

DURATION, FIRM VALUE, AND DISCOUNT RATE CHANGES

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Thomas W. Downs*
Finance Department
Boston College
Chestnut Hill, MA 02167
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*Assistant Professor of Finance at Boston College. This study represents a development of ideas originally presented in my doctoral thesis written at Purdue University under Patric Hendershott. This study has benefitted because of helpful comments by Professors Hendershott, Sheng Cheng Hu, and Dale Jorgenson. Any remaining errors are my responsibility.

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From 1965 through 1981 the real value of the New York Stock Exchange Index declined 68%. Several explanations for this phenomena have been advanced.¹ The explanation offered by Burton Malkiel (1979) is that an increase in the perceived riskiness of equities led to an increase in discount rates, thereby reducing the discounted value of expected capital income streams and lowering equity values. This hypothesis is supported by Fisher Black (1977) who finds evidence that stock returns are negatively correlated to changes in volatility and by Robert Pindyok (1984) who presents evidence that return volatility substantially increases during the 1970's. Lawrence Summers and James Poterba (1985) dispute the Malkiel contention by presenting empirical evidence that little of the real decline can be explained by volatility induced increases in risk premia.

Implicit in the Malkiel hypothesis is an acceptance that an increase in volatility, and hence discount rates, must necessarily lead to a decline in capitalized value. This study shows that the response of capitalized value to an increase in discount rates is not necessarily negative. It depends upon the relationship between the duration of the return stream the marginal investment is expected to generate and the duration of the return stream the aggregate capital stock is expected to generate. Capitalized value is positively related to the discount rate when the stock duration is less than the marginal investment's duration, and negatively related when stock duration is greater than marginal duration. The implication is that even if expected volatility in the 1970's did

lead to rising discount rates, this need not be the cause of the decline in real equity values. The equilibrium value of a firm may rise in response to an increasing discount rate.

I. Naive Models for Determining Capitalized Value

A firm's capitalized value is defined to equal the discounted sum of its expected net return stream. Notationally, let r be the discount rate by which expected returns are discounted and let $R(s+t)$ be the return net of variable cost expected at time $s+t$. Capitalized value, CV , is given by

$$(1) \quad CV(s) = \int_0^{\infty} e^{-rt} R(s+t) dt$$

Given that the net return stream is independent of r , the response of capitalized value to an infinitesimal change in the discount rate is

$$(2) \quad \frac{d CV}{d r} = - \int_0^{\infty} t e^{-rt} R(s+t) dt$$

The solution to equation 2 depends upon the shape and size of the net return distribution. When returns are expected to change at a constant perpetual rate g , such that $R(s+t) = R(s) e^{gt}$, where $g < r$, the solution is

$$(3) \quad \frac{d CV}{d r} = \frac{-R(s)}{(r - g)^2}$$

Equation 3 may be further simplified because $CV(s) = R(s)/[r-g]$. Therefore,

$$(4) \quad \frac{d CV}{d r} = \frac{-CV}{r - g}$$

The sign of 4 is negative because the growth rate of returns is less than the discount rate. This model predicts that an increase in the discount rate unequivocally leads to a decline in capitalized value.

The smaller the spread between r and g , the larger is the sensitivity of capitalized value to discount rate fluctuations.²

An alternative solution to 2 may be derived by assuming the return $R(s+t)$ can be separated into two components. One is a real component representing the level of production and the other is a profit component representing the capital income earned per unit of output. Let Q be the quantity of output produced according to the production function $Q = Q(K, L)$, where K represents capital inputs and L variable inputs, and let p' be the net profit per unit of output. The net return $R(s+t)$ equals $p'(s+t) Q(s+t)$, and CV is

$$(5) \quad CV(s) = \int_0^{\infty} e^{-rt} p'(s+t) Q(s+t) dt$$

To derive the solution say $p'(s+t) = p'(s) e^{\pi t}$ and $Q(s+t) = Q(s) e^{-\delta t}$. In a system where all prices are changing at the same rate, π represents the rate of inflation. Production declines because of the capacity depreciation that affects producer's capital; δ represents the rate of productive capacity depreciation.³ Under these conditions CV equals $p'(s)Q(s)/[r+\delta-\pi]$ and the response of capitalized value to an infinitesimal change in the discount rate is

$$(6) \quad \frac{d}{dr} \frac{CV}{r} = \frac{-CV}{r + \delta - \pi}$$

Again, this term is unambiguously negative.

Solution 6 may be derived by an alternative method. Instead of measuring the net return contributed by each unit of output it is just as feasible to measure the net return contributed by each unit of input. In competitive markets variable inputs such as labor are paid their value marginal product so that expected net returns are entirely attributable to capital assets. The return generated by

the marginal unit of capital employed is pQ' , where p is the unit price of output and Q' is the marginal physical product of capital, dQ/dK . The net return generated by the entire stock of capital at time s is $p(s)Q'(s)K(s)$.⁴ Thus, the capitalized value of the net return stream expected from the stock in place at time s is

$$(7) \quad CV(s) = \int_0^{\infty} e^{-rt} p(s+t) Q'(s+t) K(s,t) dt$$

$K(s,t)$ is the units of capital in place at time s expected to survive until time $s+t$. Say the stock is expected to depreciate at the rate δ in which case $K(s,t) = K(s) e^{-\delta t}$, and prices are expected to inflate at the rate π , and the marginal physical product is expected to remain constant. Equation 7 may be solved as

$$(8) \quad CV(s) = \frac{p(s) Q'(s) K(s)}{r + \delta - \pi}$$

and the response of CV to an infinitesimal change in r is

$$\frac{d CV}{d r} = \frac{-CV}{r + \delta - \pi}$$

This solution has a negative sign and is identical to 6.

Up to this point the sign of dCV/dr has been unambiguously negative. A rise in the discount rate leads to a fall in capitalized value. Also up to this point, the return stream $R(s+t)$ has been independent of r . Yet in the competitive setting described by equation 7 the marginal return is pQ' and the value maximizing producer selects a production plan that equates the marginal return on capital to the user cost of capital. The models discussed thus far are naive because they allow the required return and hence the user cost to rise but they do not have the producer equating marginal cost and marginal return. The next section derives a valuation model that has the producer responding to an exogenous change in discount rates by altering the production plan, thereby reestablishing equilibrium between marginal cost and marginal return.

II. An Equilibrium Model for Determining Capitalized Value

Dale Jorgenson (1963) has shown that in the absence of taxes the user cost, denoted c , is given by $c = q [r + \delta^* - \pi]$, where δ^* is the depreciation rate of the marginal capital investment and q is the supply price of capital. At equilibrium the user cost equals the value marginal product. Substituting the user cost into equation 7 shows

$$\begin{aligned}
 (9) \quad CV(s) &= \int_0^{\infty} e^{-rt} c(s+t) K(s, s+t) dt \\
 &= \int_0^{\infty} e^{-rt} q(s) e^{\pi t} [r + \delta^* - \pi] K(s) e^{-\delta t} dt \\
 &= q(s) K(s) \left[\frac{r + \delta^* - \pi}{r + \delta - \pi} \right]
 \end{aligned}$$

The term $q(s)K(s)$ is the product of the current unit price of capital times the number of units in the capital stock and is commonly known as the stock's replacement cost, denoted $RC(s)$. The capitalized value of the aggregate stock's expected net return stream equals the stock's replacement cost if the ratio in brackets is unity; i.e., if $\delta^* = \delta$. It is unlikely that CV ever exactly equals RC .⁵ M. Feldstein and M. Rothschild (1974) have shown the aggregate stock does not decline exponentially unless real capital has accumulated exponentially. So even if all capital depreciated by the same rate δ^* the stock would not depreciate at δ^* because of the volatility in historic real investment.

The ratio in brackets on the right-hand-side of 9 is related to asset duration. Duration is the weighted average maturity of a return stream where the weight applied to each period is that period's proportionate contribution to total capitalized value.⁶ Streams with a short duration generate proportionately more of their

discounted returns in the near future whereas long duration streams generate proportionately more in the remote future. At equilibrium the duration of the marginal capital investment, denoted DUR_C , may be computed as

$$\begin{aligned}
 (10) \quad DUR_C(s) &= \frac{1}{\bar{q}(\bar{s})} \int_0^{\infty} t e^{-rt} R(s+t) dt \\
 &= \frac{1}{\bar{q}(\bar{s})} \int_0^{\infty} t e^{-rt} q(s) e^{\pi t} [r + \delta^* - \pi] e^{-\delta^* t} dt \\
 &= \frac{1}{r + \delta^* - \pi}
 \end{aligned}$$

A similar computation shows the duration of the aggregate stock's expected net return stream, denoted DUR_K , is $1/[r + \delta - \pi]$.

Therefore, the ratio in 9 may be expressed as

$$\frac{[r + \delta^* - \pi]}{[r + \delta - \pi]} = \frac{DUR_K}{DUR_C}$$

The ratio DUR_K/DUR_C is hereinafter referred to as the duration ratio. CV is greater (or less) than RC as the duration ratio is greater (or less) than unity.

When the duration ratio exceeds unity the aggregate stock generates more of its return in early periods than does the marginal investment (proportionately speaking). Under the restrictive conditions used to derive 9, the stock generates proportionately more in the near term because $\delta < \delta^*$. Thus, at say time $s+1$ the aggregate stock has a larger proportion of its original productive capacity remaining than does the marginal capital investment. The aggregate stock therefore produces a larger unit gross marginal product at $s+1$ than does the surviving marginal capital investment. In total, the stock's expected net return (per original unit) is greater than the marginal capital investment's, so the aggregate

stock's capitalized value is larger too. Because the discounted value of the marginal capital investment's expected net return stream is q , the capitalized unit value for the stock is greater than q , and the aggregate stock's capitalized value is greater than qK .

The response of CV to an infinitesimal change in r is obtained by differentiating 9.

$$(11) \quad \frac{d}{dr} \frac{CV}{r} = \frac{qK}{r + \frac{\delta}{\pi}} \left\{ 1 - \frac{[r + \frac{\delta^*}{\pi}]}{[r + \frac{\delta}{\pi}]} \right\}$$

$$= \frac{qK}{r + \frac{\delta}{\pi}} \left\{ 1 - \frac{DUR_k}{DUR_C} \right\}$$

The sign of dCV/dr depends on the duration ratio. If DUR_k equals DUR_C , the duration ratio is unity, dCV/dr is zero, and firm value is unaffected by discount rate changes. An increase in required rates of return causes the producer to select a production plan that increases the marginal product of capital.⁷ The rising marginal product increases the value of the capital service stream, and the optimal adjustment leaves the discounted unit value of the aggregate stock's expected net return stream unchanged. The larger return on capital just offsets the heavier discounting; firm value is unchanged.

There is reason to believe the duration ratio is seldom unity. It is unity under the restrictive conditions of constant growth rates, smooth capital investment paths, etc. Not only do these conditions not exist, but a more realistic modeling of adjustment constraints, lagged behavior, etc., would tend to make it even more unlikely that DUR_k ever equals DUR_C . The surprising situation is when DUR_k is less than DUR_C , in which case the duration ratio is

less than unity, dCV/dr is greater than zero, and firm value rises in response to a rising discount rate. This outcome may be explained by expanding on the role of duration.

J.R. Hicks (1939) established that the elasticity of a return stream's capitalized value with respect to changes in the discount factor equals the stream's duration. A stream with a short duration responds less to a change in $1/(1+r)$ than does a long duration stream. Given a rising discount rate there is a tendency for the marginal investment's capitalized value to fall, thereby reducing its net present value. However, the attainment of equilibrium requires that the marginal capital investment has a zero net present value. A producer facing a fixed capital supply price responds to an increased discount rate by selecting an investment and production plan with an increased marginal product. The marginal product is increased to the point that it just offsets the decline in value induced by the rising discount rate, thereby reestablishing equilibrium at a zero marginal net present value. In short, DUR_0 measures the change in pQ' that exactly offsets the change in $1/(1+r)$ so that the system, at the margin, remains in equilibrium. The increased marginal product though, also causes the value of the aggregate stock's expected service stream to rise, thereby offsetting some of the decline in aggregate firm value induced by the heavier discounting. Because the aggregate duration is less than marginal duration, the change in marginal product (a change equal to DUR_0 times $\Delta(1+r)^{-1}$) restoring marginal equilibrium more than offsets the downward move of aggregate value induced by the higher discount rate. A smaller change in marginal product (a

change equal to DUR_k times $\% \Delta(1+r)^{-1}$) would have left aggregate value unchanged, but this lesser change would not have reestablished marginal equilibrium.

This equilibrium model suggests that a systematic change in discount rates tends to raise the real value of some firms while reducing the real value of others. For example, suppose two firms, A and B, face new investment alternatives with identical duration. Also suppose that the duration of the existing capital service stream for firm A is short because the capital is old, whereas for firm B the existing capital service stream has a long duration because the capital is new. Eventhough the replacement costs of firms A and B are identical, firm A's equilibrium capitalized value is less than firm B's. Suppose the equilibrium is shocked by macroeconomic phenomena resulting in an increase in shareholder systematic required rates of return. Each firm reacts to the shock the same way; they raise their marginal product by the same amounts in order to reestablish marginal equilibrium. However, this adjustment will tend to raise the aggregate value of A and lower the aggregate value of B. One might generalize the result and conclude that rising rates benefit firms with older, less productive assets while falling rates benefit firms with newer, more productive assets.

III. Empirical Evidence in a Contemporary Setting

This section demonstrates that in a contemporary setting it is empirically plausible for an asset stock's capitalized value to rise in response to an increasing discount rate. This is shown by

introducing the impact of taxes, straight line depreciation, and volatile investment paths. One effect of this introduction is that it modifies equations 9 and 11. The critical value of the duration ratio at which CV exceeds RC or at which dCV/dr switches signs is no longer unity.

The stock in place at time s is expected to generate a net return t periods in the future equal to $R(s+t)$. Once this stream is specified it is fairly simple to compute CV and DUR_k . The capitalized value of the net return stream is

$$(12) \quad CV(s) = \sum_{t=1}^{\infty} \frac{R(s+t)}{(1+r)^t}$$

and its duration is

$$(13) \quad DUR_k(s) = \sum_{t=1}^{\infty} \frac{t R(s+t)}{(1+r)^t CV(s)}$$

The net return expected at time $s+t$ is

$$(14) \quad R(s+t) = [1-\tau] NOI(s+t) + \tau D(s,s+t).$$

τ is the marginal income tax rate, NOI is net operating income (before depreciation), and $D(s,s+t)$ is the tax depreciation deduction the stock in place at time s is expected to provide at $s+t$. Thus, $R(s+t)$ equals after-tax NOI's plus the value of the depreciation tax shield.⁸

The depreciation tax shield is predetermined by the path of historic investment and the tax laws in effect at the time the investment was made. The formula used to compute $D(s,s+t)$ appears in table I, as does a summary of the notation and data sources. The NOI expected from a stock of putty-putty capital is

$$(15) \quad NOI(s+t) = c(s+t) K(s,s+t)$$

where c is the user cost and $K(s,s+t)$ is the capital in-place at time s expected to survive until $s+t$. The computation formulas for

$K(s, s+t)$ and the user cost are presented in table I. Measuring the user cost requires that the return stream expected from the marginal investment be fully specified. Once this stream has been specified, the computation of DUR_c is straightforward and follows the same methodology outlined in equation 13. Its computational formula is detailed in table I.

The sample set consists of the stock of fixed assets in the paper products industry (SIC 26) in 1976. Measures of CV , DUR_k , and DUR_c are estimated for two assets: plant and equipment. The empirical discussion hereinafter focuses on the expected net return stream for equipment because its stream is shorter than plant. However, both series are characteristically similar. The price of new equipment in 1976 is \$147 per unit. As listed in column 1a of table II, the marginal acquisition is expected to produce a net return of \$38 during its first year of use, of which \$31 is after-tax NOI and \$7 is depreciation tax savings. Because the stream's capitalized value equals \$147, the first return represents 23.6% of the stream's total discounted value (column 1b). By the tenth year of use the net return has diminished to \$12, only 3.6% of the stream's total discounted value. The duration of the marginal investment's expected net return stream is 4.6 years.

If the discount rate were to rise by 3 percentage points, say because of a rise in risk premia, the initial tendency would be for the capitalized value of the net return stream to fall. In fact, the value of the stream listed in column 1a would drop to \$131 if discounted by the larger discount rate, a fall of 13%. Because the price of the asset is fixed at \$147, investment in the asset would

represent a negative net present value alternative and a disequilibrating force would be exerted. The producer responds to this pressure by selecting a production plan which results in a larger value marginal product. This may be achieved by either raising output prices, switching to less capital intensive production techniques, or decreasing the rate of capacity utilization. Regardless, the action ultimately results in the reestablishment of marginal equilibrium and zero net present value. The amount by which the value marginal product must rise in order to offset the decline brought about by the increased discount rate is related to duration. In this situation the value marginal product must rise 24%, from \$25.57 to \$31.44.

The net return stream adjusted for the higher value marginal product is listed in column 2a. A comparison of columns 1a and 2a reveals that in every period the return is larger with the larger value marginal product. For example, in the first period the net return rises from \$37 to \$41 and in the tenth period it rises from \$12 to \$15. However, as a proportion of total capitalized value the first period's net return rises whereas the tenth period's falls. A comparison of columns 1b and 2b reveals that the discounted net returns in 2b are proportionately larger for the first four periods, whereafter they are smaller. If the intertemporal distribution of proportionate discounted returns is thought of as a schedule of weights that sum to unity, it is seen that a rise in the discount rate causes the schedule to seesaw; weights before duration rise and weights beyond duration fall.

The net return stream expected from the aggregate stock is also affected by a changing discount rate and value marginal product. The stream expected with the original discount rate is listed in column 1a of table III. The capitalized value of the stream is \$11.1 billion and its duration is 4.2 years. Notice the duration ratio is $4.2/4.6$ and is less than unity. As expected, the stock's replacement cost exceeds its capitalized value; RC is \$17.3 billion.

When the discount rate is raised three percentage points the capitalized value of the aggregate net return stream drops 10% to \$10.0 billion. However, the rising value marginal product more than offsets the drop because the duration ratio is below its critical value. Allowing the NOI to fully adjust to the new value marginal product raises CV to \$11.6 billion, or 4.5% greater than its original value. The new net return stream is listed in column 2a of table III. When the stock of plant and equipment is considered jointly, the equilibrium capitalized value of the aggregate net return stream rises by 4.7% in response to a 3 percentage point increase in the discount rate.

It is empirically feasible for a rising discount rate to lead to an increase in capitalized value. This result would likely change if the model allowed for putty-clay technologies and a general inability of the producer to equate a sluggish value marginal product to a changing user cost. However, it seems that some realities would tend to strengthen the qualitative result while others would weaken or reverse it. The main point is that firm value may be invariant to discount rate changes, it may be positively related, or it may be negatively related. Observing a

historic change in discount rates cannot, in and of itself, lead to a conclusion regarding the impact of discount rate movements on firm value.

Summary

Many valuation models have unambiguously modeled an inverse relationship between capitalized value and the discount rate because they have not had the producer adjusting the production plan in response to a changing user cost. Thus, they have been disequilibrium models because the user cost has not been equated to the value marginal product. This study shows that in a state of equilibrium where the producer equates marginal cost and marginal return, the relationship between capitalized value and the discount rate may be positive or negative and depends upon the duration of the asset stock relative to the duration of the marginal investment.

An empirical example shows that in a contemporary setting it is plausible for capitalized value to rise in response to a rising discount rate. It seems that in a mixed economy some sectors would be favorably affected by a systematic change in discount rates while other sectors might simultaneously be adversely affected. This implies that even if discount rates rose between 1965 and 1981 because of increased volatility and uncertainty, the capitalized value of the expected return streams, and hence firm values, did not necessarily fall. The existence of an enlarged risk premia is not a sufficient explanation for the decline in real equity values.

FOOTNOTES

- 1/ See Patric Hendershott (1981), Martin Feldstein (1980a, b), Lawrence Summers (1981a, b), and Franco Modigliani and Richard Cohn (1979).
- 2/ This result is derived by J.R. Hicks (1965), J. Lintner (1971), R. Haugen and D. Wichern (1974), and J. Boquist, G. Racette, and G. Schlarbaum (1975).
- 3/ Robert Coen (1975) explains the distinction between economic depreciation and productive capacity depreciation.
- 4/ For simplicity this formulation ignores monopoly and Ricardian rents. Inclusion of these rents does not affect the qualitative results.
- 5/ E. Lindenberg and S. Ross (1981) note that the market value of financial liabilities (MV) exceeds the replacement cost of capital (RC) when monopoly rents exist. L Summers (1980) and A. Auerbach (1979) note that MV may be less than RC because of the differential tax rates applied to capital gains and dividends. If MV is proportional to CV these two arguments apply to the ratio CV/RC the same as they do to MV/RC.
- 6/ The generalized notion of duration was conceived and described by J.R. Hicks (1939). Hicks referred to an asset's duration as its "average period" [p. 186] because of its conceptual similarity to the average period introduced by Eugen Bohm-Bawerk (1888). Hicks was apparently unaware of a 1938 study by Frederick Macaulay that applied the same measure to financial bonds. In any event, the literature has coined the measure as "duration" in deference to Macaulay, even though his usage was restricted to bonds and he did

not espouse the connection to general equilibrium processes envisioned by Hicks and Bohm-Bawerk.

7 The partial equilibrium nature of the model is apparent here. We allow the firm to adjust their marginal product but do not show how this adjustment occurs. The adjustment might conceivably be a substitution of labor for capital services, et al. A general equilibrium model would show how the adjustment affects the prices and demand for all factors of production.

8 The first expected return, $R(s+1)$, also includes the value of the investment tax credit.

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TABLE I

Summary of Data Sources and Notation

c : user cost of capital; computed following Downs (1986) as $c = q[r-\pi][1-v-\tau Z]/\{[1-H][1-\tau]\}$

$D(s,t)$: total tax depreciation deductions the stock in-place at time s is allowed at time t ; computed as

$$D(s,t) = \sum_{u=t} q(s+t-u) I(s+t-u) z(s+t-u,u)$$

DUR_c : duration of the marginal investment's expected net return stream: computed as $DUR_c = \sum_t t [1+r]^{-t} c [1+\pi]^{t-1} [1-h(t)] + \tau z(1976,1)$

$h(t)$: percent of capital's original productive capacity lost at the t 'th moment after acquisition; for the straight line depreciation assumes herein $h(t) = 1/L$, for $t=1, \dots, L$, and $h(t) = 0$ otherwise.

$$H : \text{equals } \sum_t [1+r-\pi]^{-t} h(t)$$

I : real investment [Bureau of Labor Statistics (BLS), 1979]

$K(s,s+t)$: the stock in place at time s which survives until $s+t$; computed as $K(s,s+t) = \sum_{u=t} I(s+t-u) [1 - \sum_{v=0} h(v)]$

L : productive service life of capital [BLS]

q : supply price of capital [BLS]

r : weighted average cost of debt and equity [based on Hendershott and Hu (H&H, 1981), Compustat (1977), CRSP(1977), and vonFurstenberg, et. al. (1979)]

v : effective rate of the investment tax credit [H&H]

$z(s,t)$: percent of supply price which, given the tax laws in effect at time s , may be deducted t years after acquisition [based on Coen (1975) and the SSRC model (1980)]

$$Z : \text{equals } \sum_t [1+r]^{-t} z(1976,t)$$

π : expected inflation rate [H&H]

τ : marginal income tax rate [H&H]

TABLE II

Net return stream for the marginal equipment investment, 1976

the	historic discount rate ::			historic rate + .03	
t'th	net	-1a- as %		net	-2a- as %
period:	return	of total		return	of total
of use:	(\$)	present value::		(\$)	present value
t=	-1a-	-1b-		-2a-	-2b-
1	\$37.94	.236		\$40.91	.247
2	23.50	.134		26.44	.143
3	21.72	.114		24.63	.119
4	20.08	.097		22.93	.099
5	18.53	.082		21.31	.082
6	17.03	.069		19.72	.068
7	16.02	.060		18.61	.057
8	14.93	.051		17.39	.048
9	13.74	.043		16.05	.040
10	12.45	.036		14.58	.032
11	9.78	.026		11.71	.023
12	6.98	.017		8.69	.015
13	5.90	.013		7.34	.012
14	4.68	.010		5.82	.008
15	3.30	.006		4.10	.005
16	1.74	.003		2.17	.002
present value				present value	
at r=.0886 is \$147				at r=.1186 is \$147	

TABLE III

Net return stream for the equipment capital stock, 1976

the : <u>historic discount rate</u> :: <u>historic rate + .03</u> .				
t'th : net : -1a- as % :: net : -2a- as %				
period: return : of total :: return : of total				
of use: (\$million): present value :: (\$million): present value				
t=	-1a-	-1b-	-2a-	-2b-
1	\$2611	.216	\$2960	.229
2	2158	.164	2488	.172
3	1954	.136	2263	.140
4	1755	.113	2041	.113
5	1560	.092	1321	.090
6	1364	.074	1600	.070
7	1175	.058	1384	.055
8	989	.045	1172	.041
9	808	.034	966	.030
10	640	.025	773	.022
11	476	.017	584	.015
12	346	.011	431	.010
13	253	.008	315	.006
14	166	.005	207	.004
15	89	.002	111	.002
16	32	.001	40	.001
present value			present value	
at r=.0886 is \$11105			at r=.1186 is \$11553	