DEBT MATURITY AND THE COST OF CAPITAL

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March 1993

Presented at the Southwestern Finance Association Annual Meetings, March 5, 1993.

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ABSTRACT: A new expression is derived in which the cost of capital is an explicit function of debt maturity structure and the underlying real asset cash flow stream. In special cases the cost of capital expression reduces to the standard weighted average cost of capital, but generally the standard formulation is invalid. Reliance on a correctly specified cost of capital expression has implications for capital budgeting and capital market studies.

In deciding whether to invest in a capital asset, the potential investor forms an expectation about the cash flows embodied in the asset, computes the present value of the cash flow stream with an "appropriate" discount rate, and considers the investment a feasible purchase if the discounted value exceeds the asset's current acquisition price. The fundamental independence between the investor's purchasing decision and the financing method is proven by Modigliani and Miller (1958) and Stiglitz (1974). Nonetheless, the relationship between the financing method and the appropriate discount rate is less clear.

The current study provides an enhanced specification of the relationship between the equilibrium discount rate, debt maturity structure, and the underlying asset cash flow stream. The analysis begins in Section I by specifying the marginal zero net present value investment in real assets for the *un*levered corporate investor. The specification stems from a production-based model employing the *user cost of capital* as a link between the asset cash flow stream (the source of funds) and the financing stream (the use of funds).[1] The user cost is the pretax cash flow generated by an asset in the zero net present value equilibrium state. The user cost expression, of course, depends on the traditional *financial cost of capital* as well as on expected changes in the value of the underlying real asset (and also tax policy parameters).[2] The discount rate in the

[1] Other financial research also relies on the user cost to link financial and real decisions. Hite (1977) and Dotan and Ravid (1985), for example, employ the user cost to show the association between debt tax shields and capital intensity.

[2] The literature about the cost of capital is large. Timely and innovative papers, however, include Booth (1991), Taggart (1991), Harris, O'Brien, and Wakeman (1989), Auerbach (1987), Yagil (1982), Miles and Ezzell (1980), Haley and Schall (1978), Taggart (1977), Myers (1974), and Stiglitz (1973).

unlevered user cost formula is isolated and constitutes the equilibrium specification for the financial cost of capital.

The zero net present value specification is expanded to reflect the maturity structure of the debt contract for a levered investor and the levered user cost is isolated. At equilibrium, the user cost equals the optimal marginal physical product of capital and is invariant to the financing method. Hence, equating the levered and unlevered user costs provides a linkage between the financial costs of capital for the levered and unlevered investors. The resultant cost of capital expression incorporates information about debt maturity structure.[3]

The remainder of the study gleans insight from the cost of capital expression. Section II restricts the general specification in order to obtain the cost of capital for a one-period model as well as for an environment characterized by a constant leverage ratio and smooth cash flow streams. For these settings, it is shown that the *weighted average cost of capital* (*WACC*) is the appropriate rate for discounting the asset cash flow stream. In Section III environments are examined in which the firm finances marginal capital investments by issuing either a corporate debenture or a fixed payment amortized loan. Typically, the cost of capital is not a simple weighted average of the debt and equity financing rates, rather it depends upon the maturity structure of the loan payment schedule.[4] This result is true even for environments in which the firm's overall debt-to-assets ratio is constant. A brief summary closes the study.

The findings presented herein are relevant for several reasons. First, they are a reminder that the *WACC* is not the preeminent, irrefutable, and absolute tool for capital budgeting analyses. It is entirely rational for investors concerned about marginal financing to incorporate into their analysis the terms and conditions relevant to the marginal financing package; it usually

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[3] The current study assumes that with equilibrated markets the market financing rates have adjusted to the point that the firm is indifferent to debt maturity structure. Several studies, including Morris (1976), Smith and Stulz (1985), Barnea, Haugen, and Senbet (1980), and Brick and Ravid (1985), examine conditions under which the firm may *prefer* financing by one maturity structure rather than another.

[4] Miles and Ezzell preview this finding, and also the challenge tackled by the current study, when they write (1980, p. 729): "Indeed, it would seem that the prospects of finding any normatively useful discount rate specified in terms of an average of equity and debt rates are remote if the firm's debt transaction schedule is exogenous with respect to realized market values."

is wrong to rely on the *WACC*. Second, an explicit specification links the cost of capital to debt maturity structure. This may prove important for studies focused on measuring the cost of capital for firms or industries characterized by differing debt maturity structures. Third, the model enhances our general understanding about the association between the financial cost of capital and underlying real assets.

I. The Model

The model described below assumes that there are no taxes in order to highlight the basic nature of the relationship between the financial cost of capital, debt maturity structure, and the underlying real asset. Also it is assumed that there is an elastic supply of real and financial capital, and that all investors take as given the price of real assets in the capital goods market. The asset's price is independent of the financing method that potential investors employ for purchasing the asset. Also, the asset generates the same operating or *asset cash flow stream* for an unlevered investor as for the investor that partially relies on debt financing. The risk associated with the asset cash flow stream determines ρ^u, the *unlevered equity cost of capital*. ρ^u is the internal rate of return for a zero net present value capital investment by an unlevered, all-equity corporate investor. At equilibrium, ρ^u represents the total return required by the financial markets and is equivalent to the traditional notion of the financial cost of capital.

The model also ignores agency costs and asymmetric information effects. Instead, real investment decisions are conditional on the following market equilibria: (1) the real asset acquisition price equals the discounted cash flows expected by an unlevered corporate investor; and (2) the real asset acquisition price equals the levered corporate investor's discounted cash flows plus the loan. With these equilibria satisfied, the values of the corporate financial claims identically sum to the present value of the expected real asset cash flow stream discounted by the unlevered equity cost of capital which, in turn, is identically equal to the current prices of the corporation's real assets in the capital goods market.[5]

Ia. Specification of asset cash flows

The specification for the real asset cash flow stream is independent and invariant to the method used for purchasing the asset. Let $c_{s,t}$ denote the expected real asset cash flow accruing at time $s+t$ from the real asset acquired at time s. Also, assume that the market price of new capital, q_s is expected to inflate at the rate π. The zero net present value equilibrium in the market for real capital goods implies

$$q_s = \Sigma_{t=1}^{\infty}\left(1+\rho^u-\pi\right)^{-t} C_{s,t} \tag{1}$$

Equation 1 shows that discounting the equilibrium real asset cash flow stream with the real unlevered equity cost of capital yields a discounted sum equal to the asset's market price.

When the product-output price is constant relative to the market price of new capital and when technology is characterized by a constant returns to scale technology, the real asset cash flow stream is shaped by changes in the asset's productive capacity. Let d_j denote the proportional decline in real asset cash flow that occurs after the j'th asset cash flow is received. The series d_j for j=0,...,∞ is the asset's capacity depreciation schedule and defines the expected real asset cash flow stream. The expected real asset cash flow accruing at time $s+t$ from the investment made at time s is

$$c_{s,t} = C_s[1-\Sigma_{j=1}^{t} d_{j-1}] \tag{2}$$

The term c_s is the *user cost of capital* and represents the operating or asset cash flow produced at time s by one unit of real capital.[6]

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[5] Downs and Shriver (1993) provide theoretical and empirical evidence that the current value of the asset in the capital goods market deviates from the asset's *current cost* (as traditionally defined by economists and the Financial Accounting Standards Board) due to tax and discounting effects. Downs (1992) provides evidence that the market value of the firm relative to the current cost of its assets (i.e., the *Q* ratio) is likewise biased beneath unity.

The specification for the unlevered user cost is obtained by substituting the asset cash flow stream (equation 2) into the equilibrium condition (equation 1), simplifying, and rearrranging. That procedure shows

$$c_s = q_s\left(\rho^u - \pi\right)\left(1 - D_s^u\right)^{-1}, \tag{3}$$

where D^u is obtained by discounting the capacity depreciation schedule as in

$$D_s^u = \sum_{t=1}^{\infty}\left(1 + \rho^u - \pi\right)^{-t} d_t. \tag{4}$$

Equation 3 specifies the user cost consistent with equilibrium in the capital goods market. As shown by Jorgenson (1963), the equilibrium user cost equals the value marginal product of capital and it varies inversely with the quantity of capital employed in the production process. There is only one equilibrium value for c when all rational entrepreneurs face similar cost curves and possess homogeneous expectations; i.e., when q, ρ^u, π, and D^u are the same for everyone, then c is the same for everyone too, regardless of the financing mix relied upon to purchase the marginal investment.

Ib. Integration of debt financing into the user cost framework

The expected asset cash flow stream is the same for all homogenous investors purchasing the asset, regardless of their intended method of payment, but the reliance on debt financing nonetheless alters the cash flows that accrue to the investor. The unlevered investor expects to receive the asset cash flow stream; the levered investor receives the asset cash flow net of the debt payment, referred to as the *residual cash flow stream*. Whereas ρ^u is the cost of capital appropriate for discounting the asset cash flow stream, let ρ denote the rate appropriate for discounting the residual cash flow stream. ρ is the *levered equity financing rate*. ρ and ρ^u are linked by the equilibrium condition about the irrelevance of capital structure: since the financing

[6] Jorgenson (1963) provides the seminal discourse about the association between the user cost of capital, value marginal product, and optimal capital stock.

method does not matter, then the present value of residual cash flows discounted by ρ, plus the loan, equals the present value of asset cash flows discounted by ρ^u.

Suppose that for purchasing an asset with price q_s, the investor supplies equity of $(1-\alpha_s)q_s$ and takes out a loan for $\alpha_s q_s$. At the time of investment, a loan payment schedule is established such that the lender will be repaid principal, interest, and perhaps other fees. A portion of each period's asset cash flow is used to make the debt payment, and the residual accrues to equity. The zero net present value equilibrium condition equates the funds provided by equity to the present value of the residual cash flow stream discounted by ρ:

$$\left(1-\alpha_s\right)q_s = \sum_{t=1}^{\infty}\left(\left(1+\rho-\pi\right)^{-t} c_{s,t} - \left(1+\rho\right)^{-t} B_{s,t}\right). \qquad (5)$$

α_s is the *initial* leverage ratio for the time s marginal capital investment. The evolution beyond time s of the loan-to-value ratio endogenously depends on the interaction between the asset cash flow and loan payment streams and is not restricted by the model. The absence of arbitrage opportunities is assurance that the sum of funds provided by debt, $\alpha_s q_s$, and equity, $(1-\alpha_s)q_s$, is equal to the asset's market price.

The discounted residual cash flow is contained in the curly brackets of equation 5 and is comprised of two terms. The first term, $(1+\rho-\pi)^{-t}c_{s,t}$, equals the discounted asset cash flow expected at time $s+t$ from the investment made at time s. The asset cash flow is *identical* to the one in equations 1 and 2. The second term, $(1+\rho)^{-t}B_{s,t}$, equals the discounted loan payment made at time $s+t$ for the loan financing the time s capital investment. $B_{s,t}$ is the net loan payment and it may be comprised of interest expense, principal repayment or issuance, and any other debt related fees.

The debt maturity structure is reflected in the loan payment schedule established at the time of investment. Let γ denote the net loan payment (interest, principal, and fees) that is scheduled to be paid at the end of the asset's first period of use, where γ is expressed as a proportion of the asset's market price; i.e.,

$$\gamma = B_{s,1} / q_s . \qquad (6)$$

The loan payment schedule is summarized by the series b_j for j=0,...,∞ , where b_j denotes the change in payment (as a proportion of $B_{s,1}$) that occurs after the j'th payment is made (b_0= 0). More precisely,

$$b_j \;=\; (B_{s,j} - B_{s,j+1}) \,/\, B_{s,1}\,. \tag{7}$$

Equations 6 and 7 specify the entire loan payment schedule, regardless of whether the debt contract represents a consol, a fixed payment amortized loan, a debenture with a ballon payment, or any other of many possible maturity structures.[7] The net loan payment occurring at time $s+t$ that is attributable to the loan issued to finance the time s capital investment is given by

$$B_{s,t} = \gamma\; q_s[1 - \textstyle\sum_{j=1}^{t} b_{j-1}]. \tag{8}$$

The zero net present value equilibrium with explicit specification of the asset and debt service cash flow streams is obtained by substituting equations 2 and 8 into 5. That substitution yields

$$(1-\alpha_s)q_s = \{\textstyle\sum_{t=1}^{\infty}(1+\rho-\pi)^{-t} c_s[1-\sum_{j=1}^{t} d_{j-1}]\} - \{\sum_{t=1}^{\infty}(1+\rho)^{-t}\gamma\; q_s[1-\sum_{j=1}^{t} b_{j-1}]\}. \tag{9}$$

Equation 9 shows that at equilibrium the equity provided by the investor equals the present value of the asset cash flow stream minus the present value of the net loan payment stream, each discounted by the equity financing rate.[8] The user cost is extracted from the equilibrium condition by simplifying and rearranging:

$$c_s = q_s(\rho-\pi)\left(1-\alpha_s+\gamma(1-\beta_s)\rho^{-1}\right)(1-D_s)^{-1}. \tag{10}$$

D is specified in equation 4 above; D and D^u are computationally equivalent except that the discount rates are ρ and ρ^u, respectively. β is obtained by discounting the debt maturity structure with the equity financing rate,

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[7] If debt financing is used and the first payment is zero, then γ must be redefined; for example, with zero coupon debt of term T, $\gamma = B_{s,T}/q_s$.

[8] Myers (1974) discusses a capital budgeting framework similar to the one above, in that the discounted value of asset cash flows is computed and subsequently the discounted value of financing costs is subtracted.

$$\beta_s = \sum_{t=1}^{\infty} (1+\rho)^{-t} b^t . \tag{11}$$

Equation 10 specifies the user cost for the levered investor. The levered user cost represents the operating cash flow generated by the real asset such that the present value of the residual cash flow stream discounted by ρ, plus the loan ($\alpha_s q_s$), equals the acquisition price of the real asset. If the expected operating cash flow is different than c_s as specified in equation 10, then the levered investment is inconsistent with market equilibrium because it possesses a nonzero net present value.

Ic. The equilibrium cost of capital

The equilibrium user cost of capital equals the value marginal product of capital. The value marginal product is a function of the production technology and the demand for the firm's product. Neither of these factors is affected by the capital structure of the firm; customers possess preferences about the price and quality of the product, not about the producer's leverage ratio. Due to the irrelevancy of capital structure, therefore, the user cost for the levered corporation specified in equation 10 must equal the user cost for the unlevered corporation specified in equation 3. The resulting generalized formula for the unlevered equity cost of capital is obtained by equating these user cost expressions and rearranging:

$$\rho^u = (\rho - \pi)\left(1 - \alpha_s + \gamma(1-\beta_s)\rho^{-1}\right)\left(1 - D_s^u\right)\left(1 - D_s\right)^{-1} + \pi . \tag{12}$$

Equation 12 is the primary analytical result of this study. It explicity specifies the equilibrium preserving relationship between the unlevered equity cost of capital (ρ^u) and the levered equity financing rate (ρ), the rate of expected inflation (π), the marginal leverage ratio (α), the debt maturity structure variables (γ and β), and the expected real asset cash flow stream (D_s^u and D_s).

The equilibrium cost of capital is the sum of the two right-hand-side terms in equation 12. These two terms comprise the equilibrium total return on capital. One part of the total return is due to expected inflation at the rate π. A capital gain accrues from an increase in the real asset's value. This is analogous to the dividend growth model wherein the stock price is posited to increase at the growth rate. The second part of the total return is cash flow. The equilibrium cash flow yield for the real asset is the first right-hand-side term in equation 12 and it adjusts to the real rate and the capacity depreciation schedule such that, together, the cash flow yield plus the capital gain yield sum to the return on capital required by the financial markets.

II. Relating the Equilibrium Cost of Capital to Familiar Frames of Reference

The general model given above for the equilibrium cost of capital accomodates asset cash flow streams and loan payment schedules of virtually any length (finite or infinite) or shape (rising, falling, or irregular). The following subsections show that several familiar results are special cases of the general model. First, the general model specified in equation 12 is restricted to a one-period horizon. Subsequently, the general model is restricted to an infinite horizon characterized by geometrically rising or falling cash flow streams and a constant leverage ratio.

IIa. The cost of capital in a one-period model

By restricting the model to a one-period horizon the familiar Modigliani-Miller (1958) findings are obtained. For the one-period model, the debt maturity structure variables are predetermined as follows. With a leverage ratio of α on an investment with price q, the loan at time 0 is αq. With a debt financing rate equal to i, the net loan payment at time 1 (i.e., $B_{0,1}$) is $\alpha q(1+i)$. Notice the payment includes interest plus repayment of principal *in toto*. γ, according to equation 6, equals $\alpha(1+i)$. After the first period the decline in the loan payment is 100% and so, according to equation 7, b_j=1 for j=1 and b_j=0 otherwise. β, from equation 11, is $(1+\rho)^{-1}$.

The one-period horizon also predetermines the shape of the real asset cash flow stream. In particular, the decline in productive capacity at the end of the first period is 100% and so d_j=1 for j=1 and d_j=0 otherwise. According to equation 4, therefore, $D^u=(1+\rho^u-\pi)^{-1}$ and $D=(1+\rho-\pi)^{-1}$.

Substituting into equation 12 the settings of γ, β, D^u, and D, for the "one-period model" shows[9]

$$\rho^u = (\rho-\pi)\left(1-\alpha_s+\alpha(1+i)\left(1-(1+\rho)^{-1}\right)\rho^{-1}\right)\left(1-\left(1+\rho^u-\pi\right)^{-1}\right)\left(1-\left(1+\rho-\pi\right)^{-1}\right)^{-1}+\pi,$$

$$\begin{aligned} &= (1-\alpha_s)\rho+\alpha_s i, \\ &= WACC. \end{aligned} \tag{13}$$

For the one-period model, the unlevered equity cost of capital equals the *WACC*.

Equation 13 implies that the same qualitative inference about net present value is obtained irrespective of whether the asset cash flow stream is discounted with ρ^u or *WACC*. Hence, it suggests that the *WACC* is the appropriate rate for discounting asset cash flow streams. This is potentially important when the investor possesses information about the equity financing rate (ρ), the debt financing rate (i), and the marginal leverage ratio (α_S).

IIb. The cost of capital with a constant leverage ratio and smooth cash flows

This section restricts the general model for the cost of capital given in equation 12 to an environment where the expected cash flow stream is geometrically rising or falling and the leverage ratio is perpetually constant; i.e., the loan-to-value ratio throughout the asset's service life always equals α_S. The real asset cash flow stream for this situation is modeled to reflect two potentially offsetting effects. First, upward pressure on asset cash flow is exerted from expected inflation at the rate π. Second, downward pressure on asset cash flow is exerted from capacity depreciation at the rate δ (which implies $d_j=\delta(1-\delta)^{j-1}$ for every j). When π equals δ, for example

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[9] Simplification of equation 13 actually results in $WACC+\pi(\rho-WACC)(1+\rho)^{-1}$. This extra cross-product term is trivial (less than 10 basis points for plausible settings) and it can be eliminated by modeling with the conceptually equivalent approach of discounting the real loan payment with the real equity financing rate; i.e., for equation 13 use $\alpha(1+i-\pi)(1-(1+\rho-\pi^{-1})(\rho-\pi)^{-1}$ in lieu of $\alpha(1+i)(1-(1+\rho^{-1})\rho^{-1}$. The modeling in the text is used because it is easier to visualize the loan payment stream in nominal terms.

when both equal zero, the situation characterizes a cash flow stream that is a level and constant perpetuity. Alternatively, when δ is zero but π is positive, the situation characterizes a dividend-growth model where π denotes the dividend growth rate. Finally, when both δ and π are positive, the situation characterizes the standard neoclassical environment in which nominal cash flows grow with inflation but real production declines due to capacity depreciation. Irrespective of their numerical settings, however, with smooth geometric rates π and δ, equation 4 simplifies to show $D_s = \delta(\rho + \delta - \pi)^{-1}$ and $D_s^u = \delta(\rho^u + \delta - \pi)^{-1}$.

Fixed rate debt with a coupon rate of i and principal of $\alpha_s q_s$ is issued to purchase the asset. The remainder of the purchase price, $(1\text{-}\alpha_s)q_s$, is supplied by equity. It is assumed that i is fixed irrespective of α_s. This assumption simplifies the analysis. It also implies that an increase in the leverage ratio does not lead to a higher debt financing rate (the equilibrium equity financing rate rises, however).[10]

For the environment characterized above, the real asset cash flow and the asset's value are declining (or rising) at the rate δ-π. Each loan payment includes interest plus sufficient principal such that the asset's loan-to-value ratio remains constant. Notice that for the dividend-growth framework (δ=0 and π>0), the asset's value is increasing at the rate π and new principal is issued every period. The topping-off of debt occurs because the firm can continually issue debt so as to keep constant the asset's loan-to-value ratio. With depreciating assets, on the other hand, the real asset cash flow gets progressively smaller and the net loan payment (and outstanding principal) diminishes.

The first loan payment, $B_{s,1}$, is comprised of interest and principal equal to $i\alpha_s q_s$ and $(\delta\text{-}\pi)$ $\alpha_s q_s$, respectively. Thus, γ from equation 6 equals $\alpha_s(i+\delta-\pi)$. The net loan payment changes in size over time according to $b_j=(\delta-\pi)(1-\delta+\pi)^{j-1}$ for every j. Solving equation 11 shows that β equals $(\delta-\pi)(\rho+\delta-\pi)^{-1}$.

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[10] In a more general setting, i would be endogenously determined with a lower bound of the riskless rate (when α =0).

The unlevered equity cost of capital is computed from equation 12 by substituting in the above settings for γ, β, D^u, and D. The substitution and simplification shows

$$\rho^u = (\rho-\pi)\left(1-\alpha_s+\frac{\alpha_s(i+\delta-\pi)(1-(\delta-\pi)(\rho+\delta-\pi)^{-1})}{\rho}\right)\left(\frac{1-\delta(\rho^u+\delta-\pi)^{-1}}{1-\delta(\rho+\delta-\pi)^{-1}}\right)+\pi.$$

$$\begin{aligned} &= (1-\alpha_s)\rho+\alpha_s i, \\ &= WACC \end{aligned} \qquad (14)$$

In this infinite horizon model characterized by geometric inflation and depreciation rates, the unlevered equity cost of capital equals the *WACC*.

This result merits three comments. First, it is the justification for claims such as "the *WACC* is, in fact, the discount rate that the firm uses to discount the expected cash flows of the firm" [Ross and Westerfield, 1988, p. 219]. Discounting asset cash flows with the *WACC* is appropriate to the extent that (1) asset cash flows are expected to change along geometric patterns, and (2) the firm's debt policy is focused on keeping its leverage ratio perpetually constant. Second, the qualitative result in equation 14 has been found by others, albeit in a different model. Miles and Ezzell (1980), for example, write [pp. 728-729]:

> That the textbook *WACC* yields correct valuations for either a single-period project or a project with level, perpetual cash flows is a consequence, not of project life *per se*, as has been argued in the literature, but rather of maintaining indirectly a constant leverage ratio.

Notice, as an aside, that equation 14 shows the result is not restricted to the level, perpetual cash flow stream. It applies to any situation where the cash flow changes, either positively or negatively, along a smooth geometric pattern. Third, the finding has direct implications on the measurement of the neoclassical user cost of capital. A simplification of equation 10 for this environment shows the real user cost equals $WACC+\delta-\pi$.[11] The relevant cost of funds for

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[11] Rearrangement of equation 10 with δ=0 shows $q=c(WACC-\pi)^{-1}$. This is the dividend growth model equating the price of the asset (q) to the cash flow (c) divided by the cost of funds (*WACC*) less the growth rate (π).

inclusion in the neoclassical user cost of corporate capital is the *WACC*, not exclusively the interest rate.

III. The Cost of Capital with Alternative Debt Maturity Structures

When the corporation issues debt for the sole purpose of capital investment, then the equilibrium condition in the capital goods market plays a role in determining the cost of capital. Under this scenario, the corporation has a marginalist perspective; it uses debt policy for purchasing new assets rather than for regulating the overall debt-to-assets ratio. Through time, the firm passes from one marginal condition to another, and always the cost of capital is determined from the most recent activity in the real and financial markets.

The association between the unlevered equity cost of capital (ρ^u), the levered equity financing rate (ρ), and debt maturity structure (γ and β), is generalized within equation 12. That equation simplifies under special circumstances, such as the ones described in the previous section, to show that the *WACC* is the appropriate rate for discounting the asset cash flow stream. Generally, however, it is incorrect to rely on the *WACC* as the appropriate discount rate for evaluating prospective investments. Section IIIa presents an illustration summarizing this argument. Section IIIb derives the correct cost of capital expressions for two popular debt contracts: the corporate debenture and the fixed payment amortized loan. Section IIIc discusses some of the implications for these findings.

IIIa. An illustration of the argument

Consider the case in which a corporation makes a zero net present value investment in one new asset each year. Furthermore, say that the productive capacity of the asset declines 10 percent after the first year of service, 20 percent after the second year, and after the third year the asset expires. Accordingly, the capacity depreciation schedule is given by d_1=.1, d_2=.2, and d_3=.7 (d_j=0 for every other j).[12] Suppose also that the nominal asset cash flow stream reflects a 5

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[12] This capacity depreciation schedule approximates the "beta-decay schedule" that the Bureau of Labor Statistics (1979) used for constructing stock estimates for three-year corporate equipment.

percent rate of expected inflation (π). Finally, suppose that due to the riskiness of the venture, ρ^u equals twelve percent; the required real return for the capital asset is therefore seven percent.

Table 1 describes this scenario. Panel A focuses on time s as the beginning reference point. The price of the asset is \$100 in the capital goods market. During the first year of service the real asset cash flow equals the user cost of capital which, according to equation 3, is \$43.63 (D^u is obtained by discounting the capacity depreciation schedule with the real rate of 7 percent; D^u=0.8396). The real asset cash flow during year 2 is ten percent less, or \$39.27; in year 3 it is \$30.54; after the third year the asset expires and delivers no further capital services. The present value of the real asset cash flow stream is obtained by discounting with the seven percent real rate. Column 2 shows that the present value of the three-year cash flow stream is \$100, thereby satisfying the zero net present value investment equilibrium in the real capital goods market.

Column three shows the evolution of the asset's real value as it ages. At every point in time the asset's value equals the discounted sum of remaining cash flows. The percentage change in real value (column 5) is the real capital gains yield which, together with the real cash flow yield (column 4), sum to the seven percent total real required return on capital (column 6). Columns 1-6 reflect the equilibrium asset cash flows, irrespective of the financing method used for purchasing the asset.

The corporation may decide that instead of financing its marginal investments exclusively by equity, it intends to always finance 30 percent of the asset's purchase price by issuing a 3-year debenture carrying an 8 percent coupon rate. Thus, at time s a \$30 bond is issued to finance the \$100 zero net present value investment. Due to the assumed existence of market equilibria and irrelevancy of the financing method, the present value of the residual cash flows discounted with the levered equity financing rate, plus the \$30 loan, equals \$100.

The marginal leverage ratio (α_s) is 0.30 . Column 7 lists the loan payments for the three year loan. The first loan payment, $B_{s,1}$, equals \$2.40 and, therefore, γ is 0.024 . The loan payment for the second year is unchanged, implying that b_1 is zero. The final loan payment of \$32.40 occurs at the end of the third year and includes the interest coupon and repayment of principal *in toto*; according to equation 7, $b_2 = -0.08^{-1}$ and $b_3 = (0.08^{-1} \mathrm{x} 1.08)$. The β parameter is constructed by discounting the b series with the equity financing rate ρ. Equation 12 is solved iteratively for ρ, the only unknown variable, and its resulting equilibrium value is 14.85 percent.[13] Consequently, β=-0.5651 and D=0.7849.

Column 8 of table 1 lists the cumulative present value of the residual cash flow stream when discounted as specified in equation 9.[14] The present value of the three year residual cash flow stream is \$70. This discounted sum, plus the \$30 loan, equals the asset's purchase price of \$100 and indicates that the levered financing arrangement offers no advantage relative to the unlevered arrangement. Both are zero net present value opportunities.

Let the reference point roll forward to time $s+1$, let i, ρ^u and π remain unchanged, and let the purchase price of the asset increase by the expected inflation rate of 5 percent to \$105. Panel B depicts the situation for the marginal zero net present value investment made during this year. The real asset cash flow in the asset's first year of use reflects the new user cost of \$45.81 (q_s+1 is 5 percent larger than q_s; ρ^u-π and D^u are unchanged). The present value of the real asset cash flow stream discounted with the 7 percent real unlevered equity cost of capital is \$105, thereby satisfying the zero net present value equilibrium condition in the capital goods market. With a marginal leverage ratio of 0.30, the investment is financed with equity of \$73.50 and a three-year, 8 percent debenture of \$31.50 . The debt maturity structure is the same as before. The levered equity financing rate again is found to equal 14.85 percent and the present value of the residual cash flow stream, discounted with ρ, is \$73.50, thereby evincing the irrelevance of financing method.

Panel C shows conditions for the reference point at time $s+2$. Once again the marginal investment is a zero net present value opportunity. Its purchase price of \$110.25 is financed with

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[13] The iterative solution for ρ from equation 12 is easily obtained. The left-hand-side is predetermined by ρ^u. The predetermined right-hand-side variables include α, γ, π, and the capacity depreciation (d_j) and loan payment schedules (b_j). The right-hand-side is increasing with ρ, the only unknown in the equation. For the iteration, ρ^u is used as a seed value for ρ and the latter is increased until equation 12 is satisfied.

[14] The real asset cash flow is discounted with ρ-π (9.85 percent) whereas the nominal loan payment is discounted with ρ (14.85 percent).

equity of $77.17 and a three year, 8 percent debenture of $33.07 . Again, the financing method is irrelevant; i.e., the present value of the residual cash flow stream discounted with ρ (again found to equal 14.85 percent), plus the loan, equals the acquisition price of the real asset.

At time $s+2$ the illustrative corporation enters its steady state. There now are three different debenture bonds outstanding. Since market rates have remained constant (ρ^u, ρ, i, and π are unchanged), the value of this corporation's debt is simply the sum of the bonds' face values, which is $94.57 (=30+31.50+33.07). The value of the corporation's assets is the sum of the values of the three assets it owns. The value of the new asset is $110.25; another asset has two years of service remaining and its value is $69.87; the oldest asset has one year of service remaining and its value is $31.47;[15] total market value of assets is $211.59 . The firm's overall debt-to-assets ratio is 0.4470 (=94.57/211.59), somewhat higher than the marginal leverage ratio of 0.30 . Inspection of Panel D reveals that at reference point $s+3$ the overall debt-to-assets ratio remains constant at 0.4470. A steady state has been achieved in which the firm's debt, equity, and total market value are increasing at 5 percent per year; the marginal leverage ratio and overall debt-to-assets ratio will remain perpetually constant.

This illustration reveals that even though the firm's debt ratios are constant, the rate appropriate for evaluating the marginal investment is not the *WACC*. The illustration above with ρ=.1485 and i=.08 yields a *WACC* that is either 11.79 percent (if the overall debt-to-assets ratio of .4470 is used) or 12.79 percent (if the marginal leverage ratio of .30 is used). Discounting the asset cash flow stream with 11.79 percent would indicate that the marginal investment has a positive net present value, whereas discounting with 12.79 percent would indicate its net present value is negative. Both inferences are wrong; by design this is a zero net present value investment with an internal rate of return equal to 12 percent. The rate appropriate for

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[15] This may be computed by either of two equivalent procedures: (1) This asset retains 70 percent of its original productive capacity and therefore it will generate during its last year of service real cash flow equal to 70 percent of the time $s+2$ user cost of capital. Its real cash flow is therefore $33.67 (=.7x$48.10) which, discounted at the 7 percent real rate, has a present value of $31.47. (2) The real value at time $s+2$ stated in time s constant dollars of the investment from time s is $28.54. Its nominal value at time $s+2$ equals $28.54x(1.05)^2$, or $31.47 .

discounting the marginal investment's asset cash flow stream is not the *WACC*, rather it is the blend of the debt and equity financing rates specified in equation 12 that takes into account debt maturity structure.

IIIb. The cost of capital when financing is by debenture or fixed payment amortized loan

The cost of capital appropriate for discounting the asset cash flow stream is generically specified in equation 12. The equation is tailored below by setting the debt maturity structure parameters, β and γ, to reflect the loan payment schedules for two popular debt contracts: the debenture and the fixed payment amortized loan. The cost of capital formula subsequently is simplified and insights gleaned.

Consider a corporation that finances the marginal capital investment by supplying equity of $(1-\alpha_s)q_s$. The remainder of the purchase price, $\alpha_s q_s$, is financed by issuing a debenture of term T that carries a coupon rate of i (annual coupon, no sinking fund). With debenture financing the periodic interest expense equals $i\alpha_s q_s$ and there is no repayment of principal until time $s+T$, at which point the principal is repaid *in toto*. Since the first payment, $B_{s,1}$, equals $i\alpha_s q_s$ then γ equals $i\alpha_s$. The subsequent payments $B_{s,2}$ through $B_{s,T-1}$ are the same size as the first, so b_j=0 for j=0,...,T-2. During period T, though, the payment includes the coupon as well as the repayment of principal and, according to equation 7, $b_{T-1}=-i^{-1}$. After period T, the payment drops to zero and $b_T=i^{-1}(1+i)$. The β term is computed by discounting the b_j's with ρ as specified in equation 11, showing that $\beta=-i^{-1}(\rho-i)(1+\rho)^{-T}$. Substituting β and γ into equation 12 shows

$$\begin{aligned}\rho^u &= (\rho-\pi)(1-\alpha_s+\alpha_s i(1+i^{-1}(\rho-i)(1+\rho)^{-T})\rho^{-1})(1-D_s^u)(1-D_s)^{-1}+\pi. \qquad (15)\\ &= (WACC+\alpha_s(\rho-i)(1+\rho)^{-T})(1-\pi/\rho)(1-D_s^u)(1-D_s)^{-1}+\pi\end{aligned}$$

Equation 15 implies three alternative yet qualitatively equivalent procedures for analyzing a prospective investment when the source of financing is a debenture: (1) discount the asset cash flow stream with ρ^u; (2) discount the asset cash flow stream with (*WACC* + $\alpha_s(\rho-i)(1+$

$\rho)^{-T})(1-\pi/\rho)(1-D_S^U)(1-D_s)^{-1}+\pi$; and (3) discount the residual cash flow stream with ρ. Regardless of which procedure is selected, the financing rates i, ρ, and ρ^u must be consistent with the relationship specified in equation 15 or else the levered and unlevered investments possess different net present values; violation of equation 15 implies a state of market disequilibrium.

Table 2 presents simplifications of equation 15 for differing environments. In row 2, for example, is shown the case in which a one-time debt issue is used for financing a perpetual stream of asset cash flows that are expected to rise with inflation (the real asset cash flow is constant). There is erosion in the real value of debt and the *WACC* consequently is an understatement of the total return on capital. The explicit association between the cost of capital (ρ^u) and the *WACC* is shown by the equation in row 2. With illustrative settings for the marginal leverage ratio of 40 percent, a debt financing rate of 8 percent, and an exogenous equity financing rate of 15 percent, the *WACC* is 12.20 percent. The actual cost of capital, however, is 13.94 percent; it is substantially understated by the *WACC*.

The *WACC* overstates the actual cost of capital for the environment characterized in row 3 of table 2. In that case, a 6-year debenture is used for financing an asset generating a 10-year cash flow stream. The productive capacity of the asset is constant throughout its service life, hence expected real asset cash flow is constant, yet the presence of 5 percent expected inflation causes an increase in nominal cash flow. With an 8 percent exogenous debt financing rate, 15 percent exogenous equity financing rate, and 40 percent marginal leverage ratio, the *WACC* equals 12.20 percent; the actual cost of capital is 9.72 percent. A firm using its debt policy for financing marginal investments is incorrectly analyzing prospective acquisitions if it is relying on the *WACC* (even though its overall debt-to-assets ratio may be constant).

Turn now to the corporation that finances its marginal capital investment by issuing a fixed payment amortized loan with term T, coupon rate i, and principal $\alpha_s q_s$. The first payment, $B_{s,1}$, is computed in the normal way and equals $\alpha_s q_s / PVIFA_{i,T}$, where the denominator is the standard present value interest factor for an annuity, $i^{-1}[1-(1+i)^{-T}]$. γ therefore equals α_S

$/PVIFA_{i,T}$. The subsequent payments $B_{s,2}$ through $B_{s,T}$ are the same size as the first, so b_j=0 for j=0,...,T-1. After period T, the payments drop to zero and, according to equation 7, b_T=1.0. The β term is computed by discounting the b_j's as specified in equation 11, showing that $\beta=(1+\rho)^{-T}$. Substituting β and γ into equation 12 and rearranging shows

$$\begin{aligned}\rho^u =& \left((1-\alpha_s)\rho + \alpha_s i\frac{1-(1+\rho)^{-T}}{1-(1+i)^{-T}}\right)(1-D_S^U)(1-D_S)^{-1} \\ &+\alpha_S\pi\left(1-\frac{i(1-(1+\rho)^{-T})(1-D_S^U)}{\rho(1-(1+i)^{-T})(1-D_S)}\right).\end{aligned} \tag{16}$$

The expression above represents the cost of capital that is appropriate for discounting asset cash flows when a firm finances its marginal capital investment with a fixed payment amortized loan. Table 3 presents the cost of capital from equation 16 for different environments. In all illustrative settings shown, the *WACC* always understates the cost of capital. For example, row 4 shows the case in which the real asset cash flow stream declines along a straight-line pattern throughout a 10 year service life. There is, however, 5 percent expected inflation. The investment is 40 percent financed with a 6-year, 8 percent fixed payment amortized loan, and the exogenous equity financing rate is 15 percent. The cost of capital, 12.66 percent, is found by iteratively solving the equation in row 4. It is somewhat understated by the *WACC* of 12.20 percent, again showing that analyses based upon the *WACC* may be flawed in cases for which the corporation uses its debt policy exclusively for financing marginal capital investments.

IIIc. What it means: Some implications

These findings have implications that fall into two broad categories. First, they affect the theory and application of capital budgeting. Second, they have capital market implications.

The findings establish that for capital budgeting there are three alternative yet qualitatively equivalent procedures for analyzing a prospective investment: (1) discount the asset cash flow stream with ρ^u; (2) discount the asset cash flow stream with $(\rho-\pi)(1-\alpha_s+\gamma(1-\beta_s)$

$\rho^{-1})(1-D_S^U)(1-D_s)^{-1}+\pi$; or (3) discount the residual cash flow stream with ρ. Under some conditions, the discount rate for procedure 2 is the standard *WACC*; more often, however, it is not. Even though the overall debt-to-assets ratio for the firm may be constant at one value and the marginal leverage ratio may be constant at another value, it likely is illogical to rely upon the *WACC* as the appropriate rate for discounting expected asset cash flows when a firm's debt policy focuses on financing real investments. Tables 2 and 3 show several cases in which the appropriate discount rate is a simple closed-form weighting of the exogenous debt and equity financing rates. Usually, however, the cost of capital is a blend of equity and debt financing rates that takes into account the maturity structure of the loan payment schedule and it can be obtained only through iterative solution.

It is important to learn that the *WACC* generally is incorrect; it is not a very limiting finding, though. Capital budgeting procedure 2 described above requires identification of ρ, the equity financing rate. Given that ρ is identified, however, then it is entirely plausible to apply procedure 3; i.e., discount the residual cash flow stream with ρ. This point is summarized as follows: the information requirements for procedures 2 and 3 are identical; analyses implementing procedure 2 through reliance on the *WACC* are generally wrong; it is somewhat difficult to apply procedure 2 correctly; thus, an implication of this study is that when ρ is known and exogenous then procedure 3 should be used.

These findings also have capital market implications. In particular, they establish that at equilibrium the capital income provided by a real asset equals $(\rho-\pi)(1-\alpha_s+\gamma(1-\beta_s)\rho^{-1})(1-D_S^U)(1-D_s)^{-1}+\pi$. This amount therefore represents the total return required by the financial markets; i.e., the required returns for debt and equity exactly exhaust the equilibrium capital income provided by the real asset. Consequently, an implication of these findings is that for a given debt contract and asset cash flow stream, there exists a unique equilibrium equity financing rate (ρ). Future research with this model may glean insight on the trade-offs between endogenized equity financing rates and debt maturity structure.

V. Summary and Concluding Remark

This study derives an expression for the cost of capital that is appropriate for discounting cash flows expected from prospective investments. The expression, summarized in equation 12 and simplified for alternative environments in tables 2 and 3, stems from a production based model that reflects the effects of inflation, debt maturity structure, and real asset capacity depreciation. In a few situations the cost of capital expression reduces to the standard weighted average cost of capital (*WACC*). Generally, however, when the firm uses its debt policy for financing marginal capital investments then it is incorrect to analyze prospective investments by relying on the *WACC*. The appropriate rate for discounting expected asset cash flows is a blend of equity and debt financing rates that takes into account the debt maturity structure. In some settings the true cost of capital is overstated by the *WACC*; in other settings it is understated.

This study also shows the intertwining between debt and equity capital market rates such that equilibrium in the real capital goods market is maintained. The sensitivity of the equity financing rate to the leverage ratio is dependent upon debt maturity structure. The increase in equilibrium equity financing rate in response to a rising leverage ratio is greater for some debt contracts than stipulated by Modigliani and Miller's Proposition 2; for other debt contracts it is less. Regardless, the equilibrium association between alternative debt arrangements is found, thereby enabling insights about rate spreads concomitant with different maturity structures.

The findings of this study supersede previous findings. It is widely believed, for example, that the weighted average cost of capital is the rate always appropriate for making capital budgeting decisions. It is shown herein that this tenet of corporate finance is valid only in a sterile environment. Of greater importance, a specification is provided for an environment characterized by a diversity of available methods of payment, asset cash flow patterns, and inflationary expectations. Analysis of the specification reveals that the cost of capital calculation should incorporate information about debt maturity structure.

This study opens several avenues for future research. First, the user cost framework developed herein should be extended so that it incorporates tax effects. The relatively simple cost of capital expression obtained herein likely will become more complicated when it reflects tax policy parameters. Second, the applicability of the cost of capital expression to actual capital budgeting decisions needs to be assessed. Is it feasible, for example, for a firm to analyze prospective investments conditional upon available methods of payment, and where does the equity financing rate estimate for the alternative methods come from? Third, the model yields testable propositions about equity financing rates. For example, empirical research should be able to test whether the equity financing rates for two firms differing only by their debt maturity structures align as predicted by the model.

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TABLE 1: Illustration of valuation mechanics for the marginal investment

marginal asset's service year	real asset cash flow (EOP) -1-	cumulative discounted real asset cash flow (BOP) -2-	real asset value (BOP) -3-	real cash flow yield -4-	real capital gains yield -5-	total real rate of return -6-	loan payment (EOP) -7-	cumulative discounted residual cash flow (BOP) -8-
PANEL A: Reference time is year "s"								
1	$43.63	$40.77	$100.00	43.63%	-36.63%	7.00%	$ 2.40	$37.63
2	39.27	75.07	63.37	61.96	-54.96	7.00	2.40	68.35
3	30.54	100.00	28.54	107.00	-100.00	7.00	32.40	70.00
at time *s*: marginal investment's value = $100.00 (equity financing = $70.00; debt financing = $30.00; a_s = 0.30)								
PANEL B: Reference time is year "s+1"								
1	$45.81	$42.81	$105.00	43.63%	-36.63%	7.00%	$ 2.52	$39.51
2	41.23	78.82	66.54	61.96	-54.96	7.00	2.52	71.77
3	32.07	105.00	29.97	107.00	-100.00	7.00	34.02	73.50
at time *s+1*: marginal investment's value = $105.00 (equity financing = $73.50; debt financing = $31.50; a_{s+1} = 0.30) total debt = $61.50; total assets = $171.54; debt/assets = 0.3585								
PANEL C: Reference time is year "s+2"								
1	$48.10	$44.95	$110.25	43.63%	-36.63%	7.00%	$ 2.65	$41.48
2	43.29	82.77	69.87	61.96	-54.96	7.00	2.65	75.35
3	33.67	110.25	31.47	107.00	-100.00	7.00	35.72	77.18
at time *s+2*: marginal investment's value = $110.25 (equity financing = $77.17; debt financing = $33.07; a_{s+2} = 0.30) total debt = $94.57; total assets = $211.58; debt/assets = 0.4470								
PANEL D: Reference time is year "s+3"								
1	$50.51	$47.20	$115.76	43.63%	-36.63%	7.00%	$ 2.78	$43.56
2	45.45	86.90	73.36	61.96	-54.96	7.00	2.78	79.12
3	35.35	115.76	33.04	107.00	-100.00	7.00	37.51	81.03
at time *s+3*: marginal investment's value = $115.76 (equity financing = $81.03; debt financing = $34.73; a_{s+3} = 0.30) total debt = $99.30; total assets = $222.16; debt/assets = 0.4470								

Notes:

BOP and EOP denote beginning-of-period and end-of-period, respectively.

The service life of the asset is 3 years and capacity depreciation is 10%, 20%, and 70% after each year of service.

The real asset cash flow equals the user cost multiplied by 1 minus accumulated capacity depreciation (equation 2).

The settings for all years are r^u=.12, p=.05, *and* i=.08 .

The purchase price of the marginal investment is $100 at time s and it rises 5% per year.

The marginal leverage ratio (a) *is always 30% and the loan is a 3-year debenture.*

The ratio of debt to total assets reaches steady state of 0.447 at time s+2.

The real and financial markets are in equilibrium yet it is incorrect to discount the marginal investment's asset cash flow stream with the WACC.

TABLE 2: Cost of capital when source of debt financing is a debenture

Conditions	Cost of capital formula r^u	Illustrative cost of capital estimate
1. No inflation, constant and perpetual real asset cash flow stream		
	$r^u = WACC + a_s(r-i)(1+r)^{-T}$	*13.41%*
2. Perpetual asset cash flow stream that rises with inflation		
	$r^u = (WACC + a_s(r-i)(1+r)^{-T})\ (1- p/r) + p$	13.94%
3. Finite asset cash flow stream that rises one-for-one with inflation		
	$PVIFA_{r^u-p,L} = PVIFA_{r-p,L}\left(1 - a_s\left(1 - \dfrac{1+r-i}{(1+r)^T}\right)\right)^{-1}$	9.72%
4. Finite real asset cash flow stream that declines by straight-line but otherwise rises with inflation		
	$\dfrac{r^u - p}{L - PVIFA_{r^u-p,L}} = (WACC + a_s(r-i)(1+r)^{-T})\ (1- p/r)\ (L - PVIFA_{r-p,L})^{-1}$	*12.71%*

Notes:
All expressions are simplifications of equation 15 when the stipulated conditions are imposed.
$PVIFA_{k,L}$ is the standard present value interest factor for an annuity, $k^{-1}(1-(1+k)^{-L})$.
WACC is the weighted average cost of capital, $(1-a_s)r + a_s i$, and it equals 12.20% for the settings below.
The illustrative estimates are computed with the following settings:

- r^u = *determined endogenously; the cost of capital*
- a_s = *.40, proportion of the investment financed by debt*
- r = *.15, equity financing rate*
- i = *.08, debt financing rate*
- p = *.05, expected inflation rate*
- L = *10, length of real asset cash flow stream*
- T = *6, term of the loan*

TABLE 3: Cost of capital when source of debt financing is a fixed payment amortized loan

Conditions	Cost of capital formula r^u	Illustrative cost of capital estimate
1. No inflation, constant and perpetual asset cash flow stream		
	$r^u = (1-a_s)r + a_s i \dfrac{1-(1+r)^{-T}}{1-(1+i)^{-T}}$	13.91%
2. Perpetual asset cash flow stream that rises with inflation		
	$r^u = \left((1-a_s)r + a_s i \dfrac{1-(1+r)^{-T}}{1-(1+i)^{-T}} \right) (1- p/r) + p$	*14.27%*
3. Finite asset cash flow stream that rises with inflation		
	$PVIFA_{r^u-p,L} = PVIFA_{r-p,L}\left(1 - a_s\left(1 - \dfrac{PVIFA_{r,T}}{PVIFA_{i,T}}\right)\right)^{-1}$	13.28%
4. Finite real asset cash flow stream that declines by straight-line but otherwise rises with inflation		
	$\dfrac{r^u - p}{L - PVIFA_{r^u-p,L}} = \left((1-a_s)r + a_s i \dfrac{1-(1+r)^{-T}}{1-(1+i)^{-T}} \right) (1- p/r) (L - PVIFA_{r-p,L})^{-1}$	*12.66%*

Notes:

All expressions are simplifications of equation 16 when the stipulated conditions are imposed.

$PVIFA_{k,L}$ is the standard present value interest factor for an annuity, $k^{-1}(1-(1+k)^{-L})$.

WACC is the weighted average cost of capital, $(1-a_s)r + a_s i$, and it equals 12.20% for the settings below.

The illustrative estimates are computed with the following settings:

- r^u = *determined endogenously; the cost of capital*
- a_s = *.40, proportion of the investment financed by debt*
- r = *.15, equity financing rate*
- i = *.08, debt financing rate*
- p = *.05, expected inflation rate*
- L = *10, length of real asset cash flow stream*
- T = *6, term of the loan*

IV. Endogenizing the Financing Rates (this section removed in January 1993)

The financing rates i, r, and r^u must be consistent with the relationship specified in equation 12; violation of this equality indicates a state of disequilibrium because the net present value of the marginal investment would depend on the financing method. To the extent that the real and financial markets are equilibrated, the interrelationship among the financing rates satisfy the following rearrangement of equation 12:

$$(r^u - p)(1 - D^u_s)^{-1} = (r - p)(1 - D_s)^{-1}(1 - a_s + g(1-b_s)r^{-1}) . \tag{17}$$

The g and b parameters reflect the loan payment schedule and consequently it is through them that the debt financing rate (i) exerts its influence.

An intuitive interpretation for equation 17 is obtained by considering the situation characterizing a non-depreciating real asset (D^u_s=D_s=0). The left-hand-side, r^u-p, is the real rate of return on the capital income stream and it is independent of debt policy. The right-hand-side, $(r-p)(1-a_s + g(1-b_s)r^{-1})$, also equals the real return on capital but it is expressed in terms of the capital market financing rates i and r. Equation 17 therefore shows how the debt and equity financing rates are intertwined such that they are consistent with equilibrium in the real sector.

For the analyses of equation 17 presented below the real capital income stream (i.e., r^u, p, and the asset cash flow stream) is held constant. The financing arrangements pertaining to the marginal investment are altered and the response in the equity financing rate (r) that maintains market equilibrium is deduced. Insight is gleaned about the trade-off between the equity financing rate and alternative debt policies.

IVa. Proposition 2 revisited

Consider the cases for the one period model and for the geometrically rising or falling asset cash flow with constant leverage ratio analyzed in section II. For those cases equation 17 simplifies to show that r^u equals the *WACC*. Rearrangement of this equality yields

$$r = r^u + \frac{a_s}{1-a_s}(r^u - i) . \tag{18}$$

Equation 18 is Modigliani and Miller's *Proposition 2* (1958) showing that when r^u and i are constant then the equity financing rate (r) is determined endogenously as an increasing linear function of the marginal debt-to-equity ratio.

According to Proposition 2, the determination of r given a is conditional on r^u and i. The preceding sections establish, however, that r^u and i (with a_s given) generally are insufficient for determining r; information about debt maturity structure is required. Consequently, the response of r to a given change in a_s depends not only on r^u and i, but also on the maturity structure of the debt contract.

The argument that the sensitivity of r to the leverage ratio depends upon debt maturity structure is illustrated for several of the environments described in tables 2 and 3. Consider a non-depreciating real asset that is partially financed with the one-time issuance of a consol bond (i.e., a debenture with $T\Gamma\%$). Row 2 of table 2 shows how equation 17 simplifies for this environment. The expression is solved for r, thereby showing

$$r = \frac{r^u - a_s(i+p) + \{(a_s(i+p) - r^u)^2 + 4a_s ip(1-a_s)\}^{1/2}}{2(1-a_s)} . \tag{19}$$

Notice, as an aside, that equation 19 reduces to Proposition 2 when p=0.

Equation 19 generally supercedes Proposition 2 for the case in which a non-depreciating real asset is financed by consol. Consider the settings p=.05, i=.08, and r^u=.12 . When a=0 then r, the equity financing rate, of course equals 12 percent; in the absence of leverage r and r^u are the same. As a increases then r changes so as to reestablish equilibrium. For example, when a increases from .30 to .40 then, according to equation 19, the endogenous rise in r that preserves equilibrium is 43 basis points. According to Proposition 2, however, the response of the equity financing rate to the increased leverage ratio is 95 basis points. The interplay between the equity financing rate and leverage ratio suggested by equation 19 is substantially less than suggested by Proposition 2.

For most of the other cases shown in tables 2 and 3 the equity financing rate typically can be obtained only through iterative solution of equation 17; no closed form endogenous expression for r is obtainable. Consider the finite real asset cash flow stream that declines along a straight-line pattern over a 10-year service life. Again, let p=.05, i=.08, and r^u=.12 . According to Proposition 2, of course, equilibrium r rises by 95 basis points in response to an increase in a_s from .3 to .4 . According to equation 17, however, the endogenous response of the equity financing rate to increased leverage depends on the debt maturity structure. When financing is by 5-year debenture, r responds by rising 113 basis points; with a 5-year fixed payment amortized loan r rises 39 basis points. For the former case the sensitivity of the equilibrium equity financing rate to a change in leverage is understated by Proposition 2; for the latter case it is overstated.

Proposition 2 presupposes that for a given i and r^u there is a unique response of the equity financing rate to an increase in the leverage ratio. The findings herein establish, however, that Proposition 2 is valid only under special circumstances; it goes hand-in-hand with the *WACC*. In general, the sensitivity of the equity financing rate to the leverage ratio depends on the embodied real asset cash flow stream and the maturity structure of the loan payment stream.

IVb. Intertwined debt and equity financing rates

Equation 17 shows that at equilibrium the user cost of capital is invariant to the financing method; the levered and unlevered user costs for the marginal investment are equal. Consequently, the user cost is the same regardless of whether, say, a 5-year or 10-year debenture is financing the investment. The user costs for debt contracts "*a*" and "*b*" on the same underlying real asset can be equated, thereby showing

$$(r^a - p)(1 - D^a_{\underline{s}})^{-1}(1 - a^a_{\underline{s}} + g^a(1 - b^a_{\underline{s}})/r^a) = (r^b - p)(1 - D^b_{\underline{s}})^{-1}(1 - a^b_{\underline{s}} + g^b(1 - b^b_{\underline{s}})/r^b) \,. \tag{20}$$

Equation 20 describes the intertwining between financing arrangements *a* and *b*.

The capital market financing rates i (through g and b) and r cannot be disentangled from the capacity depreciation schedule of the underlying real asset so it is difficult to make generalizations about the endogenous rate spreads implied by equation 20. Nonetheless, to illustrate the types of insights allowed by equation 20 the cases described below and summarized in table 4 impose the following conditions: (1) the real asset declines along a straight-line pattern throughout a 10-year service life; (2) the expected inflation rate (p) is 5 percent; and (3) the investment is financed with a 30 percent initial leverage ratio (a). The unlevered equity cost of capital (r^u) consistent in this setting with a zero net present value equilibrium is 12 percent.

For Panel A equation 20 is solved to find the endogenous equity financing rate given a fixed debt financing rate of 8 percent. With a 5-year debenture, for example, the endogenous equilibrium equity financing rate is 13.90 percent and with a 10-year debenture equilibrium r is 15.70 percent. Alternatively, for 5-year and 10-year fixed payment amortized loans, the endogenous equilibrium r equal 13.11 percent and 14.14 percent, respectively. When the term of the debt contract is lengthened from 5 years to 10, the endogenous equity financing rate rises in order to reestablish real and financial market equilibria. The increase in *r*, however, is 103 basis points if the debt contract is a fixed payment amortized loan and 180 basis points if it is a debenture.

One might argue from an institutional point of view that *i* is not fixed with respect to the term of the loan, but rather *r* is. For Panel B, equation 20 endogenizes the debt financing rate and fixes *r* at 13.50 percent. The endogenous equilibrium debt financing rate is 8.84 percent for a 5-year debenture and 10.46 percent for a 10-year debenture. For a fixed payment amortized loan the endogenous equilibrium debt financing rate is 6.59 percent on a 5-year loan and 9.25 percent on a 10-year loan.

These equilibria are consistent with an unlevered equity financing rate and an inflation rate that are expected to remain constant throughout the 10-year asset service life. Nonetheless, the endogenous capital market financing rates *i* and *r* increase as the term of the debt contract lengthens. In other words, equation 20 generates an upward sloped yield curve. Even though liquidity premia or segmented markets are absent, an endogenous upward sloped yield curve is implied by the model due to the interaction between the real asset's capacity depreciation schedule and the debt maturity structure.[1]

Both the debt and equity financing rates are endogenized for Panel C. An equality constraint is imposed on the debt and equity risk premia such that i=.08+P and r=.12+P . This assumes that debt and equity equally share the incremental risk added by leverage. Algebraic simplification of equation 20 shows that as the marginal leverage ratio tends to zero the premium P also converges to 0; in the absence of leverage rG.12 (=ru) and iG.08 (the latter implies a 3 percent riskless real interest rate since p=.05). As the leverage ratio rises above zero, the debt and equity financing rates endogenously rise in accordance with equation 20 and the equality constraint on the risk premia. In lieu of algebra, numerical solutions for equation 20 are reported. With a=.3, the risk premia are 129 and 196 basis points for the 5-year and 10-year debentures, respectively. On a fixed payment amortized loan the risk premia are 87 and 141 basis points for the 5-year and 10-year loans. In this particular equilibrium setting the yield curve is flatter for the fixed payment amortized loan and steeper for the debenture; any other outcome would represent a state

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[1] Even though no other studies rely on the user cost for endogenizing the yield curve, the association between the real economy and term structure is examined by, among others, Feldman (1989) and Cox, Ingersoll, and Ross (1985).

of market disequilbrium because the net present value of the marginal investment would depend on the financing method.

The intertwined real and financial variables in the equilibrium model specified in equation 20 enable insights about capital market financing rates. Under particularly restrictive conditions, the general specification reduces to Modigliani and Miller's Proposition 2. More typically, however, the financing rates are inseparable from the underlying asset cash flow stream and debt maturity structure. By specifying the asset cash flows and debt maturity structure, the financing rates can be endogenously derived and testable propositions about the capital markets formulated.

TABLE 4: Numerical solutions for endogenous capital market financing rates

endogenous rate	Debenture 5-year term	Debenture 10-year term	Fixed Payment Amortized Loan 5-year term	Fixed Payment Amortized Loan 10-year term
PANEL A:	*endogenous equity financing rate (r)* *exogenous 8% debt financing rate (i)*			
r =	13.90%	15.70%	13.11%	14.14%
PANEL B:	*endogenous debt financing rate (i)* *exogenous 13.5% equity financing rate (r)*			
i =	8.84%	10.46%	6.59%	9.25%
PANEL C:	*endogenous equity financing rate (r)* *endogenous debt financing rate (i)*			
r =	13.29%	13.96%	12.87%	13.41%
i =	9.29%	9.96%	8.87%	9.41%

Notes:
The endogenous financing rates are solutions to equation 20 conditional on the exogenous variables and stipulated debt maturity structure.
For all panels the real asset depreciates along a straight-line pattern over a 10-year service life.
For Panel C an equality constraint is forced on the risk premia for the debt and equity financing rates.
The illustrative estimates are computed with the following exogenous settings:

r^u = .12, *unlevered equity cost of capital*
a_s = .30, *proportion of the investment financed by debt*
p = .05, *expected inflation rate*
i^u = .08, *riskless interest rate (for Panel C only)*