Average Period, User Cost, and Implications for Value, Interest, and Term Structure

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ABSTRACT

This study weds the user cost of capital with average period in a modern analytical relationship and offers three implications. One, a real capital stock's fundamental value as a proportion of its current replacement cost depends on a ratio of average periods. Two, the effect on fundamental value of an increasing discount rate may be positive or negative and it too depends on a ratio of average periods. Three, an increase in average period of debt maturity allows an increase in the flow of interest from capital and, consequently, the yield curve is upward sloped irrespective of all else.

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> "The theories of early economics were necessary to reach new theories; and these, still very imperfect, will enable us to reach other theories which will be less so; and so on. Perfecting a theory is completely different from seeking to destroy it by foolish and pedantic subtleties. The first task is sensible and useful, the second is not very reasonable as well as fruitless, and someone who has no time to waste does better not to bother with it."
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> Vilfredo Pareto. *Manual of Political Economy* (1927): Chapter 3, ¶31.

1. Introduction to the Historical Backdrop

Early economics weaves explanations of value and interest around theories of capital. An innovative yet contentious contribution is Eugen von Böhm-Bawerk's *Positive Theory of Capital* (1888). Böhm-Bawerk contends real capital embodies potential interest. Intertemporal capital service streams release interest as time elapses and services flow. Böhm-Bawerk's description of real capital as the source of interest introduces two novel concepts: average period and user cost.

Irony surrounds historical development of these two concepts since their introduction by Böhm-Bawerk in 1888. Even though significant contributions by legendary economists punctuate the subsequent fifty years, it wasn't until the 1930's that average period and user cost advance intellectually. On the one hand, John Hicks (*Value and Capital*, 1939) advances the concept of average period. On the other hand, John Keynes (*The General Theory of Employment, Interest, and Money*, 1936) advances the concept of user cost. Hicks ignores user cost and Keynes ignores average period. Each proclaims rather strongly the incredible usefulness of the respective concept. Yet average period and user cost were divorced.

Lionel McKenzie and Stefano Zamagni honor Hicks and write (1991, p. xviii): “Should we not recognize *Value and Capital* to be the central document of economic theory in the twentieth century? There the main threads of the economic theory inherited from the nineteenth century are collected and from it spread the main threads of subsequent theoretical development in the twentieth.” Hicks acknowledges in *Value and Capital* the role of Böhm-Bawerk 50-years earlier:

> “Even to-day, the great name in this department of economics [that is, the study of economic dynamics] is the name of Böhm-Bawerk. This is so, not because his doctrine is generally accepted (it was not generally accepted even in his own time, and it has still fewer supporters in ours), but because it is a challenge that has somehow to be met. Nearly every one who comes to the study of capital falls a victim to Böhm-Bawerk’s theory at some stage or other. The definition of capitalistic production as time-using production; of the amount of capital employed as an indicator of the amount of time employed; of the effect of a fall in interest on the structure of production as consisting in an increase in the amount of time employed; all these ideas give to the subject an apparent clarity which is, at first sight, irresistible. The theory stands up very well to the more obvious objections which can be made against it; yet, as one goes on, difficulties mount up. The definition of the ‘time taken in production’ gets harder and harder; and so most people find themselves driven, in the end, to abandon the theory, even if they have nothing much to put in its place… Clearly Böhm-Bawerk was wrong; but there must have been something in what he said; you cannot construct such an elaborate theory as that out of nothing. The core of truth in the Austrian theory needs to be discovered before we can really claim to have a satisfactory theory of capital.”
>
> J.R. Hicks. *Value and Capital* (1939): pp. 192-193. The remark in parentheses is by Hicks, and the one in square brackets is by me.

Hicks describes a dynamic economic system in which capital promises through time a stream of returns $(x_0, x_1, x_2, \ldots, x_v)$. Let r represent the discount rate and β the discount ratio; that is, $\beta = 1/(1+r)$. The capitalized value of the stream equals $x_0+\beta x_1+\beta^2 x_2+\ldots+\beta^v x_v$. Hicks continues:

> "The elasticity of this capital value with respect to the discount ratio β is
>
> $$\frac{\beta x_1 + 2\beta^2 x_2 + 3\beta^3 x_3 + \ldots + v\beta^v x_v}{x_0 + \beta x_1 + \beta^2 x_2 + \beta^3 x_3 + \ldots + \beta^v x_v}$$
>
> (for the elasticity of a sum is the *average* of the elasticities of its parts). Now when we look at the form of this elasticity we see that it may be very properly described as the *Average Period* of the stream; for it is the *average length of time for which the various payments are deferred from the present, when the times of deferment are weighted by the discounted values of the payments.* (The reader may perhaps be angry with me for appropriating the term 'Average Period' to this quantity, since he may have in his head what appears to be a very different meaning of the term. I hope to show at a later stage, however, that the meaning I am giving it is a fair extension of the traditional meaning.)"
>
> J.R. Hicks, *op. cit.,* pp. 186-187. Parenthetical comments and italics are in original text.

Once Hicks establishes that average period equals the elasticity of capitalized value with respect to the discount ratio, the concept evolves explosively. Especially exciting to Hicks is analysis of differences in average periods across assets and agents. He uses average period to explain how discount rate changes relate to complementary and substitution effects.

The elegant analysis by Hicks of intertemporal trade-offs between components of the stream $(x_0, x_1, x_2, \ldots, x_v)$ inspires many contributions to general equilibrium theory. Roy Radner describes a common characteristic for a set of models that stem from the Hicks method of analysis.

> "Very briefly, in the ADM model [that is, the modeling framework based on Arrow (1953), Debreu (1959), and McKenzie (1963)] there are finitely many economic agents (consumers and producers), dates, and commodities at each date. In fact, to understand the beauty (and limitations) of the ADM model, it is best to think of the date of delivery or availability of a commodity as just one of its defining characteristics, along with its location and other physical characteristics. … Although the 'method' is borrowed from a 'static' picture of an economy, the predictions can be 'dynamic' in the sense that the commodities are dated."
>
> R. Radner. (1991): pp. 425-426. The remark in parentheses is by Radner, and the one in square brackets is by me.

Kenneth Arrow attributes this method of economic research to Hicks:

> "Among the many great contributions of *Value and Capital,* to my mind, the greatest of all was the representation of future goods symmetrically with present ones. At one stroke, all the *conceptual* mysteries of capital theory and the confusions about steady states were wiped out."
>
> K. Arrow. (1991): p. 42. Italics are in original text.

Despite the growth of general equilibrium theory, few studies focus directly on average period.[1] Nonetheless, to Hicks the average period seems significant:

> "This way of measuring the trend of a stream of values can be used for any stream whatsoever; it seems to have more significance than any other from the point of view of economic theory."
>
> J.R. Hicks, *op. cit.,* p. 188.

Average period embodies characteristics of dynamic equilibrium. Hicks, however, divorces average period from user cost.

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[1] Persistence of the average period concept is limited to applications involving financial securities. Frederick Macaulay (1938) computes elasticity of bond price with respect to interest rate, and coins the measure "bond duration." The more well known among the many studies that analyze bond duration include Lawrence Fisher and Roman Weil (1971), George Hopewell and Michael Kaufman (1973), and Robert Haugen and Dean Wichern (1974). A model for common stocks of the elasticity of stock price with respect to discount rate appears in John Boquist, George Racette, and Gary Schlarbaum (1975). None of these financial studies, however, pursue the equilibrium framework that Hicks postulates, nor do they wed duration and user cost.

Böhm-Bawerk introduces the user cost concept in the *Positive Theory of Capital*, wherein he refers to it as "bearer-of-the-use:"

> "In the first year of its use the owner realizes the 'current' service with its value of 100 [this equals the first cash flow in an example that Böhm-Bawerk is explaining]. Naturally this service, thus consumed or rendered, comes off the value of the machine (which we may call the 'bearer of the use'), and the good suffers a loss of value. But this loss of value cannot be quite so great as the value of the service rendered and deducted. It is partly compensated by the increased value of the services that still remain embodied in the machine."
>
> E. Böhm-Bawerk. *Positive Theory of Capital* (1888): p. 343. The remark in parentheses is by Böhm-Bawerk, and the one in square brackets is by me.

The bearer-of-the-use relates directly to x_0 in the Hicksian stream.

Keynes (1936) revisits the user cost concept nearly fifty years later:

> "Let us call this quantity … which measures the sacrifice of value involved in the production of *A*, the *user cost* of *A*. *User cost* will be written *U*. The amount paid out by the entrepreneur to the other factors of production in return for their services, which from their point of view is their income, we will call the *factor cost* of *A*. The sum of the factor cost *F* and the user cost *U* we shall call the *prime cost* of the output *A.*"
>
> J.M. Keynes. *The General Theory of Employment, Interest, and Money.* (1936): p. 53. Italics are in the original text.

Keynes links prime cost to Alfred Marshall (1890), but never acknowledges the similarity of user cost with Böhm-Bawerk's bearer-of-the-use. The following passage reveals a direct analogy:

> "It is an advantage of the concepts of user cost and supplementary cost that they are as applicable to working and liquid capital as to fixed capital. The essential difference … [is that] in the case of fixed capital, which is durable and used up gradually, the return consists of a series of user costs and profits earned in successive periods."
>
> J.M. Keynes, *op. cit.,* p. 73. The remark in square brackets is by me.

Roughly speaking, capitalized value relates according to Keynes to the discounted stream of user costs, and according to Böhm-Bawerk to the discounted stream of bearer's-of-the-use. The metrics are conceptually equivalent.

Keynes inspires research that employs user cost as a tool of analysis. Most significant is derivation by Dale Jorgenson (1963, 1967) of an empirical measure of user cost. Jorgenson wraps around the user cost a neoclassical theory of investment behavior. James Tobin comments on Jorgenson's theory:

> "As long as expectations are assumed certain, maximization of the present value of the firm is as powerful a principle for dynamic theory as profit maximization has been for static theory. A dynamic theory based on this principle has much more to say, and can handle many more complexities, than is often appreciated. Jorgenson's specific example, however, is only barely dynamic. His firm can maximize present value simply by maximizing profits at every point in time. The firm confronts no intertemporal trade-offs, in which profits now must be weighed by profits later.
> James Tobin. (1967): p. 156.

Tobin insightfully observes that user cost is only barely dynamic.[2] This temporal trait is consistent with Keynes's conception:

> "The fact that the assumptions of the static state often underlie present-day economic theory, imports into it a large element of unreality. But the introduction of the concepts of user cost and of the marginal efficiency of capital, as defined above, will have the effect, I think, of bringing it back to reality, whilst reducing to a minimum the necessary degree of adaptation.
> J.M. Keynes, *op cit.*, p. 146.

User cost embodies characteristics of static equilibrium.

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[2] Tobin's subsequent remark exemplifies how evolution of concepts over long periods of time causes arguable and subtle pedantic differences in terminology: "I would like to make a parenthetical semantic remark: Jorgenson calls the rental just discussed, specifically $q(r + \delta - \pi)$, user cost. To anyone who learned about user cost from the appendix to Chapter 6 of Keynes' *General Theory*, this terminology seems surprising. Keynes assumed that the decline in the value of a stock of goods during a period depends on the intensity of use, not just on the passage of time, hence the term *user* cost. Keynes' assumption is notably absent from most modern capital theory, including Jorgenson's. I find it confusing to see a rental which is just a time or ownership cost called user cost."

The two concepts that Böhm-Bawerk introduces, average period and user cost, differ by relativity to time. Average period relates to sensitivity of the entire cash flow stream and production plan to changes in discount rate. User cost relates to static production trade-offs inherent with the first cash flow. Yet upon user cost there exists an equilibrium condition. Not a dynamic equilibrium, but rather a temporary equilibrium. Jean-Michel Grandmont explains the nature of this equilibrium:

> "… [Grandmont cites and concisely summarizes seventeen studies] These theoretical investigations have made clear, at a formal level, that Keynesian macroeconomic models are in fact temporary equilibrium models with optimizing traders operating under conditions of imperfect (monopolistic, oligopolistic) competition, thereby confirming the intuitions of Hicks and of other early writers on that issue.
>
> J. Grandmont. (1991): p. 12. The remark in parentheses is by Grandmont, and the one in square brackets is by me.

Wedding average period to user cost enables a doubly powerful model embodying two dimensions of time. The average period embodies time in the series $(x_0, x_1, x_2, \ldots, x_v)$. This specification allows insights about dynamic general equilibrium and intertemporal trade-offs. The user cost (x_0) embodies time differently. The economic agent selects a production plan in which the most immediate cash flow, that is the value marginal product of capital, equilibrates to the user cost of capital. User cost represents the first step into the future, and the first step is different than all others. All future steps are, at any moment, simply expectations. The first step, however, is an on-going realization. The first step in many ways is most important of all – its direction and intent embodies everything known or expected about the future:

> “User cost has, I think, an importance for the classical theory of value which has been overlooked.” [p. 66] … User cost constitutes one of the links between the present and the future. [p. 69]
>
> J.M. Keynes. *op cit*.

My study consummates marriage of average period and user cost. The study proceeds as follows. Section 2 generalizes specification of the equilibrium user cost of capital. Under certain restrictive conditions, the specification reduces to the well-known Hall-Jorgenson user cost (1967). My specification, however, generalizes intertemporal dynamics pertaining to capacity depreciation and financial structure. Section 3 employs the user cost to examine implications for value. A novel finding is that a real capital stock’s fundamental value may be greater or less than the stock’s current replacement cost. The exact relation depends on average periods. Section 4 reveals that the relation between fundamental value and interest rate changes may be positive, zero, or negative. Once again, the exact relation depends on average periods. Section 5 finds that under the simplest of scenarios the term structure of interest rates, irrespective of all else, has a shape similar to the normal yield curve. Average period, just as Böhm-Bawerk hypothesizes, provides a natural explanation of interest. The study closes with a brief conclusion.

2. The generalized user cost specification

The user cost equals the pre-tax asset cash flow produced by one unit of real capital during its first period of use such that the asset represents a zero net present value investment.[3] Jorgenson (1963) introduces the user cost to modern analytical literature in his seminal investigation about fixed capital investment behavior. Jorgenson links the user cost to financing rates and tax policy parameters. He subsequently employs the user cost as an explanatory variable for net fixed investment.

Robert Hall and Jorgenson (1967) obtain a specification for the user cost at time s, denoted c_S:

$$c_s = \frac{q_s(r_s + \delta - \pi_s)(1 - \tau_s Z_s)}{(1 - \tau_s)} \qquad (1)$$

In this expression, q_S represents the supply price at time s of a new capital asset, r is the financing rate for the investment, δ is the asset's rate of decline in productive efficiency, π is the expected inflation rate, τ is the marginal corporate income tax rate, and Z is the present value of tax depreciation deductions (per dollar of asset) expected throughout the project life.[4]

Interpretations about the financing rate in the user cost framework vary. Early studies employing the user cost [e.g., Jorgenson (1963)] measure r as the long-term government bond rate. Hall (1981) argues that the short-term bond rate is

[3] Equivalent restatements of the user cost definition include (all are per unit of real capital): (a) earnings before interest and taxes plus depreciation; (b) operating income plus depreciation; (c) cash flow from operations plus taxes and interest.
[4] The Hall-Jorgenson user cost specification includes the effect of an investment tax credit at rate v. Incorporate v into the specifications herein by replacing $(1 - \tau Z)$ with $(1 - \tau Z - v)$.

appropriate. Martin Feldstein (1982) argues that these specifications ignore the cost of equity financing. He suggests the user cost should employ a measure for *r* equal to the weighted average of debt and equity financing rates. Although Jorgenson and Kun-Young Yun (1991) rely on pre-tax financing rates, most recent studies employ a weighted average of after-corporate-tax debt and equity financing rates, as in

$$r_s = (1-\alpha_s)\rho_s + (1-\tau)\alpha_s i_s \ , \qquad (2)$$

where α is the marginal debt-to-assets ratio, ρ is the levered equity financing rate, and *i* is the pre-tax debt financing rate. Equation 2 specifies *r* as the ubiquitous weighted average cost of capital. The financial cost of capital from equation 2, *r*, substitutes into the Hall-Jorgenson user cost of capital in equation 1.

The objective for the producer is maximization of discounted profits. Jorgenson (1967) shows that the marginal conditions equate the value marginal products of labor and capital, respectively, to the wage rate and user cost of capital. Alternative yet equivalent description of the equilibrating process is that the producer invests in capital whenever (a) the value marginal product of capital exceeds the user cost of capital, or (b) the internal rate of return for the after-corporate-tax (before interest) cash flow stream (also known as the Keynesian marginal efficiency of capital) exceeds the weighted average cost of capital. With each additional capital investment the marginal physical product of capital declines, thereby reducing the value marginal product and marginal efficiency of capital. Equilibrium eventually recurs when inequalities (a) and (b) are offset.

A significant strand of literature manipulates the user cost framework so that instead of solving for the equilibrium pre-tax cash flow as the unknown variable, some other variable or expression from the user cost specification serves as the unknown term. Patric Hendershott (1981) endogenizes the financing rate within the user cost framework and makes inferences about changes in risk premia and possible valuation effects of inflation. Alan Auerbach and Jorgenson (1980) invoke the assumption that risk-adjusted after-tax returns on financial and fixed assets equilibrate. They subsequently extract from the user cost framework the internal rate of return for the fixed asset's pre-tax cash flow stream and ingenuously glean insight about effective tax rates for fixed assets [other prominent studies on effective tax rates and user cost include David Bradford (1981), Jane Gravelle (1982), and Mervyn King and Don Fullerton (1984)]. Other studies rely on the user cost framework to make inferences about the financing rate; that is, they infer the equilibrium financial cost of capital (r) for fixed assets in different sectors or asset groups [see, for example, Auerbach (1987), Hans-Werner Sinn (1991), and Jorgenson and Ralph Landau (1993)].

Specification of financial structure in the preceding studies is limited. They assume explicitly, if anything, that for a firm financing by debt and equity the underlying debt-to-equity ratio is perpetually constant. The implications of this assumption are neither investigated nor relaxed by any of the studies. The model below generalizes the specification.

Suppose that for purchasing a new fixed asset at time *s* with supply price q_s the entrepreneur obtains equity financing of $(1-\alpha_s)q_s$ and takes out a loan for $\alpha_s q_s$. A loan payment schedule at time of investment establishes repayment of principal and interest. Each period the asset cash flow net of the loan payment accrues to equity; call the expected accrual the residual cash flow. The zero net present value equilibrium condition equates funds provided by equity to the present value of the expected residual cash flow stream discounted by the equity financing rate:

$$(1-\alpha_s)q_s = \sum_{t=1}^{\infty} \left\{ (1+\rho_s-\pi_s)^{-t}(1\text{-}\tau)c_{s,t} + (1+\rho_s)^{-t}\tau q_s z_{s,t} - (1+\rho_s)^{-t}B_{s,t} \right\}. \qquad (3)$$

α_s is the *initial* loan-to-value ratio for the time *s* marginal investment in real assets. The evolution beyond time *s* of the asset's loan-to-value ratio depends on the interaction between the asset cash flow and loan payment streams.

The discounted residual cash flow on the right-hand-side of equation 3 has three components. The first component, $(1\text{-}\tau)c_{s,t}$, equals the after-corporate-tax real asset cash flow expected at time $s+t$ from the time *s* investment. The second component, $\tau q_s z_{s,t}$, equals expected tax savings from depreciation deductions at time $s+t$ resulting from the time *s* investment. The third component, $B_{s,t}$, equals the after-corporate-tax loan payment made at time $s+t$ for the time *s* investment. $B_{s,t}$ may include interest, principal repayment or issuance, and any other debt related fees. Further details about cash flow components are given below.

First consider specification of the real asset cash flow stream. Let d_j denote the proportional decline in real asset cash flow that occurs after the *j*'th asset cash flow is received ($d_0 = 0$). The series d_j for $j = 0,...,\infty$ is the asset's capacity depreciation schedule.[5] Specification of the capacity depreciation schedule predetermines how much the asset incrementally contributes to potential production at every point in the useful service life. Typically, the capacity depreciation schedule is assumed exogenous and independent of utilization rates or maintenance expenditures.[6] For example, with straight-line capacity depreciation over a ten-year service life $d_j = 1/10$ for $j = 1,...,10$ and $d_j = 0$ otherwise. The expected real asset cash flow accruing at time $s+t$ from the investment made at time s is

$$c_{s,t} = c_s\left(1 - \sum_{j=1}^{t} d_{j-1}\right). \tag{4}$$

c_s is the time *s user cost of capital* and equals the asset cash flow produced by one unit of new real assets during first period of use (c_s is identical to $c_{s,1}$).

Second consider specification of the debt cash flow stream. Let γ denote the loan payment (interest, principal, and fees) to be paid at the end of the asset's

[5] Depreciation and capital stock definitions herein follow the terminology of the U.S. Bureau of Labor Statistics (1979) and U.S. Bureau of Economic Analysis (1987).
[6] Two of the few models which assume depreciation is endogenous and depends upon utilization rates and maintenance expenditures as choice variables are Larry Epstein and Michael Denny (1980) and Moshe Kim and Giora Moore (1988).

first period of use, where γ is expressed as a proportion of the asset's supply price:

$$\gamma_s = B_{s,1}/q_s\,. \qquad (5)$$

The debt cash flow stream is summarized by the series b_j for $j = 0,...,\infty$, where b_j denotes the change in cash flow (as a proportion of $B_{s,1}$) that occurs after the j'th payment is made ($b_0 = 0$). More precisely,

$$b_j = (B_{s,j} - B_{s,j+1})/B_{s,1}. \qquad (6)$$

Equations 5 and 6 specify the entire debt cash flow stream, regardless of whether the debt contract represents a consol, a fixed payment amortized loan, a debenture with a balloon payment, or any other debt maturity structure.[7] The debt cash flow at time $s+t$ attributable to the loan issued to finance the time s investment is given by

$$B_{s,t} = \gamma_s q_s \left\lfloor 1 - \sum_{j=1}^{t} b_{j-1} \right\rfloor. \qquad (7)$$

The zero net present value equilibrium with explicit specification of the asset and debt cash flow streams is obtained by substituting equations 4 and 7 into 3. That substitution yields

[7] If the first debt cash flow during the first period is zero, γ may be redefined; for example, with zero coupon debt of term T, $\gamma_s = B_{s,T}\, q_s^{-1}$.

$$(1-\alpha_s)q_s = \sum_{t=1}^{\infty}(1+\rho_s-\pi_s)^{-t}(1\text{-}\tau)c_s\left[1-\sum_{j=1}^{t}d_{j-1}\right]$$

$$+\sum_{t=1}^{\infty}(1+\rho_s)^{-t}q_s\,\tau z_{s,t} \tag{8}$$

$$-\ \sum_{t=1}^{\infty}(1+\rho_s)^{-t}q_s\,\gamma_s\left[1-\sum_{j=1}^{t}b_{j-1}\right]$$

Equation 8 shows that at equilibrium the funds provided by equity equal the expected present value of the after-tax asset cash flow stream, plus the expected present value of the depreciation tax savings, minus the present value of the debt cash flow stream, each discounted by the levered equity financing rate.[8]

Obtain the generalized user cost specification by simplifying and rearranging the equilibrium condition in equation 8:

$$c_s = \frac{q_s\left(\rho_s-\pi_s\right)\left(1-\tau_s Z_s-\Lambda_s\right)}{\left(1-\tau_s\right)\left(1-\Delta_s\right)}\ . \tag{9}$$

The two new variables in equation 9, Λ and Δ, generalize intertemporal dynamics of financial structure and capacity depreciation, respectively.[9] When equity is the only financing source (that is, debt does not exist), Λ equals zero. When capital does not depreciate, Δ equals zero. These generalized variables do not appear in the Hall-Jorgenson user cost specification. The subsections below discuss these innovations.

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[8] Stewart Myers (1974) discusses a capital budgeting framework similar to the one above, in that the discounted value of asset cash flows is computed and subsequently the discounted value of financing costs is subtracted.

[9] Thomas Downs (1988) presents a user cost specification that generalizes capacity depreciation. That specification does not explicitly model financial structure.

Capacity depreciation: Δ

Obtain Δ_s by discounting the capacity depreciation schedule with the real levered equity financing rate:

$$\Delta_s = \sum_{t=1}^{\infty} (1+\rho_s - \pi_s)^{-t} d_t \quad . \tag{10}$$

To focus on capacity depreciation suppose that equity is the only financing source. Thus, $\Lambda = 0$ and $r = \rho$. The generalized user cost specification from equation 9 reduces to equation 1 by restricting capacity depreciation to an infinite geometric time-path. This restriction implies $d_t=\delta(1-\delta)^{t-1}$ for every $t>0$. Subsequent simplification of equation 10 shows that for this special case

$$\Delta_s = \delta(\rho_s + \delta - \pi_s)^{-1} \tag{11}$$

Substitute equation 11 into 9 and obtain equation 1. The Hall-Jorgenson user cost specification is valid only when productive capacity depreciates along an infinite geometric time-path.[10] Equation 9 accommodates, however, any time-path of capacity depreciation.

Financial structure: Λ

Obtain Λ_s by discounting the loan payment schedule:

$$\Lambda_s = \alpha_s - \gamma_s \rho_s^{-1}(1-\lambda_s) \ , \tag{12}$$

where

$$\lambda_s = \sum_{t=1}^{\infty} (1+\rho_s)^{-t} b_t \quad . \tag{13}$$

[10] Feldstein and Michael Rothschild (1973) vehemently criticize the Jorgenson investment model because, in addition to the explicit assumption of geometric capacity depreciation, there is an implicit assumption that real capital investment grows along a geometric time-path.

The generalized user cost specification from equation 9 reduces to the Hall-Jorgenson specification in two special cases.

The first special case is the one period model. The following sequence of events occurs: (1) At time s investment at price q occurs with equity and debt financing equal to $(1-\alpha)q$ and αq, respectively; (2) At time $s+1$ the asset delivers pre-tax cash flow equal to the user cost, the loan is fully repaid with an after-tax debt payment of $\alpha q(1 + (1-\tau)i)$, the residual accrues to equity, and the asset expires. No other cash flows attach to the time s capital investment.

Parameterize the one period model with these settings: $d_1 = 1$ and $d_t = 0$ otherwise; $b_1 = 1$ and $b_t = 0$ otherwise; and $\gamma = \alpha(1 + (1-\tau)i)$. Solution of equations 10 and 12 shows, respectively, that $\Delta = (1+\rho-\pi)^{-1}$ and $\Lambda = \alpha[\rho - (1-\tau)i](1+\rho)^{-1}$. Substitute Δ_s and Λ_s into equation 9, simplify, and obtain the Hall-Jorgenson user cost (equation 1) containing the weighted average cost of capital (equation 2).[11]

The second special case occurs when, for the marginal capital investment, the periodic loan payment equals an amount that holds the loan-to-value ratio perpetually constant. Parameterize this scenario with geometric capacity depreciation. Thus, $d_t = \delta(1-\delta)^{t-1}$. The first loan payment, $B_{s,1}$, is comprised of interest and principal equal to $\alpha q(1-\tau)i$ and $\alpha q(\delta-\pi)$, respectively, implying that $\gamma =$

[11] Simplification results in the Hall-Jorgenson user cost with $\delta=1$, plus an extra cross-product term: $\pi(\rho - r)(1+\rho)^{-1}$, with r per equation 2. This trivial term (about 10 basis points with plausible settings) vanishes by modeling with the conceptually equivalent approach of discounting the real loan payment with the real equity financing rate. I use the modeling in the text because the loan payment schedule and tax policies typically stipulate nominal cash flows. This comment also applies to the second special case in the subsequent paragraph.

$\alpha[(1-\tau)i+\delta-\pi]$. Loan payments evolve along the time-path given by $b_t = (\delta-\pi)(1-\delta+\pi)^{t-1}$, implying that $\lambda_s = (\delta-\pi)(\rho+\delta-\pi)^{-1}$. Solution of equations 10 and 12 shows, respectively, that $\Delta = \delta(\rho+\delta-\pi)^{-1}$ and $\Lambda = \alpha[\rho - (1-\tau)i](\rho+\delta-\pi)^{-1})$. Substitute Δ and Λ into equation 9, simplify, and obtain the Hall-Jorgenson user cost (equation 1) containing the weighted average cost of capital (equation 2).[12]

The Hall-Jorgenson specification of user cost is valid only under unduly restrictive conditions. Equation 9 introduces a generalized specification that accommodates, for the marginal investment in real capital, any time-path of capacity depreciation and any financial structure.

3. Implications for Value

The Financial Accounting Standards Board [FASB (1979)] defines the current replacement cost of a real capital stock as the cost of replacing existing assets with new ones while leaving current productive capacity unchanged. Measuring current replacement cost requires modeling the history of capital accumulation. The entire capital expenditure in new capital assets at time s equals q_sI_s, where q_s is the supply price for one unit of new real capital and I_s represents the number of units of productive capacity acquired. That is, I_s represents real investment. The actual capital expenditure equals a price times a quantity, and the quantity constitutes a factor input for production [for divergent views on capital indexes

[12] James Miles and John Ezzell (1980, pp. 728-729) establish a similar finding in a more restrictive setting: "That the textbook *WACC* ['weighted average cost of capital' per equation 2] yields correct valuations for either a single-period project or a project with level, perpetual cash flows is a consequence, not of project life *per se,* as has been argued in the literature, but rather of maintaining indirectly a constant leverage ratio." The paragraph in the text explains the *WACC* is correct when the perpetual cash flow changes along any geometric time-path, even a level one.

and production functions see Joan Robinson (1953) and Edmond Malinvaud (1953)].

Obtain C_s, the current replacement cost of the time s real capital stock, by applying the capacity depreciation schedule to the history of real investments and multiplying by the current supply price of new capital:

$$\begin{aligned} C_s &= q_s \sum_{t=1}^{\infty} I_{s-t+1}\left(1 - \sum_{j=1}^{t} d_{j-1}\right) \\ &= q_s K_s \end{aligned} \quad . \qquad (14)$$

William Brainard and Tobin (1968) and Tobin (1969) argue that the firm's financial market value converges to the current replacement cost of its assets. This compelling argument led to issuance by FASB of *Statement No. 33* (1979) mandating that U.S. corporations prepare supplementary financial statements disclosing current replacement cost estimates for net fixed assets. Over 90 percent of all compliant companies followed the procedure in equation 14 [Keith Shriver (1987)].

Presumed justification for the common belief that the ratio of market value to current replacement cost, Tobin's Q-ratio, converges to unity is simply stated:

> "If markets existed for all the firm's assets and the values of all assets were recorded, the value of the firm reported on the balance sheet under current cost accounting would equal the market value of the firm's securities, because both values would reflect the expected present value of the future cash flows to be generated by the firm's assets."
>
> *In-Mu Haw and Stephen Lustgarten.* (1988): p. 332.

Future cash flows sustain value, but equation 14 is backward-looking: apply capacity depreciation schedules to historical real investments, sum through history, and multiply the sum by the current supply price. Current replacement cost relates only loosely to the expected present value of future cash flows.

User cost links capital value directly to future cash flows. Define *fundamental value* of the time *s* total capital stock as the discounted sum of expected pre-tax asset cash flows net of proportional taxes plus the discounted sum of tax savings from depreciation deductions, where the discount rate equals the weighted average cost of capital from equation 2. Obtain fundamental value, denoted V_s, as:

$$V_s = C_s \left\{ (1 - \tau Z_s) \frac{(1 - \Omega_s)}{(1 - \Delta_s)} + \tau Y_s \right\} \qquad (15)$$

Two new terms appear in equation 15.[13] Y_s is the present value for the total capital stock in-place at time *s* of the depreciation deductions promised per dollar of current cost,

$$Y_s = \sum_{t=1}^{\infty} (1 + r_s)^{-t} \sum_{u=0}^{\infty} q_{s-u} I_{s-u} z_{s-u,u+t} \div C_s \, . \qquad (16)$$

Obtain the other new term, Ω_s, by discounting the mortality distribution of the time *s* real capital stock:

$$\Omega_s = \sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} h_t \ , \qquad (17)$$

where $$h_t = (K_{s,t} - K_{s,t+1}) / K_s$$

[13] An assumption implicit with equation 15 is a constant real marginal physical product of capital beyond time *s*. In other words, the relative contribution of capital to production remains the same as at time *s*. In the framework by Arthur Thomas [1969, especially pp. 41-47], this imposes restrictions on cost and revenue functions of continuity, constant returns to scale, and simultaneous successive expansion. Setting the discount rate to the weighted average cost of capital, too, implicitly assumes that for the marginal capital investment the loan-to-value ratio remains perpetually constant (Section 5 relaxes this assumption).

and $K_{s,t}$ represents the capital services contributed at time $s{+}t$ by assets in-place at time s,

$$K_{s,t} = \sum_{u=0}^{\infty} I_{s-u}\left(1 - \sum_{j=1}^{t+u} d_{j-1}\right).$$

In order to focus on the relation between average period and fundamental value suppose that there are no taxes ($\tau = 0$). The ratio of fundamental value to current replacement cost, $V_s \div C_s$, depends on average periods:

$$\frac{V_s}{C_s} = \frac{(1-\Omega_s)}{(1-\Delta_s)} \ , \text{ and}$$

$$\frac{\begin{pmatrix} \textit{average period} \\ \textit{of total capital stock} \end{pmatrix}_s}{\begin{pmatrix} \textit{marginal} \\ \textit{average period} \end{pmatrix}_s} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1 \quad \Rightarrow \quad \frac{V_s}{C_s} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1 \ . \qquad (18)$$

The average period of the marginal capital investment depends on the capacity depreciation schedule for new assets. The average period of the total stock depends on the mortality distribution of the aggregate capital stock.

The relation in equation 18 between fundamental value and current replacement cost embodies elements of static and dynamic equilibria. The following excerpt exemplifies underpinnings of static equilibrium:

> "Since old capital and new capital are perfect substitutes in production, their net acquisition costs must be identical in equilibrium."
> Alan Auerbach and Laurence Kotlikoff. (1983): p. 127.

The statement wrongly declares that one unit of *K* has the same value as any other unit of *K*. While true that the supply price of new assets capitalizes the cash flow stream promised by one unit of new *K*, and also true that new and vintage units of *K* have the same *current* productive capacities, when their *future* productive capacities differ their fundamental capital values differ. For example, suppose that the productive capacity for an asset declines along a straight-line pattern over a 10-year service life. At conclusion of fifth year of service the asset's current cost equals 50 percent of the supply price for a new asset. Consequently, two such five-year old assets have current cost identical to one new asset. The two used assets, however, promise a five-year cash flow stream whereas the new asset promises a ten-year stream. The fundamental value of the two used assets is substantially less than the new asset, even though current replacement costs and current productive capacities are identical.[14] Current replacement cost is a generally valid valuation metric only when real capital possesses a value totally determined by current contribution to the static production process.[15] But future cash flows sustain capital value!

Average period embodies dynamic processes. The average periods of the marginal investment and total capital stock are equal when their *future* productive capacities follow coincident time-paths. An example of that special case occurs when capacity depreciation for new investments and for the total capital stock proceed at the geometric rate δ (that is, $d_t=h_t=\delta(1-\delta)^{t-1}$). For this special case the

[14] Use equation 15 to find that with a real interest rate, say, of 3 percent, fundamental value equals 57 percent of current cost. With the 10-year straight-line capacity depreciation schedule, d_j = 1/10 for j = 1,..,10. Discounting the capacity depreciation schedule shows Δ = 0.85 . For the asset stock with five years of service remaining, Ω = 0.92 (i.e., 1/5 discounted at 3 percent for 5 years). The ratio $(1 - \Omega)/(1 - \Delta)$ is 0.57. Average periods of the marginal and total cash flow streams, computed with the formula in the Hicks excerpt, equal 3.28 and 2.10 years, respectively.

[15] The presumption that the Q-ratio (i.e., market value to current cost) tends to unity is generally wrong. Among the few studies that recognize the natural divergence of Q from unity are George von Furstenburg (1977), Lawrence Summers (1981), Fumio Hayashi (1982), Andrew Abel (1982), Daniel Wildasin (1984), and Downs (1992).

supply price for new assets is a good benchmark for the discounted value of cash flows from existing assets. Generally, however, when the ratio of total-to-marginal average periods is less (greater) than unity then the marginal capital investment embodies a discounted cash flow stream that is more (less) valuable than the *per capita* average stream.

4. Implications for Interest

Average period equals the elasticity of capital value with respect to the discount ratio. The generalized user cost specification enables general insight on the relation between fundamental value, discount rate changes, and average period:

$$\frac{\begin{pmatrix} \textit{average period} \\ \textit{of total capital stock} \end{pmatrix}_s}{\begin{pmatrix} \textit{marginal} \\ \textit{average period} \end{pmatrix}_s} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1 \quad \Rightarrow \quad \frac{\partial V_s}{\partial r_s} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \, . \qquad (19)$$

The qualitative effect on fundamental value of a change in discount rate depends on a ratio of average periods. Fundamental value is invariant to a change in discount rate when the average periods of the marginal investment and total capital stock are equal. This special case of neutrality occurs, for example, when the capacity depreciation schedule for the marginal investment and mortality distribution of the total capital stock proceed along coincident time-paths (for example, $d_t = h_t = \delta(1-\delta)^{t-1}$). For this special case, two effects exactly offset. An increase in discount rate, on the one hand, reduces fundamental value because the present value of future cash flows diminishes. The producer, on the other hand, alters the production plan, increasing the marginal product of capital, and the value of capital services embodied within the total capital stock increases. A new temporary equilibrium occurs when the rising value marginal product reaches the increased user cost. For this special case of neutrality the larger return from capital exactly offsets heavier discounting, thereby leaving fundamental value invariant to the discount rate.

Fundamental value responds positively to a rising discount rate when average period of the marginal capital investment exceeds the average period of the total capital stock. The reason for the positive relation is this. A stream with long average period has high interest elasticity and responds more to a discount rate change than does a stream with short average period. A rising discount rate, all else equal, pushes the net present value of the marginal investment below zero. The producer responds by selecting an investment budget and production plan that pushes-up the marginal product of capital. The rising value marginal product eventually meets the increased user cost of capital, and re-establishes the zero net present value investment equilibrium (albeit the optimal capital stock likely is smaller than before the discount rate rise).

A relatively long marginal average period requires a relatively large adjustment to re-establish equilibrium between user cost and value marginal product of capital. A rising discount rate induces the marginal product upward by an amount at which gains from rising marginal returns from capital exactly offset, for the marginal investment, losses from a higher discounting effect. Temporary equilibrium requires that value marginal product equates to user cost. The relatively short average period of the total capital stock implies, however, that the large upward adjustment in marginal product increases total embodied cash flows substantially. The gains for the total capital stock from higher returns from capital more than offset the losses from the higher discounting effect, and aggregate fundamental value rises.

The response of fundamental value to a rising discount rate may be positive, zero, or negative. The qualitative change depends on the relation between average periods of the marginal investment and total capital stock. The user cost constitutes the path for re-establishing temporary equilibrium because, for the producer, the zero net present value condition for marginal investments is the most sensible next step.

5. Implications for Term Structure

The equilibrium user cost of capital equals the value marginal product of capital. The value marginal product is a function of production technology, input prices, and demand for the firm's product. None of these factors is affected by the capital structure of the firm; customers possess preferences about the price and quality of the product, not about the producer's leverage ratio [Joseph Stiglitz (1974)]. Due to the irrelevancy of financial structure, the user costs of capital for levered and unlevered producers must equilibrate.

The generalized user cost specification in equation 9 simplifies for the unlevered producer as follows:

$$c_s = \frac{q_s(\rho_s^u - \pi_s)(1 - \tau_s Z_s^u)}{(1-\tau_s)\,(1-\Delta_s^u)} \quad , \tag{20}$$

where ρ^u represents the unlevered equity financing rate. The terms Z^u and Δ^u are as defined previously except that the relevant discount rate now is ρ^u. Dynamic processes within the economy likely render ρ^u exogenous to any single producer who, by necessity, is a rate-taker as well as price-taker.[16] The generalized user cost for the levered producer in equation 9 equilibrates to the unlevered user cost in equation 20. Substitution and rearrangement of these two equations shows:

$$(\rho_s - \pi_s)(1 - \tau_s Z_s - \Lambda_s) \;=\; \frac{(\rho_s^u - \pi_s)(1 - \tau_s Z_s^u)(1 - \Delta_s)}{(1 - \Delta_s^u)} \quad . \tag{21}$$

Equation 21 is a financial market equilibrium condition that assures financing decisions for the marginal capital investment are consistent with zero net present value equilibrium in the capital goods market. The right-hand-side variables are largely exogenous and invariant to short-run production decisions. The left-hand side variable, Λ_s, depends on the producer's financing decisions for marginal capital investments. Λ_s equals zero in the absence of debt financing. As reliance on debt increases then Λ_s increases, too. Because the right-hand-side is basically constant, maintenance of equilibrium requires that ρ_s increase to offset

[16] The rate ρ^u actually attaches not to the producer but to the real capital good: "There are, therefore, theoretically just as many rates of interest expressed in terms of goods as there are kinds of goods diverging from one another in value." [Irving Fisher (1930): p. 42].

the rising Λ_s.[17] In other words, an increase in debt financing for the marginal capital investment leads to an increase in the marginal levered equity financing rate.

Impose on equation 21 the two special cases from section 2. That is, impose parameters for either (a) the one-period model, or (b) the loan repayment schedule for the marginal investment that makes perpetually constant its loan-to-fundamental value ratio. Simplification shows:

$$\rho_s = \rho_s^u + \left(\rho_s^u - i_s\right)\left(\frac{\alpha_s}{1-\alpha_s}\right) . \qquad (22)$$

Equation 22 is Modigliani and Miller's *Proposition 2* (1958) establishing that the levered equity financing rate is an increasing linear function of the debt-to-equity ratio.

Equation 22 neglects a significant amount of information in the financial market equilibrium condition (equation 21). There is no explicit association in equation 22 between financing rates and capacity depreciation or debt maturity structure. In the Modigliani-Miller/Jorgenson models, the geometric smoothing of the debt payment and real asset cash flow streams implies time-paths are coincident and average periods are equal. Debt maturity is irrelevant to equation 22 because α, once set, remains at its initial value. The two special cases

[17] Tax depreciation deductions retain influence through the variable Z. The levered equity financing rate depends, in other words, on the depreciation tax shield. Franco Modigliani and Merton Miller (1963) argue analogously that the levered equity financing rate depends on the interest tax shield. The discussion below, for simplicity, ignores effects of taxes on ρ.

implicitly assume that equilibrium financing rates are independent of term: they *presume* a flat yield curve.

Financial market equilibrium requires that the user cost for the marginal investment is the same regardless of whether the financing source is, say, a 5-year instead of a 15-year corporate debenture. Equation 21 implies a determinate relationship between interest rates for different financial cash flow streams – it also implies a term structure of interest rates. Extracting from the equilibrium condition the implied term structure of interest rates is a procedure analogous to several studies cited in Section 2 [e.g., Jorgenson and Auerbach (1980) and Hendershott (1981)] that extract from the user cost specification a variable besides equilibrium pre-tax cash flow. The remainder of this section determines the term structure of interest rates for debentures implied by the financial market equilibrium condition.

The debenture has a loan payment stream in which the borrower receives a lump sum from the lender, constant interest payments are made periodically throughout the life of the loan, and principal is repaid *in toto* with the last payment. This particular debt contract describes most bonds traded in the U.S. corporate and Treasury credit markets.

Consider equation 21 for a marginal investment financed by a debenture with a face value of $\alpha_s q_s$, a term of T, and a coupon rate of i_s (annual coupon, no sinking fund). Equity of $(1-\alpha_s)q_s$ finances the remainder of the purchase price. The periodic interest expense for the debenture equals $i_s \alpha_s q_s$ and there is no repayment of principal until time $s+T$ when the principal is repaid *in toto*. The first payment, $B_{s,1}$, equals $i_s \alpha_s q_s$ and γ_s equals $i_s \alpha_s$. The subsequent payments $B_{s,2}$ through $B_{s,T-1}$ are the same size as the first, so $b_j = 0$ for $j = 0,...,T-2$. During period T the payment includes the coupon as well as the repayment of principal and $b_{T-1} = -i_s^{-1}$. After period T the payment drops to zero, so $b_T = i_s^{-1}(1+i_s)$. Substitution into equation 21 shows the financial market equilibrium condition for the debenture (τ equals zero for this analysis):

$$(\rho_s - \pi_s)\left\{1 - \frac{\alpha_s}{\rho_s}(\rho_s - i_s)\left(1 - \frac{1}{(1+\rho_s)^T}\right)\right\} = \frac{(\rho_s^u - \pi_s)(1-\Delta_s)}{(1-\Delta_s^u)} \quad . \qquad (23)$$

Comparative statics for equation 23 show the effect on financing rates of changes in term *(T)*. In lieu of algebraic formulations, however, numerical findings are reported given these parameter settings: the expected inflation rate (π) is set to 5 percent, the unlevered equity cost of capital (ρ^u) is 12 percent, the initial loan-to-value ratio (α_s) is 30 percent; and productive capacity is set to decline along a 15-year double-declining-balance schedule.[18] Furthermore, an equality constraint is imposed on the debt and equity risk premia. That is, i and ρ satisfy equation 23 as well as:

$$\rho = \rho^u + P , \qquad (24a)$$

and

$$i = i^u + P , \qquad (24b)$$

[18] Charles Hulten and Frank Wykoff (1981) provide evidence this specification characterizes capacity depreciation for corporate real assets. For this setting: $d_j = (2/15)(1- 2/15)^{(j-1)}$ for $j = 1,...,14$; $d_{15} = 1-(d_1+d_2+...d_{14})$ and $d_j = 0$ otherwise.

where i^u represents the risk-free interest rate and is set to 8 percent, and *P* denotes the endogenous leverage risk premium. The equality constraint on the risk premia stipulates that i and ρ increase by the same amount.[19]

Substitute the preceding parameter settings for π, i^u, ρ^u, α, and the d_j's into equation 23. Set *T*, the debenture term, to an integer and iteratively solve equation 23, subject to constraints 24a and 24b, to find the i and ρ consistent with the temporary equilibrium. When *T* is set at one year i is found to equal 8.24 percent.[20] For *T* of 5, 10, 15, 20, and 25 years, respectively, i is 8.68, 9.11, 9.43, 9.66, and 9.82 percent, respectively.

The preceding implied interest rates are from a setting in which the expected inflation rate, the risk-free interest rate, and unlevered equity cost of capital are perpetually constant at 5, 8, and 12 percent, respectively. Consequently, the yield curves are flat for i^u and ρ^u as well as for the real rates i^u-π and ρ^u-π. Nonetheless, the yield curve for nominal interest that is consistent with market equilibrium is not flat. The implied yield curve rises steeply at first and then flattens out – it has a normal shape.

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[19] The constraint assumes that a change in debt maturity causes equal increases in the incremental debt and equity risk premia. The resultant “implied term structure of interest rates” is coincident with the “implied term structure of levered equity financing rates for different debt policies.” One could easily change the constraint away from one-to-one risk-sharing. The resultant implied term structures of debt and equity financing rates no longer would be coincident. They would, however, have an upward slope. The constraint in equations 24a and 24b, in other words, is illustrative, not essential.

[20] Due to the equality constraint on the risk premia, ρ is 12.24 percent (ρ^u is exogenously fixed at 12 percent). Both ρ and i rise 24 basis points above ρ^u and i^u, respectively.

6. Conclusion

Two capital concepts debut in *Positive Theory of Capital* by Eugen von Böhm-Bawerk in 1888: the *average period* equals the elasticity of capitalized value with respect to the discount ratio; the *user cost of capital* equates to the value marginal product of capital. Average period embodies elements of intertemporal dynamics. User cost reflects static equilibrium conditions. The current study derives a specification for the user cost of capital that generalizes intertemporal dynamics of capacity depreciation and financial structure. Analysis of the specification yields three important insights interpretable with average period.

First, fundamental value equals current replacement cost times a ratio dependent on average periods. Current replacement cost overstates fundamental value when the average period of the marginal capital investment exceeds the average period of the total capital stock. Current replacement cost assigns each unit of productive capital identical value even though their after-tax expected cash flow streams may differ. Fundamental value discounts a stream of user costs to find capitalized value. Perhaps the ratio of financial market value to fundamental value should vibrate around unity, but certainly the ratio of market value to current replacement cost should not.

Second, the equilibrium effect of an increasing discount rate on the fundamental value of a capital stock may be positive, zero, or negative. The qualitative effect depends on a ratio of average periods. When the average periods of the marginal capital investment and total capital stock are equal, fundamental value is invariant to the discount rate: an increased discount rate raises the user cost of capital; the producer responds with a production plan that raises the value marginal product of capital; zero net present value of marginal investments is maintained although the optimal quantity of capital declines; and for the aggregate stock the gain from the higher return on capital exactly offsets the loss from the higher discounting effect. Fundamental value relates negatively (positively) to the discount rate when the average period of the marginal capital investment is less than (greater than) the average period of the total capital stock.

Third, the term structure of interest rates embedded within the user cost reveals an upward sloped yield curve. Capital is a store of wealth. The wealth inside the store equals the present value of expected future cash flows to capitalists. The user cost specifies all that is known or expected about the future. The store of wealth depletes as times elapses and cash flows return to financing sources. When the average period of the marginal debt financing stream lengthens, the burden on the store diminishes and capital supports a higher rate of interest. Long-term interest rates naturally are higher than short-term rates, irrespective of all else.

"Hypotheses are invented and die every day."[21] The hypothesis that capital value depends on the relation between average period and user cost, however, has been around for quite a long time. Yet only now are the two being wed

[21] Joan Robinson (1977): p. 1323.

within a modern analytical relationship. Time and research will tell whether this marriage offers anything of lasting interest to economic literature:

> "Except for Marx's *Kapital,* no other theoretical treatise since the classical age of economics has been intellectually so stimulating [as *Positive Theory of Capital*]. It will take a great deal of time and reflection before anything like unanimity can be attained in the general verdict as to its value."
>
> Friedrich Wieser. Preface to the posthumous publication of Böhm-Bawerk's 4th edition of *Positive Theory of Capital.* (1921): p. ii.

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Technical Supplement

A1. Generalized User Cost Specification

Moving from equation 8 to 9 requires evaluation of the following summation:

$$X = \sum_{t=1}^{\infty} (1+a)^{-t} \sum_{j=1}^{t} m_{j-1} \quad . \tag{A1}$$

For the real asset and debt cash flow streams in the first and third lines of equation 8, respectively, a represents $\rho - \pi$ and ρ, respectively, while m_j represents d_j and b_j, respectively. The objective is a simpler expression for X.

Expand the summation.

$$\begin{aligned} X = {} & (1+a)^{-1} m_0 \\ & + (1+a)^{-2} m_0 + (1+a)^{-2} m_1 \\ & + (1+a)^{-3} m_0 + (1+a)^{-3} m_1 + (1+a)^{-3} m_2 \\ & + (1+a)^{-4} m_0 + (1+a)^{-4} m_1 + (1+a)^{-4} m_2 + (1+a)^{-4} m_3 \\ & + \ldots \end{aligned}$$

Introduce a new term, M, that equals the discounted sum of the series.

$$M = \sum_{t=0}^{\infty} (1+a)^{-t} m_t$$

Now factor $(1+a)^{-1}$ from each term in the expanded summation. Then re-group terms, putting together the first term from line one with the second from line two with the third from line three, etc. That group equals M. Also put together the first term from line two with the second from line three with the third from line four, etc. That group equals M. Also put together the first from line three with the second from line four, etc. That group equals M. Etc. Obtain:

$$
\begin{aligned}
X = {} & (1+a)^{-1} M \\
& + (1+a)^{-2} M \\
& + (1+a)^{-3} M \\
& + (1+a)^{-4} M \\
& + \ldots
\end{aligned}
$$

or

$$X = \frac{M}{a} \qquad \text{(A2)}$$

Substitute into equation 8 the expression in equation A2 for the real asset and debt cash flow streams, define *Z* as the discounted sum of tax depreciation deductions, and solve for *c*, the user cost of capital, as in equation 9.

A2. Elaboration on Average Period

Average period equals, as Hicks writes in an excerpt in the introductory section, the elasticity of capitalized value with respect to the discount ratio. Gain intuition about average period with examples for financial bonds. Average period in the bond literature is known as *bond duration*. The average period for a zero-coupon bond equals the number of periods to maturity. Average period increases with term and, as is well known, the effect of an interest rate change is greater on long–term than short–term bond values. For a given maturity, average period diminishes as coupon rate increases. A large coupon rate shifts discounted cash flows from the remote future toward the near-term, and average period decreases.

Consider computation of average period for the special case of all-equity financing, no taxes, and geometric capacity depreciation at rate δ. Combine the

Hicks definition from his first excerpt with the cash flow specification in equation 3 to compute average period for the marginal capital investment (for simplicity, adopt continuous time mathematical exposition and suppress time *subscripts* on q, r, and π):

$$\begin{pmatrix} \textit{marginal} \\ \textit{average period} \end{pmatrix}_s = \int_0^{\infty} t \; \frac{e^{-(r-\pi)t} c_{s,t}}{q} \, dt \, .$$

The preceding equation computes average period as the product of two terms. The first, t, represents the number of periods after investment when the respective cash flow is received; the second term is a ratio, or weight, representing that period's discounted cash flow as a percentage of total discounted value (the supply price, q, equals total discounted value). This computation is analogous to the standard definition of Macaulay duration (1938) from the bond literature. Instead of a coupon, however, the cash flow at time t equals $c_{s,t}$. The user cost of capital for this special case equals $q(r+\delta-\pi)$, and real asset cash flow declines at rate δ. Substitution and simplification shows:

$$\begin{pmatrix} \textit{marginal} \\ \textit{average period} \end{pmatrix}_s = \int_0^{\infty} t \; \frac{e^{-(r+\delta-\pi)t} q\left(r+\delta-\pi\right)}{q} \, dt$$

$$= \frac{1}{r+\delta-\pi} \, .$$

The preceding equation shows that the average period of a geometric series equals the reciprocal of the geometric rate. For a consol bond at par with yield-to-maturity r, the average period is $1/r$. For a properly priced common stock with dividend growth at constant rate g and equity financing rate r, the average period is $1/(r\text{-}g)$. For a zero net present value capital investment characterized by geometric capacity depreciation at rate δ, expected inflation at rate π, and financing rate r, the average period equals $1/(r+\delta-\pi)$. The units of measurement, or dimension, of these geometric rates is "per period." The average period, therefore, is an elasticity that is not unit-free. Rather, the average period metric is a measure of time.