short position.

Figure 10.7 illustrates several possible allocations at the minimum risk portfolio.

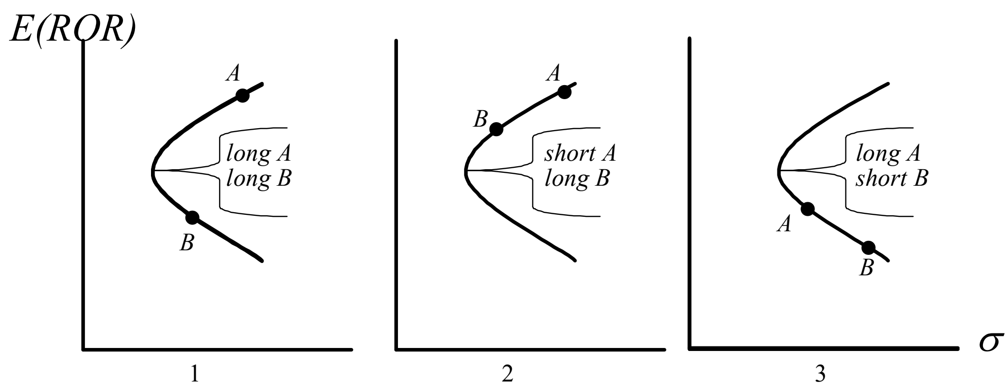


FIGURE 10.7 Risk-return profiles and allocation at the minimum risk portfolio

Graph 1 on the left is the outcome that occurs when the minimum risk portfolio includes a long position for both securities. In this case both weights w_A and w_B range between 0 and 100%. This case usually occurs when correlation among component securities is moderate, low, or negative.

The situation that graph 2 depicts usually occurs when highly correlated securities coexist as tradeoffs. For the all-*A* portfolio, that is at point *A*, w_A equals 100%. As one moves down the parabola w_A declines. For the all-*B* portfolio w_A equals 0. Beneath point *B* the weight w_A is negative. The minimum risk portfolio involves a short position in *A* ($w_A<0$) and a long position in *B* ($w_B>100\%$).

The situation that graph 3 depicts usually occurs when two securities strongly correlate and one is dominant. The minimum risk portfolio involves a long position in *A* ($w_A>100\%$) and a short position in *B* ($w_B<0$).

Rule 10.3 pertains to all risk return profiles and summarizes the preceding tendencies.

RULE 10.3 Long or short positions and the minimum risk portfolio
When the weight from formula 10.13 is between 0 and 100% then the minimum risk portfolio lies between component securities and involves a long position in both. When the weight is negative or bigger than 100% the minimum risk portfolio takes a short position on one security, long on the other.

Sometimes investment policies disallow short positions. Short positions, after all, are riskier than long positions because short positions expose investors to unlimited losses. Perhaps an institutional investor declares in their prospectus that the company does not use short sales. Alternatively, perhaps an individual investor abides a "just say no" philosophy for short sales. For those situations the finding of a negative weight simply means the asset is avoided, its weight in the portfolio is zero, and the other asset's weight becomes 100%.

Interpreting risk-return profiles provides fairly strong answers to important questions. The information requirement for making the risk-return profile includes *E(ROR)* and σ for each security as well as the correlation coefficient.

EXAMPLE 11 Find investment advice given summary statistics for large and small caps
Table 9.15 shows that since 1925 the annual rate of return for large capitalization stocks ("*LC*") averages 12.7% and the standard deviation is 20.2%. The return for small cap stocks ("*SC*") averages 17.3% and the standard deviation 33.2%. The correlation coefficient between returns for these two assets is 0.429. You believe these long-run historical statistics likely persist into the long-run future. What portfolios containing these two asset classes are in the feasible allocation set?

SOLUTION
The weight for large cap stocks at the minimum risk portfolio is found with formula 10.13

$$w_{LC}^{min\,\sigma} = \frac{.332^2 - (.332)(.202)(.429)}{.202^2 + .332^2 - 2(.332)(.202)(.429)}$$

$$= 87.1\%$$

The minimum risk portfolio from combining asset classes *LC* and *SC* invests 87.1% in *LC* and, consequently, 12.9% in *SC* (12.9% = 100% - 87.1%). Because $E(ROR_{SC}) > E(ROR_{LC})$ rule 10.1 stipulates that the *FAS* includes all portfolios allocating 12.9% or more to *SC*. Restate the findings like this:

Always allocate more than 12.9% of funds in small cap stocks.
Always allocate less than 87.1% of funds in large cap stocks.

Figure 10.8 qualitatively depicts the scenario.

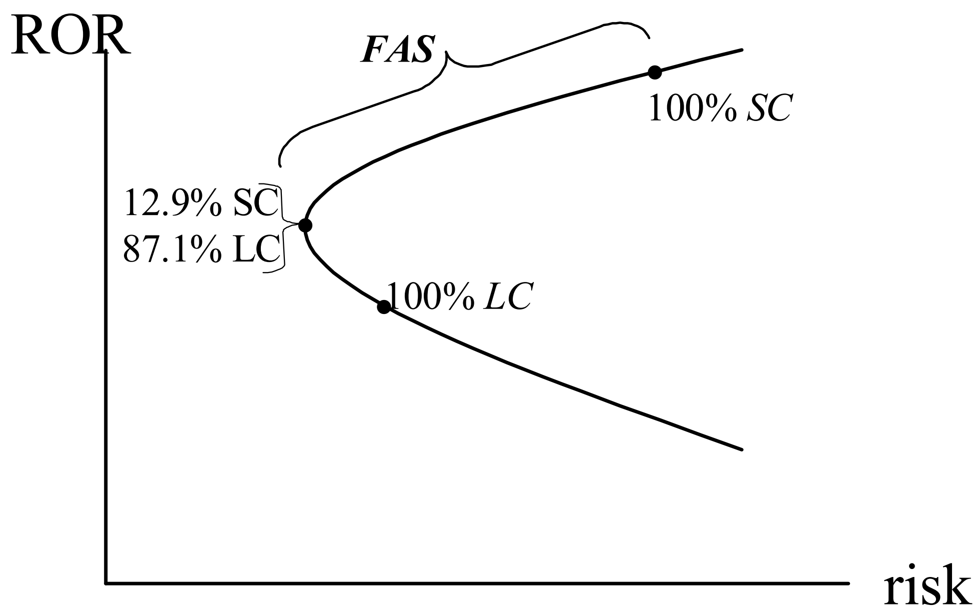


FIGURE 10.8 Risk-return profile for large (LC) and small cap (SC) stocks

Figure 10.8 offers pretty strong investment advice that is based on 75 years of history. Incredibly, too, the advice relies on <u>one</u> computation (formula 10.13) and a few rules! To assign numbers on the axis, or to compute diversification benefits, etc., requires more effort. Yet the qualitative advice flows fairly simply.

The fundamental structure underlying these lessons is well known in the investments industry. Predicting future outcomes by relying on historical statistics is a widespread practice. Formula 10.13 uses three summary statistics as inputs: σ for one security, σ for the other, and ρ. Statisticians call these "higher order" statistics; maybe you remember from your Stats class that the mean is the first moment and variance is the second moment. Studies suggest that higher order statistics are short-term stationary. This means that stocks with high standard deviation of returns for the last few years continue in the future to have high standard deviation, stocks that tended to strongly correlate during the last few years continue in the future to strongly correlate, etc. This suggests that computations of formula 10.13 for weights at the minimum risk portfolio are relatively reliable.

Unfortunately, knowing composition of the minimum risk portfolio is insufficient for formulating investment advice. Rule 10.1 clearly states that the *FAS* includes portfolios with increasing allocation to the highest returning asset. Well here is the problem – nothing predicts the highest returning asset very well. The first moment, that is *E(ROR)*, is very unstable (chapter 2 states the best among many terrible predictors of return is the equity price-to-book ratio; chapter 9 explains this instability of returns with the efficient market hypothesis)! Sometimes stocks are up, sometimes they're down, sometimes last year's winners are next year's winners, and sometimes they're losers. Predicting future stock performance is the mother of all mysteries! This example drives home the seriousness of this shortcoming for modern portfolio theory.

EXAMPLE 12 Find investment advice given σ and ρ for 2 mutual funds

You are forming a portfolio with two mutual funds. The standard deviation of returns for the HealthSciences fund ("*HS*") is 28.6%. For the Global Equities fund ("*GE*") the standard deviation is 34.4%. The correlation coefficient between returns for these two assets has been 0.107. Formulate appropriate investment advice.

SOLUTION
The weight for *HS* stocks at the minimum risk portfolio is found with formula 10.13

$$w_{HS}^{\min\sigma} = \frac{.344^2 - (.344)(.286)(.107)}{.286^2 + .344^2 - 2(.344)(.286)(.107)}$$

$$= 60.2\%$$

The minimum risk portfolio combining mutual funds *HS* and *GE* invests 60.2% in *HS* and, consequently, 39.8% in *GE* (39.8% = 100% - 60.2%). It involves a long position in each fund. The minimum risk portfolio is not "best", but it certainly is not dominated.

The setup provides no information about $E(ROR_{HS})$ or $E(ROR_{GE})$. There are only two possibilities, however. Either $E(ROR_{HS})$ is bigger or $E(ROR_{GE})$ is bigger. Figure 10.9 shows the alternative scenarios.

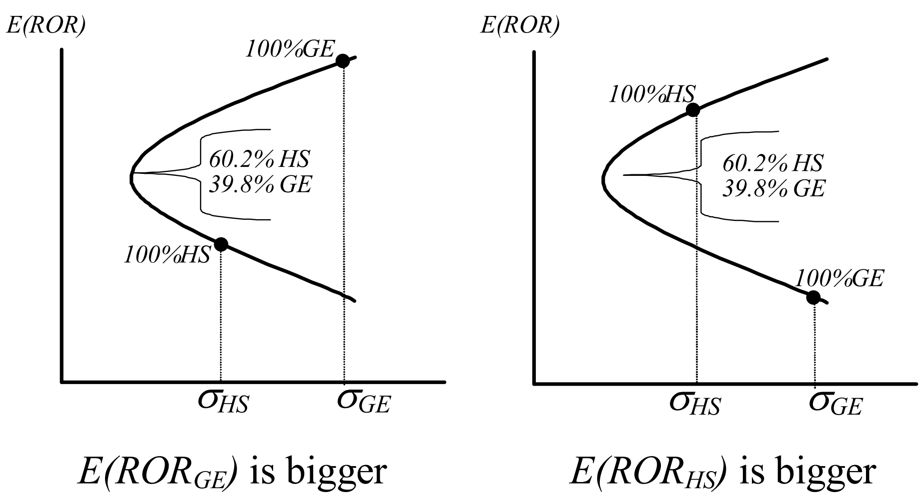


FIGURE 10.9 Two scenarios for investment advice

Both scenarios have the same ranking of standard deviations ($\sigma_{GE} > \sigma_{HS}$) and allocation at the minimum risk portfolio. The scenarios differ only because of which return is bigger.

For the scenario on the left $E(ROR_{GE})$ is bigger. The feasible allocation set includes all portfolios that allocate more than 39.8% in *GE*. The all-*GE* portfolio is in the *FAS.* The minimum risk portfolio dominates the all-*HS* portfolio.

For the scenario on the right $E(ROR_{HS})$ is bigger. The feasible allocation set includes all portfolios that allocate more than 60.2% in *HS*. The all-*HS* portfolio is in the *FAS.* The minimum risk portfolio dominates the all-*GE* portfolio.

The two scenarios generate exactly opposite investment advice! The only point in common is the minimum risk portfolio – it's not dominated and is a member of the feasible allocation set irrespective of expectations about returns.

Famed investor and very wealth man Warren Buffet once said he doesn't understand why people invest in their second, third, or even tenth best choice. He says they should just pick the winner. Consider Mr. Buffet's statement within the context of example 12 and figure 10.9.

When an investor is clueless about which asset will provide the highest return then there exists danger from putting all eggs in one basket. The danger is that *after the fact* the basket is a loser. Diversification offers protection against losers. Inclusion of the minimum risk portfolio in the feasible allocation set, irrespective of outcome, offers a convincing lesson that diversified investing assures a reasonable outcome. Extreme outcomes of component securities in diversified portfolios smooth overall portfolio returns. You may be certain, but don't be disappointed, that investment in a diversified portfolio returns less *after the fact* than if one had simply invested in winners. The problem for most investors (except apparently for Mr. Buffet!) is knowing winners *before the fact*. Hence, diversify!

Lessons in chapter 9 explain that $ROR^{expected}$ is the internal rate of return linking observed asset prices with anticipations and information. Sterile statistical studies cannot predict expected rates of return – modern portfolio theory has limitations providing investment advice. The studies convincingly establish, however, that diversification creates valuable benefits! The importance of modern portfolio theory is not for predicting $ROR^{expected}$, but rather for showing how $ROR^{required}$ relates to diversification benefits. Determination of a security's equilibrium return depends on the diversification benefit that the security contributes to a well-diversified portfolio.

EXERCISES 10.4C

Conceptual

1. You are pretty sure that security *X* will be a winner whereas *Y* will be a loser. You intend to buy one or both of these (but no others). What does the story about "feasible allocation set" imply about investment advice for you?

2. Suppose that you are interested in two securities, call them *X* and *Y*. You have no idea about their expected returns or correlation. You are confident, however, that their risks are identical. You want to make a portfolio from either or both securities. Your only criterion is to create a portfolio that is not dominated, that is, choose a portfolio in the feasible allocation set. Describe what you should do.

3. According to formula 10.13, under what conditions does the minimum risk portfolio involve a short position in one security?

Numerical quickies

4. Throughout the past the return for type *X* stocks has averaged 11.9% and the standard deviation has been 27.6%. For type *Y* stocks the return has averaged 16.2% and the standard deviation 34.8%. The correlation between the returns for these two assets has been 0.13. You expect these tendencies to persist into the future. What is the most comprehensive allocation rule that correctly describes all portfolios in the feasible allocation set? ©MR5

5. The standard deviation of expected returns for investments *X* and *Y* equal 18.0% and 11.0% , respectively. The correlation between returns for *X* and *Y* is 0.40 . Find the combination of *X* and *Y* that yield the minimum risk portfolio. If your objective is to form a portfolio with these two securities that is not dominated by any other combination, what

should you do? ©MR3cm

Challengers

6. Each pair of outcomes listed below is equally likely.

%return Alpha	4.0%	10.0%	24.2%	14.5%
%return Zed	22.4%	26.2%	13.1%	-7.4%

Describe the portfolios that are in the feasible allocation set. ©MR1em

ANSWERS TO CHAPTER 10 EXERCISES

EXERCISES 10.1

1. *Y* is better with regards to *E(ROR)* because it's bigger. *Y* is better with regards to σ because it's smaller. *Y* dominates *X* because it's better in every dimension.

2. For these three securities there are 3 possible pairs of securities to compare.

Compare *X* to *Y*. *X's* risk of 20 is worse than *Y's* risk of 4 (for convenience use "worse" to denote "less preferable"; "better" denotes "more preferable"). *X's* return of 9 is better than *Y's* return of 6. *X* is better in some ways whereas *Y* is better in other ways. *X* and *Y* coexist as tradeoffs.

Compare *X* to *Z*. *X's* risk of 20 is worse than *Z's* risk of 11. *X's* return of 9 is better than *Z's* return of 7. *X* is better in some ways whereas *Z* is better in other ways. *X* and *Z* coexist as tradeoffs.

Compare *Y* to *Z*. *Y's* risk of 4 is better than *Z's* risk of 11. *Y's* return of 6 is worse than *Z's* return of 7. *Y* is better in some ways whereas *Z* is better in other ways. *Y* and *Z* coexist as tradeoffs.

The three comparisons show that each coexists with the other as a tradeoff. That's the bottom line inference. A more thoughtful analysis doesn't change the answer, but it adds a qualification. Movement from *X* to *Z* involves a decline in risk of 9 units and a decline in return of 2 units. That implies a 4.50 tradeoff in risk per unit of return (4.50 = 9/2). Movement from *Z* to *Y* implies a 7 unit decline in risk and 1 unit decline in return, or 7.00 tradeoff of risk per unit of return. An investor at *X* that prefers to reduce risk finds that movement to *Z* reduces risk quite a bit, but that further movement to *Y* provides a lot of risk reduction with very little additional reduction in return. In other words, *X* is the best high risk, high return choice. As far as the low risk, low return choice goes, *Y* and *Z* coexist as tradeoffs, but *Y* is likely (but not definitely) a better choice than *Z*.

3. For this problem there are two kinds of risk and each is bad. Rank the three choices from best to worst for the two risk and 1 return measures (for convenience use "better" to denote "more preferable"; "worse" denotes "less preferable").

	Risk$_1$	Risk$_2$	Return
best	*A & B*	*B*	*C*
middle		*A*	*B*
worst	*C*	*C*	*A*

Now compare the 3 possible pairs of securities:

Compare *A* to *C*. In some ways *A* is better but in other ways *C* is better. *A* and

C coexist as tradeoffs.

Compare *B* to *C.* In some ways *B* is better but in other ways *C* is better. *B* and *C* coexist as tradeoffs.

Compare *A* to *B*. With regards to $Risk_1$ there is a tie between *A* and *B*. *B* is better than *A* with regards to $Risk_2$ and *B* is better with regards to *Return*. Thus, *B* is as at least as good or better than *A* in every way. *B* dominates *A*.

EXERCISES 10.2

1. The *DOL, DFL,* and *DTL* are similar formulas because each is a ratio such as x/(x-y). The variable x equals *(p-v)Q* for the DOL, *EBIT* for the *DFL*, etc. The variable y equals *Total fixed costs* for the DOL, *Interest* for the *DFL*, etc. There seems to be an economic restriction that $y \geq 0$. For example, fixed costs or interest cannot be negative. This means that as the x variable gets larger the ratio converges to 1.0. That is, eventually the x gets so big that y gets swamped. At the upper limit, then, for companies operating well beyond breakeven the effect of a 1% increase in *Sales revenue* is a 1% increase in *EBIT* and a 1% increase in *EAC*.

Consider what happens to the formulas when y consumes more and more of x. For example, as *Total fixed costs* approaches (*Sales revenue – Total variable costs)* then the denominator of the *DOL* formula approaches zero. The *DOL* approaches infinity. But the economic significance of a *DOL* equal to 10.0 probably is the same as for a *DOL* equal to 10 million. Both are relatively large. Furthermore, when x equals y the *DOL* is mathematically undefined. That means it's economically undefined, too, except to say that it's relatively big!

Another noteworthy property of these formulas occurs when the ratio is negative. For example, a company with *Sales revenue* less than operating costs may have a negative *DOL.* A negative *DOL* suggests that a 1% increase in sales leads to a decrease in *EBIT* - but that is false when $p>v$. The upshot is that the *DOL, DFL,* and *DTL* formulas may have no economic meaning when they result in negative numbers.

Despite these impracticalities, the stylized income statement yields some pretty valuable insights for assessing idiosyncratic company risk!

EXERCISES 10.3A

1. $ROR_{win} = \$1,300/\$900 – 1; = 44.44\%$. $ROR_{lose} = -100\%$.. Now find the weighted average: $E(ROR) = 0.50(44.44\%) + 0.50(-100\%); =$ -27.78%

2. First find the rates of return based on number of adoptees. $ROR_0 = -100\%$. $ROR_1 = \$11/\$6 – 1; = 83.33\%$. $ROR_2 = \$17/\$6 – 1; = 183.33\%$. Now find the weighted average: $E(ROR) = 0.40(-100\%) + 0.40(83.33\%) + 0.20(183.33\%); = 30.0\%$. Now find $\sigma = [\ 0.4(-1.0 – 0.30)^2 + 0.4(0.8333 – 0.30)^2 + .2(1.8333 – 0.30)^2\ ]^{0.5}; = 112\%$.

3. This problem is analogous to example 5. Use formulas 10.5 and 10.6 to compute (σ,*E(ROR)*) for company X:(9.77,12.44); and Y:(6.41,14.41). Y is dominant because it's got more return and less risk.

4. This problem is analogous to example 6. Use formulas 10.7 and 10.8 or, preferably, allow the BAII Plus calculator to perform the calculations. Find that (Risk, return) equals (9.09%,10.68%) for X and (7.12%,16.88%) for Y; also Y dominates X.

EXERCISES 10.3B

1. Use formula 10.9 to compute the weighted average (notice that the allocation for stock *Z* equals 30%): $E(ROR_{portfolio})$ = 40%(-12%) + 30%(20%) + 30%(-24%); = -6.0%

2. Use formula 10.9 and solve for the weight in *X*. Notice that the weight for *Y* equals $(1 - w_x)$.

$E(ROR_{portfolio}) = w_x(ROR_x) + (1 - w_x)(ROR_Y)$

$0.152 = w_x(0.059) + (1 - w_x)(0.235)$

$-0.0830 = -0.176w_x$

$w_x = 47.16\%$

The portfolio that allocates 47.16% in *X* and 52.84% in *Y* has expected return of 15.2%.

3. Find the portfolio *ROR* for the different outcomes given a 30% weight in X, 70% in Z. For outcome 1 the portfolio *ROR* = *.30(.10) + .70(.38); = 29.60%.* For outcome 2 the portfolio *ROR* = *.30(.45) + .70(.08); = 19.10%.* Now apply formulas 10.5 and 10.6 to the preceding portfolio *ROR* given that the probability of state 1 is 30% and state 2 is 70%. Compute that $E(ROR_{portfolio})$ = 22.3% and $\sigma_{portfolio}$ = 4.81%.

4. Find the portfolio *ROR* for the different outcomes given a 35% weight in X, 65% in Y. For outcome "declining" the portfolio *ROR* = *.35(-0.019) + .65(-0.065); = -4.89%.* For outcome "flat" the portfolio *ROR* = *.35(0.054) + .65(0.154); = 11.90%.* For outcome "rising" the portfolio *ROR* = *.35(0.177) + .65(0.021); = 7.56%.* Now use formulas 10.5 and 10.6 and apply the state probabilities of 35%, 30%, and 35%, respectively, to the preceding portfolio *ROR.* Compute that $E(ROR_{portfolio})$ = 4.50% and $\sigma_{portfolio}$ = 7.11%.

5. Find the portfolio *ROR* for the different outcomes given a 25% weight in X, 75% in Y. For outcomes in column 1 the portfolio *ROR* = *.25(-0.039) + .75(0.134); = 9.08%.* For outcomes in column 2 the portfolio *ROR* = *.25(0.046) + .75(0.134); = 11.20%.* For outcomes in column 3 the portfolio *ROR* = *.25(0.219) + .75(0.125); = 14.85%.* For outcomes in column 4 the portfolio *ROR* = *.25(0.176) + .75(0.067); = 9.43%.* Now use formulas 10.5 and 10.6 with probabilities of ¼ for each of the preceding portfolio *ROR.* Compute that $E(ROR_{portfolio})$ = *11.14%* and $\sigma_{portfolio}$ = 2.3%.

EXERCISES 10.4A

1. Find below arrows indicating whether the rates of return rise or fall relative to the preceding period. For example, the *ROR* for X is -3.9% in period 1, 4.6% in period 2 (this is ↑ from period 1) and 21.9% in period 3 (this is ↑ from period 2).

	period 1	2	3	4	5
X:		↑	↑	↓	↓
Y:		↓	↓	↑	↑
Z:		↑	↓	↓	↑

Comparison of X and Y shows that in 4-of-4 cases the arrows are in opposite directions. This demonstrates extreme negative correlation. Notice that negative correlation exists between X and Y even though the movement in Y is very slight. The standard deviation of Y is relatively tiny, but its correlation with X is a relatively huge negative number.

Comparison of Y and Z shows that half the time the arrows are in the same direction, half the time they are different. This one is hard to call; computation shows correlation is moderately positive.

Comparison of X and Z shows that in 3-4 cases the arrows are in opposite directions. This suggests moderate-to-strong negative correlation.

2. Just plug the numbers into formula 10.11 and find $\sigma_{portfolio} = \{.8^2(.145^2) + .2^2(.095^2) + 2(.8)(.145)(.2)(.095)(.3)\}^{1/2}$; = 12.3%.

3. Just plug the numbers into formula 10.11 and find the actual risk, $\sigma_{portfolio} = \{.4^2(.21^2) + .6^2(.121^2) + 0\}^{1/2}$; = 11.1%. Use formula 10.10 to find the average risk, $\sigma_{average} = .4(.21) + .6(.121)$; = 15.7%. The diversification benefit, measured as percent reduction in risk, equals 15.7% - 11.1%, or 4.6%.

4. Follow the "Calculator clue" following example 9 and let the calculator automatically compute that σ_x= 9.81%; σ_y= 5.72%; and ρ= -0.9099. Use formula 10.10 to find $\sigma_{average} = .3(.0981) + .7(.0572)$; = 6.94%. Use formula 10.11 to find $\sigma_{portfolio} = \{.3^2(.0981^2) + .7^2(.0572^2) + 2(.3)(.0981)(.7)(.0572)(-0.9099)\}^{1/2}$ 1.80%. The diversification benefit equals *6.94% - 1.80%,* or 514 BP.

5. Apply formula 10.6 to the given *ROR* and find that $\sigma_X = 7.1\%$ and $\sigma_Z = 8.9\%$. Use formula 10.10 to find $\sigma_{average} = .6(.071) + .4(.089)$; = 7.85%. Now obtain the actual portfolio risk. First find the portfolio *ROR* for the different outcomes given a 60% weight in X, 40% in Z. For outcome 1 the portfolio *ROR* = *.60(.10) + .40(.37); = 20.80%.* For outcome 2 the portfolio *ROR* = *.60(.30) + .40(.12); = 22.80%.* Now apply formula 10.6 to the preceding portfolio *ROR* given that the probability of state 1 is 15% and state 2 is 85%. Compute that $\sigma_{portfolio}$ = 0.71%. The diversification benefit is almost total risk reduction, that is, *DB = 7.85% - 0.71%;* = 714 BP.

6. Just plug the numbers into formula 10.11 and find the actual risk, $\sigma_{portfolio} = \{.75^2(.105^2) + .25^2(.215^2) + 2(.75)(.105)(.25)(.215)(-0.20)\}^{1/2}$; = 8.60%. Use formula 10.10 to find the average risk, $\sigma_{average} = .75(.105) + .25(.215)$; = 13.25%. The diversification benefit, measured as percent reduction in risk, equals *13.25% - 8.60%*, or 4.65%.

EXERCISES 10.4B

1. Examples 9 and 10 use the same data. Example 9 finds that with a 40:60% allocation in X:Y the diversification benefit equals 213 basis points. Example 10 finds that the minimum risk portfolio allocates 21.12%X, 78.88% Y and attains diversification benefit of 208 basis points. The objective "minimizing portfolio risk" is *NOT* the same as "maximizing diversification benefit". The minimum risk portfolio occurs at the leftmost tip of the parabola. The maximum diversification benefit occurs where the slope of the parabola exactly equals the slope of the $\sigma_{average}$ line that connects X and Y.

2. Simplify formula 10.11 with $\rho=1$.

$$\sigma_{portfolio} = \{w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + 2\, w_X\sigma_X\, w_Y\sigma_Y)\}^{1/2}$$
$$= \{\, (w_X\sigma_X + w_Y\sigma_Y)(w_X\sigma_X + w_Y\sigma_Y)\,\}^{1/2}$$
$$= w_X\sigma_X + w_Y\sigma_Y$$

Formula 10.10 is

$$\sigma_{average} = w_X\sigma_X + w_Y\sigma_Y$$

Definition 10.6 shows

$$DB = \sigma_{average} - \sigma_{portfolio}$$
$$= w_X\sigma_X + w_Y\sigma_Y - (w_X\sigma_X + w_Y\sigma_Y)$$
$$= 0$$

The diversification benefit from combining two perfectly correlated securities is zero.

3. Plug numbers into formula 10.13 for Large cap *("LC")* and International Stocks *("IS")*.

$$w_{LC}^{min\ \sigma} = \{0.371^2 – (0.328)(0.371)(-0.10)\} / \{0.328^2 + 0.371^2 – 2(0.328)(0.371)(-0.10)\}; \ = 55.6\%$$

For these data the minimum risk portfolio allocates 55.6% in Large caps and 44.4% in International stocks.

4. Plug numbers into formula 10.13 for X and Y.

$w_X^{min\ \sigma} = \{0.22^2 - (0.14)(0.22)(-0.50)\} / \{0.14^2 + 0.22^2 - 2(0.14)(0.22)(-0.50)\}; = 64.6\%.$
The minimum risk portfolio allocates 64.6% in X and 35.4% in Y.

Now plug into formula 10.11 and find the actual risk, $\sigma_{portfolio} = \{.646^2(.14^2) + .354^2(.22^2) + 2(.646)(.14)(.354)(.22)(-0.50)\}^{1/2}; = 8.50\%$. Use formula 10.10 to find the average risk, $\sigma_{average} = .646(.14) + .354(.22); = 16.85\%$. The diversification benefit, measured as percent reduction in risk, equals *16.85% - 8.50%*, or 835 BP.

5a. Let your calculator automatically compute that $\sigma_{Alpha} = 8.46\%; \sigma_{Zed} = 11.86\%; \rho = -0.6803.$ Plug numbers into formula 10.13 for Alpha and Zed.

$w_{Alpha}^{min\ \sigma} = \{0.1186^2 - (0.0846)(0.1186)(-0.6803)\} / \{0.0846^2 + 0.1186^2 - 2(0.0846)(0.1186)(-0.6803)\}; = 59.9\%.$
The minimum risk portfolio allocates 59.9% in Alpha and 40.1% in Zed.

5b. Now plug into formula 10.9 and find the actual return, $E(ROR_{portfolio}) = .599(.1030) + .401(.1298); = 11.40\%.$

Now plug into formula 10.11 and find the actual risk, $\sigma_{portfolio} = \{.599^2(.0846^2) + .401^2(.1186^2) + 2(.599)(.0846)(.401)(.1186)(-0.6803)\}^{1/2}; = 3.90\%.$

5c. Use formula 10.10 to find the average risk, $\sigma_{average} = .599(.0846) + .401(.1186); = 9.82\%$. The diversification benefit, measured as percent reduction in risk, equals *9.82% - 3.90%*, or 592 BP.

EXERCISES 10.4C

1. Financial science suggests that you dig deeper into the facts. The feasible allocation set includes portfolios that prudently tradeoff risk for return. The facts above state absolutely nothing about risk. Reconsider the possible outcomes. If you are 100% confident that *X* will be winner and *Y* a loser then definitely *X* is in the feasible allocation set. Go for it! But are you really 100% confident?

2. All that you know is $\sigma_X = \sigma_Y$, just call it σ. Solve formula 10.13

$$w_X^{min\ \sigma} = \{\sigma^2 - \sigma\,\sigma\,\rho\} / \{\sigma^2 + \sigma^2 - 2\sigma\,\sigma\,\rho\}$$
$$= \sigma^2\{1 - \rho\} / \{2\sigma^2(1 - \rho)\}$$
$$= 1/2$$

The minimum risk portfolio invests half all assets in *X,* half in *Y.* That strategy is the only one that for sure is not dominated. When you're uncertain how to invest then diversification is wise.

3. Solve formula 10.13 by setting $w_X^{min\ \sigma}$ equal to 1. This is a sort of breakeven point beyond which one security is *super*long and the other is *short*.

$$1.0 = \{\sigma_Y^2 - \sigma_X\,\sigma_Y\,\rho\} / \{\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\,\sigma_Y\,\rho\}$$
$$\{\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\,\sigma_Y\,\rho\} = \{\sigma_Y^2 - \sigma_X\,\sigma_Y\,\rho\}$$
$$\sigma_X^2 = \sigma_X\,\sigma_Y\,\rho$$
$$\rho = \sigma_X \div \sigma_Y$$

When the correlation coefficient equals the ratio of standard deviations then the allocation at the minimum risk portfolio is 100% for one security, 0% for the other. When ρ exceeds the ratio of standard deviations then the minimum risk portfolio involves a short position in one security.

For example, suppose $\sigma_X = 12\%$ and $\sigma_Y = 15\%$. The ratio of standard deviations is 12/15, or 0.80 (put the largest on bottom so that the ratio, like ρ, is less than one). If ρ were 0.80, solve for the allocation to *X* at the minimum risk portfolio.

$$w_X^{min\ \sigma} = \{.15^2 - (.12)(.15)(0.8)\} / \{(.12^2) + (.15^2) - 2(.12)(.15)(0.8)\}$$
$$= 100\%$$

The weight $w_Y^{min\ \sigma}$ is 0%. When ρ equals the ratio of standard deviations the minimum risk portfolio includes only one security (the tip of the parabola is the point representing the smallest σ security).

Suppose that with $\sigma_X = 12\%$ and $\sigma_Y = 15\%$ the ρ were 0.90. Solve for the allocation to *X* at the minimum risk portfolio.

$$w_X^{min\ \sigma} = \{.15^2 - (.12)(.15)(0.9)\} / \{(.12^2) + (.15^2) - 2(.12)(.15)(0.9)\}$$
$$= 140\%$$

The weight $w_Y^{min\ \sigma}$ is -40%; the negative sign means a short position. With a relatively large ρ the minimum risk portfolio involves a short position in one security.

4. Solve for the allocation to *X* at the minimum risk portfolio.

$$w_X^{min\ \sigma} = \{.348^2 - (.276)(.348)(0.13)\} / \{(.276^2) + (.348^2) - 2(.276)(.348)(0.13)\}$$
$$= 63.2\%$$

The weight $w_Y^{min\ \sigma}$ is *36.8*%. Because the setup states $E(ROR_X) < E(ROR_Y)$ then always allocate 63.2% or less in *X*. Stating this same rule from the other perspective, always allocate 36.8% or more in *Y*.

5. Solve for the allocation to *X* at the minimum risk portfolio.

$$w_X^{min\ \sigma} = \{.11^2 - (.18)(.11)(0.4)\} / \{(.18^2) + (.11^2) - 2(.18)(.11)(0.4)\}$$
$$= 14.6\%$$

The weight $w_Y^{min\ \sigma}$ is 85.4%. The setup does not provide the expected returns. Definitely the allocation (w_X, w_Y) of (14.6%,85.4%) is in the feasible allocation set. Otherwise, weighting one or the other security more heavily requires confidence. With confidence that $E(ROR_X) > E(ROR_Y)$ then always allocate 14.6% or more in *X*. Conversely, with confidence that $E(ROR_X) < E(ROR_Y)$ then always allocate 85.4% or more in *Y*. Moving away from the a well-diversified portfolio requires confidence in the ability to pick high return assets.

6. Let the calculator compute all statistics for you: $E(ROR_{Alpha}) = 13.18\%$; $\sigma_{Alpha} = 7.38\%$; $E(ROR_{Zed}) = 13.58\%$; $\sigma_{Zed} = 13.01\%$ and $\rho = -.4013\%$. Solve for the allocation to *Alpha* at the minimum risk portfolio.

$$w_{Alpha}^{min\ \sigma} = \{.1301^2 - (.0738)(.1301)(-.4013)\} / \{(.0738^2) + (.1301^2) - 2(.0738)(.1301)(-.4013)\}$$
$$= 69.1\%$$

The weight $w_{Zed}^{min\ \sigma}$ is 30.9%. Because $E(ROR_{Alpha}) > E(ROR_{Zed})$ then always allocate 69.1% or more in *Alpha*. Stating this same rule from the other perspective, always allocate 30.9% or less in *Zed*. Note in passing, however, that $E(ROR_{Alpha}) \approx E(ROR_{Zed})$ so, given these expectations, all portfolios containing these two securities have approximately the same return. For a given return investors generally should minimize risk.

CHAPTER 11: THE PRICE OF RISK, COST OF CAPITAL, AND RISK-ADJUSTED ROR

Companies and households allocate resources toward many different types of capital investments. Companies may own a variety of brands, patents, factories, or even subsidiary companies. Mutual funds own many diverse securities. Households own human capital, housing, and an array of financial securities. For any company, regardless of definition, their set of capital investments represents a diversified portfolio. Diversification reduces or eliminates idiosyncratic risk. Diversification benefit *DB* equals difference between actual portfolio risk and average risk of components. *DB* is a measurement for risk reduction. Because companies and investors are risk averse then risk reduction is valuable. In competitive and perfect capital markets there exists an equilibrium price for risk reduction. This chapter investigates the relation between diversification benefits, equilibrium risk premia and determination of equilibrium financial rates of return.

1. The efficient frontier and the market price for risk

Figure 11.1 shows in panel 1 the risk-return profile for a two security portfolio. The profile shows that between A and B there is no dominance. Instead, risk trades-off for return between relatively low risk B and high risk A. The *feasible allocation set* of non-dominated portfolios containing securities A and B is the upper surface of the parabola. The profile also shows that the 100% B portfolio is not in the feasible allocation set. Instead, the investor that prefers a low risk and return position, say because of life-cycle effects, is better off to create a diversified portfolio by selling a little bit of low risk B and buying high risk A. Overall portfolio risk actually declines while return increases.

There are a lot more than two securities available for investment. Panel 2 overlays the risk-return profile for securities C and D. That risk-return profile has its own feasible allocation set. Similarly, there exist risk-return profiles and feasible allocation sets for two-security portfolios containing exclusively A and C, or A and D, or B and C. Combine 3 securities, say A+B+C, and there exists yet another feasible allocation set that projects a parabola onto panel 2. There are more than 10,000 publicly traded equities in the U.S.A., plus thousands of different bonds, and literally untold capital investment opportunities. For all of them exist little parabolas that map the feasible allocation sets. Panel 3 figuratively shows all of them. Panel 3 also shows that despite their number there nonetheless exists a *top of tops*.

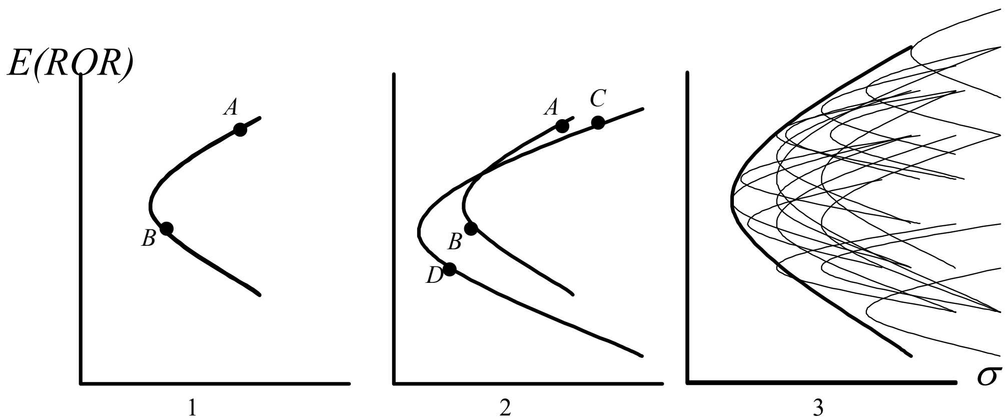


FIGURE 11.1 Risk-return profiles and the Efficient frontier

The set of points along the upper surface in panel 3 made from the highest reaches of all component feasible allocation sets is the *efficient frontier*.

DEFINITION 11.1 *Efficient frontier*
The *efficient frontier* is the set of portfolios that is not dominated and is formed by allocating across all possible risky capital investments in varying weights.

Discoverer of the efficient frontier concept is Harry Markowitz. For this work and subsequent development of *modern portfolio theory* Harry Markowitz received the 1990 Nobel Prize in Economic Science. Investment performance measurements often compare risk and return of specific investment strategies with the efficient frontier. A money manager is aware that obtaining a return and subjecting clients to an amount of risk that places his or her fund in the interior of the efficient frontier is not very good. Portfolios in the interior are dominated because their return is subpar compensation for their risk. Performance anywhere on the efficient frontier is excellent. Interpretation thus far suggests that all portfolios on the frontier coexist as trade-offs, none is better than the other, and the one that is best for any specific investor depends on personal risk-return preferences.

Next stop in development of modern portfolio theory is introduction of the risk-free asset. Figure 11.2 shows how this seemingly trivial technicality dramatically alters interpretations.

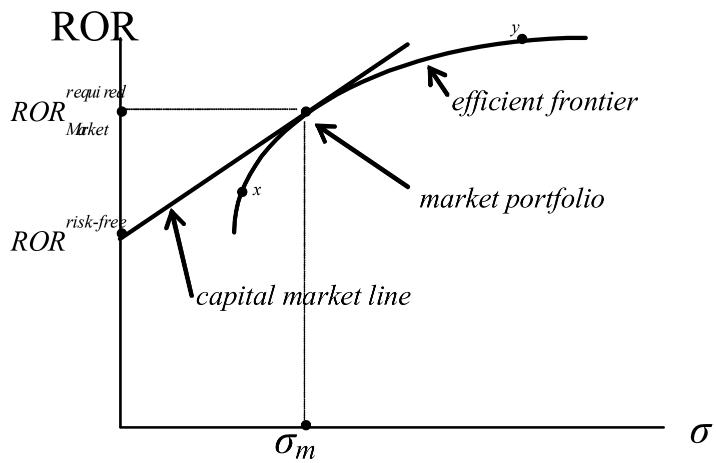


FIGURE 11.2 Efficient frontier and the Capital market line

The risk-free rate of return $ROR^{risk\text{-}free}$ is a certain outcome that is uncorrelated with anything in the short-term. Chapter 9 reports that U.S. Treasury bills often are regarded as the world's most risk-free security. Purchase a T-bill and be assured of the rate of return irrespective of whether the stock market is up or down, or interest rates rise or fall, or economic activity swoons or booms. For the risk-free asset $\sigma_{rf} = 0$.

Consider an investment that forms a two-asset portfolio by combining the risk-free asset with portfolio *X* on the efficient frontier. Find shape of the resultant risk-return profile by applying formula 10.11 for actual portfolio risk:

$$\sigma_{portfolio} = \left\{ w_X^2 \sigma_X^2 + w_{rf}^2 \sigma_{rf}^2 + 2 w_X \sigma_X w_{rf} \sigma_{rf}\, \rho_{X,rf} \right\}^{1/2} .$$

But the risk-free asset has σ_{rf} of zero and furthermore risk-free return is uncorrelated with anything so $\rho_{x,rf}$ equals zero, too. Simplify the preceding formula and find

$$\sigma_{portfolio} = w_X \sigma_X \quad .$$

Allocate 20% of funds to *X* and 80% at $ROR^{risk\text{-}free}$. Actual portfolio risk equals 20% of σ_x. A 50% allocation gets 50% of *X*'s risk; 90% in *X* gets 90% of *X*'s risk. In other words, combining the risk-free asset with risky investment *X* yields a risk-return profile that is a straight line-of-averages connecting points $ROR^{risk\text{-}free}$ and *X*.

Notice in figure 11.2 that the straight-line connecting points $ROR^{risk\text{-}free}$ and *X* (not shown) lies beneath the bold line coming out of $ROR^{risk\text{-}free}$ that is shown. Notice also that the straight-line connecting points $ROR^{risk\text{-}free}$ and *Y* (not shown) lies beneath the bold line. The bold line extending from $ROR^{risk\text{-}free}$ and *tangent* to the efficient frontier is special because it contains portfolios dominating all others on the diagram. This line is the *capital market line*.

DEFINITION 11.2 ***Capital market line***
The *capital market line* is the set of portfolios that combine the risky portfolio at point of tangency on the efficient frontier with the risk-free asset. Portfolios on the capital market line are not dominated by any other capital investments.

Interpretation now suggests that one specific portfolio on the efficient frontier is better than all others. That specific portfolio at the point of tangency is the *market portfolio.* Market portfolio components include all possible risky capital assets where the weight in any one equals that asset's total market capitalization as a proportion of total global market cap. Nobody in the world really thinks that the *true* market portfolio actually exists in anybody's holdings. Investment constraints and a litany of causes suggest that mixing the market portfolio with the risk-free asset is an ideal but impractical outcome. Practically speaking, however, many financial professionals think of the market portfolio as a broad collection of securities like those in the SP500. Despite its impracticalities the capital market line concept enables important insights.

Let σ_m and $ROR^{required}_{Market}$ denote risk and required rate of return for the market portfolio. Slope of the capital market line is easily found as rise over run.

FORMULA 11.1 Market price for risk ©AP11b .
The slope of the capital market line measures the equilibrium *price for risk*.

$$\begin{pmatrix} slope\ of\ the \\ capital\ market\ line \end{pmatrix} = \frac{ROR^{required}_{Market} - ROR^{risk-free}}{\sigma_m}$$

$$= \frac{market\ risk\ premium}{\%total\ market\ risk}\ .$$

Recall formula 10.1 defining $ROR^{required}$ and realize that the numerator of formula 11.1 measures required *risk premium* for the market portfolio. The denominator measures market portfolio risk. Slope of the capital market line measures required risk premium per unit of risk for the market portfolio and represents the equilibrium *price for risk*.

Suppose the risk-free rate on T-bills, for example, is 5% and the required return on the market portfolio is 12%. The market risk premium equals 7% (= 12% - 5%). Investors for this scenario choose between investing in risk-free T-bills offering 5% or investing in the risky market portfolio returning 12% (or some combination of both). The 7% risk premium is the consensus required compensation for bearing extra risk. Suppose also that σ_m = 21%. Slope of the capital market line equals 1/3 (= 7% ÷ 21%). The market price for one percentage point of risk is 33 basis points (= 1/3 × 1%). Equilibrium trade-off on a $100 marginal investment for accepting an extra percentage point of risk is 33 cents of extra return (= $100 × 0.0033).

Risk-free rate of return $ROR^{risk\text{-}free}$ has a special place on the capital market line and plays a significant role for financial equilibrium. $ROR^{required}$ for any capital investment equals risk-free rate $ROR^{risk\text{-}free}$ plus a risk premium (see formula 10.1). Chapter 10 also reports that at the limit, two types of risk exist: risk that can be managed or eliminated (idiosyncratic) and risk that can't (systematic). Formula 11.2 restates $ROR^{required}$ as a function of idiosyncratic and systematic risk.

FORMULA 11.2 Required rate of return $ROR^{required}$ and component risk premia

The "required rate of return" is the minimum discount rate that an investor willingly accepts for computing intrinsic value. The required rate of return for any capital investment *A* denoted $ROR_A^{required}$ equals the risk-free rate plus that asset's risk premium:

$$ROR_A^{required} = ROR^{risk\text{–}free} + (security\ risk\ premium)_A$$

where $ROR^{risk\text{-}free}$ is the short-term risk-free rate and

$$\begin{pmatrix} security \\ risk\ \ premium \end{pmatrix}_A = f\left\{ \begin{pmatrix} systematic \\ risk \\ premium \end{pmatrix}_A, \overbrace{\begin{pmatrix} liquidity \\ risk \\ premium \end{pmatrix}_A, \begin{pmatrix} term \\ risk \\ premium \end{pmatrix}_A, \begin{pmatrix} default \\ risk \\ premium \end{pmatrix}_A}^{sources\ of\ idiosyncratic\ risk} \right\}$$

The implicit function *f{.}* depends upon all possible sources of systematic and idiosyncratic risk in ways that are not fully understood. The function shows one source of systematic risk (but there may be more) and three sources of idiosyncratic risk (these are the main ones but there may be more of these, too). The systematic risk premium relates directly to the market price for risk from the *Capital market line.* Security investment occurs when $ROR^{required}$ is less than $ROR^{expected}$ (see rule 9.1).

Formula 11.2 expresses the risk premium as a function of different risk sources. The explicit mechanism by which risk premium depends on risk sources is unknown. Hence, the function is *implicit*. Many times finance practitioners adopt an *ad hoc* approach and simply add together premia for different factors that affect a specific security. For example, perhaps an analyst concludes that a specific small cap stock commands a 7% systematic risk premium and 1½% liquidity risk premium for a total risk premium of 850 basis points. Find $ROR^{required}$ by adding 8.5% to the risk-free rate and consider the investment a buy if $ROR^{expected} > ROR^{required}$.

While much remains unknown about how different risk sources determine the security risk premium a lot nonetheless is known. Later lessons relate systematic risk premia to the market price of risk. First, however, consider determination of the benchmark risk-free rate of return.

EXERCISES 11.1

1. Analysts tell you that the risk-free rate of return equals 5.0% and the market portfolio's required rate of return and risk (standard deviation) equal 11.0% and 24%, respectively. Compute according to the *Capital market line* the equilibrium price for risk. For an increase in personal portfolio risk of five percentage points (and no extra diversification benefit) how much is the increase in required risk premium. ©AP11b .

2. The risk-free rate of return equals 3.0% and the market portfolio's required rate of return and risk (standard deviation) equal 12.5% and 14.0%, respectively. Suppose that the equilibrium price for risk computes according to the *Capital market line*. Your objective is to combine the risk-free asset with the market portfolio in order to create a portfolio with required return equal to 4.9%. Find the allocation that satisfies your objective. ©AP12 .

3. The risk-free rate of return equals 3.5% and the market portfolio's required rate of return and risk (standard deviation) equal 8.0% and 17.0%, respectively. Suppose that the equilibrium price for risk computes according to the *Capital market line*. Your objective is to combine the risk-free asset with the market portfolio in order to create a portfolio with standard deviation of returns equal to 12%. Find the allocation that satisfies your objective. ©AP13 .

4. The risk-free rate of return equals 4.0% and the market portfolio's required rate of return and risk (standard deviation) equal 8.5% and 16.5%, respectively. Suppose that the equilibrium price for risk computes according to the *Capital market line*. Your objective is to combine the risk-free asset with the market portfolio in order to create a portfolio that earns a risk premium of 3.9%. Find the allocation that satisfies your objective. ©AP14 .

2. Equilibrium rates of return for credit market securities

Financial studies establish that over time market rates of return and risk premia change. Causes relate to shifts in investor confidence, overall market liquidity, and any of myriad factors influencing buy-side demand or sell-side supply for financial securities. Preceding lessons establish that $ROR^{risk\text{-}free}$ is a core rate important for explaining financial equilibrium. Figure 11.3 shows supply and demand schedules driving determination of the short-term risk-free rate $ROR^{risk\text{-}free}$. The explanation below is adaptation of a theory described by Irving Fisher in 1930 and coined the *loanable funds theory of interest*.

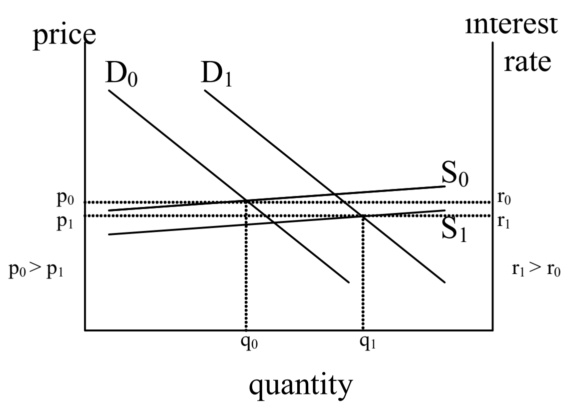


FIGURE 11.3 Supply and demand schedules for risk-free credit market securities

Buy-side demand for securities is shown as demand schedule D_0. Buy-side demand emanates from institutional investors plus households. Participants on the buy-side for securities are investors, also known as savers. The buy-side for *securities* is another way of describing the supply-side for *loanable funds*.

Sell-side supply for credit market *securities* generally includes the companies and government entities that issue securities for borrowing money. For exclusive consideration of risk-free securities, however, the only supplier is the U.S. Treasury. The Treasury competes with private sector companies for *loanable funds*. Supply schedule S_o for risk-free securities represents demand for loanable funds. Slope of the supply schedule is important for quantitative insights but for most qualitative lessons below the steepness is irrelevant. It's drawn flat (elastic) to look similar to figure 9.1 in the discussion about buy-side demand and sell-side supply for financial securities.

With original supply and demand schedules S_0 and D_o the equilibrium price and quantity of risk-free securities equal p_0 and q_0, respectively. Time value lessons teach that for a given cash flow stream there is a one-to-one relation between price and rate of return. Risk-free rate r_0 associates with price p_0. For example, with a risk-free security maturing in 1 year and equilibrium price p_0 of $950 (for principal of $1,000) equilibrium risk-free rate r_0 equals 5.26% (= $1,000 ÷ $950 – 1, see formula 4.5).

Over time stuff happens that shifts supply and/or demand schedules. For the moment suppose they shift to S_1 and D_1. The figure illustrates that the equilibrium price and quantity change to p_1 and q_1, respectively. For the way these curves are drawn (supply is pretty flat) there is a relatively large increase in equilibrium quantity. Equilibrium price, however, declines only slightly ($p_0 > p_1$). Suppose, for example, that price declines from p_0 of $950 to p_1 of $940. The new equilibrium risk-free rate r_1 equals 6.38% (= $1,000 ÷ $940 – 1). Price relates inversely with interest rates, $r_1 > r_0$, and numbers along the right "Interest rate" vertical axis get bigger toward the bottom.

Chapter 9 discusses buy-side participants and factors affecting them. A few buy-side considerations are mentioned here.

Credit market competition for loanable funds: During an economic expansion companies borrow money for purchasing plant and equipment and other factors of production from stakeholders. Households, too, borrow money for improvements and competition for loanable funds heats up. Private sector borrowing crowds out the government and the demand curve D_0 shifts left as private credit market securities substitute in portfolios for risk-free securities. For a given supply schedule S_0 the equilibrium security price declines and equilibrium $ROR^{risk\text{-}free}$ increases (irrespective of whether the supply schedule slope is steep or gradual). Conversely, during economic contractions there are not many private credit market securities, risk-free securities become the only game in town, demand curve shifts right toward D_1 and equilibrium $ROR^{risk\text{-}free}$ declines.

Confidence or nervousness: Political and economic world events sometimes trigger a flight to quality. Risk-free Treasury securities are the safest, highest quality possible investment. Nervousness causes the demand curve to shift right, say to D_1. For a given supply schedule S_0 the equilibrium price rises and short-term risk-free rate $ROR^{risk\text{-}free}$ declines (irrespective of supply schedule slope). Conversely, an increase in buy-side confidence for alternative investments shifts the demand curve for risk-free securities left, equilibrium price falls, and equilibrium $ROR^{risk\text{-}free}$ rises.

Now consider outcomes when there is a shift in sell-side supply of risk-free credit market securities.

Government surpluses and deficits: When tax revenues are insufficient for paying government expenditures then the Treasury issues securities and borrows loanable funds. This shifts the supply schedule for risk-free credit market securities rightward toward S_1. For a given demand schedule D_0 the equilibrium price declines and risk-free rate $ROR^{risk\text{-}free}$ rises (irrespective of supply schedule slope). Conversely, when the rare surplus occurs then the supply curve shifts left, equilibrium price rises, and equilibrium $ROR^{risk\text{-}free}$ falls.

Preceding explanations are simplifications of complex economic phenomena about which many complications exist. Especially confusing is when both supply and demand curves shift or when feedback effects exist. When both schedules shift then qualitative outcomes depend on relative steepness of the schedules. Consider complicating feedback effects from government deficits. Perhaps Congress cuts taxes and the deficit grows (*S* shifts right and $ROR^{risk\text{-}free}$ rises) but the tax cut stimulates private sector economic activity (*D* shifts left and $ROR^{risk\text{-}free}$ rises) and then eventually tax revenues rise (and *S* shifts left) thereby pushing $ROR^{risk\text{-}free}$ lower than it began. There are many stories that one can tell about complex economic phenomena, some more plausible than others. Seldom are outcomes definite and certain.

2.A. Risk-free rate of return and the term premium

By definition the rate $ROR^{risk\text{-}free}$ has zero risk premium. Satisfying the definition requires a very short-term security that is very liquid with absolutely no default risk. For most applications short-term Treasury bills satisfy the definition. Financial markets received a tiny jolt toward year-end 2004 when political deadlock in Congress led to inaction raising the government debt ceiling. That led to the remote possibility that the Treasury would lack authority to borrow additional money necessary for repaying principal coming due. Inability to repay principal means *default!* Financial mavens around the globe questioned whether U.S. Treasury securities should continue to be considered default free or whether instead investors should insist on a default risk premium. A rising default risk premium increases government borrowing costs. Congress quit bickering and raised the debt ceiling, the Treasury issued securities to satisfy current obligations, and the question receded to the back burner. U.S. government shutdowns of recent years have not defaulted on full faith and credit issuances of interest or principal payments. Competitive markets, not the government, determine T-bill rates (the government decides how much to borrow). Perhaps buy-side demand for Treasury securities requires a tiny default premium but $ROR^{risk\text{-}free}$ does not.

Formula 11.2 shows that $ROR^{required}$ depends on systematic and idiosyncratic risk sources. For the risk-free rate the default and liquidity risk premia are nil and, for reasons discussed later, the systematic risk premium also is zero. There is, however, a term risk premium within $ROR^{risk\text{-}free}$. Figure 11.4 allows comparison of interest rates for 20-year Treasury bonds and 30-day bills with the inflation rate and high-quality corporate bond rates. Rates on the 20-year Treasury bond average about 1.25% higher than the 30-day rate. The term premium, as chapter 9 explains, depends on many dynamic factors among which inflation and future expectations are most important. The 30-day T-bill rate is an annual rate and even though inflation over 30 days is not “huge” the inflation premium gets big when annualized. With double-digit inflation around 1980 annual rates on 30-day T-bills exceeded 14½%.

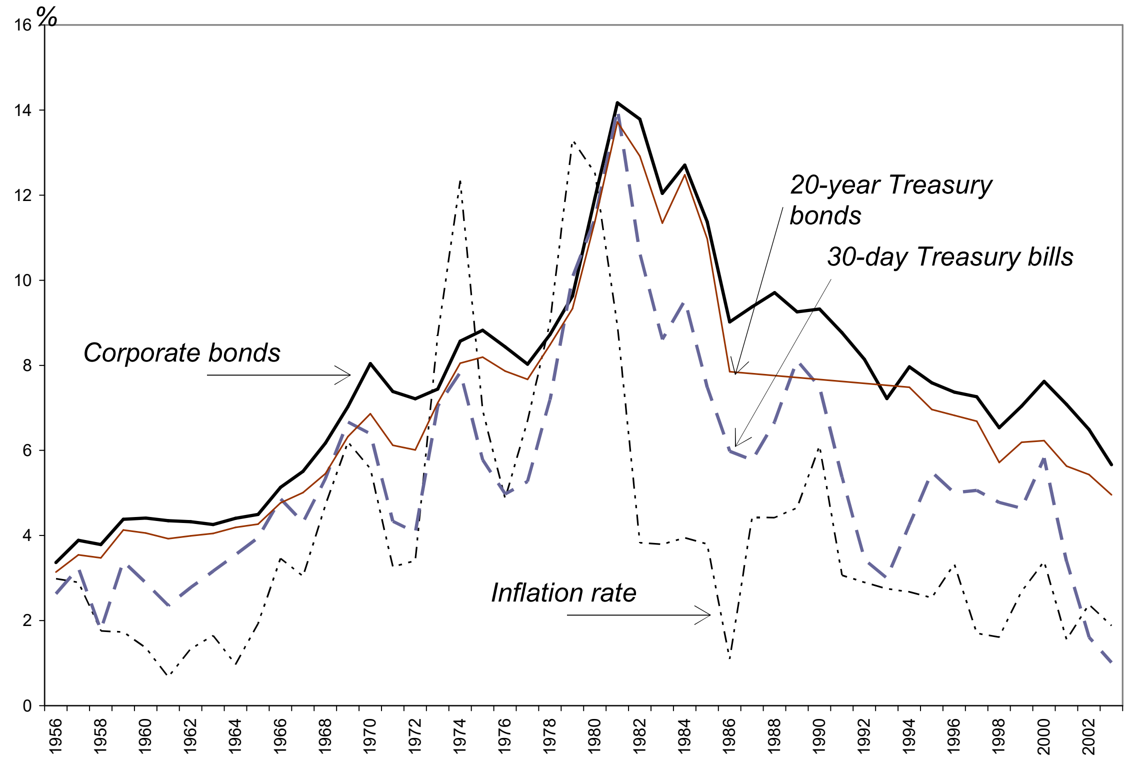


FIGURE 11.4 Inflation and interest rates for corporate bonds and U.S. government securities.
Notes: The *inflation rate* is the annual percentage change in Consumer Price Index for All Urban Consumers: All Items. *Corporate bond rate* is the annual average of a monthly top grade bond yield.

Risk-free rates exclude risk premia for all sources of systematic and idiosyncratic risk *except* term. The term risk premium relates to *time* and risk-free securities embody time so they cannot be *totally* risk-free. They are, however, free of default risk, liquidity risk, etc. Throughout the past half-century variation in short-term risk-free rates occurs largely because of the inflation component in the term premium. Formula 11.3 shows $ROR_N^{risk\text{-}free}$, the equilibrium risk-free rate for a highly liquid and default-free security that matures in *N* periods.

FORMULA 11.3 Nominal risk-free rate $ROR^{risk\text{-}free}$ and the *inflation premium*
The observable short-term, very liquid, zero coupon, default free interest rate is *nominal* $ROR^{risk\text{-}free}$. The relation between the very short risk-free rate and maturity in *N* periods ($N \geq 1$), denoted $ROR_N^{risk\text{-}free}$, approximates as follows:

$$ROR_N^{risk-free} = \begin{pmatrix} short-term \\ real\ risk-free \\ interest\ rate \end{pmatrix} + \begin{pmatrix} term\ risk\ premium \\ for\ N-period \\ horizon \end{pmatrix}$$

For the special case when the inflation premium is the only component in the term risk premium:

$$ROR_N^{risk-free} = \begin{pmatrix} short-term \\ real\ \ risk-free \\ interest\ \ rate \end{pmatrix} + \left(\sum_{t=1}^{N} \frac{(inflation\ \ rate)_t}{N} \right).$$

The *inflation premium* equals the arithmetic average periodic inflation rate expected throughout term *N*. Short-term nominal risk-free interest rate $ROR^{risk\text{-}free}$ is identical to $ROR_1^{risk\text{-}free}$.

Today a significant amount of complex financial research is investigating determinants of term risk. Future research undoubtedly will reveal many important and fundamental lessons about the term structure of risk premia. For current purposes, however, consider the special case when inflation is the only component in the term risk premium.

EXAMPLE 1 Find risk-free interest rates given the real rate and expected inflation rates

©AP15 The short-term real risk-free interest rate averages 4%. Suppose that expected inflation is 3% over the next year, 6% during the second year, and 8% thereafter for 2 years after which it drops to 5% per year perpetually. Given that inflation is the only component of the term premium for risk-free securities find today's interest rates for risk-free securities with terms of 1 to 5 years. Also, what is the rate on a 20-year risk-free security?

SOLUTION

The risk-free rate $ROR_N^{risk\text{-}free}$ equals 4% plus the inflation premium over the next *N* years. That means $ROR^{risk\text{-}free}$ is 7% (that is, $ROR_1^{risk\text{-}free}$ = 4% + 3%). Inflation premium on a two-year bond is 4.5% [= (3% + 6%)/2] so the two-year bond rate $ROR_2^{risk\text{-}free}$ is 8.5%. Inflation premium on a three-year bond is 5.7% [= (3% + 6% + 8%)/3] so the three-year bond rate $ROR_3^{risk\text{-}free}$ is 9.7%. Likewise, for 4 and 5-year risk-free securities the rates equal 10.25% [= 4% + (3% + 6% + 8%+ 8%)/4] and 10.0% [= 4% + (3% + 6% + 8%+ 8% + 5%)/5], respectively. The 20-year risk-free rate $ROR_{20}^{risk\text{-}free}$ is 9.25% [= 4% + (3% + 6% + 8%+ 8% + 16 × 5%)/20].

2.B. Other credit market securities

Chapter 9 discusses the many types of credit market securities and figure 11.5 shows their share of the U.S. credit market. Two types of securities sum to more than half the credit market: U.S. government securities (35%) and mortgages (23%). Already the preceding subsection discusses required returns for risk-free Treasury securities. Financial analysts often compute required rates of return for credit market securities by adding risk premia to the risk-free return.

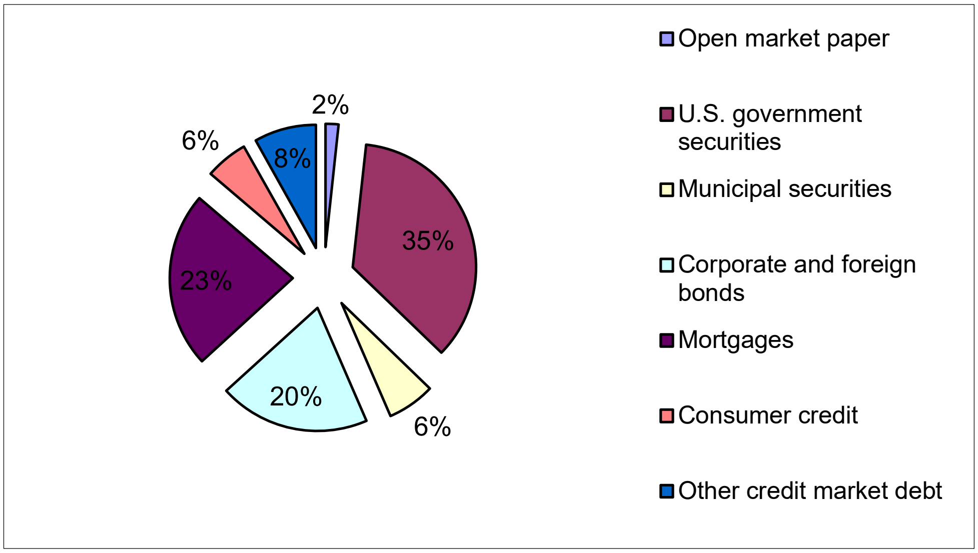


FIGURE 11.5 Components of U.S. credit market securities

Notes: ***The credit market total is $57,982 billion at 9/30/2014. Data are from the Board of Governors of the Federal Reserve System, "Flow of Funds Accounts for the United States", table L.4.***

Two procedures exist for adding risk premia to the risk-free rate and they differ quantitatively. One approach adds risk premia to short-term $ROR^{risk\text{-}free}$ per formula 11.2. The other approach adds risk premia to $ROR_N^{risk\text{-}free}$ where N is the term of the credit market security under analysis. The difference pertains exclusively to handling the term risk premium. Strict application of formula 11.2 requires measuring the term premium for the security under analysis. For example, the term premium for a 20-year mortgage equals the excess by which the 20-year mortgage rate exceeds the rate on an otherwise identical short-term mortgage security. On the other hand if the analyst believes that $ROR_{20}^{risk\text{-}free}$ properly embodies the term risk premium applicable to a 20-year mortgage then the analyst adds to the 20-year Treasury rate a default risk premium, liquidity risk premium, etc. The spread between long-term and short-term mortgages often correlates over time with the spread between long-term and short-term Treasury securities. But sometimes it doesn't. An internet search for "risk premium" finds hundreds of thousands of hits. The issue is extremely important to companies as well as to finance professors – much remains unknown about the behavior and determination of risk premia.

EXERCISES 11.2

1. The short-term real risk-free interest rate averages 3.0%. Suppose that expected inflation is 5.6% over the next year, 5.9% during the second year, and 6.2% thereafter perpetually. Inflation is the only component of the term premium for risk-free securities. Find today's interest rates for risk-free securities with terms of 2 years, 4 years and 20-years. ©AP15 .

3. Equilibrium rates of return for equity market securities

Formula 11.2 properly specifies the required rate of return and risk premium for any and all capital investments. For equity securities previous lessons on diversification benefits are very helpful. For the special limiting case when all sources of idiosyncratic risk distribute like white-noise across all securities then diversification completely eliminates idiosyncratic risk. Only systematic risk remains in the market portfolio. For that special limiting case formula 11.2 simplifies as shown below.

FORMULA 11.4 Systematic risk premium and the market price for risk

The required rate of return for any capital investment *A* denoted $ROR_A^{required}$ equals short-term risk-free rate $ROR^{risk\text{-}free}$ plus that asset's risk premium. When idiosyncratic risk may be eliminated completely through diversification and only one source of systematic risk exists then:

$$ROR_A^{required} = ROR^{risk-free} + \overbrace{\rho_{A,Market}\ \sigma_A \left(\frac{ROR_{market}^{required} - ROR^{risk-free}}{\sigma_m} \right)}^{systematic\ risk\ premium\ for\ security\ A}$$

$$= ROR^{risk-free} + \rho_{A,Market}\ \sigma_A \begin{pmatrix} market\ price \\ for\ risk \end{pmatrix}.$$

Rates of return for security *A* carry risk σ_A. Correlation between rates of return for *A* and the market portfolio equals $\rho_{A,Market}$. The *market price for risk* equals slope of the *Capital market line* and represents required return per unit of risk. The correlation coefficient $\rho_{A,Market}$ measures the proportion of A's risk that requires the market price for risk.

When all idiosyncratic risk vanishes through diversification then idiosyncratic risk is irrelevant for determining required rates of return. This does not mean that idiosyncratic risk does not exist. Rather, idiosyncratic risk merits zero compensation because it vanishes in well-diversified portfolios. Definition 10.2 states that systematic risk is unaffected by diversification strategies but that security sensitivities to systematic risk vary. Correlation coefficient $\rho_{A,Market}$ directly measures that sensitivity.

Risk for security *A* equals σ_A. Lessons from chapter 9 show that combining two assets creates diversification benefit *DB*. As correlation between assets diminishes then *DB* increases. Formula 11.4 shows that all of σ_A does not merit compensation because adding *A* to the market portfolio possibly creates *DB* and some of *A's* risk vanishes. Correlation coefficient $\rho_{A,Market}$ measures proportion of σ_A that receives the market price for risk.

EXAMPLE 2 Find required rate of return given the risk-free rate and summary statistics

Suppose the risk-free rate on T-bills is 5% and the required return on the market portfolio is 12%. Suppose also that σ_m = 21% and that risk for security *X* is σ_x = 27%. Correlation $\rho_{X,Market}$ between *X* and the market portfolio is 0.40. Find the risk premium and required rate of return for security *X.*

SOLUTION

The market price for risk equals the market risk premium of 7% (= 12% – 5%) divided by market risk σ_m and equals 1/3. Risk σ_x equals 27% and if each percentage point of risk received the equilibrium market price for risk then the risk premium for *X* would equal 9% (= 27% × 1/3). Because correlation $\rho_{X,Market}$ equals 0.40, however, the risk premium only equals 3.6% (= 9% × 0.40). Thus, $ROR_X^{required}$ is 8.6% (= 5% + 3.6%).

Formula 11.4 shows that the systematic risk premium for security *A* equals σ_A times $\rho_{A,Market}$ times market price for risk. When $\rho_{A,Market}$ equals zero then systematic risk premium equals zero. Solve formula 11.4 with $\rho_{A,Market}$ = 0 and find that $ROR_A^{required}$ equals $ROR^{risk\text{-}free}$ irrespective of security risk. Similarly set $\rho_{A,Market}$ = 1 and solve formula 11.4. Find that securities that correlate perfectly with the market portfolio receive the full market price of risk for every unit of risk σ_A.

Figure 11.6 illustrates rays for different correlation coefficients. The topmost line passing from $ROR^{risk\text{-}free}$ through $ROR_{Market}^{required}$ is the capital market line. Its slope measures the market price for risk as in formula 11.1. The figure labels the market portfolio. The efficient frontier (not shown) is tangent to the capital market line at the market portfolio. All securities that correlate perfectly with the market portfolio lie on the ray coincident with the capital market line. No specific security lies on this ray further out than the market portfolio, however, because all specific capital investments lie on or inside the efficient frontier. No portfolio on the efficient frontier is dominated by any specific capital investment (panel 3 in figure 11.1 illustrates this principle).

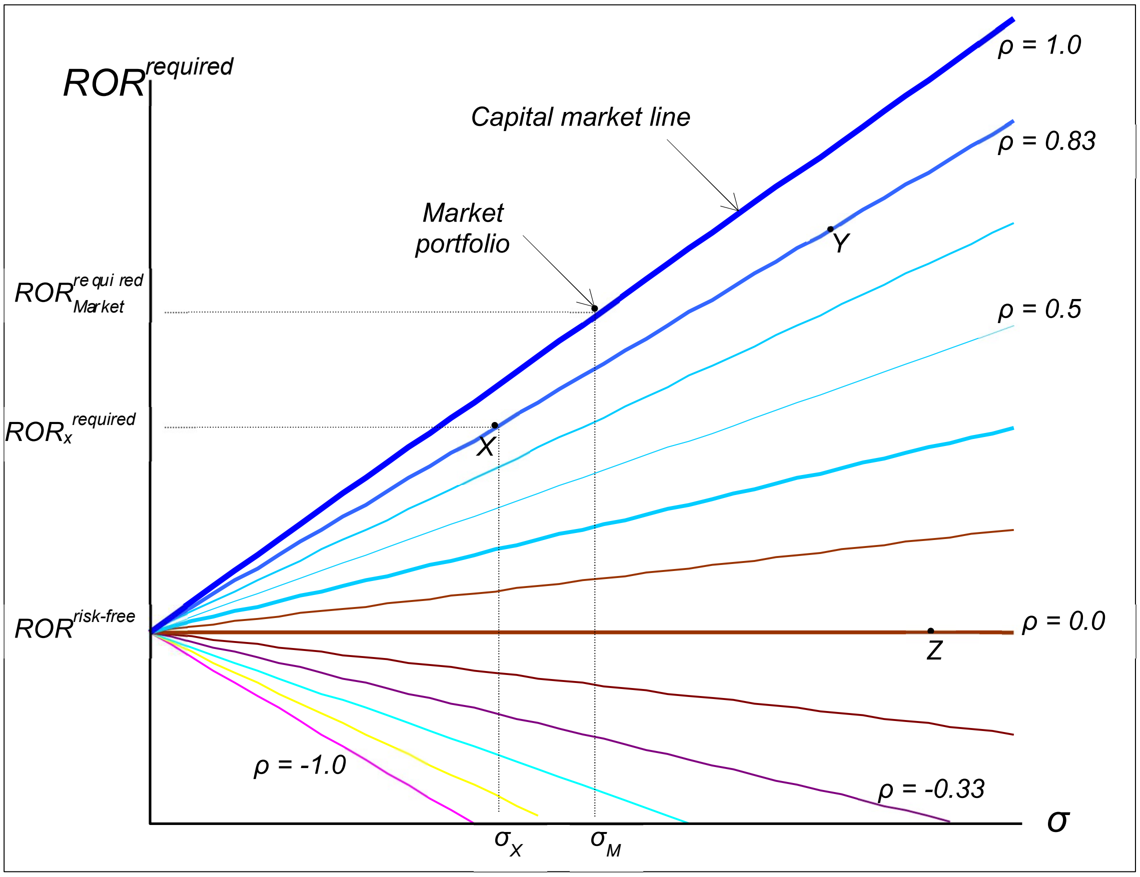


FIGURE 11.6 Capital market line and the rays of correlation
Notes: Coefficient ρ measures correlation between rates of return for the market portfolio and a specific security. All securities with the same ρ are pushed onto the respective ray of correlation. Correlation coefficients between the market portfolio and securities *X* and *Y* equal 0.83. Consequently, their required rates of return align onto the same ray. Eighty-three percent of their respective risks, σ_x and σ_y, require compensation at the market price for risk. Because $\sigma_x < \sigma_y$ then $ROR_X^{required} < ROR_Y^{required}$. For security *Z* the risk σ_z is even higher but $ROR_Z^{required}$ equals $ROR^{risk\text{-}free}$ because *Z* is uncorrelated with the market portfolio and merits zero risk premium.

Securities *X* and *Y* lie on the same ray because their correlation coefficients with the market portfolio are equal at 0.83. Eighty-three percent of their respective risks, σ_x and σ_y, command the market price for risk. Slope for this particular ray containing securities *X* and *Y* equals 83 percent of the capital market line slope. Along any ray of correlation for which ρ exceeds zero the required rate of return increases with security risk. But when $\rho \leq 0$ the situation changes. Security *Z* in figure 11.6 is uncorrelated with the market portfolio ($\rho_{Z,Market} = 0$) and hence the risk premium for *Z* is zero. The ray stretching horizontally from $ROR^{risk\text{-}free}$ contains all securities that are uncorrelated with the market portfolio. Some securities may have lower σ than others and vice versa. For all of them, however, the required rate of return equals the short-term risk-free rate. Securities with $\rho = 0$ contribute absolutely zero risk to a well-diversified market portfolio so zero percent of their risk merits a premium.

Many securities possess high risk σ while also having low required rates of return. Any security *A* for which $\sigma_A > \sigma_M$ and $ROR_A^{required} < ROR_{Market}^{required}$ is dominated by the market portfolio. The entire area southwest of the market portfolio represents bad stand-alone investments. Investors with a poorly diversified portfolio that hold high-risk securities with low market correlation subject themselves to subpar compensation for amount of risk carried.

Securities that correlate negatively with the market portfolio would lie on rays that slope downward. Required rates of return for these securities would be less than the risk-free rate (systematic risk premia would be negative). But if such securities existed then this specific source of systematic risk could be eliminated by diversification. This specific systematic risk factor would become just another type of idiosyncratic risk that could be diversified away. By definition, however, systematic risk does not vanish through diversification. Perhaps today there are several systematic risk factors, some affecting credit markets and others equity markets. Financial research that study different sources of systematic risk are *multifactor* risk models. As financial markets evolve and create new securities then arguably the number of systematic risk sources declines. Almost certainly there were many more types of systematic nondiversifiable risk factors a century ago than exist today. The many sources of idiosyncratic risk that exist today probably were systematic factors long ago – growth of financial markets creates diversification benefits.

EXAMPLE 3 Analyze two securities for by comparing $ROR^{required}$ with $ROR^{expected}$

You want to add one additional stock to your well-diversified portfolio and are considering two alternatives. Information about stock *A* leads you to believe that its expected rate of return is 10%, the standard deviation of expected returns is 38%, and that its correlation with the market portfolio is 0.35. For stock *B* those figures are $ROR^{expected}$ = 12%, σ_B = 32%, and $\rho_{B,Market}$ = 0.90. The risk-free rate is 5%, the required return for the market portfolio is 11%, and σ_M = 22%. Determine which one of these securities, if either, should be added to your portfolio.

SOLUTION

Rule 9.1 provides the investment decision rule to invest when expected rate of return exceeds required rate of return. For this example notice that stock *A* has lower expected return than *B* and also $\sigma_A > \sigma_B$. Figure 11.7 shows this situation.

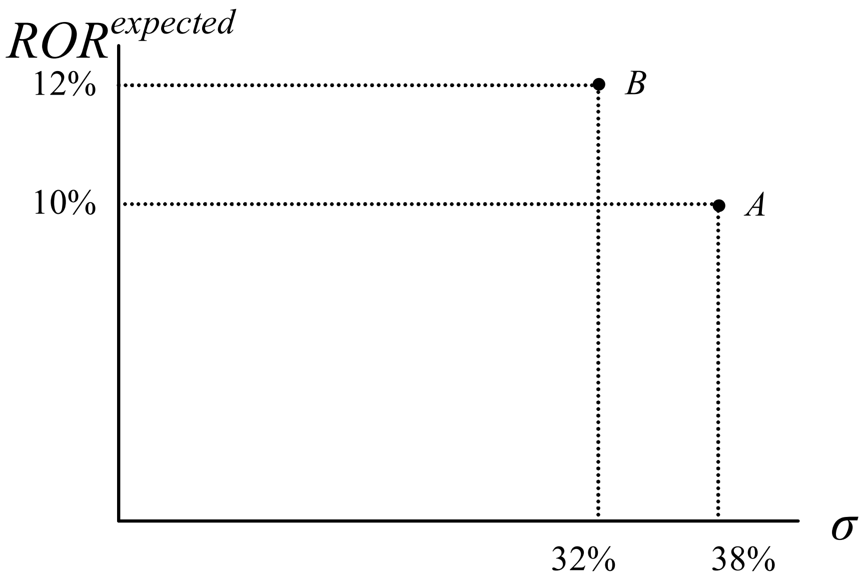


FIGURE 11.7 Expected return and σ for example 3

Comparison of *A* directly with *B* suggests that *B* dominates *A* and that therefore *B* is a better choice for addition to the portfolio. That analysis is incomplete, possibly wrong!

Lessons above establish that σ does not properly measure risk relevant for determining risk premia due to existence of diversification benefits. The illustration of dominance in figure 11.7 is misleading. The portion of σ that represents relevant risk is measured by the correlation coefficient with the market portfolio. Use formula 11.4 to find *required* rates of return and subsequently compare them to *expected* rates of return.

Compute that the market price for risk equals the market risk premium of 6% (= 11% – 5%) divided by market risk σ_m and equals 0.2727. Compute that $ROR_A^{required}$ equals 8.63% [= 5% + (0.35 × 38% × 0.2727]. Security *A* is a reasonable choice for addition to a well-diversified portfolio because $ROR_A^{expected} > ROR_A^{required}$. Compute that $ROR_B^{required}$ equals 12.85% [= 5% + (0.90 × 32% × 0.2727]. The 12% expected rate of return for *B* does not fully compensate for its relevant risk; that is, $ROR_B^{expected} < ROR_B^{required}$ so do not buy *B*. Add security *A* to your portfolio. The impression from figure 11.7 that *B* dominates *A* is a mirage.

3.A. Beta and the Capital asset pricing model

The maxim that high risk gets high return does not apply when using the total risk measure σ as the number for risk. Correlation between security and market rates of return reduce risk exposure and create diversification benefits. Rearrange formula 11.4 for $ROR^{required}$ and find an alternative risk measure: *beta (β).*

FORMULA 11.5 Beta and the *Capital asset pricing model* ("*CAPM*") AP6b

The required risk premium for any capital investment *A* equals required rate of return $ROR_A^{required}$ minus short-term risk-free rate $ROR^{risk\text{-}free}$. When idiosyncratic risk may be eliminated completely through diversification and only one source of systematic risk exists then the ratio of security to market risk premia is

$$\beta_A = \frac{systematic\ \ risk\ \ premium\ \ for\ \ security\ \ A}{risk\ \ premium\ \ for\ \ market\ \ portfolio}$$

$$= \frac{covariance_{A,Market}}{\sigma^2_{market}} . \qquad (11.5a)$$

Rearrange the top line and obtain a formula known as the *Capital asset pricing model:*

$$ROR_A^{required} = ROR^{risk-free} + \overbrace{\beta_A \left(ROR_{market}^{required} - ROR^{risk-free} \right)}^{systematic\ \ risk\ \ premium\ \ for\ \ security\ \ A} .$$
(11.5b)

β_A measures proportion of market risk premium applicable to security *A*.

Inspection of *CAPM* formula 11.5b shows that required returns increase with beta. When $\beta_A = 0$ then the security systematic risk premium is zero and $ROR_A^{required} = ROR^{risk\text{-}free}$. When β is 1.0 then the security risk premium equals 100% the market portfolio risk premium. This linear relation between security required rate of return and beta is shown in figure 11.8 as the *Security market line*.

DEFINITION 11.3 *Security market line (SML)*

The *Security market line* is the graph showing the required rate of return as a linear function of the equity β. Slope of the SML equals the required risk premium for the market portfolio, $ROR_{Market}^{required} - ROR^{risk\text{-}free}$.

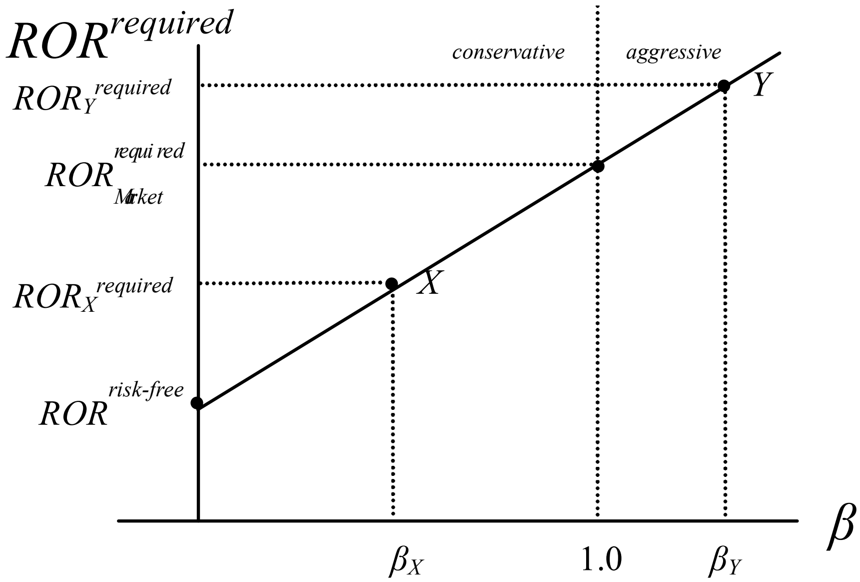


FIGURE 11.8 The *Security market line*

When idiosyncratic risk may be eliminated completely through diversification and only one source of systematic risk exists then the security risk premium rises proportionately with β. The slope of the security market line equals the required risk premium for the market portfolio. Securities with $\beta < 1$, such as security X in figure 11.8, possess relatively low systematic risk. These securities are known as *conservative* securities and observations show that they tend to be less volatile than the overall market. Securities with $\beta > 1$, such as security Y in figure 11.8, possess relatively high systematic risk and require risk premium bigger than the market portfolio. These relatively high-risk securities are known as *aggressive* securities and they tend to be more volatile than market. Some conservative securities may have low σ and others have high σ – likewise for aggressive securities. Some conservative securities may have higher σ than aggressive securities! It is β and not σ that measures the risk relevant for the systematic security risk premium.

EXAMPLE 4 Revisit example 3 that compares for two securities $ROR^{required}$ with $ROR^{expected}$

The previous example says that for stock A the expected rate of return is 10%, σ_A = 38%, and $\rho_{A,Market}$ = 0.35. For stock B those figures are $ROR^{expected}$ = 12%, σ_B = 32%, and $\rho_{B,Market}$ = 0.90. The risk-free rate is 5%, the required return for the market portfolio is 11%, and σ_M = 22%. Find betas and required rates of return for these securities.

SOLUTION

Formula 11.5a computes β as the ratio of covariance between security and market returns divided by variance of market returns. Covariance between any two variables always has the same sign as their correlation coefficient (see discussion at formula 10.12). The difference is that ρ takes on values exclusively between -1 and +1 whereas covariance takes on a number about as big as the variable measurements squared. Formula 10.12 shows that $covariance_{x,y} = \rho_{x,y} \times \sigma_x \times \sigma_y$. Substitute this into formula 11.5a:

$$\beta_A = \frac{\overbrace{0.35 \times 0.38 \times 0.22}^{covariance_{A,Market}}}{0.22^2} = 0.6045.$$

Security *A* has a beta smaller than 1 and is therefore a conservative security. Investment in *A* requires a risk premium equal to 60.45% the market premium. That is,

$$ROR_A^{required} = 5\% + 0.6045\,(11\% - 5\%) = 8.63\%.$$

The expected rate of return for security *A* of 10% exceeds the required return so it's a *buy*. Perform analogous computations for security *B.*

$$\beta_B = \frac{\overbrace{0.90 \times 0.32 \times 0.22}^{covariance_{B,Market}}}{0.22^2} = 1.3091,$$

and

$$ROR_B^{required} = 5\% + 1.3091\,(11\% - 5\%) = 12.85\%.$$

The 12% expected rate of return for *B* does not fully compensate for its relevant risk; that is, $ROR_B^{expected} < ROR_B^{required}$ so put *B* on the *sell* list.

Numbers obtained here support identical inferences as example 3. Add security *A* to your portfolio and avoid *B*. The illusion from figure 11.7 that *B* dominates *A* is a mirage that easily corrects by substituting measurement of risk by *β* instead of *σ*. See figure 11.9.

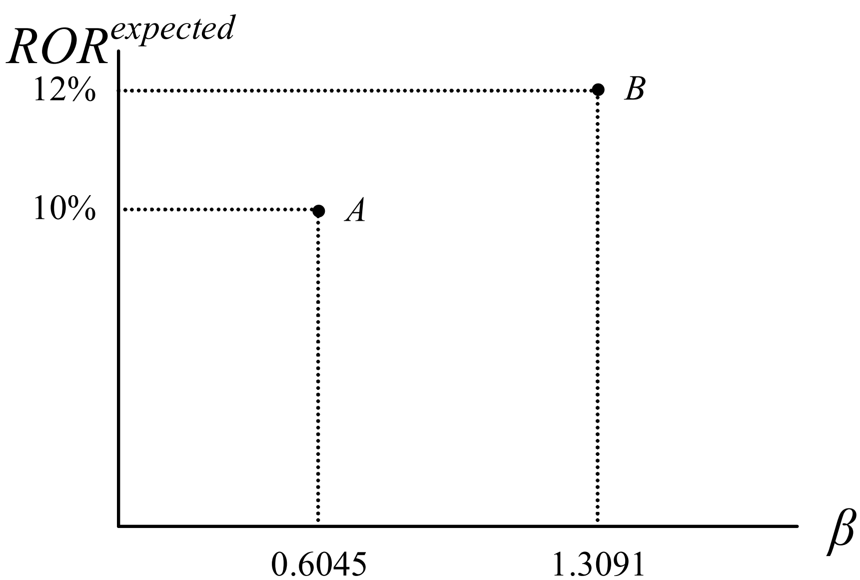


FIGURE 11.9 Expected return and *β* for examples 3 & 4

β measures systematic risk and so figure 11.9 is a risk-return graph. Definitely there is no dominance in figure 11.9 between securities *A* and *B*. Instead, the decision about which security is best depends on something that is not on the graph. It depends on the equilibrium price for risk and market risk premium. When we take those factors into consideration as formula 11.5b stipulates then a clear decision emerges: security *A* is worth adding to a well-diversified portfolio and *B* is not.

The required rate of return is the minimum discount rate an investor willingly accepts for computing intrinsic value. Once the investor obtains $ROR^{required}$ then information about expected cash flows may be collected and intrinsic value computed. This chapter presents many lessons about finding required rates of return for different situations. When the *CAPM* situation fits, namely idiosyncratic risk is perfectly diversifiable and only one source of systematic risk exists, then formula 11.5b provides a discount rate useful for finding intrinsic value. The example below combines the *CAPM* with the constant growth dividend valuation model from chapter 8.

EXAMPLE 5 Effect of company policy change on $ROR^{required}$ and intrinsic value

The company conducts a study that finds their digital products division is less volatile than their traditional magazine division. The study concludes that expansion of the digital products division will diminish company beta to 0.80 from 1.20. The profit margin is lower for digital products, however, and the expansion will diminish dividend growth rate to 5% from 7%. Suppose that the required rate of return for the market portfolio is 14%, that the risk-free rate is 6%, and that a $2.00 dividend was just paid. Assume that the stock price equals the present value of dividends when discounted by the required rate of return from the *CAPM*. Find the effect of the policy change on the stock price.

SOLUTION

This solution relies on dividend valuation formula 8.8 for finding intrinsic value:

$$V_0 \quad = \quad div_0(1+g)/(r-g)$$

The solution strategy computes intrinsic value before and after the policy change. The setup directly states all variables for V_o except for the discount rate *r* which obtains from the *CAPM* formula. Before the policy change find the required rate of return:

$$ROR^{required} \quad = \quad 6\% \; + \; 1.20\,(14\% - 6\%) \quad = \quad 15.6\%.$$

Find intrinsic value before the policy change

$$V_o^{before} \quad = \quad \$2.00(1 + 0.07) / (0.156 - 0.07) \quad = \quad \$24.88.$$

After the policy change the dividend growth rate drops to 5% and β falls to 0.80. Find the new required rate of return

$$ROR^{required} \quad = \quad 6\% \; + \; 0.80\,(14\% - 6\%) \quad = \quad 12.4\%$$

and new intrinsic value

$$V_o^{after} \quad = \quad \$2.00(1 + 0.05) / (0.124 - 0.05) \quad = \quad \$28.38.$$

This policy change increases the price by 14.0% (= $28.38 ÷ $24.88 – 1).

Financial market equilibrium occurs when required and expected returns are equal. The required return includes the risk premium. The expected return is the internal rate of return that equates actual price with expected cash flows. The example below applies these fundamental lessons in a simply powerful problem.

EXAMPLE 6 Compute $ROR^{expected}$ from information and $ROR^{required}$ from *CAPM* and buy or sell

The company beta is 1.25, its dividend growth rate is 6%, just yesterday it paid a dividend of $0.70, and today's share price is $12.50. Most likely today's share price equals today's intrinsic value. Furthermore, the share price moves in accordance with the dividend constant growth model. The economy wide risk free interest rate is 5% and the required risk premium for the market portfolio is 8%. The stock represents a good investment if the expected total rate of return implied by the dividend constant growth model exceeds the required rate of return implied by the capital asset pricing model. Determine whether the stock is a *buy* or *sell.*

SOLUTION

This solution relies on dividend valuation formula 8.8 and solves for the unknown discount rate *r*. That discount rate is the *expected* rate of return from the stock investment. Substitute numerical settings into the valuation formula:

$$V_0 = div_0(1+g)/(r-g),$$

$$\$12.50 = \$0.70\ (1 + 0.06)\ /\ (r - 0.06) \quad or \quad r = 11.94\%.$$

$ROR^{expected}$ is 6 basis points shy of 12%. Find $ROR^{required}$ with the *CAPM* formula:

$$ROR^{required} = 5\% + 1.25\ (8\%) = 15.0\%.$$

When the stock is added to a well-diversified portfolio the required return is 15%. The expected return is about 12% and does not compensate for risk. That puts the stock on the *sell* list. Realize that the stock fares even worse in a poorly diversified portfolio.

A1. Estimating β and risk-adjusted returns

Common and well-tested approaches for estimating company beta rely on historical stock returns. Typically a broad index such as the SP500 proxies for return on the market portfolio. That market rate of return serves as explanatory variable and security stock returns serve as the dependent variable. Estimation procedures usually do not include data on $ROR^{risk\text{-}free}$ short-term T-bill is very low. A typical sample period for estimating β may include 30 monthly or 120 daily observations. Fortunately, the BAIIPlus© calculator allows simple computation of β from a few observations and teaches procedural fundamentals.

Suppose that five periodic rates of return for security *A* and for the overall market portfolio are these: ©AP2d

	ROR1	ROR2	ROR3	ROR4	ROR5
Security A	18%	-4%	6%	21%	5%
Market	15%	-2%	7%	16%	5%

Use the accompanying *Calculator clue* and easily compute that $\beta_A = 1.36$. This particular application of the *CAPM* that regresses security returns on market returns is *the market model.*

CALCULATOR CLUE 10.7 The BAII Plus© calculator contains a spreadsheet that enables easy estimation of β that best fits formula 11.5b. On the BAII Plus© type 2nd DATA. Clear unwanted numbers already stored in the data worksheet by typing 2nd CE/C. For estimating β column X contains the market return series and column Y contains security returns. Enter this problem's data by typing

15 ENTER ↓ 18 ENTER
↓ 2 +/- ENTER ↓ 4 +/- ENTER
↓ 7 ENTER ↓ 6 ENTER
↓ 16 ENTER ↓ 21 ENTER
↓ 5 ENTER ↓ 5 ENTER

The preceding keystrokes set $X_1 = 15$ and $Y_1 = 18, \ldots, X_5 = 5$ and $Y_5 = 5$. Find the statistical estimate of beta from the data in memory by typing 2nd STAT . Now hit 2nd SET repeatedly until the display says "LIN". With this setting the calculator finds the best linear fit between the Y and X columns. The statistics course calls this procedure an ordinary least squares linear regression of Y on X.

To find the beta estimate hit ↓ repeatedly (probably 9 times) until the display shows "b = 1.3636." This number equals β.

As an aside notice that the STAT function also displays $\sigma_x = 6.6753$ (that equals market standard deviation σ_M), $\sigma_y = 9.1520$ (that equals security standard deviation σ_A), and r = 0.9946 (that is the correlation $\rho_{A,M}$). Use these numbers and verify that formula 11.5a also computes that β is 1.3636 (= $0.9946 \times 9.1520 \times 6.6753 \div 6.6753^2$). The number on the display for "a = –1.9811" is the intercept (*alpha*) for the regression and appears in subsequent discussions.

The best estimate for β from the market model suggests that security *A* requires a systematic risk premium that is 136.36% as large as the required return for the market portfolio. Investment equilibrium prevails when required and expected returns are equal. Estimation of the market model assumes an equilibrium relation between security and market rates of return as in:

$$ROR_A^{equilibrium} = alpha + (\, beta \times ROR_{Market}^{actual} \,),$$

where *alpha* and *beta* are the best estimates of the intercept (= *alpha*) and slope (= *beta*) from the market model. The difference between actual and equilibrium security rates of return represents economic profit (or loss) and is the *risk-adjusted return*.

FORMULA 11.6 Risk-adjusted return is economic profit

The risk-adjusted rate of return for any capital investment *A,* denoted $ROR_A^{risk\text{-}adjusted}$ equals the actual rate of return minus the equilibrium rate of return $ROR_A^{equilibrium}$: The risk adjusted return equals economic profit or loss accruing during the period.

$$ROR_A^{Risk\text{-}adjusted} = ROR_A^{actual} - ROR_A^{equilibrium}.$$

The risk-adjusted return equals economic profit or loss accruing during the period.

When idiosyncratic risk may be eliminated completely through diversification and only one source of systematic risk exists then, subject to standard statistical assumptions, the *CAPM* provides a procedure through the market model for estimating risk-adjusted returns.

Historical data for the on-going illustration with security *A* and the market portfolio suggest that β_A equals 1.3636. Suppose that for the most recent period the market was up 3% while security *A* was down 6%. Let's compute the periodic risk-adjusted return for security *A*. For the numbers already in the calculator *alpha* = –1.9811 and *beta* = 1.3636. Because the actual market return was 3% find best estimate of the equilibrium return:

$$ROR_A^{equilibrium} = -1.9811 + (1.3636 \times 3\%), = 2.11\%$$

The stock *should* have been up 2.11% when the market was up 3%. But the stock fell 6%. From formula 11.6 find the risk-adjusted return:

$$ROR_A^{risk\text{-}adjusted} = -6\% - 2.11\% = -8.11\%.$$

Shareholders bore the burden of an 8.11% economic loss.

CALCULATOR CLUE 10.8 Enter data for security *A* and the market portfolio into the BAII Plus© calculator as the previous *Calculator clue* describes. Next find the equilibrium return given a 3% market return. While in the STAT function hit ↓ until the display shows "X' = ". Type

3 ENTER ↓ CPT .

The display shows that Y' is 2.1095%; this equals the security equilibrium return given the 3% market return. Now subtract the entry on the display from the actual security return of –6% with these keystrokes:

+/- – 6 = .

The display shows risk-adjusted return equals -8.1095%.

Risk-adjusted returns are useful for many purposes. Regulated utility companies such as Edison Electric often present risk-adjusted returns to public commissions when arguing for electric utility rates. Sometimes class-action shareholder suits against management present risk-adjusted returns to bolster argument of managerial malfeasance. Figure 9.5 illustrates the efficient market hypothesis by relying on *cumulative abnormal returns*. The periodic abnormal return is identical to the risk-adjusted return. Literally thousands of studies by companies and professors in finance, accounting, and economics use the preceding methodology for computing effects of events on company economic profit. Shareholders, like everyone else, are recipients of unexpected windfall gains and losses. Risk-adjusted returns measure economic profit.

EXERCISES 11.3

1. The company's beta is 1.25 and the required risk premium for the market portfolio is 8.0%. What equilibrium risk premium for the company's stock is implied by the Capital Asset Pricing Model? ©AP7 .

2. The economy wide risk free interest rate is 5.0% and the required risk premium for the market portfolio is 7.5%. At the same time, the company's required risk premium according to the Capital Asset Pricing Model is 9.0%. What is the company's β?
©AP4a .

3. You want to add an additional stock to your portfolio and will rely on the *Capital asset pricing model* to determine whether a security should be added to your portfolio. The risk-free rate is 4.6% and the required return on the market portfolio is 10.7%. For stock X the expected return is 10.70% and the beta is 0.94.
3a. Should stock X be added to your well-diversified portfolio?
3b. For stock Y the expected return is 12.60% and the beta is 0.61. Between X and Y determine which one of these securities should be added to your portfolio. ©AP1a .
3c. For stock Z the expected return is 10.25% and the beta is 0.85. Between X and Z determine which one of these securities should be added to your portfolio.

4. The company beta is 1.50, its dividend growth rate is 11.2%, and just yesterday it paid a dividend of $1.30. The economy wide risk free interest rate is 5.0% and the expected risk premium for the market portfolio is 8.0%. Find the stock's intrinsic value using the dividend constant growth model and the equilibrium total rate of return implied by Capital Asset Pricing Model. ©AP6a .

5. The company beta is 1.10, its dividend growth rate is 7.4%, just yesterday it paid a dividend of $1.60, and today's share price is $17. Most likely today's share price equals today's intrinsic value. Furthermore, the share price moves in accordance with the dividend constant growth model. The economy wide risk free interest rate is 4% and the required risk premium for the market portfolio is 7%. The stock represents a good investment if the expected total rate of return implied by the dividend constant growth model exceeds the required rate of return implied by the capital asset pricing model. Determine whether the stock is a *buy* or *sell.* ©AP5b .

6. The economy wide real risk-free rate is 2.5%, the inflation premium is 1.5%, and the market risk premium is 7.5%. At the same time, the company beta is 0.90, its dividend growth rate is 5.5%, and it just paid a dividend of $0.75 per share. Due to sudden and unexpected political events, the market risk premium increases by 100 basis points. What is the likely resultant percentage change in the intrinsic value of the company's shares? ©AP8 .

7. The economy wide real risk free interest rate is 3.5%, the inflation premium is 2.5%, and the market risk premium is 10.0%. At the same time, the company beta is 0.85, its dividend growth rate is 7.0%, and it just paid a dividend of $1.10 per share. The company anticipates a change in production plan that should affect its beta and dividend growth rate. The new beta becomes 0.70 and the growth rate becomes 8.70%. What is the likely resultant percentage change in the intrinsic value of the company's shares?
©AP9 .

8. You have the following information about equity rates of returns for the past 5 periods.

	obs 1	obs 2	obs 3	obs 4	obs 5
company ROR	19%	15%	-2%	18%	8%
market ROR	15%	15%	-6%	20%	11%

8a. Based on the above observations, what is the company's β? ©AP2am .
8b. The most recent information suggests that the current period market return is -7% and the security return is 5%. Use the market model to find the company risk-adjusted rate of return? ©AP2dm .

4. The company financial cost of capital

Rule 6.4 states that companies assess profitability of investment opportunities by discounting incremental cash flow from assets. When net present value of asset cash flow is positive the project captures economic profit for the company. Management distributes the resultant economic profit to capitalists and stakeholders in accordance with relative strength of principal-agent relationships. Conversely, undertaking negative net present value investments destroys wealth and is to be avoided. The discount rate for finding present values obviously influences computations. Selection of the proper discount rate relies on some of the most important concepts in finance.

DEFINITION 11.3 Company cost of capital ©CC2

The company cost of capital is the discount rate for computing net present value of company investment opportunities.

Inspection of any balance sheet shows that a variety of debt and equity financing sources support company assets. For pursuit of entrepreneurial activities companies obtain financing from many sources. Some projects borrow from the bank or issue bonds or rely on trade credit to obtain financing. Other projects rely exclusively on cash already in the checking account. Sometimes the company issues equity to raise capital. For many companies, especially large on-going concerns, the process of raising capital does not uniformly match the process of making investments. There is a separation between investment and financing decisions. Irrespective of a particular project's financing method the company cost of capital for discounting asset cash flows is a weighted average of financing rates for all financing sources.

FORMULA 11.7 Weighted average cost of capital ROR^{wacc}

The discount rate appropriate for assessing net present value of investment opportunities for company *A* equals a weighted average of debt and equity financing rates. Compute that discount rate, denoted ROR_A^{wacc}, as follows:

$$ROR_A^{wacc} = w_{debt}\ i_A \left(1 - \frac{tax}{rate}\right) + w_{equity}\ ROR_A^{required}.$$

Weights w_{debt} and w_{equity} sum to 100% and measure the proportion of existing long-term financing provided by debt and equity. The pretax interest rate i_A is the yield-to-maturity on company long-term debt. ROR_A^{wacc} applies to any capital investment for company *A* irrespective of actual financing method for that specific project.

Formula 11.7 ignores many technical complications in order to focus on the fundamental lesson that the cost of capital is an average of all company financing costs. A short list of significant technicalities (among many, many that exist) includes the following.

Project specific risk: The most significant technicality is implicit assumption that risk of the investment opportunity approximately equals average risk of the company's existing assets. For opportunities with above average risk the cost of capital should include an additional risk premium and vice versa.

Capital structure weights: Irrespective of a particular project's financing method the company cost of capital for discounting asset cash flows is a weighted average of financing rates for all financing sources. Important technicalities pertain to measuring the weights. Liability side of the balance sheet lists historical financing sources. Arguably, however, there is divergence between book values and market values for long-term liabilities. Weights may be based upon either; book values are handier but economic arguments suggest market values may be better. Another difficulty is whether weights should be based on actual measurements or on target measurements. For example, a company actively reducing their debt-ratio may prefer to use target instead of actual weights for computing the company cost of capital.

EXAMPLE 7 Find the weighted average cost of capital for a simple setting

The company is evaluating profitability of a long-term investment opportunity. Their balance sheet shows that 35% of long term financing relies on debt at a 7.25% pretax interest rate. The other 65% is *Stockholders equity*. The company marginal tax rate is 34%. Statistical estimates find that company β is 1.40. The short-term risk-free rate currently is 4.5% and the company believes that the required risk premium for the market portfolio is 8%. Find the company's weighted average cost of capital appropriate for computing net present value of investment opportunities.

SOLUTION

The setup provides all numbers that formula 11.7 needs except for the cost of equity financing. Find this financing rate with *CAPM* formula 11.5b (naturally this is valid only when idiosyncratic risk is perfectly diversifiable and only one source of systematic risk exists):

$$ROR^{required} = 4.5\% + 1.40\ (8\%) = 15.70\%.$$

Now substitute all numbers into formula 11.7 and compute the *WACC:*

$$ROR^{wacc} = 0.35 \times 7.25\% \times (1 - 0.34) + 0.65 \times 15.70\% = 11.88\%.$$

If the company insists that all investment opportunities possess internal rates of return exceeding 11.88% then net present values will be positive. Often the company cost of capital is referred to as the *hurdle rate* for investment.

Public utilities companies often justify pricing policies by documenting the weighted average cost of capital to state regulatory commissions. The snippet below from table 2.1 shows the biggest electric power holding company in the list of 11,000 U.S. companies circa beginning of year 2014.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
AFL	$121,307	9	$3,158	$23,939	$ 30,689
COP	$118,057	18	9,156	54,413	86,613
RF	$117,396	24	1,122	5,602	13,626
DUK	$114,779	28	2,665	24,549	48,721
BAM	$112,745	28	2,120	20,166	23,899
GOOG	$110,920	48	12,920	59,825	376,370

SNIPPET from table 2.1 in chapter 2: DUK

Duke Energy Corp. (DUK, 4th row at rank 61 in table 2.1) gained the distinction of nation's largest electric utility after takeover of Progress Energy Inc in 2012. When the takeover closed DUK executed a 1-for-3 *reverse stock split*. With a 1-for-3 reverse stock split, shareholders got one DUK share for every three DUK shares they already owned. The market determined stock price for each share approximately tripled so that the overall market cap and portfolio values remained the same. In late 2013 the DUK management finalized a rate case settlement agreement with the South Carolina Office of Regulatory Staff request for its Carolinas electricity company to raise customer rates by 5.53 percent the first year ($220 million, DUK requested 15.1 percent). The settlement approved the DUK ROR^{WACC} with weights of 53 percent on debt, 47 percent on equity, and a cost of equity of 10.2 percent. Had the Commission settled on 8.5 percent, according to news reports, then customer rates would not have risen at all. 170 basis points was worth $220 million in this case. The financial contracting between a nonfinancial corporation like DUK and its inflow stream from customers relies on measures that must be made for sound financial management and public finance decisions to occur.

EXERCISES 11.4

1. The company is evaluating profitability of a long-term investment opportunity. Their balance sheet shows that 65% of long term financing relies on debt at a 8.5% pretax interest rate. The other 35% is *Stockholders equity*. The company marginal tax rate is 34%. Statistical estimates find that company β is 0.85. The short-term risk-free rate currently is 5% and the company believes that the required risk premium for the market portfolio is 7.5%. Find the company's weighted average cost of capital appropriate for computing net present value of investment opportunities. ©CC1 .

2. A company pursues a cost-cutting initiative that costs $27,000 to implement. Thereafter, however, the initiative reduces after-tax costs by $5,000 per year perpetually. The company relies on 29% debt financing at a 7.2% pretax interest rate. The company marginal tax rate is 37%. The company β is 1.39, short-term risk-free rate is 4.0%, and required risk premium for the market portfolio is 10.5%. Find the project's net present value. ©CC2 .

ANSWERS TO CHAPTER 11 EXERCISES

EXERCISES 11.1

1. ©AP11b The market risk premium equals 6% (= 11% - 5%). Also, σ_m = 24% so slope of the capital market line equals 1/4 (= 6% ÷ 24%). The market price for one percentage point of risk is 25 basis points (= 1/4 × 1%) and for extra risk of 5% the increase in required risk premium is 1.25% (= 1/4 × 5%).

2. The risk-return profile for combinations of the risk-free asset and the market portfolio lies along the line of averages connecting those two points. Let w_{rf} and $(1 – w_{rf})$ equal the proportions invested in the risk-free asset and the market portfolio, respectively. The portfolio rate of return always is an average of component returns. Thus, in order to obtain a 4.9% rate of return write:

$$0.049 = w_{rf} \times 0.030 + (1 - w_{rf}) \times 0.125; \text{ or } w_{rf} = 0.80.$$

Investing 80% of capital in the risk-free asset and 20% in the market portfolio satisfies the objective.

3. ©AP13 Let w_{rf} and $(1 – w_{rf})$ equal the proportions invested in the risk-free asset and the market portfolio, respectively. The portfolio risk *when the correlation between components equals zero* is an average of component risks. Thus, in order to obtain a portfolio risk of 12% write:

$0.12 = w_{rf} \times 0.0 + (1 – w_{rf}) \times 0.17$; or $w_{rf} = 0.2941$.

Investing 29.4% of capital in the risk-free asset and 70.6% in the market portfolio satisfies the objective.

4. The risk-return profile for combinations of the risk-free asset and the market portfolio lies along the line of averages connecting those two points. Let w_{rf} and $(1 – w_{rf})$ equal the proportions invested in the risk-free asset and the market portfolio, respectively. To obtain a 3.9% risk premium requires a 7.9% portfolio rate of return. Write:

$0.079 = w_{rf} \times 0.04 + (1 – w_{rf}) \times 0.085$; or $w_{rf} = 0.1333$.

Investing 13.3% of capital in the risk-free asset and 86.7% in the market portfolio satisfies the objective.

EXERCISES 11.2

1. ©AP15 The risk-free rate $ROR_2^{risk\text{-}free}$ equals 3% plus the inflation premium of 5.75% [= (5.6% + 5.9%)/2] which is 8.75%. Inflation premium on a four-year bond is 5.7% [= (5.6% + 5.9% + 2 × 6.2%)/4] so $ROR_4^{risk\text{-}free}$ is 8.97%. The 20-year risk-free rate $ROR_{20}^{risk\text{-}free}$ is 9.15% [= 3% + (5.6% + 5.9% + 18 × 6.2%)/20].

EXERCISES 11.3

1. β equals the proportion of market risk premium that the security garners and is 10.0% (= 1.25 × 8%).

2. Ratio of security to market risk premium equals β and is 1.20 (= 9.0% ÷ 7.5%).

3a. Use *CAPM* formula 11.5b to find required return and consider the security a buy if $ROR^{required} < ROR^{expected}$. For security X required return is 10.33% [= 4.6% + 0.94 × (10.7% – 4.6%)] and its expected return is bigger so it's a buy.

3b. For security Y required return is 8.32% [= 4.6% + 0.61 × (10.7% – 4.6%)] and its expected return is bigger so it too is a buy. For X and Y expected returns exceed required returns so they are both buys. But which one is best? Notice that on a graph of $ROR^{expected}$ versus β like figure 11.9 that security Y dominates X so Y is the one to add.

3c. For security Z required return is 9.78% [= 4.6% + 0.85 × (10.7% – 4.6%)] and its expected return is bigger so it too is a buy. For X and Y expected returns exceed required returns so they are both buys. But which one is best? Notice that on a graph of $ROR^{expected}$ versus β like figure 11.9 that there is a trade-off between relatively high risk/return X and low risk/return Z; there is no dominance. We could, however, apply the market price for risk from the *Capital market line* to determine whether the extra return for X compensates for its extra risk (that deduction also requires information about correlation coefficient ρ_{xy}, too). The problem does not give enough information to make that deduction, however. Bottom line is this: both X and Z are good but we can't tell which is best without additional information.

4. Use the *CAPM* to compute that $ROR^{required}$ equals 17% [= 5% + (1.50 × 8%)]. At equilibrium required and expected returns are equal. Hence, use 17% in the intrinsic

valuation formula for constant growth dividends. Intrinsic value equals $24.92 [= $1.30 × (1 + 0.112) ÷ (0.17 – 0.112).

5. Solve for the unknown discount rate *r* from the intrinsic value formula with constant growth of dividends. Compute that this $ROR^{expected}$ equals 17.51% [= $1.60 × 1.074 ÷ $17) + 7.4%. Use the *CAPM* to compute that $ROR^{required}$ equals 11.7% [= 4% + (1.10 × 7%)]. The expected return more than compensates for risk and places the stock on the *buy* list.

6. The real risk-free rate is 2.5% and the inflation premium is 1.5%. Use formula 11.3 to find that the nominal risk-free rate is 4.0% (= 2.5% + 1.5%). Now use the *CAPM* to compute that $ROR^{required}$ before the political crisis equals 10.75% [= 4.0% + (0.90 × 7.5%)]. At equilibrium required and expected returns are equal. Hence, use 10.75% in the formula for constant growth dividends to find that intrinsic value equals $15.07 [= $0.75 × (1 + 0.055) ÷ (0.1075 – 0.055)]. After the political crisis use the *CAPM* to compute that $ROR^{required}$ equals 11.65% [= 4.0% + (0.90 × 8.5%)]. The intrinsic value moves to $12.87 [= $0.75 × (1 + 0.055) ÷ (0.1165 – 0.055)], a decline of 14.6% (= $12.87 ÷ $15.07 – 1).

7. Use formula 11.3 to find that the nominal risk-free rate is 6.0% (= 3.5% + 2.5%). Now use the *CAPM* to compute that $ROR^{required}$ before change in production plan equals 14.5% [= 6.0% + (0.85 × 10%)]. Use 14.5% in the formula for constant growth dividends to find that intrinsic value equals $15.69 [= $1.10 × (1 + 0.07) ÷ (0.145 – 0.07)]. After the change compute that $ROR^{required}$ equals 13.0% [= 6.0% + (0.70 × 10%)]. The intrinsic value moves to $27.81 [= $1.10 × (1 + 0.087) ÷ (0.13 – 0.087)], an increase of 77.2% (= $27.81 ÷ $15.69 – 1).

8a. Use the DATA and STAT functions on the calculator to find that "b = 0.8259." The beta equals 0.8259. Notice as an aside that the calculator also displays σ_x = 8.97% (that equals market standard deviation σ_M), σ_y = 7.81% (that equals security standard deviation σ_A), and r = 0.95 (that is the correlation $\rho_{A,M}$). Use these numbers and verify that formula 11.5a also computes that β = 0.83. The number on the display for "a = 2.5154" is the intercept (*alpha*) for the regression and appears below.

8b. ©AP2d Easily compute this answer by following the *Calculator clue* in the text. Find from the calculator that alpha = 2.5154, beta = 0.8259, and compute that when the market return is –7% the security equilibrium return is –3.2% [= 2.5154 + (0.8259 × -7%)]. Compute with formula 11.6 that $ROR^{risk\text{-}adjusted}$ equals 8.27% [= 5% – (-3.27%)].

EXERCISES 11.4

1. Use formula 11.5b to find $ROR^{required}$ is 11.375% [= 5% + 0.85 (7.5%)]. Now substitute all numbers into formula 11.7 and compute the *WACC* is 7.63% [= 0.65 × 8.5% × (1 – 0.34) + 0.35 × 11.375%].

2. ©CC2 Use formula 11.5b to find $ROR^{required}$ is 18.60% [= 4% + 1.39 (10.5%)]. Now substitute all numbers into formula 11.7 and compute the *WACC* is 14.52% [= 0.29 × 7.2% × (1 – 0.37) + 0.71 × 18.60%]. Now find the NPV of the perpetuity is $7,440 [= $5,000/0.145 – $27,000].

PART 3: ECONOMIC PROFIT#3; ARBITRAGE AND RISK MANAGEMENT

Increasing maturation of financial markets makes arbitrage opportunities possible. Recall this definition from the preface.

> *Arbitrage value* exists when prices or rates in different markets misalign, thereby providing a temporary opportunity for instantaneous profit.

The essence of arbitrage is that when different financial securities have cash flow streams that somehow interact then the interaction sometimes imposes restrictions on the security prices. For example, consider arbitrage possibilities when a share of Microsoft may be bought in one market for $45 and, at the same time, may be sold in another market for $47. The cash flow stream from one share of Microsoft promises to be identical with every other share's. These shares consequently should trade for the same price even though they are in different markets. Traders with access to both markets will capture value by taking advantage of this fleeting price discrepancy – buy at $45 and instantly sell at $47. Arbitrage pressures force these prices into narrow tolerance ranges. Part 3 of *Lessons about the Structure of Finance* examines financial arbitrage and shows the usefulness of several risk management applications.

CHAPTER 12: THE NO-ARBITRAGE EQUILIBRIUM OF THE FINANCIAL ECONOMY

In well-developed financial markets securities allow businesses to manage risk. Risk management strategies often involve hedging, a process involving two or more simultaneous positions in different markets. The ability to manage risk occurs because multiple positions potentially eliminate uncertainty. By eliminating uncertainty the outcomes no longer depend upon subsequent price movements.

The futures markets were the first risk management markets to develop. For millennia sea captains have plied the oceans transporting scarce commodities. The uncertainty facing merchants needing those commodities was rather substantial. What price would the sea captains charge? Was the ship even carrying the commodity, or had unimaginable storms or other calamity decimated crops in some distant land? The sea captains worried whether there was a ready market for the product in port. In the midst of this substantial uncertainty, a few entrepreneurs saw significant opportunity.

Middlemen began writing contracts guaranteeing the right to buy or sell the commodity in the future. Merchants were able to lock in the buy price long before the ship arrived. Sea captain backers no longer had to worry whether some other ship arrived a week earlier, thereby glutting the commodity market. The sea venture could lock in the sell price before arrival by signing a contract with the middlemen. The middlemen sometimes would guess right and make money. Other times they would lose money. But they were providing a valuable service. They were absorbing risk. Economic forces dictate that providing a valuable service merits fair compensation. The middlemen flourished. Futures markets moved society forward.

This chapter formally but simply introduces fundamental lessons about arbitrage. The arbitrage process is the third and final source of value in *Structure of Finance*. It is perhaps most powerful of all financial forces and merits study in a survey course. Section 1 explains the arbitrage concept and provides brief examples. Sections 2 and 3 examine risk management strategies associating with futures and options markets, respectively. Section 4 focuses on very powerful international arbitrage relationships that bind global financial markets.

1. The arbitrage concept

An arbitrage opportunity exists when prices in several markets misalign thereby providing temporary potential for certain profit. Consider a simple example in which there exist two markets for gold: the spot market and the futures market. A spot market is a "cash and carry" market. Most consumer markets are spot markets. The grocery store is a spot market because the price on the shelf represents the price to take immediate possession of the commodity. The stock exchange is a spot market because when you pay the asked price you immediately get ownership in the stock. For gold the spot and futures markets are large, active, and competitive.

A futures contract locks in today the price to be paid in the future for buying or selling a commodity. The payment occurs at time of delivery and the contract is free to enter. Existence of complete and competitive markets means that prices in spot and futures markets align in accordance with the formula below.

FORMULA 12.1 No-arbitrage equilibrium, simplest scenario

When interim cash flows such as warehouse fees, adjustment costs, dividends, etc., equal zero then prices in spot and futures markets satisfy this condition:

$$\begin{pmatrix} no-arbitrage \\ spot\ price \end{pmatrix} = \begin{pmatrix} no-arbitrage \\ futures\ price \end{pmatrix} \Big/ (1+r)^N$$

There are four variables in formula 12.1. Given values for any three, the fourth may be determined. Usually, however, length of futures contract N and discount rate r are known and predetermined.

Competitive behavior of market participants determines spot and futures prices. The prices, however, align as formula 12.1 stipulates or else arbitrage profit exists.

DEFINITION 12.1 Arbitrage profit and the no-arbitrage equilibrium

Arbitrage profit is a certain return exceeding the risk-free return. Arbitrage profit is economic profit and represents a form of wealth creation. At equilibrium arbitrage profit equals zero because market forces drive certain returns to the risk-free return. Prices and rates force a *no-arbitrage* equilibrium in which arbitrage profits vibrate around zero.

No-arbitrage equilibrium formula 12.1 is analogous to lump-sum time value formula 4.6. Supply the actual value for the futures price in formula 12.1 (analogous to *FV*) and solve for the no-arbitrage spot price (analogous to *PV*). If the actual spot price differs from the no-arbitrage spot price then an opportunity exists to capture arbitrage profit. Present value of arbitrage profit equals the difference between actual and no-arbitrage spot prices.

Likewise in formula 12.1 supply the actual value for the spot price and solve for the no-arbitrage futures price. If the actual futures price differs from the no-arbitrage futures price then an arbitrage opportunity exists. Future value of arbitrage profit equals the difference between actual and no-arbitrage futures prices.

In reality an actual price may deviate very slightly from its underlying no-arbitrage price. Perhaps the difference is so slight that capturing the resultant arbitrage profit is not worth the time, trouble, or transaction cost. Once the arbitrage profit is large enough, however, competitive behavior is assurance that prices snap back into place as formula 12.1 stipulates. Arbitrage profit is like money lying on the sidewalk of the busiest street in town – someone picks it up. The example below illustrates the arbitrage process.

EXAMPLE 1 Futures Arbitrage ©FT2b

Your company is a middleman that buys, sells, and stores warehouses full of gold. Today's spot price for gold is $310 per ounce. Your company is able to invest and borrow at the risk-free interest rate of 9.9%. Furthermore, your company can store and safeguard, if necessary, additional gold for virtually free. Describe the arbitrage forces if today's futures price for delivery in one year is (a) $275, or (b) $350.

SOLUTION

Use formula 12.1 given that *r* equals 0.099 and *N* equals 1. For part *a*, take as given today's futures price of $275. This means that today the company can lock in the right to buy or sell gold in one year for a price of $275. Use the formula to find the corresponding no-arbitrage spot price.

no-arbitrage spot price = *$275 / (1.099)*
= *$250.23*

The actual spot price of $310 differs from the no-arbitrage spot price of $250.23. This temporary mispricing by $59.77 (= $310 - $250.23) represents present value of arbitrage profit. The following strategy captures the arbitrage profit.

Gold is overvalued in the spot market because its actual spot price of $310 is larger than its no-arbitrage price of $250.23. The no-arbitrage price is somewhat analogous to the gold's "intrinsic value." Always sell an overvalued asset and buy an undervalued one.

Sell gold in the spot market. That is, sell today an ounce of gold from your warehouse and collect $310. Subsequently invest the $310 for one year at 9.9%. Simultaneously take a position in the futures market whereby you agree to buy gold in one year at today's futures price of $275. These actions suggest you have taken a short position in the spot market and a long position in the futures market. When an asset is overvalued in the spot market then the asset is undervalued in the futures market and vice versa.

One year from today the financial investment of $310 matures and returns $340.69 (= $310 x 1.099). Fulfill obligations for your long futures position by buying gold at $275 per ounce. You are left in one year with $65.69 (=$340.69–$275) and exactly the same amount of gold in the warehouse as today. That $65.69 equals future value of arbitrage profit. Its present value is $59.77 (=$65.69 ÷ 1.099). Arbitrage profit exactly equals the amount of the mispricing.

For the preceding strategy the future value of arbitrage profit equals $65.69 and accrues at end of the investment horizon. An alternative approach capitalizes arbitrage profit, collecting its present value at the beginning. Sell gold today in the spot market and collect $310. Go long in the futures market and obligate to buy an ounce for $275 in one year (thereby replacing the ounce you sell today). Instead of investing the full $310 at 9.9%, simply invest $250.23. Pocket the other $59.77 (= $310 - $250.23) and use it for whatever purpose you want. This pocket money is “free” arbitrage profit. In one year the investment grows and returns $275 (= $250.23 x 1.099) which is exactly the amount required for fulfilling the long futures obligation.

b. For this question today's futures price equals $350. This means that today the company can lock in the right to buy or sell gold in one year for a price of $350. Use formula 12.1 to find the corresponding no-arbitrage spot price.

no-arbitrage spot price = *$350 / (1.099)*

= *$318.47*

The actual spot price of $310 differs from the no-arbitrage spot price of $318.47. This temporary mispricing by $8.47 represents arbitrage profit. Use the following arbitrage strategy to capture the arbitrage profit.

Gold is undervalued in the spot market because its actual spot price of $310 is smaller than its no-arbitrage price of $318.47. Borrow $310 at 9.9 percent to buy gold in the spot market. Put the gold in your warehouse. Simultaneously take a position in the futures market whereby you agree to sell gold in one year at today's futures price of \$350. These actions suggest you have taken a long position in the spot market and a short position in the futures market.

One year from today fulfill obligations for your short futures position by taking an ounce of gold out of the warehouse and selling it for \$350 per ounce. Repay the loan's \$340.69 principal and interest (=\$310x1.099). You are left in one year with \$9.31 (=\$350–\$340.69) and exactly the same amount of gold in the warehouse as today. That \$9.31 equals the future value of the arbitrage profit. Its present value is \$8.47 (=\$9.31 ÷ 1.099). The arbitrage profit exactly equals the amount of the mispricing.

The example above illustrates that arbitrage profit exists when prices in the spot and futures markets improperly align. The example below reinforces the irrelevance of expectations on the no-arbitrage equilibrium.

EXAMPLE 2 Stock Index Arbitrage

The SP2 Stock Index equals the sum of stock prices for companies A and Z. These companies never pay dividends. Today's stock prices equal \$37 and \$78 for stocks A and Z, respectively. The interest rate at which you may borrow and invest is 9.4%. What is today's prudent futures price for a one year contract on the SP2 index if you expect stock prices (a) to rise 25%, or (b) to fall 25%?

SOLUTION

The SP2 today equals the sum of 37 and 78. The SP2 in the spot market is 115. Use formula 12.1 to find the no-arbitrage futures price.

No-arbitrage spot price = No-arbitrage futures price / (1 + r)

115 = No-arbitrage futures price / (1.094).

No-arbitrage futures price = 125.81 .

a. A portfolio containing one share of each stock today is worth \$115. You expect the stock prices to rise 25 percent above their current level. Thus, in one year you expect the portfolio to be worth \$143.75. Suppose that today you observe the futures price is 135. You view this as a keen opportunity to take a long position in the futures market because you would love next year to buy stocks for \$135 that would be worth \$143.75. You reason that next year you could buy at \$135 and instantaneously sell at \$143.75 in the stock market. This strategy sounds sensible but is inherent with risk.

The actual futures price of \$135 overvalues the no-arbitrage futures price of \$125.81. Capture the arbitrage profit by today going short in the futures market and long in the spot market. Borrow \$115 at 9.4 percent and immediately buy the stocks. Simultaneously go short in the futures market thereby guaranteeing sale of the stock in one year at \$135.

One year from today fulfill obligations of the futures contract by selling the stocks for \$135. Take the proceeds and repay the loan's \$125.81 principal and interest (= \$115 x 1.094). You are left in one year with \$9.19 (=\$135–\$125.81). That sum equals the future value of the arbitrage profit. Its present value is \$8.40 (=\$9.19 ÷ 1.094). Present value of arbitrage profit exactly equals the difference between actual and no-arbitrage spot prices (= \$135/1.094 - \$115). You make the arbitrage profit without any risk and without any cost. Capitalize the arbitrage profit, if you wish, by immediately borrowing \$123.40 and buying the stocks for \$115 and pocketing \$8.40.

Notice that to capture the arbitrage profit you take a short position in the futures market, even though today's futures price for next year is less than the spot price you expect to prevail next year.

b. A portfolio worth \$115 today containing one share of each stock is expected to be worth \$86.25 in one year (= \$115 x (1-.25)). Suppose that today you observe the futures price is 100. First reaction might be taking a short position in the futures market because you would love next year to sell stocks for \$100 that could be purchased at that instant for \$86.25. This strategy is not taking advantage of the apparent arbitrage opportunity.

The actual futures price of \$100 undervalues the no-arbitrage futures price of \$125.81. Capture the arbitrage profit by today going long in the futures market and short in the spot market. Reinvest the proceeds from the \$115 short sale at 9.4 percent.

One year from today the loan matures and returns \$125.81. Fulfill obligations of the futures contract by buying the stocks for \$100. You are left in one year with \$25.81 (=\$125.81–\$100). That sum equals the future value of the arbitrage profit. Its present value is \$23.59 (=\$25.81 ÷ 1.094). Present value of arbitrage profit exactly equals the difference between actual and no-arbitrage spot prices (= \$115 – \$100/1.094). You make the arbitrage profit without any risk and without any cost.

Expectations about spot prices in the future have little relation with today's futures prices. This uncoupling of subsequent price movements from financial outcomes provides internal structure for most risk management decisions.

EXERCISES 12.1

Numerical quickies

1. Your company buys, sells, and stores warehouses full of gold. Today, the futures price for gold with delivery in one year is \$370 per ounce. The spot price is \$390 per ounce. Your company is able to invest and borrow at the risk-free interest rate of 6.9%. Furthermore, your company can, if necessary, store and safeguard an additional 1000 ounces of gold for virtually free. Find the present value of arbitrage profits on 1000 ounces of gold and describe the strategy for capitalizing that profit. ©FT2b .

2. The SP2 Index equals the sum of stock prices for companies Y and Z. Today's stock prices equal \$48 and \$40 for stocks Y and Z, respectively. Today's futures price for the SP2 with delivery in 1-year is \$110.58. The interest rate at which you may borrow and invest is 11.2%. Find the present value of stock index arbitrage profits and describe the strategy for capitalizing that profit. ©TQ17 .

2. Futures contracts

By taking a position on a futures contract an investor agrees to either buy or sell an underlying commodity at a specified future date. The investor agreeing to sell is taking a “short” position. The investor agreeing to buy is taking a “long” position. Each party entering the contract agrees today upon a price to be paid in the future. Table 12.1 lists active U.S. futures exchanges and commodities that they trade. Recent establishment or reorganization of several exchanges attests to dramatic changes occurring in financial markets. The variety of commodities traded attests to usefulness of futures contracts for pursuing business objectives. These exchanges do not exist solely to serve as speculative dins of greed. Instead, they exist and thrive because they allow companies and investors to pursue diverse investment and business objectives.

Exchange	Major Commodities	Remarks
CBOE Futures Exchange, LLC (CFE)	Volatility Indexes	CFE is a subsidiary of the Chicago Board Options Exchange (CBOE). Established 2003.
Chicago Mercantile Exchange (CME)	Livestock, dairy and agricultural products, stock indexes, Eurodollars and other interest rates, currencies. Grains, soybeans, US Treasury notes and bonds, other interest rates, and stock indexes.	CME was originally known as the Chicago Butter and Egg Board, which was formed in 1898. It became the CME in 1919, trading futures on a variety of agricultural products. In 2010 the CME merged with the CBOT, the older Chicago Board of Trade. Organized as a grain cash market in 1848, the CBOT is generally considered to be the oldest organized futures exchange. While experts disagree about the exact date when "true" futures trading began, CBOT cash contracts evolved into what are now considered futures contracts. Shortly before the civil war, traders at the CBOT began trading "to-arrive" or forward contracts in agricultural commodities including wheat, corn, and oats. In 1859, the CBOT was granted a charter by the Illinois legislature which, among other things, standardized grades and provided for inspectors of grain to be appointed by the CBOT, whose decisions were binding on members. In 1865, formal trading rules were instituted particularly concerning margin and delivery procedures. In 1877 the CBOT began publishing futures prices, and in 1883 the first clearing organization was established to clear CBOT contracts, initially on a voluntary basis.
Kansas City Board of Trade (KCBT)	Wheat, natural gas, and stock indexes	KCBT was established by local Kansas City merchants in 1869 as a means of trading grain. Futures trading in grains began in 1876.
Minneapolis Grain Exchange (MGE)	Spring wheat	MGE was established by the Minneapolis Chamber of Commerce in 1881 as an organization designed to promote trade in grains and to prevent abuses. In 1947, it became the MGE.
NQLX LLC Futures Exchange (NQLX)	Security futures products	NQLX originally established in 2001 as the Nasdaq LIFFE LLC Futures Exchange, and it operated as a joint venture of the Nasdaq Stock Market and the London International Financial Futures and Options Exchange (LIFFE). NQLX's relationship with Nasdaq ended on July 24, 2003, and it was renamed as NQLX.
New York Board of Trade (NYBOT)	Coffee, sugar, cocoa, cotton, frozen concentrated orange juice, currencies.	NYBOT was formed in 1998 when the Coffee, Sugar and Cocoa Exchange (CSCE) and the New York Cotton Exchange (NYCE) entered into a merger agreement, which was to occur in several stages. In June 2004 when the merger was completed, the CSCE's and NYCE's contract market designations were extinguished and transferred to NYBOT.
New York Mercantile Exchange (NYMEX)	Energy products	NYMEX was founded in 1872 as the Butter and Cheese Exchange of New York and became the New York Mercantile Exchange in 1882. COMEX was founded in 1933 from the merger of the National Metal Exchange, the Rubber Exchange of New York, the National Raw Silk Exchange, and the New York Hide Exchange (the oldest of these exchanges was founded in 1882). Since 1994, COMEX has operated as a subsidiary of NYMEX.
The COMEX Division (COMEX)	Metals	
Philadelphia Board of Trade (PBOT)	Currencies	The PBOT was established in 1985 as a subsidiary of the Philadelphia Stock Exchange.
U.S. Futures Exchange, LLC (Eurex US)	US Treasury Notes and Bonds	Eurex US formed in 2004 and is owned 80% by U.S. Exchange Holdings, Inc., a Delaware corporation that is a separately capitalized wholly-owned subsidiary of Eurex Frankfurt, AG, and 20% by Exchange Place Holdings, L.P., a Delaware limited partnership.

12.1

importance of the underlying commodity price	movements in the commodity price have modest effect on the overall outcome	movements in the commodity price almost totally determine the outcome
importance of the profit or loss on the futures contract	whether the futures contract turns a profit or a loss is largely irrelevant to the overall outcome	the outcome depends exclusively on whether the futures contract makes a profit or a loss

TABLE 12.2 Comparison of hedging and speculative motives for trading

A hedger is, for example, a farmer growing cotton that uses the futures contract to lock-in the selling price of cotton. They certainly care to trade a cotton contract rather than, say, a contract on silver. To a speculator, however, there is not much difference between futures contracts on silver or cotton since they have no real interest in the underlying commodity. Likewise, once the hedger locks in the price with the futures contract there is some indifference about subsequent movements in the price (at least as far as the current deal goes; for the long-run movements in price may matter a lot). The futures position may turn a profit or loss that is of little relevance since the delivery price is fixed. The profit or loss on a futures contract for a hedger is largely irrelevant. To a speculator, however, favorable price movement and profit from the position motivates the trade. The example below considers a speculative position for a futures contract.

EXAMPLE 3 Speculator rate of return ©FT1b

The *Wall Street Journal* shows that silver futures for delivery in 4 months have a margin requirement of one percent. They are quoted as follows:

SILVER 5,000 oz., cents per oz; last = 540.60

You anticipate a rise in the price of silver and therefore enter long on one contract. What is your rate of return if six weeks later when you close the contract the futures price is (a) 547.20 cents per ounce, or (b) 531.30 cents per ounce?

SOLUTION

Regardless of motive, hedging or speculating, the initial margin requirement from formula 12.3 per contract equals \$270.30 (= 0.01 *x* \$5.406 *x* 5,000). Each investor, long side or short, submits to the futures exchange \$270.30 per contract.

a. Six weeks later the futures price equals 547.2 cents per oz. Thus, enter long on one contract when futures price is \$540.6 cents and close when futures price equals 547.2 cents. Profit per contract from a long position is \$330.00 [= (\$5.472 – \$5.406) *x* 5,000)]. When you close your position at the futures exchange you receive a check for \$330 plus you get a refund of the \$270.30 initial margin. Your rate of return is determined from the usual formula that

$$\begin{aligned} \text{rate-of-return} &= (\text{ending wealth} - \text{beginning wealth}) \div \text{beginning wealth} \\ &= \text{profit} \div \text{beginning wealth} \\ &= \$330.00 \div \$270.30 \\ &= 122.09\% \end{aligned}$$

The rate of return exceeds 100% implying that your money more than doubles. Notice that to take a position in the contract you had to ante \$270.30. Although this money is not a cost, rather it is collateral, this sum nonetheless represents beginning wealth. Ending wealth equals return of collateral, \$270.30, plus profit from the futures position, \$330.00. Ending wealth of \$600.30 is certainly double beginning wealth.

b. For this problem enter long on one contract when futures price is $540.6 cents and close your position when futures price equals 531.3 cents. Profit per contract from your long position is $–465.00 [= ($5.313 – $5.406) *x* 5,000]. When you close your position at the futures exchange you must send them a check for $465; you do get a refund of the $270.30 initial margin. Your rate of return is –172% (= $–465.00 ÷ $270.30).

The rate of return exceeds –100% implying that you lose more than you put up. Indeed this is true. You ante $270.30 as initial margin. Upon closing your position send in another $465 which is partially offset by the refund of $270.30. Nonetheless, your net cash flow upon closing your position is $-194.70 (= $–465 + $270.30). This investment is so bad that you pay to get in and you pay to get out. With standard investments it typically is impossible to lose more than 100%. Speculative investments on futures are especially risky, though, because your potential losses often are unlimited — you can lose more than 100%.

EXAMPLE 4 Hedging with commodity futures contracts

The *Wall Street Journal* shows that cotton futures for delivery in 3 months have a margin requirement of 3/4s of one percent. They are quoted as follows:

COTTON 50,000 lbs., cents per lb; settle = 89.43

You are in the cotton-growing business and expect to deliver 100,000 lbs of cotton to the local gin in 3 months. You enter short on 2 contracts so that you can “guarantee” the price. Describe the cash flows for the following scenarios:

a. the spot price at the time of delivery is 80 cents per pound.

b. the spot price at the time of delivery is $1.00 per pound.

c. ALL your crops are destroyed by drought and the spot price at the time of delivery skyrockets to $2 per pound.

SOLUTION

a. The quote shows that each party agrees upon a futures price of 89.43 cents per pound. “Contract value” is from formula 12.2 and equals $44,715 (= $0.8943 x 50,000). The contract value represents the amount of money that in three months the long side pays to take delivery of the cotton. Alternatively, this is the amount that the short side receives upon making delivery. Formula 12.3 finds that the margin requirement equals $335.36 (= 0.0075 x $44,715). Each investor, long side as well as short, submits to the futures exchange $335.36 per contract. For two contracts, the margin requirement is $670.72

At delivery, the futures and spot prices converge. For this problem, enter short on two contracts when the futures price is 89.43 cents and close when the futures price equals 80 cents. Profit per contract upon closing the short position is

profit per contract = *($0.8943 – $0.80) (50,000)*

= *$4,715*

Profit for two contracts is $9,430. Effectively, you make money from this position because you agree to sell cotton for 89.43 cents and then, three months later, the price of cotton falls to 80 cents. Your right-to-sell cotton at a high price is valuable and you profit from owning this right!

You receive \$9,430 from the exchange upon closing your position. You also get back the margin requirement of \$670.72 that you paid at the beginning, but the margin "washes out."

At the local gin deliver 100,000 pounds of cotton and receive the spot price of 80 cents per pound for a total of \$80,000. The \$9,430 profit from the futures position brings ending financial wealth (ignoring the irrelevant margin) to \$89,430.

By relying on futures contracts you effectively "locked-in" the futures price of 89.43 cents prevalent three months before your harvest. By locking-in the price, you actually enhanced your position relative to what it would have been had you simply "floated" with the spot price.

b. For this problem futures and spot prices converge at delivery to \$1 per pound. Enter short on two contracts when futures price is 89.43 cents, and close when futures price equals 100 cents. Profit per contract upon closing the short position is

$$\textit{profit per contract} = (\$0.8943 - \$1.00)\,(50{,}000)$$

$$= \$-5{,}285$$

Cover your position at the futures exchange by sending them a check for \$5,285 per contract. For two contracts, the total loss is \$10,570. Effectively, you lose money from this position because you agree to sell cotton for 89.43 cents and then, three months later, the price of cotton rises to 100 cents per pound. Your obligation-to-sell cotton at a relatively low price is costly!

You pay \$10,570 to the futures exchange upon closing your position. You get back from the exchange the margin requirement of \$670.72 that you paid at the beginning, but the margin "washes out."

At the local gin deliver 100,000 pounds of cotton and receive the spot price of 100 cents per pound for a total of \$100,000. Subtract the \$10,570 loss from the futures position and find that ending wealth (ignoring the irrelevant margin) equals \$89,430.

Once again, the futures contract allows you to "lock-in" the futures price of 89.43 cents prevalent three months before your harvest. In this situation, however, you would have been better off had you "floated" with the spot price.

There is no way to be certain whether prices will be rising or falling. This uncertainty about future prices implies risk. Many businesses realize that by utilizing futures contracts they can effectively control the risk associated with price fluctuations. Futures contracts enhance entrepreneurial opportunities for locking-in outcomes.

c. Drought is devastating cotton crops, its price is skyrocketing, and this is really bad. Because you are short, the rising price means you are losing money in the futures market. Enter short on two contracts when the futures price is 89.43 cents and close when the futures and spot prices converge at 200 cents. Profit per contract upon closing the short position is

$$\text{profit per contract} = (\$0.8943 - \$2.00)\,(50{,}000)$$

$$= \$-55{,}285$$

Cover your position at the futures exchange by sending them a check for \$55,285 per contract. For two contracts, the total loss is \$110,570. This outcome might be tolerable if you could sell your cotton at the local gin for \$200,000; that is, 100,000 pounds at \$2 per pound. But your crop was destroyed. Somehow you have to pay off the futures exchange or they will send their lawyers after you and foreclose on the farm. Seldom is hedging totally risk-free. When fuel prices plummeted 50% in 2014 airline companies with long-term hedging arrangements had to continue paying high fuel prices. Companies with relatively short 30-day contracts could pay the lower prices giving these airlines a huge competitive advantage.

2.A. Currency transactions

U.S. companies could insist on working with a foreign company only if the foreign company agrees to transact with U.S. dollars. That shortsighted approach doesn't work well in today's globally competitive marketplace. If the U.S. company wants the job then they must do what it takes to obtain the job even if that means working with foreign currency.

More than a hundred currencies trade in the global market place. Table 12.3 lists countries, name of their currency, and currency prices measured in U.S. dollars. Some currencies are seldom heard about, such as the *dongo* of Vietnam. Other currency names, such as *dollar*, are very common. A very short list of countries, however, use the dollar with George Washington's picture (U.S.A. and its territories, Ecuador, Liberia, and Turk Islands). Otherwise, there is big difference in the value of a Canadian dollar versus a Singapore dollar – the currency name is simply a label.

TABLE 12.3 World currency prices measured in U.S. dollars
Notes: The table shows a historical snapshot of a foreign-exchange quotation table http://online.wsj.com/documents/mktindex.htm?worldval.htm.

Currency prices move as wildly as stock prices. Uncertainty about exchange rate movements introduces risk. Futures contracts on currencies provide a mechanism for business to manage exchange rate risk because they hedge against currency depreciation or appreciation. Currency depreciation or appreciation signifies that a currency's value is declining or increasing. The numeraire is the currency measuring the price of another currency. Most but not all transactions rely on the U.S. dollar as numeraire. For this lesson suppose one unit of foreign currency named *FC* has a price measured by a numeraire currency named *NC* that equals *a*, as in:

$$1 \ FC = \quad a \ NC.$$

For example, in Table 12.3 the price in dollars of one Mexican peso is 8.89 cents and therefore

$$1 \ \text{Mexican peso} = \quad 0.0889 \ \text{U.S. dollars.}$$

Rule 12.1 summarizes effect of currency appreciation and depreciation on currency price.

RULE 12.1 Currency appreciation and depreciation
Let *a* equal initial price measured in numeraire currency called *NC* for one foreign currency called *FC*. Let *x* equal the percentage appreciation ($x > 0$) or depreciation ($x < 0$) that occurs in the currency price. The price of one *FC* after the change is:

$$1 \ FC = \ a\,(1 + x) \ NC \ .$$

When x is positive the new price of one *FC* is bigger than the old price; this signifies the *FC* has appreciated (or strengthened) and NC depreciated (or weakened). Conversely, a declining price for *FC* implies a depreciating *FC* and appreciating numeraire.

For example, if the Mexican peso appreciates 4 percent relative to the U.S. dollar then find the new price as follows:

$$1 \ \text{Mexican peso} = \quad 0.0889\,(1 + 0.04) \ \text{U.S. dollars,}$$

or

$$1 \ \text{Mexican peso} = \quad 0.0925 \ \text{U.S. dollars.}$$

The price for 1 peso after it appreciates is 9 ¼ U.S. cents. The peso strengthened and the dollar weakened. The example below applies rule 12.1 and shows how exchange rate risk possibly hurts company performance.

EXAMPLE 5 Effect of currency exchange rate risk on the pretax rate of return ©FT4cm

Williams Imagineering Co. of Huntsville is preparing a bid to deliver expert software to the Riccar Co. of Lucerne, Switzerland.. Williams estimates that they can produce the software over the next 4 months at a pretax cost of 80,000 dollars ("USD").

a. If Williams believes that Riccar were willing to pay for the software in US dollars, how much should Williams bid on the project such that they obtain a 20% pretax rate of return (= Pretax profit ÷ Sales revenue)?
b. Williams anticipates they have a higher likelihood of winning the job if they accept payment from Riccar denominated in Swiss francs ("CHF"). Currently, the spot exchange rate is 1 CHF= 0.8433 USD. What should the bid equal in CHF if Williams expects exchange rates to remain constant.
c. Find effect on pretax rate of return if Williams makes the bid from part b and the exchange rate fluctuates so that in 4 months price of the CHF (i) depreciates or (ii) appreciates 15% relative to the dollar.

SOLUTION

a. The pretax cost is $80,000 and Williams wants a 20% pretax rate of return. Thus,

$$\textit{Pretax rate of return} = \textit{Pretax profit} \div \textit{Sales revenue}$$

$$0.20 = (\textit{Sales Revenue} - \$80{,}000) \div \textit{Sales revenue}$$

$$\textit{Sales revenue} = \$100{,}000$$

The bid should equal $100,000. Subtraction of the $80,000 pretax costs nets pretax profit of $20,000 and 20% pretax rate of return.

b. The problem tells us that the current spot price is:

$$1\ CHF = 0.8433\ USD\ .$$

Williams wants to receive $100,000 USD so its equivalent value in CHF is

$$(100{,}000 / 0.8433)\ CHF = 100{,}000\ USD$$

or

$$118{,}582\ CHF = 100{,}000\ USD$$

If Williams does the job for 118,582 CHF and exchange rates remain constant then Williams is able to convert their CHF into $100,000. This yields $20,000 pretax profit and a 20% pretax rate of return (= $20,000 ÷ $100,000).

c (i). For this problem price of the CHF depreciates 15% relative to the USD and the new currency price in four months for one franc becomes 71.68 US cents (= 0.8433 (1 – 0.15)). The company receives 118,582 CHF for doing the job and exchanges their francs as follows:

$$118{,}582\ CHF = 118{,}582\ (0.7168)\ USD$$

$$= 85{,}000\ USD$$

The 118,582 CHF that Williams receives from Riccar exchanges into 85,000 USD. Actual *Sales revenue* is 15% less than the target of \$100,000. Subtraction of \$80,000 in pretax costs from actual *Sales revenue* of yields \$5,000 of pretax profit and 5.88% pretax rate of return (= \$5,000 ÷ \$85,000). Effectively, Williams is hurt because the dollar strengthens and they are paid in a foreign currency worth less USD than expected.

c (ii). For this problem price of the CHF appreciates 15% relative to the USD and the new currency price becomes 96.98 US cents (= 0.8433 (1 + 0.15)). The company exchanges 118,582 francs into 115,000 USD (= 118,582 × 0.9698). Actual *Sales revenue* is 15% larger than target. Subtraction of pretax costs from actual *Sales revenue* yields \$35,000 of pretax profit and 30.4% pretax rate of return (= \$35,000 ÷ \$115,000). Williams is helped because the dollar weakens and they are being paid in a foreign currency that is worth more USD than expected.

Company management may make excellent plans for producing, marketing, and distributing products to clients. Because of factors beyond company control, however, exchange rate risk may sink the best laid plans. Currency prices change as wildly as stock prices and effects can be disastrous for companies conducting international commerce. Almost every large multinational company hedges currency exchange rate risk. The example below considers such a scenario.

EXAMPLE 6 Hedging currency exchange rate risk with a futures contract

The Company produces specialized equipment at a cost of \$275,000. They wish to sell the equipment in June to a British client paying with sterling pounds ("GBP") so that the pretax rate of return (= *Pretax profit* ÷ *Sales revenue*) is 16%. Today's exchange rate quoted in USD per GBP is \$1.8879 in the spot market and \$1.9450 in the futures market for June delivery. The Company today hedges the revenue by taking an appropriate position on currency futures contracts (assume contracts exist for fractional quantities). By June the spot price of the pound has depreciated 8% relative to the USD; the June futures and spot prices converge. In June the company sells the GBP in the local spot market and closes its futures position with a cash settlement. Find the Company's net revenue associated with the sale. Is the hedge beneficial?

SOLUTION

The pretax cost is \$275,000 and target pretax rate of return equals 16%. Thus,

$$0.16 = (\text{Sales Revenue} - \$275{,}000) \div \text{Sales revenue}$$

or $$\text{Sales revenue} = \$327{,}381.$$

The company plans to sell the equipment for \$327,381. The buyer, however, expects to pay in British pounds. The company uses a futures contract to lock-in the selling price of 1 GBP at \$1.9450. The company takes a *short* position in the futures market. The company immediately knows that the selling price in GBP is:

$$1\ \text{GBP} = 1.9450\ \text{USD},$$

$$(327{,}381 / 1.945)\ \text{GBP} = 327{,}381\ \text{USD},$$

$$168{,}319\ \text{GBP} = 327{,}381\ \text{USD}.$$

The company immediately informs the British client that the selling price for the equipment is 168,319 pounds payable in June upon delivery.

The spot price for 1 GBP in June is \$1.7369 (= \$1.8879 × (1 – 0.08)) because the spot price of the pound depreciates 8%. The company exchanges pounds at the local bank for \$292,348 (= \$1.7369 × 168,319). They close their futures position on 168,319 GBP at a futures price of \$1.7369 (futures price converges to spot price). Beginning futures price was \$1.9450. Profit for the short futures position from formula 12.4 equals \$35,033 (= 168,319 × (\$1.9450 – \$1.7369)). The currency exchange at the local bank plus profit on the futures contract totals \$327,381. Subtract the pretax cost of \$275,000 and find pretax rate of return equals the 16% target.

The futures market hedge is beneficial. The company receives payment in June in British pounds that are worth less USD than expected because the pound depreciated. The hedge let them avoid an unexpected loss.

EXERCISES 12.2

Numerical quickies

1. Awhile ago futures contracts for crawdads (1250 lbs. per contract) traded at a futures price of \$3.90 per lb. Today the futures price is \$3.71. The margin on the contract is 2.25%. Find for a speculative investor that was long one contract during this period the profit (or loss) and rate of return. ©FT1b .

2. The Company hopes to win a job for delivering its product to an overseas client. The Company must submit a bid to the client stating the cost of the job and the client decides whether or not to hire the Company. The Company estimates they can produce the product over the next few months at a pretax cost of \$110,000. Their target pretax profit margin (= Pretax profit ÷ Sales revenue) for this job is 12%. The Company is willing to accept payment from the client in foreign currency (krone). The spot exchange rate today is 1 USD = 1.35 krone.

2a. The company makes a bid such that if exchange rates remain constant the company gets the target pretax profit margin. How much in krone does the bid equal? ©FT4am .

2b. The client agrees to pay the Company its requested bid, but by the time the Company receives the payment the price of the krone has appreciated by 10 percent relative to the dollar. Find the actual pretax profit margin. ©FT4cm .

3. A futures contract provides the opportunity to lock-in the exchange rate at which you can buy or sell 100,000 pesos. The futures price quoted in U.S. cents per peso currently is 76.60. The margin requirement is 1.75%. You enter short on one contract. Thereafter, the price of the peso appreciates 6% relative to the USD. You then close your futures position. Find your profit (or loss) and rate of return. ©FT5a .

4. This is January and the Company plans on harvesting 40,000 bushels of soybeans in October. Currently, soybeans cost \$3.00 per bushel in the cash market and \$3.10 in the futures market for November delivery. The Company today goes short on 4 contracts (10,000 bushels each). In October, the Company delivers 40,000 bushels in the local market for the cash price of \$4.00. Also in October the Company closes its futures position on the November contracts at a futures price of \$4.30. Find the Company's net revenue and the benefit of the hedging strategy. ©FT3a .

5. This is January and the Company plans on receiving from its foreign subsidiary 120,000 sucre in June. Today's exchange rate quoted in U.S. cents per sucre is 74.20 in the local spot market and 74.50 in the futures market for July delivery. The Company

today goes short on 5 contracts (24,000 sucre each). By June the price of the sucre has depreciated 4% relative to the USD; this percentage change is reflected in both the spot and futures exchange rates. So in June the Company sells in the local spot market and, also, the Company closes its futures position with a cash settlement. What is the Company's net revenue associated with these transactions? ©FT6a ©FT6b .

3. Option contracts

Increasing sophistication of financial markets led to development within recent decades of another widely available financial security: the *option.*

DEFINITION 12.2 Long option position
A long option position is the right but not the obligation to buy (*call* option) or sell (*put* option) an underlying asset (the *underlier*) at a fixed price (the *strike* price) on or before an expiration date (*expiry*).

Options exist on many different types of underlying assets: stocks, currencies, indexes, commodities, interest rates, etc. Some options are private business-to-business contracts with substantial counterparty risk. Other options are issued by companies to key stakeholders or capitalists. Yet other options are standardized and marketable securities that trade on exchanges such as the American Stock Exchange (AMEX), Chicago Board Options Exchange (CBOE), or the International Securities Exchange (ISE). Lessons herein focus on marketable options.

Marketable stock options are especially popular. A call option contract on IBM with strike price of $50 and expiry next January, for example, provides the owner of the option the right to buy 100 shares of IBM for $50 per share anytime on or before expiration next January. The call option is especially valuable when IBM trades for a price bigger than $50. If IBM were trading at $80, for example, the call option provides its owner with a $30 advantage for purchasing a share. If on the other hand the price of IBM were below $50 at expiry then the owner of the call option throws away the option because it offers no advantage.

A long put option position provides the right to sell the underlier at the strike price on or before expiry. A Microsoft put option with strike of $40 is especially valuable when, for example, Microsoft trades for $25. Owner of the put option contract can sell 100 shares for $40 each, an advantage of $15 per share.

Option contracts are valuable tools. Getting into a long option position requires purchasing the option and paying the market price. The price of an option is called the *premium*. Losses on long option positions are limited because, unlike futures contracts which entail an obligation, a long option position always may be discarded. Table 12.4 compares characteristics of futures and options.

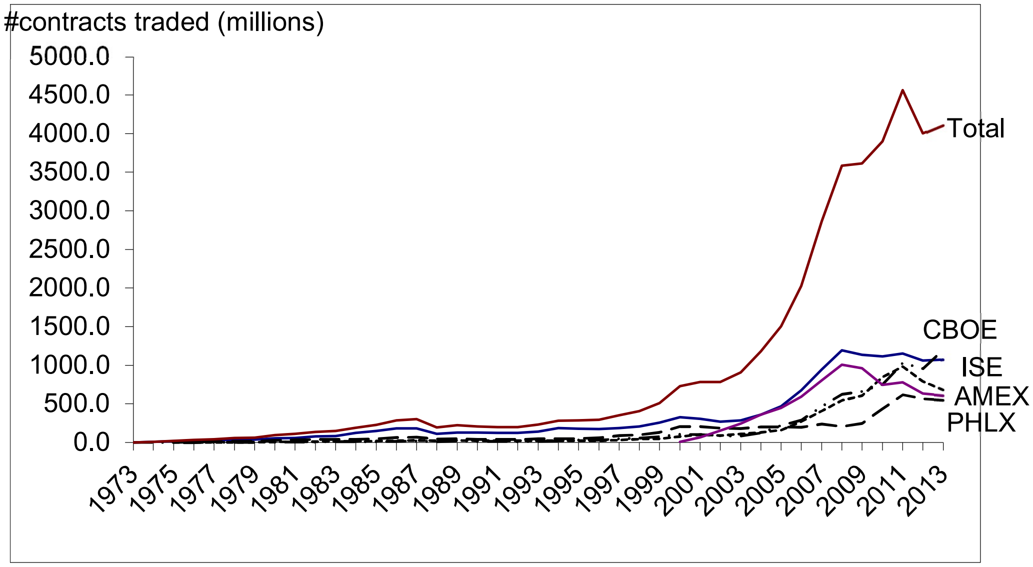
	futures contract (long or short)	*long option position (call or put)*	*short option position (call or put)*
initial cash flow	free to enter but requires deposit of refundable collateral (*margin*)	requires payment of purchase price (*premium*) to the short-side	receives purchase price (*premium*) from the long-side
terminal cash flow	profit increases as underlier price rises (long-side) or falls (short-side); otherwise loss is unlimited	profit increases as underlier price rises (call) or falls (put); otherwise loss is limited to premium	when long-side discards option terminal *CF*= 0; otherwise loss is unlimited
obligation	either buy (long-side) or sell (short-side) underlier at fixed futures price or buyout contract	pay the purchase price and have right to exercise or discard the option	fulfill wish of long-side and sell (call) or buy (put) underlier at the strike price

TABLE 12.4 Characteristics of futures and option contracts

Figure 12.1 charts the number of option contracts traded on major exchanges since 1973 when the CBOE began trading call options. In 1977 the CBOE started trading put options. Prior to the CBOE the trading of options was off-exchange, peer-to-peer trading with substantial counterparty risk and non-standard contracts. An exchange provides a place for the market to thrive and serve its purpose. By 1990 more than 200 million option contracts traded. By 2004 the number had grown to 1.2 billion contracts per year. In 2011 more than 4.5 billion option contracts on a diversity of products transferred ownership and wealth. The 2013 annual number is 4.1 billion contracts traded in U.S. option exchanges.

Trades of all option contracts at the CBOE in 2013 represent about 26% of U.S. option trading (ISE is 15% in 2013). That obviously is down from 100% of total options trading when the CBOE was the only forum in town. CBOE still is the largest U.S. options exchange and more than a half-dozen other U.S. option exchanges are growing rapidly, too. The dollar volume of premiums paid for CBOE options in 2013 is $552 billion of which 59% are for call contracts and 41% for put contracts. The average premium per contract in 2013 at the CBOE is $515. The CBOE trades options for stocks (39%), indexes (35%), and exchange traded funds (26%).

FIGURE 12.1 Number of option contracts traded per year on U.S. exchanges.



Chicago Board Options Exchange (CBOE), International Securities Exchange (ISE), American Stock Exchange (AMEX), and Philadelphia Stock Exchange (PHLX), and Pacific Stock Exchange (PCX). Compiled by author. http://www.cboe.com/data/marketstats-2013.pdf

Trading volume and the market growth of option exchanges for more than a decade has been phenomenal. Surely some options trading is speculative in nature but options nonetheless provide companies with opportunities for implementing risk management strategies. Tens of thousands of workers pursuing company objectives to use financial tools provided by the modern world for managing wealth. Its like a giant chess game in which the players shape balance sheets with the flow of funds. Sections below offer lessons on how companies and investors employ options to attain diverse objectives.

3.A. Call options

A call option provides the owner with the right to buy an underlying asset on or before expiry at a fixed strike price. Purchase of a call option requires paying its market price to the seller (the seller is the short-side for this trade). The option price, also known as the premium, rises and falls in response to market demand. The single most important factor driving the premium is the price of the underlier. A call option to buy IBM for a $50 strike price, for example, is more valuable when IBM trades at $80 than at $51. Option contracts, like futures, derive value based upon price of the underlying asset. Futures and options for this reason are known as derivative securities.

The payoff on a call option at expiry equals:

FORMULA 12.5 Call option payoff for a stock option

$$\begin{pmatrix} call\ option \\ payoff \\ at\ expiry \end{pmatrix} = maximum\left[0\, , \begin{pmatrix} stock & - & strike \\ price & & price \end{pmatrix} \right]$$

The option payoff is zero if the stock price at expiry is less than the strike price. Conversely, the payoff surpasses zero as the stock price rises above the strike price. Terminal payoff for a long call option position increases with price of the underlier.

The option investment yields a profit of $0, and hence a 0% rate of return, when the payoff just equals the initial cost of purchasing the option. The stock price at expiry that yields the 0% rate of return for the option is called the "break-even stock price." Solve for this break-even stock price from the following definition for a non-zero payoff:

payoff for an in-the-money call option = stock price − strike price .

Set the payoff equal to the initial cost of the option, and solve for the break-even stock price:

$$\begin{pmatrix} initial \\ cost\ of \\ call\ option \end{pmatrix} = \begin{pmatrix} break-even \\ stock\ price \end{pmatrix} - \begin{pmatrix} strike \\ price \end{pmatrix} .$$

$$\begin{pmatrix} break-even \\ stock\ price \end{pmatrix} = \begin{pmatrix} strike \\ price \end{pmatrix} + \begin{pmatrix} initial \\ cost\ of \\ call\ option \end{pmatrix}$$

The break-even stock price equals the sum of strike price and initial cost of the option.

EXAMPLE 7 Call option rate of return

Common stock for Microsoft Co. is currently trading at $83.50 . A call option on the stock with expiry in three months and strike of $90 is priced at $7 1/8. Explain whether the following statements are accurate?

a. If the stock price at expiry is $87 then investment in the stock would have provided a lower rate of return than investment in the option.

b. If the stock price at expiry is $94 then investment in the option would have provided a lower rate of return than investment in the stock.

c. If the stock price at expiry is $100 then investment in the option would have provided a greater rate of return than investment in the stock.

SOLUTION

a. The option's rate of return is –100% because the stock price, $87, is less than the strike, $90. The stock price ends slightly (+4.19%) higher than its beginning price of $83.50. Without making any computations whatsoever, we see the stock rate of return exceeds the option rate of return so the statement is false.

b. For this problem, the strike is 90 and the original option cost is 7 1/8. The break-even stock price is:

$$\begin{pmatrix} break-even \\ stock\ price \end{pmatrix} = 90 + 7.125$$

$$= \$97.125$$

When the stock price at expiry is $97 1/8 the payoff equals $7 1/8. This exactly equals the initial cost of the option, and implies the profit and rate of return from the option investment equals zero.

The actual stock price at expiry, $94, is less than the break-even stock price of $97. Computations are unnecessary to realize that the rate of return on the option investment is negative. The positive stock rate of return exceeds the negative option rate of return so the statement is true.

c. This problem requires a few computations in order to determine whether the statement is true or false. The percentage change in the stock price is 19.76%; the stock begins at $83.50 and ends at $100.00. Compare this to the option rate of return. First, find the payoff when the stock price at expiry is $100:

$$\begin{aligned} \textit{call option payoff} &= \textit{maximum } [\,0\,,\ 100 - 90\,] \\ &= \$10\,. \end{aligned}$$

For the option investment, the payoff is $10 whereas the initial cost is $7 1/8. The rate of return is 40.35% (= ($10.00 ÷ $7.125) − 1). The option rate of return exceeds the stock rate of return so the statement is true.

EXAMPLE 8 Find stock price giving call option ROR

You buy 1 share of stock at $41 and also purchase one call option with strike of $40 at a price of $1.50 . What is the stock price at expiry such that your overall rate of return on the position is: (a) 0%; (b) 10%; (c) −10%?

SOLUTION

a. You buy 1 share for $41 and one call option for $1.50. Your total initial cost therefore equals $42.50. The rate of return on your investment in this position compares the beginning wealth of $42.50 with your ending wealth. To receive a zero percent rate of return, as in this problem, your ending wealth equals $42.50.

The ending wealth equals the sum of the stock price at expiry plus the payoff on the call option. The call option payoff is

$$\textit{call option payoff} = \textit{maximum } [\,0\,,\ \textit{stock price} - 40\,]$$

Setting the desired ending wealth to its components shows

$$\begin{aligned} \textit{ending wealth} &= \textit{stock price} + \textit{maximum } [\,0\,,\ \textit{stock price} - 40\,] \\ \$42.50 &= \textit{stock price} + \textit{maximum } [\,0\,,\ \textit{stock price} - 40\,] \end{aligned}$$

To solve for the stock price we must check each of the two conditions in the payoff function. If the resultant stock price is logically consistent with the stipulated condition, then it is the answer.

To check the first condition in the payoff function, suppose the stock price is less than $40. The call option payoff subsequently equals zero. The equation for ending wealth given this condition is

$$\$42.50 = \textit{stock price} + 0\,,$$

or

$$\textit{stock price} = \$42.50\,.$$

The resultant stock price is *in*consistent with the stipulated condition. Therefore, this is <u>not</u> the answer.

To check the second condition in the payoff function, suppose the stock price exceeds $40. The call option payoff subsequently is positive. The equation for ending wealth given this condition is

$$\$42.50 = \textit{stock price} + \textit{stock price} - 40\ ,$$
or
$$\textit{stock price} = \$41.25\ .$$

The resultant stock price is consistent with the stipulated condition. Therefore, this is the answer.

When the stock price at expiry equals $41.25, the call option payoff is $1.25. The call option payoff, together with the share that you own, have a total value of $42.50 — an amount exactly equal to your initial investment.

b. Because your rate of return is 10%, and your beginning wealth is $42.50, your ending wealth is $46.75 (= $42.50 × 1.10). As in the previous problem, equate the ending wealth to the sum of the stock price at expiry plus the call option payoff. Solve for the stock price for both conditions in the payoff function.

To check the first condition in the payoff function, suppose the stock price is less than $40. The call option payoff subsequently equals zero. The equation for ending wealth given this condition is

$$\$46.75 = \textit{stock price} + 0\ ,$$
or
$$\textit{stock price} = \$46.75\ .$$

The resultant stock price is *in*consistent with the stipulated condition. Therefore, this is not the answer.

To check the second condition in the payoff function, suppose the stock price exceeds $40. The call option payoff subsequently is positive. The equation for ending wealth given this condition is

$$\$46.75 = \textit{stock price} + \textit{stock price} - 40\ ,$$
or
$$\textit{stock price} = \$43.375\ .$$

The resultant stock price is consistent with the stipulated condition. Therefore, this is the answer.

c. Solve for the ending wealth given an initial wealth of $42.50 and –10% rate of return.

$$-0.10 = (\textit{ending wealth} \div \$42.50) - 1$$
$$\textit{ending wealth} = \$38.25\ .$$

Equate the ending wealth to the sum of the stock price at expiry plus the call option payoff, and check for both conditions.

To check the first condition in the payoff function, suppose the stock price is less than $40. The call option payoff subsequently equals zero. The equation for ending wealth given this condition is

$$\$38.25 = \textit{stock price} + 0\ , \quad \text{or} \quad \textit{stock price} = \$38.25\ .$$

The resultant stock price is consistent with the stipulated condition. Therefore, this is a correct answer. To be complete, check the second condition that the stock price exceeds $40. The call option payoff subsequently is positive. The equation for ending wealth given this condition is

$$\$38.25 = \textit{stock price} + \textit{stock price} - 40\ , \quad \text{or} \quad \textit{stock price} = \$39.125\ .$$

The resultant stock price is *in*consistent with the stipulated condition. Therefore, this is not an answer.

EXAMPLE 9 Call option portfolio insurance

You pursue a 2-year investment strategy that is tied to the good fortunes of the General Electric company. Instead of buying shares in GE, however, you instead pursue a portfolio insurance strategy that invests 90% of your $10,000 in a safe money market account earning 7.8% compounded monthly. The remainder of your funds is allocated to long term call options ("LEAPS") on General Electric. The options expire 24 months from now, have a strike of 110, and cost $12.50 . The GE shareprice is 103. Suppose you can buy fractional options.

a. How much is invested in options and how much in the safe asset?
b. What is the floor below which your wealth will not dip?
c. If at expiry GE is up 30%, what is your position's rate of return?

SOLUTION

a. Ninety percent of your $10,000 is invested in the money market; this is $9,000. The remaining $1,000 is invested in call options that cost $12.50 each. You buy 80 call options.

b. You will not lose the $9,000 invested in the money market regardless of occurrences in the stock market. In fact, you also are certainly going to earn interest on your money market account, although there is uncertainty about exactly what the rate may equal. Given that the interest rate is 7.8% compounded monthly, and your money is invested for 24 months, then your ending balance for the money market account is determined as follows:

$$\begin{aligned} FV &= 9000x(1 + .078/12)^{24} \\ &= \$10,514 . \end{aligned}$$

No matter what, you at least end with $10,514 after 24 months. Notice that in the worst case you lose the $1,000 invested in options. This worst case occurs if the GE shareprice in 2-years is less than the strike of $110. Still, however, your rate of return for this worst case scenario is 5.14%.

c. When GE rises 30%, it closes at $133.90. The payoff on each option is $23.90. For your 80 options, the total payoff is $1,912. Add this to the money market terminal value of $10,514 and the total position is worth $12,426. This is a 24.26% return. It is less than the 30% movement in shareprice, but then again this portfolio insurance strategy limits downside risk — your downside loss is a +5.14% gain, and your upside potential is unlimited.

3.B. Put options

The put option payoff is

FORMULA 12.6 Put option payoff

$$\begin{pmatrix} put\ option \\ payoff \\ at\ expiry \end{pmatrix} = maximum\left[0\ ,\ \begin{pmatrix} strike \\ price \end{pmatrix} - \begin{pmatrix} stock \\ price \end{pmatrix} \right]$$

The option payoff is zero if the stock price at expiry exceeds the strike price. Conversely, the payoff surpasses zero as the stock price falls below the strike price — the put option gains as the stock price falls.

EXAMPLE 10 Put option portfolio insurance

You buy 100 shares of Pacific Dunlop at $43 per share and also purchase 100 put options with strike of $40 at a price of $ 3/8 .
a. If the stock price at expiry equals $46 then what is your overall rate of return?
b. What is the stock price at expiry such that your overall rate of return is 0%?
c. How does your portfolio perform if the share price falls below $40?

SOLUTION

a. You buy 100 shares for $4,300 and 100 put options for $37.50 . Your total initial cost therefore equals $4,337.50. The rate of return on your investment in this position compares the beginning wealth of $4,337.50 with your ending wealth.

The ending wealth equals the sum of the stock price at expiry plus the payoff on the put option. For this problem, the stock price at expiry is $46. The put option payoff is zero because the actual stock price exceeds the strike. Consequently, the only asset of value at expiry is the 100 shares. The shares are worth a total of $4,600. The rate of return is therefore 6.04% [= ($4,600 / $4,337.50) – 1].

b. A zero rate of return occurs when your ending wealth is $4337.50. Setting the desired ending wealth to the sum of its components shows that for all 100 shares:

$$\begin{aligned} ending\ wealth &= 100 \times \{\ stock\ price\ +\ maximum\ [\ 0\ ,\ 40 - stock\ price\ ]\ \} \\ \$4{,}337.50 &= 100 \times \{\ stock\ price\ +\ maximum\ [\ 0\ ,\ 40 - stock\ price\ ]\ \} \\ \$43.375 &= stock\ price\ +\ maximum\ [\ 0\ ,\ 40 - stock\ price\ ] \end{aligned}$$

To solve for the stock price we must check each of the two conditions in the payoff function. If the resultant stock price is logically consistent with the stipulated condition, then it is the answer.

To check the first condition in the payoff function, suppose the stock price exceeds $40. The put option payoff subsequently equals zero. The equation for ending wealth given this condition is

$$\$43.375 = stock\ price\ +\ 0\ ,$$

or

$$stock\ price = \$43.375\ .$$

The resultant stock price is consistent with the stipulated condition. Therefore, this is an answer.

For completeness, check the second condition in the payoff function. Suppose the stock price is less than $40. The put option payoff subsequently is positive. The equation for ending wealth given this condition is

$$\$43.375 = stock\ price\ +\ 40 - stock\ price\ ,$$

or $43.375 = 40$

This answer is *in*consistent. Therefore, this is not the answer.

c. Below a shareprice of \$40, the put offers a positive payoff equal to 40 – P, where P is the shareprice. Your share is worth P, and your option pays off 40 – P. Your ending wealth is the sum of these two:

$$\text{ending wealth} = 100 \times (P + 40 - P) = \$4{,}000$$

No matter how low the shareprice falls, you always end up with \$4,000. Your worst case rate of return therefore equals

$$\text{rate of return} = (\text{ending wealth} \div \text{beginning wealth}) - 1 = (\$4{,}000 \div \$4{,}337.50) - 1 = -7.79\%$$

Many strategies use combinations of options, like buy both a put and a call, to structure the payoffs toward specialized targets. The example below illustrates the well known straddle combination.

EXAMPLE 11 Straddle

Due to political events you believe that the markets are unusually nervous. You also believe that the uncertainty will be resolved in the near future and that consequently stocks either will rise or fall, but they likely will not remain unchanged. You therefore enter long in a "straddle" on IBM stock options. In particular, you go long on one put option with strike of 60 and price of \$0.13, and you go long on one call with strike of 60 and price of \$1.25. The share currently is trading at \$61.

a. What is the maximum possible loss?

b. Suppose the stock price at expiry is \$65 . What is the rate of return for the straddle?

c. What is the stock price at expiry such that the rate of return on the straddle is 0%?

SOLUTION

a. The initial investment is \$0.13 for the put and \$1.25 for the call, totaling \$1.38. This is the most you can lose. The payoff equals zero for both the call and the put when the stock price at expiry equals the strike price. For both options, the strike price is \$60. Thus, you'll lose everything if the shareprice at expiry is \$60.

When the stock price at expiry is below \$60, your put generates a payoff whereas the call is worthless. When the stock price at expiry exceeds \$60 the call generates a payoff whereas the put is worthless. The worst case outcome is a \$60 stock price.

b. When the stock price at expiry equals \$65 the put is worthless whereas the call generates a \$5 payoff per share. Your ending wealth therefore is \$5. Your rate of return is 262% [= (\$5.00 – \$1.38) / \$1.38].

c. The rate of return is zero when the total payoff equals the initial cost of \$1.38 . There are two possible scenarios. One possibility occurs when the put generates the \$1.38

payoff while the call is worthless. The other possibility occurs when the call generates the $1.38 payoff while the put is worthless.

When the stock price at expiry is less than the $60 strike by $1.38, the put generates a payoff of $1.38. Thus, at a stock price of $58.62 (= $60 – $1.38) the rate of return on the straddle is zero. Any stock price less than $58.62 implies a positive rate of return for the straddle.

When the stock price at expiry exceeds the $60 strike by $1.38, the call generates a payoff of $1.38. Thus, at a stock price of $61.38 (= $60 + $1.38) the rate of return on the straddle is zero. Any stock price greater than $61.38 implies a positive rate of return for the straddle.

EXERCISES 12.3

Numerical quickies

1. A common stock has a current share price of $10.80. A call option on the stock with strike of $2.50 has an option price of $5.00. How much is the arbitrage profit? ©DS10 .

2. Company shares have a current market price of $69.00. A call option on the shares has a strike of $70 and a price of $1.00 . Find the rate of return on the call option investment if at expiry the percentage change in shareprice is 21%. ©DS2a .

3. Suppose a call option with strike of 60 costs $5.60.
3a. Find the underlying stock price at expiry such that the profit is zero. ©DS3a .
3b. Find the underlying stock price at expiry such that the rate of return on the option investment is 160%. ©DS3b .

4. You buy 1 share of stock at $20.35 and also purchase one put option with strike of $25.00 and option price of $7.75. Find the stock price at expiry such that your overall rate of return on the position is 21%. ©DS22 .

5. You buy 1 share of stock at $33.20 and also purchase one put option with strike of $35 and price of $4.00.
5a. Find the overall rate of return on your position for the worst-case outcome. ©DS4bm .
5b. Suppose the percentage change in stock price at expiry is 8%. Find the overall rate of return from the position. ©DS4am .

6. You buy a put option on 3,000 Euros with a strike of 1.10 USD for a price of 0.0560 USD per Euro. At expiry, the spot exchange rate is 1 Euro = 0.89 USD and you cash in the options at their payoff value (if any). Find the rate of return from this speculative transaction. ©DS5a .

Numerical challengers

7. Your company purchases a call option on Corn in order to hedge its production costs. The quote for the option looks like this:

CORN 5,000 bushels; cents per bushel

strike	call last
208	14.10

The spot price of corn at expiry is $2.55 per bushel and the option price converged to its intrinsic value. How much money did this strategy save your company? ©DS24 .

8. You have $14,000 to invest in Company shares that currently trade at $25.20. You choose to invest 10% of your funds in long-term call options with a strike of 30 that currently are quoted at $0.70. The options expire in 10 months. The other funds will be placed into a money market earning 5.5% compounded monthly.
8a. Find the rate of return for the holding period in the worst-case outcome. ©DS6bm .
8b. Find the rate of return for the holding period on the total investment position if the share price is up 28% at expiry. ©DS6am .

9. The price of Company stock currently is $38.60. You have $10,000 available for investing in the good fortunes of the Company. Instead of buying the stock, however, you pursue a portfolio insurance strategy that invests in a money market account earning 7.20% compounded monthly. Also, you invest in call options on the Company stock with a strike of $47.50 and option price of $3.50 (assume you can buy fractions of options). Your allocation assures you that, even in a worst-case scenario, you will not lose more than $2,000 of your original principal. Suppose that at the conclusion of your 28 month investment horizon the Company stock has risen 30%. Find the ending wealth and rate of return for the investment strategy. ©DS17a .

10. Many individual investors employ a “buy-and-write” investment strategy that involves a long stock position and short call position. You implement the strategy by buying a stock at price $40.65 and writing a call with strike of $50.00 and option price of $5.60. In one year you receive the stock’s annual dividend of $2.85.
10a. Find the maximum annual rate of return that you can possibly earn from this buy-and-write strategy. ©DS23 .
10b. Suppose that in one year the stock price has increased 11%. Find the amount by which the rate of return for this buy-and-write strategy exceeds the rate of return for the stock-only strategy. ©DS19b .
10c. Find the worst case outcome, i.e., the minimum annual rate of return that you can possibly earn from this buy-and-write strategy. ©DS21 .

11. Today is Jan. 2, 2525 and the Company plans to exchange 4,000 Euros with its international subsidiary in 3 months. A put option on Euros with a strike of 1.12 USD and expiry in 3 months costs 0.043 USD per Euro. The Company buys put options on 4,000 Euros. In 3 months, just prior to expiration of the options, the spot exchange rate is 1 Euro = 0.92 USD. The Company cashes in the options at their intrinsic value and sells 4,000 Euros at the spot exchange rate. Find the net revenue in USD of exchanging the Euros. ©DS5b .

12. The share price of Company stock currently is $54.00. Due to a pending court case there is a lot of uncertainty about the Company. You believe the share price might either rise a lot or fall a lot. You do not buy the share. Instead, you buy one call option that costs $9.80 and you buy one put option that costs $0.90. For both options, the strike is 45.
12a. Describe how possible rates of return depend on the stock price at expiry. ©DS8 .
12b. Describe the outcome(s) that generate a 30% overall rate of return from investment in the options. ©DS7c .
12c. If at expiry the share price is $36.60 what is your overall rate of return? ©DS7b .

13. You have accumulated 1,800 shares of company stock because of a generous employee stock ownership plan. Today’s share price is $32.50. You use a collar to lock-in the value 8 months from now of today’s stock holdings. The collar takes a long position on 1,800 put options with strike of 35 and per unit option price of $7.50. Also you take a short position on 1,800 call options with strike of 45 and per unit option price of

$4.00. Compute the initial cash flow for the collar, the maximum ending wealth, and the minimum ending wealth. ©DS9 .

4. Important financial economic arbitrage relationships

Currency flows lubricate global and local business transactions. Currencies are commodities. Like all other commodities they have prices. When a business wishes to exchange, say, $100 for British pounds the business effectively sells US dollars and buys British pounds. The exchange rate represents the currency price. This currency exchange in every way is analogous to a trip to the grocery store to buy milk. You sell your dollars and buy the milk. The price of milk represents the exchange rate between milk and dollars. Likewise, between all currency pairs there exists a price representing the exchange rate.

The currency *fo*reign *ex*change market, known as the *forex* market, began maturing in 1971 when major world economies adopted floating exchange rate systems. A floating exchange rate is one where supply and demand by traders determines currency price. Currency traders include multinational companies like General Electric or Toyota and financial institutions such as Compass Bank or J.P. Morgan Chase. Even some households trade currencies through internet accounts (an internet search for "forex" finds hoards of companies wanting your business). Other currency traders include government central banks such as the Federal Reserve Bank of the U.S.A. or the Bank of England or the European Central Bank or the Bank of Japan. Around the globe more than 50 central banks hold membership in the Bank for International Settlements (see www.bis.org) and exert significant influence on currency supply and demand.

Prior to 1971 exchange rates did not float. Instead, for the decades following World War II exchange rates were set at fixed prices in accordance with the Bretton-Woods Agreement of 1944 (not all governments became members of the system). This fixed exchange rate system forced currency exchanges at prices established by central banks through a committee within the newly established International Monetary Fund ("IMF"). U.S. companies making sales in France and receiving French francs, for example, exchanged francs into dollars at local banks with fixed prices set by the IMF. Banks and businesses around the world used these fixed exchange rates for converting currencies. Quite often, however, forces of supply and demand caused a currency's black market unofficial rate to diverge from the official government exchange rate.

Currencies are widely held by everybody (check your wallet) and even though government central banks exert significant influence other traders also affect supply and demand. A hotel in Paris, for example, would give a businessman a hotel bill denominated in francs. The hotel allowed the businessman to pay in either French francs or U.S. dollars. But the hotel was forced by law to use the official government exchange rate. Out on the street money-changers made a living by trading currencies at black-market floating exchange rates related to currency supply and demand. The hotel guest found advantageous prices for currency exchange in the black market. The businessman, and companies too, could rely on black-market money-changers to exchange currency and thereby reduce costs or increase revenue.

Currency black-markets grew. Governments increasingly had trouble holding the line on fixed currency prices. For a government to set the price of its currency is analogous to IBM setting the price of its stock – that's fine as long as everyone agrees but when supply and demand shift then economic forces cause the price to float away from its official fixed point. In 1971 the Bretton-Woods system of fixed prices disbanded and member central banks officially accepted free-market floating exchange rates. A flood of financial companies moved into the currency exchange line of business. The forex market flourished – and continues to do so. Currency prices in today's world, like prices for stocks, bonds, and milk, are set by competitive free-market forces of supply and demand.

The currency exchange market is the world's largest financial market. Table 12.5 shows that average trading volume for 2013 exceeds $5,300 billion dollars *per day*, that is nearly $300 billion per hour! Look back to chapter 7 and find that Table 7.1 shows average daily trading volume for the New York Stock Exchange is about $50 billion on a good day. Certainly the NYSE is huge and influential. But the difference between $50 and $5,300 per day is huge, especially when you are measuring a billion dollars! This remarkable fact attests to the power and pervasiveness of economic globalization. The forex market is by far the most liquid and active financial market in the world. Prices in this market float on global consensus estimates of currency values. It is impossible for any individual, company, central bank, or even any government, to persistently resist the free-market price for a dollar or dinar, rupee or ruble, yen or euro, or any other world currency.

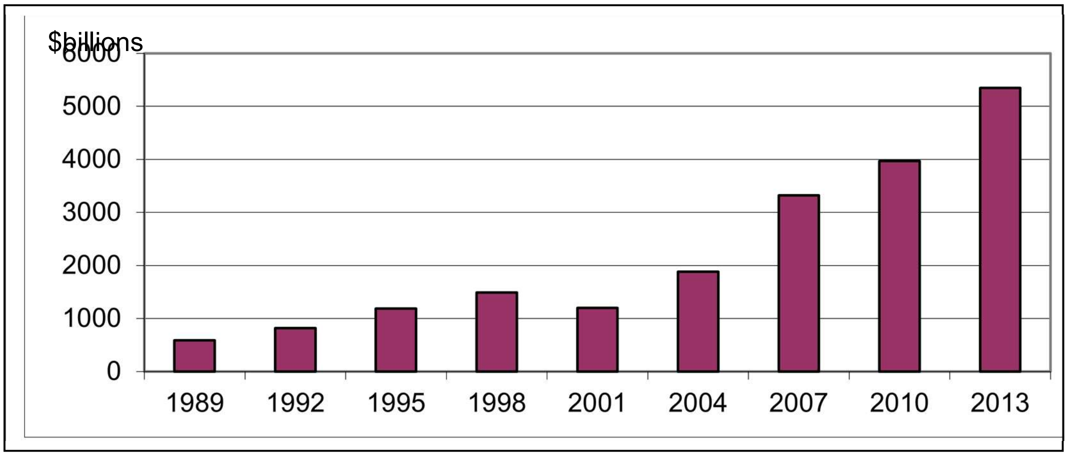


TABLE 12.5 Average daily trading volume in the currency foreign exchange market (measured in billions of U.S. dollars).
Source: Bank for International Settlements. http://www.bis.org/publ/qtrpdf/r_qt0412f.pdf

More than three-quarters of all forex activity involves the euro (EUR), Japanese yen (JPY), Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), or British pound (GBP). Most trades involve the U.S. dollar (USD) on one side. Trading between two non-dollar currencies usually occurs by first trading one against the USD and then trading the USD against the second non-dollar currency. The USD is indeed the numeraire for measuring the price of all other currencies (this stems from the Bretton-Woods proclamation that $35 USD is "good-as-gold"). Direct and heavily traded non-USD currency pairs include GBP-to-EUR, EUR-to-CHF, EUR-to-JPY, and AUD-to-JPY.

Trading in the forex market is decentralized without exchanges or physical trading floors. Trades occur between any two counterparties agreeing to trade via telephone or electronic networks such as Reuters.com. Still, forex businesses concentrate geographically and London is site for about 30% of the market, New York handles 20%, Tokyo 12%, Zurich, Frankfurt, Hong Kong and Singapore each represent about 7% of the forex market, and Paris and Sydney host about 3% each.

4.A. Triangle arbitrage and currency prices

The term *cross-rate* refers to the price between two non-USD currencies. The currency price measured in British pounds for buying one Swiss franc, for example, is a cross-rate. Between the GBP, CHF, and USD there exist three currency pairs (GBP-to-USD, CHF-to-USD, and GBP-to-CHF) and three relative prices. Relations between prices for <u>any</u> 3 currencies abide by a triangle arbitrage equilibrium condition. Suppose for this lesson that numbers *a* and *b* measure prices in numeraire currency *NC* for one unit of foreign currencies *FCA* and *FCB*, respectively:

$$1 \ FCA = a \ NC$$

and

$$1 \ FCB = b \ NC \,.$$

Rearrange and find that:

$$1/a \ FCA = 1/b \ FCB \,.$$

The preceding formula implies the equilibrium cross-rate for one unit of currency *FCA* as measured by *FCB* equals *a/b*. Formula 12.7 summarizes this lesson.

FORMULA 12.7 Triangle arbitrage equilibrium cross-rate

Prices for 1 unit of foreign currencies *FCA* and *FCB* as measured by numeraire *NC* equal *a* and *b* respectively. Equilibrium cross-rate between currencies *FCA* and *FCB* is given by:

$$\begin{pmatrix} \text{equilibrium price} \\ \text{measured in } FCB \\ \text{for 1 unit of } FCA \end{pmatrix} = a/b \,.$$

Violation of formula 12.7 implies existence of arbitrage profits.

When the actual cross-rate for 1 unit of *FCA* differs from the no-arbitrage equilibrium cross-rate in formula 12.7 then arbitrage profit exists. Capture the arbitrage profit by executing three simultaneous trades between numeraire, *FCA*, and *FCB*. The exact sequence of trades depends on whether the actual cross-rate exceeds (overvalues) or is less (undervalues) than equilibrium. Rule 12.2 describes the triangle arbitrage trading strategy. The rule writes three trades involving three currency pairs wherein the middle trade is sale of overvalued and purchase of undervalued foreign currencies, respectively. First and last currency pairs involve the numeraire.

RULE 12.2 Triangle arbitrage trading strategy

Prices for 1 unit of foreign currencies *FCA* and *FCB* as measured by numeraire equal *a* and *b* respectively. Equilibrium cross-rate for one unit of currency *FCA* as measured by *FCB* equals *a/b*. When actual cross-rate exceeds *a/b* then *FCA* is overvalued relative to *FCB* and vice versa. This trading strategy captures the arbitrage profit:

$$\begin{matrix} \text{TRADE 1} & \text{TRADE 2} & \text{TRADE 3} \\ \begin{pmatrix} \text{sell} & \text{buy} \\ \text{numeraire} & \text{overvalued} \\ & FC \end{pmatrix} & \begin{pmatrix} \text{sell} & \text{buy} \\ \text{overvalued} & \text{undervalued} \\ FC & FC \end{pmatrix} & \begin{pmatrix} \text{sell} & \text{buy} \\ \text{undervalued} & \text{numeraire} \\ FC & \end{pmatrix} \end{matrix} .$$

Percentage profit from the arbitrage strategy equals percentage mispricing between actual and no-arbitrage cross-rates.

The example below illustrates why currency exchange rates must stay within rather tight tolerance limits.

EXAMPLE 12 Triangle arbitrage with USD as numeraire

The *Wall Street Journal* shows the following for currency exchange rates

Country	USD equivalent	Currency per USD
Britain (Pound, "GBP")	1.8879	0.5297
Switzerland (Franc, "CHF")	0.8433	1.1858

Suppose you begin with $10,000 USD.

a. How much of each currency can you purchase?
b. What does the table imply about the GBP-to-CHF cross-rate?
c. Describe the possible triangle arbitrage if the actual cross-rate between GBP and CHF is 1 GBP = 2.50 CHF.

SOLUTION

a. To find how much GBP equals $10,000 express the GBP-to-USD currency pair as an equation:

$$0.5297 \text{ GBP} = 1 \text{ USD}$$

The *Wall Street Journal*, like many other sources, reports two numbers between USD and GBP. The "USD equivalent" of 1.8879 is the price measured in dollars of 1 British pound. The "Currency per USD" of 0.5297 is reciprocal of 1.8879 and measures price in GBP of 1 USD. Multiply each side of the equation by 10,000.

$$5{,}297 \text{ GBP} = 10{,}000 \text{ USD.}$$

Computations for the CHF show

$$1.1858 \text{ CHF} = 1 \text{ USD}$$

or

$$11{,}858 \text{ CHF} = 10{,}000 \text{ USD.}$$

$10,000 is equivalent in value to 11,858 Swiss francs or 5,297 British pounds.

b. Because one USD is the same as any other USD the following should hold true for the GBP-to-CHF currency pair:

$$0.5297 \text{ GBP} = 1.1858 \text{ CHF.}$$

Solve for price of British pound as measured by Swiss franc:

$$1 \text{ GBP} = 2.2387 \text{ CHF.}$$

2.2387 Swiss francs represents equilibrium cross-rate for 1 British pound. This result is consistent with formula 12.7 wherein prices in USD for 1 GBP is a = $1.8879 and for 1 CHF is b = $0.8433; equilibrium cross-rate a/b equals 2.2387.

c. The problem states that the actual cross-rate is:

$$1 \text{ GBP} = 2.50 \text{ CHF.}$$

The actual price for 1 GBP of 2.50 Swiss francs is bigger than the no-arbitrage price from part b of 2.2387. The GBP is overvalued relative to the CHF.

Actual and equilibrium cross-rates differ thereby signaling existence of arbitrage profit. Rule 12.2 says to sell overvalued GBP and buy undervalued CHF. First, however, exchange beginning wealth of 10,000 USD into GBP. Second, exchange GBP into CHF. And third, exchange CHF into USD. A successful strategy finishes with more than 10,000 USD. The increase equals arbitrage profit. Definition 12.1 states that arbitrage profit equals a certain return exceeding the risk-free return. These trades occur instantaneously and therefore the risk-free return is zero — any risk-free increase in wealth is therefore arbitrage profit.

The three trades are:

$$\begin{matrix} \textit{TRADE 1} & \textit{TRADE 2} & \textit{TRADE 3} \\ \begin{pmatrix} sell & buy \\ USD & GBP \end{pmatrix} & \begin{pmatrix} sell & buy \\ GBP & CHF \end{pmatrix} & \begin{pmatrix} sell & buy \\ CHF & USD \end{pmatrix} \end{matrix}$$

Trade 1 exchanges 10,000 USD into 5,297 GBP. Trade 2 exchanges 5,297 overvalued GBP into undervalued CHF by using the actual cross-rate:

$$1\ GBP = 2.50\ CHF$$

or

$$5{,}297\ GBP = 13{,}242\ CHF.$$

Trade 3 exchanges 13,242 CHF into USD by using the actual price of the CHF:

$$1\ CHF = 0.8433\ USD$$

$$13{,}242\ CHF = 11{,}167\ USD.$$

Ending wealth of 11,167 USD exceeds beginning wealth of 10,000 USD. Arbitrage profit therefore equals 1,167 USD. Notice that the percentage increase is 11.67%. This exactly equals the percentage by which the actual cross-rate overstates the equilibrium price (= 2.50 / 2.2387 – 1). If the arbitrageur were to begin with $100 million then arbitrage profit equals $11,670,000. Large multinational banks use arbitrage profit for maintaining global offices and staff for exploiting this type of transaction.

Triangle arbitrage is a powerful force that does not require the USD as numeraire. Between any three currencies, as the next example shows, exchange rates must align as formula 12.7 shows or else arbitrage profit exists.

EXAMPLE 13 Triangle arbitrage without the USD

The *Wall Street Journal* shows key cross-rates are

	EUR	CHF
European Union (euro “EUR”)	…	0.6464
Japan (yen “JPY”)	134.86	87.1720
Switzerland (franc “CHF”)	1.5470	…

Each number is the amount of currency in each row that may be purchased by using one unit of the currency in the column heading.

a. Find the JPY-to-CHF cross-rate implied by EUR-to-JPY and EUR-to-CHF cross-rates. Is triangle arbitrage possible?

b. Suppose that European problems cause price of the EUR to depreciate 10% relative to the CHF. Relation between CHF and JPY, however, remains unchanged. Re-write the above table with resultant cross- rates.

SOLUTION

a. There are several ways to begin this answer, but all lead to the same place. Suppose you have 1 EUR. You may buy 1.5470 CHF or 134.86 JPY. Thus,

1.5470 CHF = 134.86 JPY.

This implies the following equilibrium cross-rate between JPY and CHF:

1 CHF = 87.1752 JPY.

The actual cross-rate in the table is that 1 CHF = 87.1720 JPY. Arbitrage profit exists because 87.1752 ≠ 87.1720. Notice though, that the percentage mispricing is less than one basis point. The arbitrage opportunity is so minuscule as to be practically unprofitable.

b. The table shows initial CHF-to-EUR cross-rate:

1 EUR = 1.5470 CHF.

The statement “price of the EUR depreciates 10% relative to the CHF” implies a new cross-rate:

1 EUR = 1.5470 (1 – 0.10) CHF

or *0.7182 EUR = 1 CHF.*

Relation between CHF and JPY remains unchanged at 1 CHF = 134.86 JPY. Thus, equilibrium EUR-to-JPY cross-rate is:

0.7182 EUR = 134.86 JPY

or *1 EUR = 187.77 JPY.*

In summary, the new cross-rate table looks as follows:

	EUR	CHF
Euroland	…	0.7182
Japan	187.77	87.1720
Suisse	1.3923	…

4.B. Relative purchasing power parity

Why do currencies have different values? The answer depends fundamentally on the historical concept of purchasing power parity. Centuries ago political economists saw merchants trading gold across international borders. They also saw merchants trading currencies. Very quickly they realized that if an ounce of gold cost two British pounds in London and eight Danish krone in Copenhagen then the exchange rate tended to equate 2 British pounds with 8 krone; that is, the price of one krone in London approximately equaled one-quarter pound (5 shillings). Economists saw that exchange rates move in tandem with currency purchasing power. The concept of purchasing power parity argues that a currency's value depends on the quantity of goods that the currency buys.

Inflation erodes currency purchasing power. Rising prices in the U.S.A. means that one USD buys less milk, less gold, and less foreign currency. For this lesson suppose that today an ounce of gold costs $400 in New York. Also, say that today's spot price in USD for one Swiss franc is $0.8433.

$$1\ CHF = 0.8433\ USD$$

Strict purchasing power parity for gold worth $400 an ounce suggests this relation:

$$(400 / 0.8433)\ CHF = (400 \times 0.8433 / 0.8433)\ USD$$

or

$$\overbrace{474.33\ CHF}^{\substack{\text{purchases one}\\ \text{ounce of gold}\\ \text{today in Geneva}}} = \overbrace{400\ USD}^{\substack{\text{purchases one}\\ \text{ounce of gold}\\ \text{today in New York}}}\ .$$

Today an ounce of gold in Geneva probably costs about 474.33 Swiss francs.

Now suppose that over the next year inflation, that is the percentage change in prices of goods including gold, is 6% in the U.S.A. and 3% in Switzerland. Then next year the equality becomes

$$\overbrace{474.33 \times (1 + 0.03)\ CHF}^{\substack{\text{purchases one}\\ \text{ounce of gold}\\ \text{next year in Geneva}}} = \overbrace{400 \times (1 + 0.06)\ USD}^{\substack{\text{purchases one}\\ \text{ounce of gold}\\ \text{next year in New York}}}$$

or $488.56\ CHF = 424\ USD$, or $1\ CHF = 0.8679\ USD$.

The spot price expected in one year for 1 Swiss franc is $0.8679 (= 424 / 488.56). Price of the CHF appreciates relative to the USD over the next year because inflation is bigger in the U.S.A. than Switzerland and the dollar erodes more than the franc.

Predicting price movements is important in any financial market, especially one as large as the foreign exchange market. Formula 12.8 summarizes the relation between today's currency spot price, expected inflation, and the expected currency spot price in the future.

FORMULA 12.8 Relative purchasing power parity ("PPP")

Let *a* equal initial price measured in numeraire currency called *NC* for one foreign currency called *FC*. The inflation rates applicable to *NC* and *FC* equal $inflation_{NC}$ and $inflation_{FC}$. Purchasing power parity pushes for this alignment of inflation rates with current and future currency spot prices:

$$\begin{pmatrix} \text{next year's} \\ \text{spot price for} \\ \text{1 unit of FC} \end{pmatrix} = a \times \left(\frac{1 + inflation_{NC}}{1 + inflation_{FC}} \right).$$

Formula 12.8 reveals that foreign currency price relates directly with inflation of the numeraire currency. As inflation in the U.S.A. increases, for example, the prices for all other currencies increase. From one perspective this suggests the values of other currencies increase. A more realistic perspective, however, is that inflation erodes numeraire currency value (after all, it buys less milk too).

EXAMPLE 14 Purchasing power parity and equilibrium cross-rates

A BigMac costs 3.90 Australian dollars (“AUD”) in Sydney and 1.95 euros (“EUR”) in Paris. Table 12.3 lists today’s spot prices in USD for the AUD and EUR equal $0.7748 and $1.3046, respectively. Suppose that inflation is 9% down-under, 4% in Europe, and 6% in the U.S.A. Assume that strict purchasing power parity for BigMacs exists and that production costs for making them in Paris and Australia are identical. Find AUD and EUR currency prices expected next year and comment on movement in the AUD-to-EUR equilibrium cross-rate.

SOLUTION

Apply formula 12.8 to find next year’s expected price of the euro given that European and U.S. inflation rates equal 4% and 6%, respectively:

next year’s spot price for 1 EUR = *$1.3046 (1.06 / 1.04),*

= 1.3297 USD.

Higher inflation in the U.S. than Europe means price of the euro appreciates almost 2% relative to the USD. Similar computation for the aussie dollar finds:

next year’s spot price for 1 AUD = *$0.7748 (1.06 / 1.09),*

= 0.7535 USD.

Higher inflation in Australia than U.S.A. means price of the AUD depreciates almost 3% relative to the USD.

Consider now the AUD-to-EUR equilibrium cross-rate. Apply formula 12.7 to today’s spot prices and find:

today’s price in EUR for 1 AUD = *$0.7748 / $1.3046,*

= 0.5939 EUR.

Price today for 1 AUD is 59.39 European cents. Apply formula 12.7 with next year’s spot prices.

next year’s price in EUR for 1 AUD = *$0.7535 / $1.3297,*

= 0.5667.

This price also obtains from application of formula 12.8 with inflation rates in the PPP relation:

next year’s price in EUR for 1 AUD = *0.5939 EUR (1.04 / 1.09),*

= 0.5667 EUR.

Higher inflation in Australia than Europe means price of the AUD depreciates almost 5% relative to the euro.

Currency prices float in response to forces of supply and demand. Inflation influences currency supply and demand. Inflation usually results from rapid growth in money supply and, just as for any other commodity, an increase in supply causes a fall in equilibrium price. Formula 12.8 for purchasing power parity captures powerful economic relationships.

Financial markets establish futures prices for currencies. For the preceding example today's spot price for one euro is $1.3046. Given U.S. and European inflation rates of 6% and 4%, respectively, strict adherence to PPP implies that next year's expected spot price for one euro is $1.3297. Suppose that today's futures price for one euro with delivery in one year differs from $1.3297. Does this signal an arbitrage opportunity? The answer is no.

Purchasing power parity is a fundamental relationship that for several reasons does not give rise to financial arbitrage opportunity. Foremost is ambiguity about *inflation*. A currency buys so many things from BigMacs to milk to gold to buildings to labor services, etc. Inflation is a general concept yet financial arbitrage requires contract specificity. Another reason that explains why PPP does not lead to a no-arbitrage equilibrium is incompleteness of financial markets. In the example that opens this chapter arbitrage with gold is possible because spot and futures markets for gold exist. Futures markets for most commodities do not exist – nobody wants a BigMac that's been in storage a year! Currencies store easier than BigMacs, however, and lessons in the next section explain how a similar parity relationship between currency prices and interest rates indeed gives rise to a no-arbitrage financial equilibrium.

4.C. Interest rate parity and covered interest arbitrage

Currency spot and futures markets exist and also interest rates link currency today with currency tomorrow. Currency prices and interest rates come together in a powerful parity relation constituting the biggest no-arbitrage financial equilibrium on the globe. Formula 12.9 summarizes the *interest rate parity* relationship.

FORMULA 12.9 Interest rate parity

Let *a* equal today's spot price measured in numeraire currency *NC* for one foreign currency *FC*. Let f_T equal today's futures price for 1 *FC* with delivery at time *T*. Interest rates applicable to *NC* and *FC* equal i_{NC} and i_{FC}. Interest rate parity pushes for this alignment of today's spot and futures currency prices with interest rates:

$$f_T = a \times \left(\frac{1 + i_{NC}}{1 + i_{FC}} \right)^T .$$

Violation of formula 12.9 implies existence of arbitrage profits.

Interest rate parity formula 12.9 is analogous to purchasing power parity formula 12.8 except that violation of 12.9 means arbitrage profit exists. There are five variables in formula 12.9. Usually length of futures contract *T* and interest rates i_{NC} and i_{FC} are known and predetermined. Supply actual value f_T for the futures price in formula 12.9 and solve for no-arbitrage spot price. If actual spot price *a* differs from no-arbitrage spot price then an arbitrage opportunity exists. Present value of arbitrage profit equals difference between actual and no-arbitrage spot prices.

Likewise in formula 12.9 supply actual value *a* for spot price and solve for no-arbitrage futures price. If actual futures price f_T differs from no-arbitrage futures price then an arbitrage opportunity exists. Future value of arbitrage profit equals difference between actual and no-arbitrage futures prices.

Capture arbitrage profit by exchanging *NC* and *FC* in the spot market, and by taking an opposite position in the futures market, and by borrowing and investing at interest rates i_{NC} and i_{FC}. Precise direction of transactions depends on whether *FC* is over or undervalued. Notice that formula 12.9 easily rearranges to solve for the no-arbitrage spot price:

$$\begin{pmatrix} no-arbitrage \\ spot\ \ price \\ for\ 1\ FC \end{pmatrix} = f_T \times \left(\frac{1 + i_{FC}}{1 + i_{NC}} \right)^T .$$

When actual spot price *a* exceeds no-arbitrage spot price then *FC* is relatively overvalued in the spot market and *NC* relatively undervalued. Rule 12.3 describes the *covered interest arbitrage* trading strategy for capturing arbitrage profit. The rule writes transactions in the top row that occur immediately at time *0* and in the bottom row transactions that occur at time *T*. At each time three steps occur.

RULE 12.3 Covered interest arbitrage trading strategy

For one unit of *FC* as measured by numeraire *NC* let *a* equal today's spot price and f_T today's futures price with delivery at time *T*. Credit market interest rates applicable to *NC* and *FC* equal i_{NC} and i_{FC}. When $a > f_T\,((1+i_{FC})/(1+i_{NC}))^T$ then *FC* is overvalued in the spot market relative to *NC* and vice versa. This trading strategy captures arbitrage profit from the mispricing:

transactions today at time 0

STEP 1	STEP 2	STEP 3
borrow overvalued currency with loan of term T	*in the spot market*: sell overvalued currency, buy undervalued currency; *in the futures market*: short undervalued currency	invest undervalued currency for horizon of term T

unwind transactions at time T

STEP 1	STEP 2	STEP 3
redeem investment from undervalued currency	*close the futures position*: deliver undervalued and receive overvalued currency	repay loan on overvalued currency.

Percentage profit from the arbitrage strategy equals percentage mispricing between actual and no-arbitrage prices.

The first step today identifies currency overvalued in the spot market and takes out a credit market loan for that currency. The loan principal and applicable interest rate are repayable at time *T*. Step 2 exchanges loan proceeds for currency undervalued in the spot market. Receive undervalued currency and invest at its applicable credit market interest rate. Also take a position in the futures market to deliver at time *T* currency that the investment promises to return.

Unwind the strategy at time *T* by receiving proceeds from the maturing investment and then by delivering the proceeds and exchanging them for currency originally overvalued by the spot market. Repay the loan and realize arbitrage profit. The example below implements the strategy.

EXAMPLE 15 Covered interest arbitrage

Table 12.3 shows today's price for one Australian dollar ("AUD") is \$0.7748. Suppose the actual futures price for the AUD with delivery in one year is \$0.8526. Applicable one year credit market interest rates equal 9.84% and 5.72% in Australia and U.S.A., respectively. Find the no-arbitrage spot price that *interest rate parity* implies and explain the trading strategy for capturing arbitrage profit.

SOLUTION

Actual spot price *a* for 1 AUD equals \$0.7748. No-arbitrage spot price equals \$0.8858 (= \$0.8526 × (1.0984/1.0572)]. Actual spot price for the aussie dollar is less than no-arbitrage price implying that the AUD is relatively undervalued and USD relatively overvalued: *sell USD and buy AUD in the spot market!*

First step is borrow overvalued currency, say, one million USD at 5.72% for one year. This obligates repayment of principal and interest next year equal to 1,057,200 USD (= 1,000,000 × 1.0572). Immediately exchange the million USD into AUD at the actual spot rate:

1 AUD = \$0.7748 USD,

and *1,290,656 AUD = 1,000,000 USD.*

Immediately invest 1,290,656 AUD for one year at 9.84% and expect to receive 1,417,656 AUD next year (= 1,290,656 × 1.0984).

Immediately take a short position in the futures market obligating to sell 1,417,656 AUD next year at today's actual futures price f_1 of \$0.8526. Note that today's net cash flows cancel and therefore cost of implementing arbitrage is zero.

One year from today redeem the 1,417,656 AUD and deliver to fulfill the futures obligation:

1 AUD = \$0.8526 USD,

and *1,417,656 AUD = 1,208,694 USD.*

Repay the loan principal and interest of 1,057,200 USD and realize future value of arbitrage profit equal to \$151,494 (= \$1,208,694 – \$1,057,200).

Notice that present value of arbitrage profit is \$143,297 (= \$151,494 ÷ 1.0572). Present value of arbitrage profit is 14.3% of the \$1,000,000 initial loan. This exactly equals percentage spot market mispricing [= (\$0.8858 – \$0.7748) ÷ \$0.7748].

Initial out-of-pocket net cash flow is zero. Without cost the preceding arbitrageur creates at time *T* net cash flow of \$143,297. Arbitrage continues in foreign exchange and credit markets until interest rates and currency prices align as in interest parity formula 12.9.

The arbitrageur may capitalize arbitrage profit to "get it all and get it now." Borrow one million USD today to begin the covered interest arbitrage trading strategy plus borrow an additional \$143,297 to immediately pocket and use for whatever purpose desired. This pocket money is present value of arbitrage profit. Principal plus interest for the one-year loan of \$1,143,297 at 5.72% equals \$1,208,694 (= \$1,143,297 × 1.0572). Sell 1 million USD for AUD in the spot market, invest those AUD at 9.84%, take a short futures position on the expected AUD returns, and net cash flow upon unwinding at time *T* equals zero. The free pocket money is like currency lying on the sidewalk of the busiest street in town. And there is no risk (except counterparty) because the outcome is contractually guaranteed. Covered interest arbitrage induces globalization of financial markets with a force as powerful for financial science as is the force of a black hole for physical science – absolutely irresistible.

Volume in the forex market averages more than $5,300 billion *per day* during year 2013. Even tiny deviations from interest rate parity create huge profit potential. Unlike entrepreneurial factors driving hopeful profits in the company cash flow cycle, arbitrage profit is not a hope but a certainty. This powerful force binds wealth of nations, companies, and households, in a complex interdependent web.

Derivatives activity in the U.S. banking system continues to be dominated by a small group of large financial institutions. Four large commercial banks represented 93% of the total banking industry notional amount of derivative products during 2013. The snippet below from table 2.1 shows the biggest from the list of 11,000 U.S. companies circa beginning of year 2014 with $1 trillion USD *Total assets*.

Ticker Symbol	Total Assets $millions	Employees Thousands	Net Income (Loss) $millions	Sales/Turnover (Net) $millions	Market Capitalization $millions
FNMA	$3,270,108	7	$83,963	$122,606	$ 17,332
JPM	2,415,689	251	17,923	105,790	219,657
DB	2,220,348	98	918	58,812	49,172
BAC	2,102,273	242	11,431	101,697	164,914
FMCC	1,966,061	5	48,668	75,311	9,425
C	1,880,382	251	13,673	92,543	157,854
SAN	1,537,238	183	6,021	91,876	102,794
WFC	1,527,015	265	21,878	88,069	238,675
UBS	1,134,164	60	3,562	39,549	72,538
MET	885,296	65	3,368	68,180	60,500

SNIPPET from table 2.1 in chapter 2: JPM and the trillion dollar club

J.P. Morgan Chase Co. is the largest U.S. private depository institution. The market cap of $219.7 billion may look small relative to the $700 billion market cap that nonfinancial corporate Apple Inc. reached in spring 2015. The *Total assets* of $2.4 trillion are colossal, however (AAPL *Total assets* were $207 billion). The fab four banks mentioned above include not only JPM but also Deutsche Bank (DB), Bank of America (BAC), CitiGroup (C), joined by other trillion dollar club members Banco Santander (SAN), Wells Fargo (WFC), and UBS. The financial contracts that these and many other balance sheets write link the present and future wealth for many. Financial markets spur creation and recognition of real and common interests. Perhaps from lessons about the structure of finance we can learn policies that maximize global wealth creation and that distribute economic profits and losses in response to principal-agent and moral persuasions.

EXERCISES 12.4

Numerical quickies

1. Suppose a company in the USA has a chance to sell its product internationally for either (i) 36,000 bhat or (ii) 44,600 krone. Shipping and other costs are identical. The company bases its decisions on today's currency exchange rates: 1 USD = 4.10 bhat, and 1 USD = 3.15 krone. Which deal generates the most USD? ©CR1a .

2. The table below indicates the quantity of currency in each row required to purchase 1 unit of the currency in each column.

	dinar	yuan
koruna	4.00	3.08
yuan	1.61	

2a. How much can you increase your wealth by executing a triangle arbitrage? ©CR3c
.

2b. Compare each pair of currencies and state which is overvalued. ©CR3c ©CR6 .

3. The company requires revenue of $175,000 USD on a particular export sale in order to cover costs and fair profit. The company accepts payment in the purchaser's local currency, which is peso. Today's spot rate is that 1 USD = 7.80 peso. Suppose the company makes a bid in peso to sell the product such that at today's spot rate the required revenue is obtained. The purchaser agrees to pay the bid. Several weeks later at time of delivery the purchaser makes the agreed upon payment in peso. By that time, however, price of the peso depreciates 8% relative to the USD. How much does the company receive in USD from the sale? ©CR4bm .

4. A BigMac costs 21.3 yuan in China and 128.2 dinar in Bahrain. Suppose that business and other costs for the McDonalds in both countries are identical. Inflation over the next year is 24% in China and 31% in Bahrain. If exchange rate movements over the next year reestablish the Purchasing Power Parity relation, what is the exchange rate (dinar per yuan) for next year? ©PR2 .

5. Today's spot exchange rate is that 1 koruna = 6.90 zloty. Everybody correctly knows that over the next year inflation will equal 20% in Slovakia (koruna) and 26% in Poland (zloty). If exchange rate movements strictly adhere to the Purchasing Power Parity relation (PPP), complete the following statement:

> PPP implies an exchange rate next year of ____(i)_______ zloty per koruna, and the price of the koruna is expected to _____(ii)______ relative to the zloty. ©PR1b .

Numerical challengers

6. The one-year risk-free interest rate is 7.25% in Slovakia (currency is the koruna) and 9.40% in Bahrain (currency is the dinar). Today's spot price for 1 koruna is 11.20 dinar. The futures price of 1 koruna with delivery in one-year is 9.52 dinar. Which currency is overvalued in the spot market and describe the trading strategy for capturing the arbitrage profit (assume complete futures markets)? ©PR3b .

7. Listed below is the quantity of krone required today to purchase one unit of the currency in each column heading.

	peso	bhat
krone	5.30	7.58

Because of differences in economic prospects, the price of the peso is expected to depreciate 30% relative to the krone, and price of the bhat is expected to appreciate 20% relative to the krone.

7a. Find today's equilibrium peso-to-bhat cross-rate that is most consistent with the triangle arbitrage concept. ©CR2am .

7b. Rewrite the table entries and explain movement in the peso-to-bhat cross-rate. ©CR2bm .

ANSWERS TO CHAPTER 12 EXERCISES

EXERCISES 12.1

1. ©FT2b Use formula 12.1 to find the no-arbitrage spot price is $346.12 per ounce (= $370 / 1.069). Arbitrage profit equals absolute value of difference between actual and no-arbitrage spot prices. That equals $43.88 per ounce, or $43,882 for 1000 ounces. The actual spot overvalues the no-arb spot so short gold in the spot market, receive $390,000 (= 1000 x $390), go long in the futures obligating to buy 1000 ounces in one year for $370,000 (= 1000 x $370), immediately pocket the arbitrage profit of $43,882 for whatever purpose desired, and immediately invest the remaining $346,118 (= $390,000 – $43,882) for one year at 6.9%. In one year redeem the investment for $370,000 (= $346,118 x 1.069) and use the proceeds to purchase the 1000 ounces of gold, thereby satisfying the futures obligation and replacing the previously-sold gold.

2. The actual SP2 spot equals 88 (= 48 + 40). The no-arb SP2 spot equals 99.44 (= 110.58 / 1.112). The difference of $11.44 is the present value of arbitrage profits. Actual spot undervalues no-arb spot so take a long position in the spot market, short position in the futures market. Immediately borrow $99.44 at 11.2 percent for one year. Use $88 to buy the two stocks and immediately pocket the arbitrage profit of $11.44 for whatever purpose desired. In one year sell the stocks for $110.58 thereby satisfying the futures obligation. Use the proceeds to repay the loan principal plus interest (= $99.44 x 1.112).

EXERCISES 12.2

1. ©FT1b Compute with formula 12.4 that for a long position,

profit = ($3.71 – $3.90) x 1250 ; = $-238.

This represents a loss. Figuring the rate of return requires the margin. Compute with formula 12.3 that

margin = 0.0225 x $3.90 x 1250; = $110.

For a speculative position the margin represents initial cost of investment (even though it surely shall be returned). Rate of return equals profit divided by initial cost. The *ROR* is -217% (= -$238 /$110).

2a. The pretax cost is $110,000 and the company wants a 12% pretax rate of return.

0.12 = (Sales Bid - $110,000) ÷ Sales Bid
Sales Bid = $125,000

The problem tells us that the spot exchange rate today is 1 USD = 1.35 krone. Thus,

125,000 USD = 1.35 x 125,000 krone
= 168,750 krone

If the company does the job for 168,750 krone and the exchange rate remains constant the pretax rate of return equals 12%.

2b. ©FT4cm Find the original price of the krone as measured by the USD numeraire:

1 krone = (1/1.35) USD.

The price of the krone appreciates by 10 percent so that the new price is:

1 krone = (1/1.35) x (1 + 0.10) USD
= 0.8148 USD.

The company receives 168,750 krone. Thus,

168,750 krone = 168,750 x 0.8148 USD
= 137,500 USD.

The company exchanges its krone for $137,500 and the actual pretax profit is $27,500 (= $137,500 - $110,000). The pretax profit margin is 20% (= $27,500 / $137,500).

3. The price of the peso becomes:
1 peso = 0.7660 x (1 + 0.06) USD
= 0.8120 USD.
Use formula 12.4 for the short position and find
profit = ($0.7660 – $0.8120) x 100,000
= $-4,596.
This represents a loss. To find the rate of return requires finding the margin.
margin = 0.0175 x $0.7660 x 100,000
= $1,340.
ROR equals profit divided by cost and is -343% (= $-4,596 / $1,340).

4. Use formula 12.4 for the short position and find the profit on the futures contract is
profit = ($3.10 - $4.30) x 40,000
= $-48,000.
The company sends the futures exchange $48,000 (the margin is a wash and is irrelevant). They sell their soybeans for $160,000 (= 40,000 x $4.00). Net revenues therefore equal $112,000 (= $160,000 - $48,000). In this case the hedging strategy costs the company money ($48,000). That's because the price of soybeans rises. On the other hand, if the price had fallen the hedge would have saved money.

5. ©FT6a ©FT6b Find the June price of the sucre:
spot market: 1 sucre = 0.7420 (1 – 0.04) USD; = $0.7123,
futures market 1 sucre = 0.7450 (1 – 0.04) USD; = $0.7152.
The company sells sucre in the spot market for $85,478 (= 120,000 x $0.7123). The profit on the short futures position is
profit = ($0.7450 - $0.7152) x (5 x 24,000); = $3,576.
The net revenue is $89,054 (= $85,478 + $3,576).

EXERCISES 12.3

1. Intrinsic value of the call is $8.30 (= $10.80 - $2.50). That's the payoff at the current stock price and represents a lower bound on the option price. The arbitrage profit equals the amount by which actual option price is less than $8.30. The arbitrage profit is $3.30 (= $8.30 - $5.00).

2. Ending stock price is $83.49 (= $69.00 x 1.21). Payoff on the call is $13.49 (= $83.49 - $70). Rate of return is 1,249% (= $13.49 / $1 - 1).

3a. With stock price of $65.60 the profit is zero.
3b. Getting a 160% rate of return given a beginning investment of $5.60 requires an ending wealth of $14.56 (= $5.60 x (1 + 1.60)). Obtain a payoff of $14.56 when the stock price exceeds the strike by $14.56. A stock price of $74.56 (= $14.56 + $60) generates a 160% rate of return.

4. Apply a 21% rate of return to beginning wealth of $28.10 (= $20.35 + $7.75) and compute that ending wealth equals $34.00 (= $28.10 x 1.21). An ending stock price of $34.00 yields ending wealth of $34 (the put is out-of-the-money and worthless at that price).

5a. The initial investment is expenditure of $37.20 (= $33.20 + $4). Worst-case outcome is that the stock price ends below $35 in which case you exercise the put and sell the stock for $35. The rate of return for that outcome is -5.9% (= $35 / $37.20 – 1).

5b. Ending share price is $35.86 (= $33.20 x 1.08). The put is out-of-the-money and therefore ending wealth equals $35.86. Rate of return is -3.6% (= $35.86 / $37.20 – 1).

6. ©DS5a The cost of the put option is $168 (= 3,000 x $0.0560). The payoff equals $630 (= 3,000 x ($1.10 - $0.89)). The rate of return equals 275% (= $630 / $168 – 1).

7. An option with total cost of $705 (= 5,000 x $0.1410) provides the right to buy 5,000 bushels of corn at the strike of $2.08 each. The company buys 5,000 bushel for $10,400 with the option, much less than the cost of $12,750 (= 5,000 x $2.55) without the option. The net savings to the company of the strategy is $1,645 (= $12,750 - $10,400 - $705).

8a. ©DS6a For the worst-case outcome options expire out-of-the-money and are worthless. You still have, however, the 90% invested in the money market plus interest. The ending wealth for the worst-case outcome equals $13,190 (= 0.90 x $14,000 x (1 + .055/12)^{10}. The rate of return is -5.79% (= $13,190/$14,000 – 1).
8b. For this case the ending wealth equals $13,190 from the money market investment plus the payoff on the options. Number of options purchased at $0.70 each is 2,000 (= 0.10 x $14,000 / $0.70). Ending share price is $32.26 (= $25.20 x 1.28). The payoff on 2,000 options with strike of 30 is $4,520 (= 2,000 x ($32.26 - $30)). Total ending wealth is $17,710 (= $13,190 + $4,520) and the rate of return is 26.5% (= $17,710 / $14,000 – 1).

9. ©DS17a You begin with $10,000 and the worst-case outcome is that ending wealth equals $8,000 (= $10,000 - $2,000). The future value of your money market investment equals $8,000 which means the present value is $6,766 (= $8,000 / (1 + .072/12)^{28}. You initially invest $6,766 in the money market and $3,234 in call options (= $10,000 - $6,766). Number of options purchased at $3.50 each is 924 (= $3,234 / $3.50). Ending share price is $50.18 (= $38.60 x 1.30). The payoff on 924 options with strike of $47.50 is $2,476 (= 924 x ($50.18 - $47.50)). Total ending wealth is $10,476 (= $8,000 + $2,476) and the rate of return is 4.8% (= $10,476 / $10,000 – 1).

10a. Purchase the stock and spend $40.65 and write the call option and receive $5.60. Your initial net investment therefore equals $35.05. In one year receive $2.85 plus proceeds from the stock. If the stock exceeds the strike of $50 then the long-side of the call option buys the stock from you for $50. For that best-case scenario your ending wealth is $52.85 and your annual rate of return is 50.8% (= $52.85 / $35.05 - 1).
10b. Ending share price is $45.12 (= $40.65 x 1.11). The call option expires out of the money and so that obligation disappears without cost. Ending wealth equals $47.97 (= $45.12 + $2.85). Rate of return equals 36.9% (= $47.97 / $35.05 – 1). For the pure stock strategy buy the stock for $40.65. In one year receive a dividend of $2.85 and sell for $45.12, thus W^{end} = $47.97. The ROR is 18%. The buy and write ROR equals twice the pure stock strategy.

11. The cost of the put option is $172 (= 4,000 x $0.043). The intrinsic value at expiry equals $800 (= 4,000 x ($1.12 - $0.92)). The company receives $3,680 from selling the 4,000 Euros at the spot exchange rate (= 4,000 x $0.92). Total outflows equal $172, total inflows $4,480, and net cash flow from the transactions equal $4,308.

12a. ©DS8 Your initial investment cost equals $10.70 (= $9.80 + $0.90). For any stock price at expiry that is lower than $34.30 (= $45 - $10.70) then payoff on the put exceeds the investment cost and the strategy returns a profit (the call is worthless at those prices). For any stock price at expiry greater than $55.70 (= $45 + $10.70) then payoff on the call exceeds the investment cost and the strategy returns a profit (the put is worthless at those prices). For any stock price between $34.30 and $55.70 the straddle generates a loss. Maximum loss of $10.70 occurs at $45.

12b. Apply a 30% rate of return to beginning wealth of $10.70 and compute that ending wealth is $13.91. Two outcomes generate that amount of payoff. When the stock price at expiry equals $31.09 (= $45 - $13.91) then the put payoff is $13.91 and call is worthless. When the stock price at expiry equals $58.91 (= $45 + $13.91) then the call payoff is $13.91 and put is worthless.
12c. The put is in-the-money and generates a payoff of $8.40 (= $45 - $36.60). The call is worthless. The rate of return is -21.5% (= $8.40 / $10.70 - 1).

13. Initial cash flow equals the revenue of $7,200 (= 1,800 x $4.00) from writing the calls minus the expenditure of $13,500 from purchasing the put options. Initial cash flow is $-6,300. Minimum ending wealth occurs when the stock price collapses in which case the put is in-the-money and you sell the stock for $35. The minimum ending wealth is $63,000 (= 1,800 x $35). Maximum ending wealth occurs when the stock price runs up in which case the call is in-the-money and you are forced to sell the stock for $45. The maximum ending wealth is $81,000 (= 1,800 x $45).

EXERCISES 12.4

1. CR1a Apply the exchange rates and find:
(1 / 4.10) x 36,000 USD = 36,000 bhat; = 8,780 USD.
(1 / 3.15) x 44,600 USD = 44,600 krone; = 14,159 USD.
The company receives $5,378 more USD if they make the deal in krone.

2a. CR3b Rewrite the table as formulas for the price of 1 dinar:
1 dinar = 4.00 koruna
1 dinar = 1.61 yuan
This implies that the no-arbitrage cross-rate between koruna and yuan:
1 yuan = (4.00/1.61) koruna; = 2.4845 koruna.
Equilibrium price for 1 yuan equals 2.4845 koruna. This is consistent with formula 12.7 wherein numeraire price *a* for 1 yuan equals (1/1.61) dinar and price *b* for 1 koruna equals ¼ dinar . Equilibrium cross-rate *a/b* equals 2.4845 [= (1/1.61) ÷ ¼].
Actual cross-rate for 1 yuan from the table is 3.08 koruna. Actual price of one yuan exceeds the no-arb price. Percentage overvaluation is 24.0% (= 3.08/2.4845 - 1). Execution of triangle arbitrage increases wealth by 24.0%.

2b. The yuan costs more koruna than is consistent with no-arbitrage equilibrium. This means the yuan is overvalued relative to the koruna. Use rule 12.2 to write three currency pairs wherein middle pair is the sale of overvalued yuan and purchase of undervalued koruna. First and last currency pairs match the middle as shown by rule 12.2:

$$\begin{pmatrix} \text{sell} & \text{buy} \\ \text{dinar} & \text{yuan} \end{pmatrix} \begin{pmatrix} \text{sell} & \text{buy} \\ \text{overvalued} & \text{undervalued} \\ \text{yuan} & \text{koruna} \end{pmatrix} \begin{pmatrix} \text{sell} & \text{buy} \\ \text{koruna} & \text{dinar} \end{pmatrix}$$

Part *a* takes as given the dinar-to-yuan and koruna-to-dinar cross-rates and finds actual price of 1 yuan measured in koruna is bigger than equilibrium cross-rate. Within each currency pair, however, there is a relative mispricing – none of them is properly priced when one is mispriced. This is analogous to a triangle with 3 sides that don't connect and leave a gap at one corner, there is not just one side out of alignment – they are all misaligned.

Find relative mispricing between any currency pair by taking as given the other two. For example, take as given the yuan-to-koruna and koruna-to-dinar cross-rates. Find that the dinar is overvalued relative to the yuan. Between each currency pair the currency on left is relatively overvalued and currency on right relatively undervalued.

Each of the three transactions involves selling the overvalued and buying the undervalued currency. An implication is that a specific currency is overvalued relative to one currency yet undervalued relative to the other. For example, the yuan is overvalued relative to the koruna and at the same time the yuan is undervalued relative to the dinar.

Another implication is that the triangle arbitrage trading strategy may rely on any one of three currencies as numeraire. Two trading sequences shown below work as well as the one shown above

$$\begin{pmatrix} \text{sell} & \text{buy} \\ \text{yuan} & \text{koruna} \end{pmatrix}\begin{pmatrix} \text{sell} & \text{buy} \\ \text{overvalued} & \text{undervalued} \\ \text{koruna} & \text{dinar} \end{pmatrix}\begin{pmatrix} \text{sell} & \text{buy} \\ \text{dinar} & \text{yuan} \end{pmatrix},$$

$$\text{or } \begin{pmatrix} \text{sell} & \text{buy} \\ \text{koruna} & \text{dinar} \end{pmatrix}\begin{pmatrix} \text{sell} & \text{buy} \\ \text{overvalued} & \text{undervalued} \\ \text{dinar} & \text{yuan} \end{pmatrix}\begin{pmatrix} \text{sell} & \text{buy} \\ \text{yuan} & \text{koruna} \end{pmatrix}.$$

Even though an arbitrageur holds only one of these currencies in inventory the trading strategy may be started with that currency in order to capture arbitrage profit. In summary, use formula 12.7 to find equilibrium cross-rate and overvalued currency. Use rule 12.2 to write the three currency pairs. Execute the strategy by proceeding rightward through the three transactions.

3. The company bid for the job is 1,365,000 pesos (= $175,000 × 7.80). Today's spot price for 1 peso is $0.1282 (= 1/7.80). Price of the peso depreciates 8% so that its new price is $0.1179. The company receives 1,365,000 pesos and sells them for $161,000 (= 1,365,000 × $0.1179). The revenue is 8 percent less than their target.

4. PR2 .Write today's exchange rate

21.3 yuan = 128.2 dinar or 1 yuan = 6.02 dinar.

The dinar is numeraire *NC* measuring price of *FC* yuan. Apply formula 12.8 and find that next year's price for 1 yuan is 6.36 dinar [= 6.02 × (1.31/1.24)]. Price of the yuan appreciates 5.65% relative to the dinar (= 6.36/6.02 – 1).

5. The numeraire currency measuring price is zloty. Apply PPP formula 12.8 and find that next year's price for the koruna is 7.245 zloty [= 6.90 (1.26/1.20)]. Price for one koruna appreciates 5.0 percent relative to the zloty.

6. PR3b Apply interest rate parity formula 12.9 wherein dinar is *NC* and koruna *FC*. Solve for the no-arbitrage spot price for 1 koruna.

no-arb spot price for 1 koruna = 9.52 (1.0940 / 1.0940) dinar,

= 9.3329 dinar.

Actual spot price of 11.20 overvalues by 20% (= 11.20/9.3329 – 1) the no-arb spot price implying the koruna is overvalued relative to the dinar: *sell overvalued koruna and buy undervalued dinar in the spot market!*

Immediately borrow, say, 100,000 koruna at 7.25% for one year to start covered interest arbitrage. Also borrow an additional 20%, that is 20,000 koruna, to capitalize the arbitrage profit for whatever use desired. The loan of 120,000 koruna at 7.25% obligates repayment of principal and interest next year equal to 128,706 koruna (= 120,000 × 1.0725). Immediately exchange 100,000 koruna into dinar at the actual spot rate and receive 1,120,000 dinar (= 100,000 × 11.20). Invest the dinar at 9.4% and expect returns in one year of 1,225,280 dinar (= 1,120,000 × 1.0940). Immediately take a futures market position to exchange 1,225,280 dinar into koruna at the actual futures price of 9.52 dinar per koruna.

Unwind the position next year and expect zero net cash flow. Exchange the 1,225,280 dinar into 128,706 koruna (= 1,225,280 ÷ 9.52). This sum exactly repays the loan and implies net cash flow at time *T* equals 0. The strategy nonetheless created wealth at time *0* of 20,000 koruna at absolutely no cost.

7a. Rewrite the table entries as formulas:

1 peso = 5.30 krone
1 bhat = 7.58 krone

This implies that the no-arbitrage cross-rate between peso and bhat:

(1/5.30) peso = (1/7.58) bhat.

Solve today's equilibrium price for 1 peso as measured in bhat:

1 peso = (5.30/7.58) bhat; = 0.6992 bhat.

This price is consistent with formula 12.7 wherein numeraire price *a* for 1 peso equals 5.30 krone and price *b* for 1 bhat equals 7.58 krone. Equilibrium cross-rate *a/b* equals 0.6992 bhat.

7b. Today's price for one peso measured in krone is 5.30. Depreciate the price 30% and find the new price for one peso is 3.71 krone (= 5.30 ×(1 – 0.30). This is the table entry for the first column. Today's price for one bhat measured in krone is 7.58. Appreciate the price 20% and find the new price for one bhat is 9.0960 krone (= 7.58 ×(1 + 0.20). This is the table entry for the second column.

The new equilibrium cross-rate *a/b* equals 0.4079 bhat (= 3.71 / 9.0960). Price for one peso measured in bhat begins at 0.6992 and ends at 0.4079 implying that the peso depreciates 41.7 percent relative to the bhat (= 0.4079 / 0.6992 – 1).

CHAPTER EXERCISES & PROBLEMS WITH ANSWERS

Chapter 2 Exercises with answers

EXERCISES 2.1

Return to text @Ex. 2.1

1. Consider the items on your personal balance sheet showing all that you own today and all that you owe. Suppose you own a car. Is it represented on one side or both sides of the balance sheet? Explain whether or not you have paid off the loan used for buying the car affects the balance sheet.

2. Discuss whether the following items are flows or balances.
a. the semiannual insurance payment
b. an inheritance
c. the purchase of a car
d. the car
e. a stock investment
f. Grand Central Park
g. a battleship
h. a battle

3. Consider the items on your personal balance sheet showing today all that you own and all that you owe. Suppose your only assets include cash accounts, a car and personal effects. Contrast the balance sheet this year with next year's. Explain how the changes in these 3 balances are sources or uses, realized or accrued. What happens to the liability and equity accounts during the same year and are those cash flows realized or accrued?

4. Explain and justify whether each is true: a. An increase in a liability account represents a source of funds; b. An increase in an asset account represents a use of funds; c. A decrease in an asset account represents a source of funds ©FF22

EXERCISES 2.2A

Return to text @Ex. 2.2A

Conceptual
1. The Company has Stockholders Equity equal to $7,100 and there are 250 shares outstanding. The market shareprice for their stock is $24.30. The price-to-book ratio for this company's peer group equals 0.67. How does the company's price-to-book ratio compare to its peer group, and what might this possibly mean? ©FA4

2. The latest news is that a $42 billion *market cap* company merged with a $5 billion company to create a $47 billion company. The target shareholder return was 25%. Find the Raider shareholder rate of return according to Identity 2.1.
2a. How might the ratios tilt away from the identity 2.1 requirement in response to high versus low principal-agent costs?
2b. How might the ratios tilt away from the identity 2.1 requirement in response to high versus low barriers to entry costs? How about great versus lousy investment opportunities for shareholders?

Numerical quickies

3. The company share price in the stock market is $41. The equity book value per share according to the balance sheet is $35. There are 490 million shares outstanding. Find the company market capitalization and equity price to book ratio. ©FA5

4. The company stock price yesterday was $23 a share. Suppose that today the share price increases by 3.2%. There are 260 million shares outstanding. Find the change in company market capitalization. ©FA10

5. The latest news that a $42 billion *market cap* company merged with a $5 billion company to create a $47 billion company means what for Raider shareholders given that there was a $1.2 billion wealth transfer from Raider to Target?

6. Target shareholders made a 30% overnight return and Raider shareholder wealth was down 2%. How much was the wealth transfer if the market cap for the combined companies was a “tiny” $1 billion? What if it were a huge $100B combined market cap?

7. Target shareholders made a 30% overnight return and the wealth transfer to Target from Raider claimaints was $100 million. If the market cap for the combined companies was a “tiny” $1 billion then how much is the percentage change in Raider wealth balances? What if it were a huge $100 billion deal?

Challengers: problems 8-12 refer to these balance sheets and setup

The balance sheets for the Raider Company and the Target Company appear below:

Raider Balance Sheet

$3,900	Curr. Assets	$3,900	Debt
$9,500	PP&E	$9,500	Stockholders Equity
$13,400	Total Assets	$13,400	

Target Balance Sheet

$2,000	Curr. Assets	$2,600	Debt
$4,200	PP&E	$3,600	Stockholders Equity
$6,200	Total Assets	$6,200	

The Raider Company plans to takeover the Target Company. The Raider Company has 820 common shares outstanding and their equity price-to-book ratio is 3.80. The Target Company has 770 common shares outstanding and their equity price-to-book ratio is 1.50. The Raider Company offers 1 shares of Raider stock to Target shareholders that tender 5 Target shares (the exchange ratio is 0.20; assume fractional shares can be exchanged). Suppose tax effects and synergistic gains and losses equal zero; that is, sales and profits remain the same.

8. Find the market capitalization for the new conglomerate company. ©FA3 j

9. Find the shareprice for the new conglomerate company. ©FA3c

10. Find the equity price-to-book ratio for the new conglomerate company. ©FA3a

11. Find the total transfer of wealth between Raider and Target shareholders. ©FA3g

12. Find the percentage change in wealth for each shareholder. ©FA3f

EXERCISES 2.2B

Return to text @Ex. 2.2B

Conceptual

1. The Company balance sheet lists Stockholders' Equity at $12 million. Does this necessarily imply that the company has cash available for paying its bills?

Numerical quickies

2. The Company had quite a few changes during the past year. On the balance sheet, for example, Cash is $6,800 at year-end 2525, and $7,900 at year-end 2526. Explain whether this change represents a source or use of funds.

3. The Company had quite a few changes during the past year. The changes for their different balance sheet items from last year to this year were (the changes in parentheses are declines; otherwise the changes are increases) : ($4,400) for Receivables; $6,800 for Payables; $6,100 for Cash; $5,600 for Short-term Notes; $6,000 for Plant, Property, & Equipment; and ($6,800) for Long-Term Debt. Was the change in net working capital a source or use of funds? ©FA1

APPENDIX 1: FUTURE AND PRESENT VALUE FACTORS OF ANNUITIES

Panel A: Future Value Interest Factor of Annuities *(FVIFA).*

Each entry is the future value of $1 deposits made for N consecutive periods that earn the periodic discount rate *r* computed with this formula:

$$FVIFA_{r,N} = \frac{(1+r)^N - 1}{r}.$$

Column headings list periodic rate *r*, rows list *N*. The table omits entries larger than 10,000.

	0.5%	1.0%	3.0%	5.0%	7.5%	10.0%	12.5%	15.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0050	2.0100	2.0300	2.0500	2.0750	2.1000	2.1250	2.1500
3	3.0150	3.0301	3.0909	3.1525	3.2306	3.3100	3.3906	3.4725
4	4.0301	4.0604	4.1836	4.3101	4.4729	4.6410	4.8145	4.9934
5	5.0503	5.1010	5.3091	5.5256	5.8084	6.1051	6.4163	6.7424
10	10.2280	10.4622	11.4639	12.5779	14.1471	15.9374	17.9786	20.3037
20	20.9791	22.0190	26.8704	33.0660	43.3047	57.2750	76.3608	102.44
25	26.5591	28.2432	36.4593	47.7271	67.9779	98.3471	144.02	212.79
30	32.2800	34.7849	47.5754	66.4388	103.40	164.49	265.95	434.75
90	113.31	144.86	443.35	1594.60	8934.14			
120	163.88	230.04	1123.70	6958.23				
240	462.04	989.26						
360	1004.52	3494.96						

Panel B: Present Value Interest Factor of Annuities *(PVIFA).*

Each entry is the initial deposit earning interest at the periodic rate *r* that perfectly finances a series of *N* consecutive $1 withdrawals computed with this formula:

$$PVIFA_{r,N} = \frac{1 - (1+r)^{-N}}{r}.$$

Column headings list periodic rate *r*, rows list *N*.

	0.5%	1.0%	3.0%	5.0%	7.5%	10.0%	12.5%	15.0%
1	0.9950	0.9901	0.9709	0.9524	0.9302	0.9091	0.8889	0.8696
2	1.9851	1.9704	1.9135	1.8594	1.7956	1.7355	1.6790	1.6257
3	2.9702	2.9410	2.8286	2.7232	2.6005	2.4869	2.3813	2.2832
4	3.9505	3.9020	3.7171	3.5460	3.3493	3.1699	3.0056	2.8550
5	4.9259	4.8534	4.5797	4.3295	4.0459	3.7908	3.5606	3.3522
10	9.7304	9.4713	8.5302	7.7217	6.8641	6.1446	5.5364	5.0188
20	18.9874	18.0456	14.8775	12.4622	10.1945	8.5136	7.2414	6.2593
25	23.4456	22.0232	17.4131	14.0939	11.1469	9.0770	7.5790	6.4641
30	27.7941	25.8077	19.6004	15.3725	11.8104	9.4269	7.7664	6.5660
90	72.3313	59.1609	31.0024	19.7523	13.3135	9.9981	7.9998	6.6666
120	90.0735	69.7005	32.3730	19.9427	13.3311	9.9999	8.0000	6.6667
240	139.5808	90.8194	33.3057	19.9998	13.3333	10.0000	8.0000	6.6667
360	166.7916	97.2183	33.3325	20.0000	13.3333	10.0000	8.0000	6.6667

Endnote A: Author's note

Every science has its golden age and today is the golden age for finance. Perhaps a century ago was the age of the economist and everyone benefited by discovery of key insights on how government policies affect business and household financial economic behavior. It must have been exciting a century and a half ago, too, as physicists and chemists discovered fundamental laws describing the behavior of atoms and molecules, of light and energy. Biologists have been at work for centuries and their accumulated efforts are providing bigger dividends today than ever-dreamed possible. Mathematics transcends millennia. These historic sciences shape daily life in today's financial economy and promise to change the future even more.

Finance is a relatively new but rapidly growing science. The rapid growth in financial science is due to dramatic increases in financial applications and the resultant demand for financial answers. Recurrent financial crises remind government and households that capital flows throughout the financial economy are as important as water for a garden. Through time a capital balance expands or consolidates spewing economic income and capital flows as it wiggles and beats. Sound public policy requires measuring and acting upon the forces that pull and push capital flows through the distribution system we call the real financial economy. The diversity of markets and methods of payment boggle the imagination. The distribution system of wealth that pulsates throughout the financial beast is hugely complex.

Each day more than the equivalent of 8 trillion U.S. dollars flows through global financial transactions. Explosive growth over the past 8 decades in the volume of financial market transactions was possible because technological innovations coincided with maturing political economies to create a world ready for finance. As the volume of transactions explodes, the characteristics of markets, securities, and in recent years the methods of payment or investment, evolve further and faster. Increasing sophistication of financial applications reveals novel and sometimes puzzling observations about financial relations. The existence of unexplained observation drives scientific discovery. In today's world financial observations abound even though during the new millineaum many previously publicly available de pursued pata sources hav. Financial science is quickly evolving because the world demands answers for financial questions.

A single unifying theme underlies all topics in finance: *the creation and management of value*. Finance has everything to do with studying value! Financial science describes three sources of economic profit: time value which tends to accrue over time, transformation value which also often takes time to capture, and the instanteous value realized from arbitrage, an irresistibally powerful economic force.

Time value is the simple worth of an asset and is sustained as the present value of future service or cash flow streams.

Transformation value is the value-added by combining different inputs to produce a unique output. A special case of transformation value is the diversification benefit from combining cash flow streams.

Arbitrage value exists when prices or rates in different markets misalign, thereby providing a temporary opportunity for instantaneous profit.

Valuation principles underlying the three sources of value follow a natural progression from simple to complex. Time value is the simplest yet most common source of value because all economic assets at a given moment possess time value. Entrepreneurs have known throughout *the age of commodities* that a sure dollar in the hand is worth more than a probable dollar in the future. In primitive economies, such as largely barter and yesteryear pre-mercantilist economies, the exclusive source of value often relates to simple time value concepts. Austrian economist Eugen von Böhm-Bäwerk intent on explaining the determination of interest rates formally links the consumer and producer sides of the financial economy through time value relationships and arguments (*Positive Theory of Capital* 1888). His structural foundation supports today's producer and consumer theories of value.

As financial economies morphed into complex webs the need to understand economic principles of production grew. Industrial companies add value by transforming land, labor, and capital into a product that clients demand. Transportation companies, likewise, add value by transforming a product at location A into a product at location B. Hairdressers transform a bad-hair day into a proud moment. In the perfect information competive equilibrium clients willingly pay a price to providers of goods and services that equals transformation value plus production cost. Real economic equilibrium assures fair compensation for productive processes that create transformation value. Economist John Keynes (*The General Theory of Employment, Interest, and Money* 1936) intent on explaining producer investment behavior parameterizes an equilibrium financial economy with zero net present value cash flow streams populating the markets. The entrepreneur and company, Keynes explains, willingly invests when the contribution margin from capital exceeds the user cost of capital. Modern economists, policymakers, and politicians sometimes build upon this structural foundation, the producer theory of value, to explain fixed investment behavior, national business cycles, and public policy.

Growth of financial markets in the second half of the twentieth century revealed a novel perplexing situation. Institutions joined governments as dominant forces in financial markets, collecting money from investors, buying securities, and passing profits back to investors. Often investors had the same access as institutions to investment opportunities. Sometimes, too, investors had as much skill as institutions picking securities. Rapidly growing institutions were adding value to the world, employing people and pouring out cash like a fireman's hose at a fire. Yet why? In some situations an institution was little more than a paper-shuffling pyramidist taking advantage of uninformed customers, perhaps providing few apparent incremental services worth valuing. Still, they were collections of contractual streams flowing with cash, royalties, or other property rights that the times delivered huge growth in those balance sheet numbers.

Financial economists discovered that even though an institution may not produce a tangible good and sometimes their services are nil even predatory, they nonetheless garner transformation value. Institutional investors provide clients with the diversification benefits accruing from the divisibility of property or cash flow rights. Clients willingly pay for *diversification benefits* a price that equals the transformation value of combining cash flow streams. Transformation value accrues when inputs are combined to form a unique output. Sometimes inputs are tangible land, labor, and capital. Sometimes inputs are intangible expected cash flow streams from different securities packed in a portfolio. Transformation value accrues in either case.

Insights about diversification benefits gleaned through financial science apply universally to many situations, even non-financial ones. Companies today routinely use diversification principles to manage bundles of assets and multiple product lines. British economist and Nobel laureate John Hicks (*Value and Capital* 1939) intent on explaining consumer behavior describes an equilibrium financial economy in which households compare returns from consuming income versus investing income with returns from investment. Hicks dedicates his book to Böhm-Bäwerk calling him the founder of economic dynamics. Modern portfolio practitioners and asset pricing theorists build the consumer theory of value upon the structural foundation from Hicks. Marginal utilities align with certainty equivalents such that the yield curve for no-arbitrage real risk-adjusted rates of return is horizontal at zero percent. The delivery date of a commodity in a complete market becomes but a simple property much like size or color and the spot and forward yield curves are mirror images.

Events of the past half-century reveal the existence for a global financial force that distributes huge quantities of arbitrage value. Technological possibility of instantaneous information flow throughout global markets made possible only in the last quarter century means that financial rates and prices must align within certain tolerance limits. Movement outside those limits creates arbitrage value. Multinational institutional investors have a keen sense for arbitrage opportunities. When opportunistic capitalized balance sheets see arbitrage profit they capture it. They prudently spend money going after it, too, hiring knowledgeable workers and setting up offices and computer lines to disparate parts of the globe. The strong attractive force of arbitrage economic profit causes development of markets and economies in ways unthinkable two generations ago. Arbitrage value is economic profit, a force driving globalization processes in the modern financial economy. Irrespective of policymaking the financial economy is driven toward creation of assets returning economic income much like the real utility from the bridge by the bay. Some modern commodities are big and marvelously complex, others as small as seeds.

Movement toward a unitary financial science coincides with curriculum changes at business colleges. Until the late 1990s most business students were required to study two finance courses. One course was “Corporate Finance” and the other was “Financial Markets and Institutions.” Finance textbooks of that era gave rise to today’s leading books. Business schools now generally require one finance course. This one course is the only one many business students ever take. A modern finance business school textbook, instead of focusing largely on corporate or financial markets, must offer lessons on the structure of the financial forces that shape the cash flows for companies and households in the modern financial economy. Maturation of the discipline makes possible such a presentation. This book delivers it through the 12 chapters shown in figure 1 on the Permissions page (see here).

Endnote B: Acknowledgement

To many I owe lifetime gratitude for revealing lessons on time dependencies. I appreciate and acknowledge Prof. Chuck Stasek for the 3 quarters of invertebrate zoology (FSU 1973-74). He taught how the phyla of life weave a miraculous fabric by intertwining running time with sideways time. Werner Herz in organic chemistry taught how mirror image stereoisomers reveal cross-section moments are like a half-life with a twin. George Glatthar (College of the Virgin Islands 1977) introduced me to Paul Samuelson *Economics* (9th edition), John Keynes *The General Theory*, and John Hicks *Value and Capital*, an experience still inspiring. Patric Hendershott directed my thesis (Purdue University Ph.D. Economics 1982). His perspective of the user cost of capital moved the flow of funds over the terrain of the real economy through formulaic tunnels. Committee members Hu Sheng Cheng and James Moore tried their best to direct my crazy tangents toward real targets. The 7 years at Boston College (thanks Jerry Viscione) provided time for meeting with Dale Jorgenson, finder of the key that Keynes calls user cost. BC colleagues Nikolaos Travlos, Betty Stowe, Kathy Hevert and Hamid Mehran give more than memories. UA colleague Pat Rudolph (1994) showed how *Word* and *Excel* can create algorithmic problems like those herein. Eric Baklanoff for a decade shared selected books that continue to inspire appreciation for the history of us in the political financial economy. Until my sabbatical year 1995-96 at Bond University in Gold Coast Australia (thanks Tony Hall) my time allocated say 70% to research and 30% to teaching. Since Australia I've chosen to rebalance weights at 80% for teaching/book purposes and 20% for research (service is institutionalized). Early book focus was read the *WSJ* and *Financial Times* and *Investors Business Daily* then translate and parameterize a' la Hendershott problems for notes and software. Tom Ash initiated the dynamic textbook when he (Dryden Press 1999) encouraged binding camera ready chapters. Tom said that Gene Brigham told him "the book is built around the problems." Thanks for that truth! Editions later editor Mike Widra found the first of a few Golden Gate pictures for the book cover. Keith Devlin forever captured my attention during his math show (2005 NPR) teaching a formula new to me, $e^{i\pi}=-1$. Jon and Jadonna Robinson showed that the period length of time affects flip to flip-side force of rotation. Billy Helms offered and delivered through coaching and team play an opportunity for me and many others to grow fairy tale relationships in Aladambama the beautiful and beyond. My father Samuel Downs taught me something on almost everything I know, including the link of the bridge by the bay to finance. Shoulders and work from before provide giant returns today and tomorrow that enrich our consolidated balance sheet inclusive of planet earth. I hope lessons that I build are useful for someone, maybe many. Any such benefits are returns from impossible to repay gratitude for sharing time with Denise. Since *m* breaths ago, her first day of Fall term (*Universidad Católica de Puerto Rico* 1971) when she walked into freshman World Literature silhouetted by the glorious sun, then till now and forevermore, I acknowledge that Denise is the sunshine of my life. Together never a dull moment, light enough to see so much like Tad, Taylor, and Whitney brighter still for great lessons from Cynthia, John, Everett, Josie, my glimpse toward infinity.

By one reckoning the first third of my academic career built a research structure of lifetime interest and the second third authored a book and processes for packaging and delivering meaningful algorithmic content for teachers and students. Hopefully during the final third weights rebalance allowing me opportunity to connect old interests $(1+r)^{-t}$ and e^{-rt} with old soul $e^{i\pi}$. \\=\\tom downs, tuscaloosa

Endnote C: Dedication

iOverLogI

Together *u* and *i* dance forever through endless hustle and flow.
Hand to hand, cheek to cheek, beautiful eyes revealing for *u* and *i* what and why we think.
Love binds *u* with *i* like sunshine coupling with moon howls, lights afire along all horizons.
Tendrils of perfect spin on our planet in space, *u* and *i*, watch time swing!

i^{1} is a dancing child between parents, a soul nonsingular never unattached,
bouncing between hands, back and forth, the wiggle-worm, time treasures we want and must embrace.
i^{0} and i^{4} stand stunningly tall, giant double pines flowing arms with love and distanced image.
i^{2} is the gain from common ground, molten fuel financing takeaway for real permanent common good.

LogI^{1} is a force from our child, the integral meld of use with source, a ratio always whole with 1.
LogI^{0} is 50/50 Duda, a charlatan's chance, singular give plus take, nothing times everything, good luck all!
LogI^{4} is firm Grandma, filling station for the fandamily giving wisdom and love for our sideways drive in time.
LogI^{2} is *u* over *i*, balanced spin, loss and gain, infinity separate from never, the flip shimmying with flip-side.
*i*OverLog*I* is real as 7 over 11, the rhythm and beat as *u* and *i* finger paint primes on skies of time.

\=\ Dedicated by Tom, the author, to Denise his bride in celebration of 4 decades of partnering from the slopes on Sugar Hill above Dorsch Beach, St. Croix, amidst One love with friends Jerome and Betty Weninger.
September 15, 2013

Photo by Grandma of author returning with dogwalkabout companion Lamb and grandson June 24, 2011
Tuscaloosa, Aladambama the Beautiful, USA, elementsoffinance.net LLC.

Endnote D: References and Additional Readings

… new for the Elements of Finance 1st edition

Index

… the index by the author also is new


भारतीय रिज़र्व बैंक
RESERVE BANK OF INDIA
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100
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GOVERNOR
THE UNIVERSITY OF ALABAMA
TEN RUPEES
BANKA SLOVENIJE
20
DVAJSET TOLARJEV
FEDERAL RESERVE NOTE
KG 11750439 A
G7
UNITED STATES OF AMERICA
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