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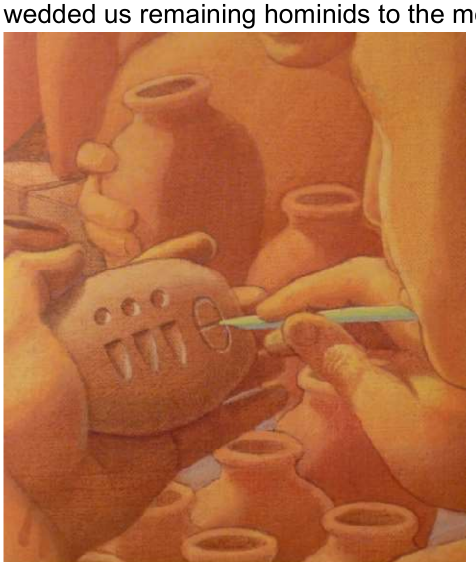
4. The financial economic foundation of the exchange economy

By prof-td viewable at http://elementsoffinance.net/palette.pdf **v20230220**

5, 4, 3, 2, 1, *blast-off!* Numbers grab the attention of the new brain neocortex in present day humans at undoubtedly the highest message processing rate *per capita per diem* ever in history. The power or durability of numerical incremental information processing crosses a broad spectrum: (i) An echocardiogram displaying a patient heart image transmits digitized dynamic data read remotely as images by a human specialist showing an actively tearing aorta thereby enabling a successful remedy that is applied by a specialist team with practiced learning mindful during surgery and recovery of relevant digitized information thus returning immeasurable outcomes of consequence for a 41-year old living person and a community of families; (ii) An instant message with a simple number enables completing one step, maybe opening a keypad and gate thus permitting continuation onto the next step; (iii) A number enables figuring a different number thereby transferring incremental information into the newly figured compound number.

Communication mechanisms among *homo sapiens* depend on brain and bodily foundations for speech and spoken language that evolved throughout 2 million years (20,000 centuries), likely longer since whales and elephants communicate with complex auditory languages, have similar brains and DNA, and physical anthropological artefacts indicate separation of hominid ancestors from cetaceans and elephantids about 50 to 60 million years ago. Throughout the last 300,000 years (3,000 centuries) the neocortical processing of seemingly “flat” written or etched symbols or markings such as {‖®¤≭≣⁒ÞØϴ©½X2andΔ] evolved extensively in six tissue layers of the hominid neocortex. The approximate 6×150,000 polyhedral neocortex cellular tissue grid organizes some cognitive information processing by function or purpose into multilayer cortical column groups (Hawkins 2021). Functions mapped thus far find thousands of cortical columns involved in object oriented processing, some groups larger than others depending on task complexity. Column members share input data then return coordinated electrical impulse(s) to downstream brain and body parts, even perceived limbs lost by injury.

The new brain interprets symbology like numbers, letters, logograms and memes, predicts the object being touched by a hidden hand or how an event might proceed, stores and retrieves memories, even remembers thinking about thinking. The new brain figures out schemes, some good and some bad. Symbological processing enables human choices that over the past 240 centuries for better and for worse have wedded us remaining hominids to the modern constructed environment on natural Earth.



Reprint from illustration in *The History of Counting* by Denise Schmandt-Besserat, snippet of a full-page (p. 7) workroom by illustrator Michael Hays, Morrow Press 1999. TWD PERMISSION REQUESTED?

Information processing rates for the new brain accelerated rapidly following the retreat of the last ice age from 160 to120 centuries ago. Illustration 1 depicts how the capacity for storing incremental information with words and numbers takes a giant step forward about 55 centuries ago in the Fertile Crescent. The scribe holds a wet clay tablet writing θ, a symbol known by many in the community to mean *sheep*. Each inscribed dot ● communicates the count of *ten*. Three tens ● ● ● thus equal 30. Each wedge mark ▼ is the symbol for *one*. The incremental information recorded is *thirty three sheep*. Were the count 32 instead of 33 sheep then the symbols comprise •••▼▼θ. The written word and the written number share a common instantiation.

Dried clay tablets storing useful incremental information written by scribes 55 centuries ago provided decision-makers guidance for choosing actions. The same is true now except that today's electronic or paper records, symbols, and sounds move more precisely and speedily than numbers and nouns in wet clay. Many of the 8 billion humans on earth today process numbers daily, especially when exchanging money for goods, services, and time savers. People generally move more precisely, speedily, and generally further for longer today than in antiquity. Some people thankfully have access to specialized training that sometimes provide durable return streams for humanity.

The power of potential incremental information to compound within numbers is infinite when the message includes exclusively numbers. Introduction of words diminishes the *signal-to-noise* ratio of the alphanumeric message due to the natural ambiguity intrinsic with words and utterances. Words, similarly art and music, possess a message that the receiver interprets with context sensitive individual specific response functions. Even the binary YES or NO when spoken or read may be communicated with such sarcasm that nearly every observer interprets the opposite of what was said, heard, or seen.

Numbers move perfectly with each other for they care not at all about emotion or context. Equivalencing a simple number to a numerical count tends to generate a high signal-to-noise ratio outcome. That is, within the context of a truthful scribe writing *33 sheep* 55 centuries ago then the message almost surely provides specific incremental information perfectly understood by the reader as intended by the scribe. The number 33 is definite, unambiguous. The word *sheep* 55 centuries ago for a simple count of herd size satisfied needs. For modern financial contracts for sheep, however, properties like delivery time, delivery location, even quality standards such as weight, age, health, or wool type may require specification. A modern commodities trader might find that one number is an incomplete information set for a sheep description. The signal-to-noise ratio attached to *33 sheep* nears zero as the ambiguity increases. The trader, rancher, or decision maker may seek clarification from additional incremental information which often means equivalencing more properties to numbers.

Numbers possess the property to compound incremental information. The power of incremental information that compounds into a human message has grown in the present day to where mass extinction events for earthly life may depend on how one or relatively few decision-makers respond to relatively few words or numbers. Words and numbers have superpowers that can blast the planet to bits or can build mindful best effort remedies for managing planetary challenges.

Artefacts dated to 36 centuries before present day show that Fertile Crescent scribes recorded marks in wet clay tablets interpreted by scholars today as 1.41421296, a number with nine digits. *Multiplying* the number by itself returns 2:

$$1.41421296 \times 1.41421296 = 2!$$

Nameless scholars of antiquity learned to compute the square root of 2, a cognitive feat that few today could figure without an electronic device. This root number with its infinite digits right of the decimal point, abbreviated herein as 1.41, also serves as c in the $a^2+b^2=c^2$ formula for $1^2+1^2=1.41^2$. That is, for the *right triangle* with leg lengths a and b of 1 unit then the hypotenuse length c equals 1.41 units.

Specifying a number with 9 digits (the rightmost 3 digits, *296*, were wrong and actually equal *356*) is especially remarkable when the counts of the day were much smaller than today. By that time, 36 centuries ago, counting sheep had been going on for about 70 centuries. Sheep domestication launched in the Fertile Crescent about 110 centuries ago after the last Ice Age. After the <u>next to last</u> Ice Age about 240 centuries ago one or more of the maybe four hominid species roaming around the planet had figured how to make sharp flake stone tools. They hunted in groups for large wooly mammoths. Artefacts for one particular society on the ancient Thames River finds rough hewn sail powered watercraft used for transportation services. The complexity of it all almost surely required the intergenerational transfer of learned behavior. Similar events were occurring in other places worldwide.

Fertile Crescent innovations 110 centuries ago concurrent with animal domestication include crop cultivation, food processing, granary storage facilities, and a thoughtful system for storing, retrieving, and sharing incremental information. Various goods like sheep or a bushel of chickpeas or a jar of olive oil were counted with dried clay tokens when the farming and ranching enterprises launched. A uniform interpretation of tokens existed throughout millinea even though spoken languages, rulers, enterprises, and implicit social contracts changed.

> "From 9000 to 3500 BC, tokens represented mostly products of the farm, such as measures of cereals, oil and small cattle. … What is remarkable about 'plain tokens' is that they remained the same during their 5,500-year lifespan. Whereas coexistent cultural artefacts, such as pottery, never ceased to evolve, plain tokens continued to be modelled from the same material, in the same shapes and sizes. For example, in 9000 BC Mureybet (Syria) and Tell Asiab (Iran) yielded plain tokens before pottery ever appeared. Plain tokens were numerous during the Jarmo Culture of 6000 BC, when pottery was crude and barely fired. One thousand years later, the same tokens were also present in the Halaf Culture (ca. 5000 BC), when pottery was at its finest, perfectly fired and decorated with harmonious complex compositions. Again, one thousand years later (ca. 4000 BC) the same smooth faced cones, spheres and discs were still used in the Ubaid Culture, when pottery was over-fired and sloppily decorated. Contrary to all other contemporaneous Neolithic artefacts, which were in constant flux, plain tokens remained for five and a half millennia persistently and obstinately identical."
>
> Denise Schmandt-Besserat 2019, p. 13.

In the present day when the definition of normal daily activities changes in 30 years, when in 200 years nation states come and go, to contemplate a semblance of stasis for 5,000+ years (about 300 successive human generations), that stability seems quite simply beyond comprehension for modern minds. Fodder for myth and legend, for sure.

By 36 centuries ago in Babylon one of the two remaining species of hominids roaming the planet repurposed the function of numbers from simple counting to complex reasoning. Imaginable possibilities became as infinite as the numbers. Construction of a constructed world accelerated in earnest. Advantages from the intergenerational accumulation and transfer of tangible goods plus an intangible ever expanding knowledge base increased. Scientists largely agree that human brain capacity and processing potential 36 centuries ago was the same as today. By 36 centuries ago the modern *homo sapiens* brain had arrived – a new brain with alphanumeric processors laying atop an older brain full of motivations and survival processes.

Packaging incremental information into a number is a cognitive property of human behavior, deliberate and thoughtful, storable and recoverable. The potential power of embodied information increases with the specialized uses for which numbers may apply. During the current half-century numbers embody incremental information like color, sound, even robotic action. Electronic devices depend upon numerical processing to function. The prevalence of numbers that count into the millions and trillions in the present day, as well as millionths and trillionths, attests to an increasing maximum for message incremental information content. From learning then following the numbers we humans operate, improve or replace fancy tools and systems made by our many, many predecessors.

Human utterances of flowing vowel sounds, constricted consonants, moans, grunts, pure emotion and spirit communicate much that is beyond the concrete information of numbers. The language tool is inherently context sensitive making ambiguity or misinterpretation difficult to avoid. For perhaps 1/3 to 1/6 of human communication the message intent when sent possibly is not identical to the interpretation rendered. Learnt numbers aren't ambiguous: 1+1=2 every time.

The concrete firmness of numbers and their absolute immunity to preference or intention creates a firm foundation for an exchange economy. Measurement errors and noise impose systemic hurdle costs when equivalencing a number to intangible properties, for sure. Still, the capacity for numbers to compound incremental information from other simple numbers enables a truthful auditable foundation. Audit and carry costs impose systemic hurdle costs, too. The assignment of meaning to a number carrying the real compound interests of different incremental information is subject to the natural ambiguity of words, preference, and human machination.

Whole numbers like 1, 2 and 8 relate to one another in firmly fixed relationships that are the same today as in prehistory. The choice between this much of this for that much of that drives a lot of behavior. The most compact symbology of the number tool equates the markings shown in formula 1.1.

FORMULA 1.1 The compact number tool

The number tool collapses into a convenient equality of a few symbols at two numerical grid layers, the *exponent* layer and the *base* layer:

$$2.7183^{3.1416\,i} = i^2 \qquad (1.1)$$

where $i \times i = i^2 = -1$

That is, "*i* squared equals *-1" and* $i^2 \times i^2 = -1 \times -1 = i^4 = +1$ (and $i^6 = -1$, $i^8 = +1$). The number at right in the exponent layer is 2 and number *i* is the base.

The 3.14159… number in the left exponent layer is commonly known as *pi* or *π.* The number in the left base layer 2.71828… is commonly known as the *natural number* or *e*. That symbology after rearrangement of i^2 shows $e^{i\pi} + i^4 = 0$. For presentation purposes *π, e*, and 1.41 are abbreviated to 2 digits right of the decimal point though actually infinite correct precisely ordered digits tag along right of the decimal point. An extension of 1.1 is:

$$2.72^{\,3.14\,i} = i^2 \times \left(\frac{a}{b}\right) . \qquad (1.1b)$$

The ratio (*a/b*) represents any identity that always equals 1 since multiplying a number by 1 does not change the number. Examples of *a* and *b* may be the length of sides on an isoceles right triangle or *Total Assets* ÷ (*Total Liabilities* + *Stockholders Equity*), or even %Δ*TA* ÷ %Δ(*TL+SE)* since that ratio of percentage changes also equals 1.

The 3.14 and 2.72 are *accumulative* numbers that square or invert and generally respond to the *rules* of exponents the same as any other number, e.g.,

$$2.72 \times 2.72 = 2.72^2 = 7.39 \quad \textit{and} \quad 3.14 \times 3.14 = 3.14^2 = 9.87$$

$$\sqrt[2]{2.72} = 2.72^{½} = 1.65 \quad \textit{and} \quad \sqrt[2]{3.14} == 3.14^{½} = 1.77$$

$$\frac{1}{2.72} = 2.72^{-1} = 0.39 \quad \textit{and} \quad \frac{1}{3.14} = 3.14^{-1} = 0.37$$

As the 3.14 and 2.72 numbers swing through the above transformations each is carrying an infinite tail of correctly specifed digits gripped eternally to the decimal point like stacked trapeze artists as they fly from one side of the double horizontal marks = to the other. The 1.41 number also swings with an infinite tail of accurately ordered symbols. The 1.41 artist joins hand-to-hand with *π* and *e* through bindings made from 1, 2 and i^2. These latter 3 whole numbers (1, 2, -1), as do all whole numbers, carry an implicit decimal dot with a tail infinite zeroes. Notable moves that 1.41 displays is how squaring its reciprocal lands on ½, the fraction able to split anything except 0:

$$1.41^{-1} \times 1.41^{-1} = 1.41^{-2} = 2^{-1} = ½ = 0.50000…$$

When the reciprocal is added rather than squared then the exponent layer on 1.41 makes an amazing leap over 0:

$$1.41^{-1} + 1.41^{-1} = 1.41^{+1}$$

Probably most remarkable is how the same outcome for 1.41 occurs when two seemingly opposite operations shakes its stem: *halving* 1.41 is the same as *inverting* it!

$$\frac{1.41}{2} = 1.41 \times 2^{-1} = 0.71 \quad \textit{and} \quad \frac{1}{1.41} = 1.41^{-1} = 0.71$$

Because i^2 equals negative 1 then sometimes *i* is described as the square root of -1. That's rather confusing, however, since one negative number multiplied by any other negative number *always* equals a positive number; -1 certainly is negative. Professional mathematicians, and my profession is finance, write that *i* is neither positive or negative but i^2 definitely is *-1.* Hence, *i* is like a placeholder that possesses many properties of 1 plus i^2 self-multiplies into a periodic motion that you can count on since i^2=-1 followed by i^4=+1 then i^6=-1 etc. The fixed number 2 combines with accumulative numbers 2.72 and 3.14 by forcing evermore smaller mini-triangles into a circle with 2 bisecting chords, inflating the interior area until the tiniest hypotenuse overlaps the tiniest slice of π, a point where area and length share common ground and the exponent gap from 1.41^{-1} to 1.41^{+1} arcs across 0.

The numerical values of π and *e* do not levitate above zero 0 like pie in the sky. The numbers 3.14 and 2.72, accumulate from parts. The number written 36 centuries ago, 1.41, plays a part in every slice of π. That capture process proceeds as written about 5 centuries ago by scholar Francois Vieta:

$$\frac{\pi}{2} = \underbrace{\frac{2}{\sqrt[2]{2}}}_{\text{term } a} \times \underbrace{\frac{2}{\sqrt[2]{2+\sqrt[2]{2}}}}_{\text{term } b} \times \underbrace{\frac{2}{\sqrt[2]{2+\sqrt[2]{2+\sqrt[2]{2}}}}}_{\text{term } c} \times \underbrace{\frac{2}{\sqrt[2]{2+\sqrt[2]{2+\sqrt[2]{2+\sqrt[2]{2}}}}}}_{\text{term } d} \times \underbrace{ad\ infinitum}_{\text{terms } efgh\ldots} \quad (1.2)$$

This formula easily models across a spreadsheet row since the downstairs of each right-hand side term (or spreadsheet cell) comprises √2 plus the previous term's downstairs. The spreadsheet formula easily fills cells rightward showing that by term *d* the accumulation equals 3.1365…, a number already roundable to the recognizable 3.14! Multiplication of terms *a-to-e* puts π at 3.1403… Evermore multiplications turn evermore digits to correct digits. The product of the first ten terms put π at 3.141591…, within a millionth of the correct accumulation after term *k* of 3.141592… . Present day scholars (e.g. Kreminski 2008) and electronic processors use the Vieta formula or enhancements (e.g., Chudnovsky 1987) to calculate 3.14 with trillions of correct digits to the right of the decimal dot!

The Vieta formula 1.2 puts *half* a π on the left-hand side. The numerically equal right-hand-side *multiplies* an infinite series of numbers, one term at a time. Each term is a ratio with 2 in the upstairs. The number tool uses 2 to scoop the mappable space from the expanse of infinity. That is, the downstairs acts as a counterweight that keep all of the upstairs 2's from blasting off to infinity since

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times \textit{ad infinitum} \ldots \rightarrow \infty$$

Term *a*, 2 ÷ √2, equals 1.41 × 1.41 ÷ 1.41 which is 1.41, the square root of 2 number written about by the Babylonians. Since *half* a π equals 1.41 times the rest of the terms then the *whole* π equals 2.82 (=2 × 1.41) times the rest of the terms. Below find the outcome of multiplying both sides of formula 1.2 by 2:

$$\pi = 2.82 \times \frac{2}{\sqrt[2]{2+1.41}} \times \frac{2}{\sqrt[2]{2+\sqrt[2]{2+1.41}}} \times \frac{2}{\sqrt[2]{2+\sqrt[2]{2+\sqrt[2]{2+1.41}}}} \times ad\ infinitum$$

$$\pi = 2.82 \times \frac{2}{1.84} \times \frac{2}{\sqrt[2]{2+\sqrt[2]{3.41}}} \times \frac{2}{\sqrt[2]{2+\sqrt[2]{3.84}}} \times ad\ infinitum$$ TWDverifytermD

$$\pi = 2.82 \times 1.08 \times 1.02 \times 1.004 \times 1.0012 \times 1.0003 \times 1.00007 \times \textit{ad infinitum}$$

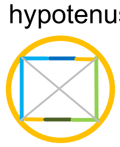
Term *b*, 2 ÷ $(1.41+2)^{½}$, equals 1.082392 (= 2/1.847759). The infinite terms of formula 1.2 start with term *b* a *run of* <u>*m*</u> *consecutive zeroes* that is countable and accumulates incrementally with each multiplication, it *moves*, to become in the mind's eye a numberline filled with endless zeroes that begins from a truncated *point*: the decimal dot ●. Term *b* equals 2 ÷ √3.41 showing that the the 41.42% rate is influential with the whole number 3 as well as the 1.

The run of 1 zero persists through *term c* (= 1.02) but following multiplication by *term d* another digit flips to zero (= 1.004) extending the length of the *zero run* to 2. The terms *efgh…* tend quickly toward the *limit* of 1 yet never arrive to join the whole number 1.0000…. With the 14th multiplicative term the zero run length equals 8 and the numerical value of accumulative *π* is 3.1415926**49**. The more accumulated thus more accurate *π* is 3.1415926**53**…

Whole numbers claim the domain left of the decimal dot and require nothing as the remainder – zeroes all the way. An accumulative number like *π* or *e* demands that digits right of decimal fit perfectly, each like a unique piece in an infinite jig saw puzzle. The jagged looking line of numbers right of the decimal point admits a permissible remainder, the next digit to flip, that is dependent on the number of symbols in the system. For the ten numerals 0 to 9 of the modern base 10 system the tail for *π* found by numerical processing is ●1415926536….

Since 1×*anything* equals *samething* then multiplying infinite terms together has diminishing change on the accumulation when the term is 1 with a trillion zeroes beyond the decimal point when finally appear the unflipped nonzero digit remaining from the previous multiplication. The potential trail of correct digits is endless, a string with limitless length like the edge of two endless faces at the cliff edge. The perfect *π* attains perfection not by initial endowment but rather through accumulation. Accumulation is a process that uses time. A company announcement by Google in 2019 publicly releases 31 trillion accurate digits trailing behind *π*, a new record besting 23 trillion correct digits set a few years earlier. Lead chef cooking the 2019 *π* accumulated correct digits on numerical processors of 25 virtual cloud machines for 121 days (Porter 2019). The zero run length therein is pretty long, certainly less than 31 trillion, definitely less than halfway to infinity! Only the mind's eye cortical processor fully envisions infinity.

Exhibit 1.1 shows a square on the interior of a circle with a diameter of 1, a radius of ½, and a circumference of 3.14. The 2.82 combined length of the 4 sides of the interior square equals 2×1•41, identical to <u>twice</u> *term a* from above. This 2.82 comprises the first accumulative slice of *pi*. The 4 triangles within this *pi* circle form 2 containments that accumulate *interest on interest* within 2 of the 4 mini-hypotenuses. Each mini-hypotenuse *is* a side of the *pi circle interior square* with a length of 0.7071 (=$1.414\underline{2}^{-1}$)



standard units of any measurable *x*, say a $*wealth* metric since wealth management motivated learning from numbers (construction, too). The 2 main diagonals ✖ of the square bisect at a right angle in the circle center. Each half-diagonal is of length $1.414\underline{2}^{-2}$, that is ½, e.g., $0.50, and is a leg that supports the hypotenuse. Let two opposite square sides, say <u>vertical</u> sides and as shown at left, equal two separate but equal accounts of 0.7071 units of *Beginning wealth* that each earn 41.4<u>2</u>% interest per period. After one period the *interest on principal* posts 0.2929 wealth units (= $0.7071×0.414<u>2</u>) to each account balance, a sum illustrated as and for the top and bottom edges of the square. At this point each account balance equals $1.0000000…, the whole number 1, length of the main diagonal of the square *and* also the bisecting chords of the circle. There are a lot of overlapping positions here that wind or unwind in ways of natural interest.

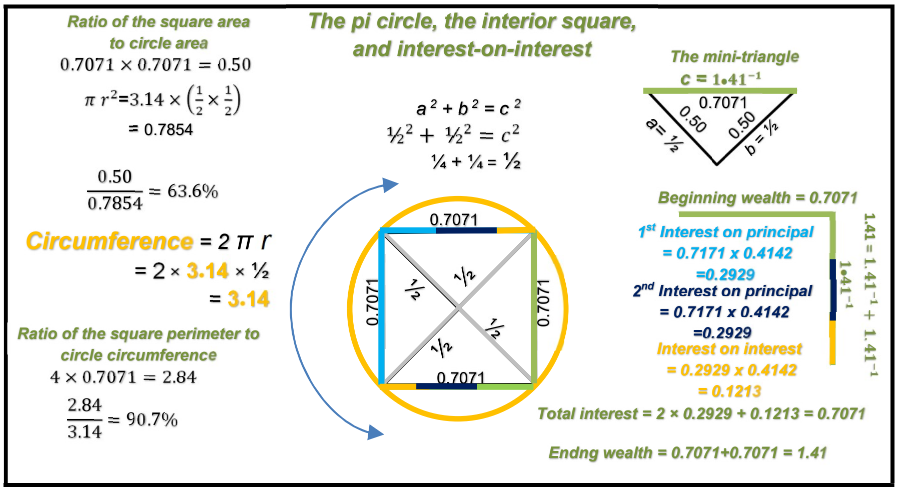


EXHIBIT 1.1 The *pi circle*, the interior square, and interest on interest

The reciprocal of 1.41, that is to say 0.7071, is the hypotenuse for the isocoles triangle with leg lengths of ½. Summing this 0.7071, for the 4 encircled hypotenuses that form the square above gives a perimeter of 2.82 that accounts for 90.7% of the circle circumference length of 3.14. Two sides representing beginning wealth of $0.7071 each accruing interest for two periods at a periodic rate of 41.42% each grow entirely new sides thus completing the square.

During the second period an additional 0.2929 units of *interest on principal* accrues, designated ▬ for both account balances. Plus, the first period posting of interest earns 41.42% *interest on interest*, that is 0.13 units (= $0.29×0.4142), designated ▬ for both accounts. This brings the closing accumulation for each account to $1.4142, exactly double the beginning wealth of $0.7071 as *total interest* is $0.7071 (= 2×$0.29 + $0.13). The *cumulative rate of return* for the *two* period investment horizon equals 100%, a doubling of wealth. The periodic average rate of return is 50% (= 100% ÷ 2) by the *arithmetic* averaging procedure and 41.42% $(=1.41/0.7071)^{1/2}$ by the *geometric* averaging procedure. Were the period of the scenario analysis defined as a year with annual compounding, that is a 2-year investment horizon, then the *annual percentage rate* (*APR*) would be 41.42%. Were the period defined as a half-year with semiannual compounding and 1-year investment horizon then the *APR* would be 81.84% and the *effective annual rate* would be 100%. For all interpretations, the *ending wealth* equals the reciprocal of the beginning wealth since $\$0.7071^{-1} = \$1.4142$, a very unique event indeed!

Follow the money and the baseline for the numbers is \$1.41. See in column A of the exhibit below *exponents,* a superscript above the base number 1.41, *one dot forty one raised to the m* as *m* steps from zero to infinity. The exponent means the number of self-multiplications that occur, such as in row *c*, 1.41^2 of the exhibit at right: $1.41^2 = 1.41 \times 1.41 = 2$. Immediately scan down column A, see exponents 0 1 2 3 4 and on and on, the numbers in uniform standard step. Easily visualize how this list expands beyond 1.41^{16} (row q), an endless parade of ordered whole numbers 1 to ∞.

	A	B	C	D
	1.41^m	$(1.41^2)^{m/2}$		*doublings of 1*
a	$\$1.41^0$	$= 2^0$	$= 1.41^{-1} + 1.41 \times 0.41 = 1$	1
b	1.41^1	$= 2^{1/2}$	$= 1.41^{-1} + 1.41^{-1} = 1.41$	$1.41^2 = 2$
c	1.41^2	$= 2^1$	$= 1.41 \times 1.41 = 2$	$2^2 = 4$
d	1.41^3	$= 2^{3/2}$	$= 2 \times 1.41 = 2.82$	$2.82^2 = 8$
e	1.41^4	$= 2^2$	$= 4$	$4^2 = 16$
f	1.41^5	$= 2^{5/2}$	$= 4 \times 1.41 = 5.65$	$5.65^2 = 32$
g	1.41^6	$= 2^3$	$= 8$	$8^2 = 64$
h	1.41^7	$= 2^{7/2}$	$= 8 \times 1.41 = 11.31$	$11.31^2 = 128$
i	1.41^8	$= 2^4$	$= 16$	$16^2 = 256$
j	1.41^9	$= 2^{9/2}$	$= 16 \times 1.41 = 22.62$	$22.62^2 = 512$
k	1.41^{10}	$= 2^5$	$= 32$	$32^2 = 1024$
l	1.41^{11}	$= 2^{11/2}$	$= 32 \times 1.41 = 45.25$	$45.25^2 = 2048$
m	1.41^{12}	$= 2^6$	$= 64$	$64^2 = 4096$
n	1.41^{13}	$= 2^{13/2}$	$= 64 \times 1.41 = 90.51$	$90.51^2 = 8192$
o	1.41^{14}	$= 2^7$	$= 128$	$128^2 = 16384$
p	1.41^{15}	$= 2^{15/2}$	$= 128 \times 1.41 = 181.02$	$181.02^2 = 32768$

Exhibit 1.2 The endless parade of whole numbers (col. A) as exponents for the base 1.41 giving rise to the doublings of one

Column A above lists in rows *a b c… p* the whole number *exponent m* for the base 1.41^m as *m* increments by 1 from 0 to 15. The numerical value of 1.14^m in column C is the rightmost number. That same value in column D left of “=” is squared giving the *doublings of one*: 1 2 4 8 16 32 64 128 *ad infinitum*. These doublings equal 1.14^{2m} which is the same as $(1.14^m)^2$. Columns C & D show that the square root for *every* doubling equals the previous doubling’s square root multiplied by 1.4142…., an expansion of 41.42% per doubling. At this rate (41.42%=1.4142-1), *interest on interest* accumulates to complete two of the 4 corners on the interior square within the *pi* circle of exhibit 1.1. The doubling in row *b* column D shows $1.41^2 = 2$ and thus 1.41 is the square root of 2, a square root that is 41.42% larger than 1. The doubling of 4 in row *c* has a square root of 2 which is 41.42% larger than the previous doubling’s square root. Likewise, for the next doubling of 8 the square root is 41.42% larger than 2 (the previous square root) which is 2.82, twice *term a* (1.41) from formula 1.2 that slices *π*. Thrice as nice, 1.41^3 equals 2.82, the square root of 8 as shown in row *d* column D since $(1.41^3)^2 = 1.41^{2\times3} = 8$. The subsequent doubling of 16 has a square root that is 41.42% larger than 2.82 which is 4. *Ad infinitum.*