The “Embodied Equity” Theory of Term Structure

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ABSTRACT

This study analytically specifies the residual cash flow stream that real capital embodies. The specification separates debt cash flows from equity and obtains an equilibrium condition equating marginal physical product and real user cost of capital. Analysis of the user cost specification reveals that the equilibrium interest rate is an increasing function of the debt contract's loan-to-value ratio and average period of debt. The basic reason why the interest rate increases with average period is this: the equity financing rate exceeds the interest rate, a lengthening debt average period reduces to equity the discounted cost of debt, the financing rate increases to re-establish equilibrium. The “embodied equity” hypothesis advanced herein joins the expectations hypothesis, the liquidity preference hypothesis, and the market segmentation hypothesis as a fundamental explanation for the upward slope on the yield curve.

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I. Introduction

Theories from early economics explain interest by attaching expected income streams to real capital. Eugen von Böhm-Bawerk hypothesizes in *Positive Theory of Capital* (1888) that the value of real capital embodies the expected capital income stream. Capital releases interest as time passes, according to Böhm-Bawerk. One of his novel and enduring contributions is recognition that the first cash flow contains information about the entire cash flow stream. He refers to the first cash flow as "bearer of the use." John Keynes in *General Theory of Employment, Interest, and Money* (1936) explains that the first cash flow, re-coined the "user cost of capital," directs producer incentives because value marginal product equilibrates to user cost of capital. A second contribution of Böhm-Bawerk's theory is that the temporal length of the capital income stream is important to determination of the interest rate. He refers to the stream's temporal length as "average period." John Hicks in *Value and Capital* (1939) hypothesizes that fluctuation in average periods of capital income streams explains variation in interest rates.

My study employs average period and user cost in an analytical model and finds a new answer for an old question: Why does the yield curve usually slope upward? Existing hypothetical answers to this question fall into four basic categories. (1) The unbiased expectations hypothesis asserts that the yield curve is shaped by market forces equating implied forward rates to expected spot rates. (2) The liquidity preference hypothesis explains that long-term debt contracts offer investors fewer reinvestment options than short-term contracts and, all else equal, long-term yields are higher in order to induce investment. (3) The preferred habitat hypothesis, also called the modified market segmentation hypothesis, suggests that for institutional reasons investors prefer one term to another, bonds of differing term are not good substitutes for one another, and yields adjust to supply and demand conditions in various market segments. (4) Stochastic interest rate hypotheses show that uncertainty about possible outcomes generally increases with term; risk premia are a direct function of uncertainty and consequently they too increase with term.[1]

The preceding hypotheses are not mutually exclusive. Each presents a fundamental explanation for the upward slope on the yield curve. When real capital embodies the expected capital income stream, however, there exists an additional fundamental explanation. Gist of the explanation is this: equity is residual claimant on the embodied capital income stream; the equity financing rate exceeds the interest rate on debt; an increase in average period of the debt payment stream reduces to equity the discounted cost of debt; user cost of capital is invariant to leverage so something must offset reduction in the discounted cost of debt; the interest rate consistent with equilibrium rises. Capital naturally releases embodied interest at a higher rate when average period of debt is long, so long-term interest rates naturally are higher than short-term rates.

A necessary condition for this term structure hypothesis is that the equity financing rate exceeds the debt interest rate. Justification for the condition is easily found. First, debt claims are senior to equity claims. Second, cash flows are more fully specified, and hence less risky, for debt than for equity. An alternative reason, however, is more parsimonious with this fundamental model of real capital. A specific unit of real capital embodies the unlevered equity financing rate. Sources of debt financing, however, hold claims on many types of real capital for many different producers. Creditors are beneficiaries of diversification

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[1] Vasicek (1977) as well as Cox, Ingersoll, and Ross (1981,1985) introduce uncertainty into the term structure literature through with intertemporal, continuous time, stochastic processes. More recent and general models of stochastic interest rates include Longstaff and Schwartz (1992) and Constantinides (1992).

benefits because they hold portfolios with many types of real capital. The existence of diversification benefits for creditors is assurance that the interest rate on debt is less than the unlevered equity financing rate, and invariance of capital's user cost to financial structure is assurance that the equilibrium interest rate is greater for long-term than for short-term debt.

The layout for this study is as follows. Section II specifies the equilibrium condition for the marginal investment in real capital. This specification includes information about the maturity structure of the debt payment stream. The invariance of the user cost to financing method enables derivation of an expression that endogenizes the interest rate. Sections III and IV provide insight about the "intrinsic yield curve" that capital embodies. Section III focuses on the nature of the debt payment stream, for example whether the loan is a debenture or fixed payment amortized loan. Section IV focuses on the importance of real capital characteristics, for example whether the economic depreciation is accelerated or decelerated. A brief conclusion in Section V closes the study.

The findings are relevant for several reasons. First, the model enables the point estimation of the intrinsic interest rate yield curve. Analysis of this specification reveals, for example, that the intrinsic yield curve is steeper for corporate debentures than for fixed payment amortized loans. Second, the framework incorporates information about debt maturity structure into the equilibrium conditions for the user cost of capital. This expands the scope of analyses in which the user cost is applicable and allows for potential reduction in measurement error. Third, the study offers a new yet fundamental explanation for the upward sloping yield curve.

II. The generalized user cost specification

The user cost equals the pre-tax cash flow produced by one unit of real capital during its first period of use such that the asset represents a zero net present value investment.[2] Jorgenson (1963) gives the user cost a modern analytical framework. He links user cost to financing rates and tax policy parameters and subsequently employs user cost as an explanatory variable for net fixed investment.

Robert Hall and Jorgenson (1967) obtain a specification for the user cost at time *s*, denoted c_S:

$$c_s = \frac{q_s(r_s + \delta - \pi_s)(1 - \tau_s Z_s)}{(1 - \tau_s)} \tag{1}$$

In this expression, q_S represents the supply price at time *s* of a new capital asset, *r* is the financing rate for the investment, δ is the asset's rate of decline in productive efficiency, π is the expected inflation rate, τ is the marginal corporate income tax rate, and *Z* is the present value of tax depreciation deductions (per dollar of asset) expected throughout the service life.[3]

Interpretations about the financing rate in the user cost framework vary. Early studies employing the user cost [e.g., Jorgenson (1963)] measure *r* as the long-term government bond rate. Hall (1981) argues that the short-term bond rate is appropriate. Martin Feldstein (1982) argues that these specifications ignore the cost of equity financing. He suggests the user cost should employ a measure for *r* equal to the weighted average of debt and equity financing rates. Although Jorgenson and Kun-Young Yun (1991) rely on pre-tax financing

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[2] Equivalent restatements of the user cost definition include (all are per unit of real capital): (a) earnings before interest and taxes plus depreciation; (b) operating income plus depreciation; (c) cash flow from operations plus taxes and interest.

[3] The Hall-Jorgenson user cost specification includes the effect of an investment tax credit at rate *v*. Incorporate *v* into the specifications herein by replacing $(1 - \tau Z)$ with $(1 - \tau Z - v)$.

rates, most recent studies employ a weighted average of after-corporate-tax debt and equity financing rates, as in

$$r_s = (1-\alpha_s)\rho_s + (1-\tau)\alpha_s i_s \ , \qquad (2)$$

where α is the marginal debt-to-assets ratio, ρ is the levered equity financing rate, and i is the pre-tax debt interest rate. Equation 2 specifies r as the ubiquitous weighted average cost of capital. The financial cost of capital from equation 2, r, substitutes into the Hall-Jorgenson user cost of capital in equation 1 in order to derive the investment equilibrium condition for real capital goods.

The objective for the producer is maximization of discounted profits. Jorgenson (1967) shows that the equilibrium conditions equate the value marginal products of labor and capital, respectively, to the wage rate and user cost of capital. Alternative yet equivalent description of the equilibrating process is that the producer invests in capital whenever (a) the value marginal product of capital exceeds the user cost of capital, or (b) the internal rate of return for the after-corporate-tax (before interest) cash flow stream (also known as the Keynesian marginal efficiency of capital) exceeds the weighted average cost of capital. With each additional capital investment the marginal physical product of capital declines, thereby reducing the value marginal product and marginal efficiency of capital. Equilibrium eventually recurs when inequalities (a) and (b) are offset.

The Hall-Jorgenson user cost specification has appeared for decades in economic analyses pursuing many different objectives. A significant strand of literature manipulates the user cost framework so that instead of solving for the equilibrium pre-tax cash flow as the unknown variable, some other variable or expression from the user cost specification serves as the unknown term. Patric Hendershott (1981) endogenizes the financing rate within the user cost framework and makes inferences about changes in risk premia and possible valuation effects of inflation. Alan Auerbach and Jorgenson (1980) invoke the assumption that risk-adjusted after-tax returns on financial and fixed assets equilibrate. They subsequently extract from the user cost framework the internal rate of return for the fixed asset's pre-tax cash flow stream and ingenuously glean insight about effective tax rates for fixed assets [other prominent studies on effective tax rates and user cost include David Bradford (1981), Jane Gravelle (1982), and Mervyn King and Don Fullerton (1984)]. Other studies rely on the user cost framework to infer the equilibrium financial cost of capital (*r*) for fixed assets in different sectors or asset groups [see, for example, Auerbach (1987), Hans-Werner Sinn (1991), and Jorgenson and Ralph Landau (1993)].

Specification of financial structure in the preceding studies is limited. They assume explicitly, if anything, that for a firm financing by debt and equity the underlying debt-to-equity ratio is perpetually constant. The implications of this assumption are neither investigated nor relaxed by any of the studies. The model below generalizes the specification by explicitly separating debt from equity.

Suppose that for purchasing a new fixed asset at time *s* with supply price q_s the economic producer supplies equity financing of $(1-\alpha_s)q_s$ and obtains debt financing for $\alpha_s q_s$. A loan payment schedule at time of investment establishes repayment of principal and interest. Each period the asset cash flow net of taxes and loan payment accrues to equity; call the expected accrual the residual cash flow. The zero net present value equilibrium condition equates funds provided by equity to the present value of the expected residual cash flow stream discounted by the equity financing rate:

$$(3) \quad (1-\alpha_s)q_s = \sum_{t=1}^{\infty} \left\{ (1+\rho_s-\pi_s)^{-t}(1-\tau)c_{s,t} + (1+\rho_s)^{-t}\tau q_s z_{s,t} - (1+\rho_s)^{-t}B_{s,t} \right\}.$$

α_s is the *initial* loan-to-value ratio for the time *s* marginal investment in real assets. The evolution beyond time *s* of the asset's loan-to-value ratio depends on the interaction between the asset cash flow and loan payment streams.

Equation 3 is the equilibrium condition for a two-market model: (1) the market for real assets, and (2) the equity market. According to Walras Law, either none or both markets are in a state of equilibrium. The incentive for equity to invest in real assets persists until equation 3 is satisfied.

The discounted residual cash flow on the right-hand-side of equation 3 has three components. The first component, $(1-\tau)c_{s,t}$, equals the after-corporate-tax real asset cash flow expected at time $s+t$ from the time *s* investment. The second component, $\tau q_s z_{s,t}$, equals expected tax savings from depreciation deductions at time $s+t$ resulting from the time *s* investment. The third component, $B_{s,t}$, equals the after-corporate-tax loan payment made at time $s+t$ for the time *s* investment. $B_{s,t}$ may include interest, principal repayment or issuance, and any other debt related fees. Further details about cash flow components appear below.

First consider specification of the real asset cash flow stream. Let d_j denote the proportional decline in real asset cash flow that occurs after the *j*'th asset cash flow is received ($d_0 = 0$). The series d_j for $j = 0,...,\infty$ is the asset's capacity depreciation schedule.[4] The capacity depreciation schedule specifies the asset's incremental contribution to potential production at every point in the useful service life. Typically, capacity depreciation is assumed exogenous and independent of utilization rates or maintenance expenditures.[5] For

[4] Depreciation and capital stock definitions herein follow the terminology of the U.S. Bureau of Labor Statistics (1979) and U.S. Bureau of Economic Analysis (1987).
[5] Two of the few models which assume depreciation is endogenous and depends upon utilization rates and maintenance expenditures as choice variables are Larry Epstein and Michael Denny (1980) and Moshe Kim and Giora Moore (1988).

example, with straight-line capacity depreciation over a ten-year service life d_j = 1/10 for j = 1,...,10 and d_j = 0 otherwise. The expected real asset cash flow accruing at time $s+t$ from the investment made at time s is

(4) $$c_{s,t} = c_s\left(1 - \sum_{j=1}^{t} d_{j-1}\right).$$

c_s is the time *s user cost of capital* and equals the asset cash flow produced by one unit of new real assets during first period of use (c_s is identical to $c_{s,1}$).

Second consider specification of the loan payment stream. Let γ denote the loan payment (interest, principal, and fees) to be paid at the end of the asset's first period of use, where γ is expressed as a proportion of the asset's supply price:

(5) $$\gamma_s = B_{s,1}/q_s .$$

The loan payment stream is summarized by the series b_j for j = 0,...,∞ , where b_j denotes the change in cash flow (as a proportion of $B_{s,1}$) that occurs after the j'th payment is made (b_0 = 0). More precisely,

(6) $$b_j = (B_{s,j} - B_{s,j+1})/B_{s,1}.$$

Equations 5 and 6 specify the entire loan payment stream, regardless of whether the debt contract represents a consol, a fixed payment amortized loan, a debenture with a balloon payment, or any other debt maturity structure.[6] The payment at time $s+t$ attributable to the loan issued to finance the time s investment is given by

(7) $$B_{s,t} = \gamma_s q_s \left[1 - \sum_{j=1}^{t} b_{j-1}\right].$$

[6] If the loan payment during the first period is zero, γ may be redefined; for example, with zero coupon debt of term T, $\gamma_s = B_{s,T}\, q_s^{-1}$.

Obtain the zero net present value equilibrium with explicit specification of the asset cash flow and loan payment streams by substituting equations 4 and 7 into 3:

$$(1-\alpha_s)q_s = \sum_{t=1}^{\infty}(1+\rho_s-\pi_s)^{-t}(1\text{-}\tau)c_s\left[1-\sum_{j=1}^{t}d_{j-1}\right]$$

$$(8) \qquad + \sum_{t=1}^{\infty}(1+\rho_s)^{-t}q_s\tau z_{s,t}$$

$$- \sum_{t=1}^{\infty}(1+\rho_s)^{-t}q_s\gamma_s\left[1-\sum_{j=1}^{t}b_{j-1}\right]$$

Equation 8 shows an equilibrium condition in which funds provided by equity equal the expected present value of the after-tax asset cash flow stream, plus the expected present value of tax savings from depreciation deductions, minus the present value of the loan payment stream, each discounted by the levered equity financing rate.[7]

Obtain the generalized user cost specification by simplifying and rearranging the equilibrium condition in equation 8:

$$(9) \qquad c_s = \frac{q_s(\rho_s-\pi_s)(1-\tau_s Z_s-\Lambda_s)}{(1-\tau_s)\,(1-\Delta_s)} \ .$$

The two new variables in equation 9, Λ and Δ, generalize intertemporal dynamics of financial structure and capacity depreciation, respectively.[8] Subsections below explore these two variables. First, however, glean general insights about user cost.

The user cost of capital, *c*, equals the immediate pre-tax cash flow an asset produces such that the net present value of the residual cash flow stream is zero. The user cost is found by specifying everything known or expected about the future. Conditioned on

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[7] Stewart Myers (1974) discusses a capital budgeting framework similar to the one above, in that the discounted value of asset cash flows is computed and subsequently the discounted value of financing costs is subtracted.
[8] Thomas Downs (1988) presents a user cost specification that generalizes capacity depreciation. That specification does not explicitly model financial structure.

expectations, then, static trade-offs exist among the user cost components. Alternative scenarios below highlight component roles.

Suppose for a one dollar asset (q = \$1) that taxes, inflation, and leverage are nil ($\tau = 0$, $\pi = 0$, and $\Lambda = 0$), and the real asset perpetually retains the same productive capacity as when new ($\Delta = 0$). For this scenario, $c = \rho$ implying that the user cost of capital equals the financial cost of capital. The only capital cost for this scenario is financing cost, so the real asset that generates a pre-tax cash flow equal to the equity financing rate (ρ) has a zero net present value.

Consider now a scenario in which inflation is positive ($\pi > 0$, $\tau = \Lambda = \Delta = 0$, and $q = \$1$). The user cost of capital equals the real equity financing rate, $\rho - \pi$. The user cost is less with inflation than without because a capital gain accrues to the real asset. For this scenario, the only capital cost still is financing cost but pre-tax cash flow of $\rho - \pi$ plus capital gain of π sum to the equity financing rate and net present value is zero.

Introduce capacity depreciation ($0 < \Delta < 1$, $\tau = \Lambda = \pi = 0$, and $q = \$1$). The user cost of capital equals $\rho/(1 - \Delta)$. An increasing depreciation rate raises Δ and increases the equilibrium user cost of capital. The most rapid depreciation occurs with the one period model in which real capital delivers one return and expires. For this scenario, as the subsequent subsection explains, $\Delta = 1/(1 + \rho)$ and $c = 1 + \rho$. Pre-tax cash flow consistent with zero net present value equals the supply price of capital plus the equity financing rate.

Finally, and most importantly, consider the effect of debt and taxes on the user cost. The term $(1 - \tau Z - \Lambda)$ from the generalized specification embodies benefits to equity of leverage and tax shields. The allowance of tax depreciation deductions reduces the equilibrium user cost by the amount of discounted tax savings (τZ). The user cost is lower for this scenario because the constant supply price equals the discounted value of after-tax cash flows, depreciation deductions shield cash flows from taxes and, in a Keynesian static equilibrium, pre-tax cash flow adjusts downward. Equilibrium pre-tax cash flow is less with tax benefits than without because the asset supply price capitalizes tax benefits.[9] Tax benefits descend like manna from heaven – they represent wealth creation.

Λ also embodies benefits pertinent to wealth creation. Debt financing, if all else were equal, increases Λ and reduces the equilibrium user cost because the debt interest rate is less than the equity financing rate. Deferral into the remote future of principal repayment reduces for equity the discounted cost of debt. The leverage benefit for equity increases as the *average period* of the loan payment stream increases.

The term $(1-\tau Z-\Lambda)$ measures for equity the reduction in capital costs from leverage and tax shields. The tax benefits descend from a government that the model leaves unspecified. Perhaps future research may introduce into this model an equilibrium condition on government and taxes, but to-date that relation remains unconstrained. The leverage benefit to equity *is* constrained, however, due to the irrelevance of financial structure to equilibrium user cost. Section III introduces an equilibrium condition on the debt market and explains trade-offs between financial structure and financing rates. Before going there, though, further explore Δ and Λ.

A. Capacity depreciation: Δ

Obtain Δ_s by discounting the capacity depreciation schedule with the real levered equity financing rate:

(10) $$\Delta_s = \sum_{t=1}^{\infty} (1+\rho_s - \pi_s)^{-t} d_t \ .$$

[9] Austan Goolsbee (1997) examines whether one dollar of investment tax benefits flows to the capital goods investor or supplier.

To focus on capacity depreciation suppose that equity is the only financing source. Thus, $\Lambda = 0$ and $r = \rho$. The generalized user cost specification from equation 9 reduces to equation 1 by restricting capacity depreciation to an infinite geometric time-path. This restriction implies $d_t=\delta(1-\delta)^{t-1}$ for every $t>0$. Subsequent simplification of equation 10 shows that for this special case

(11) $$\Delta_s = \delta(\rho_s + \delta - \pi_s)^{-1}$$

Substitute equation 11 into 9 and obtain equation 1. The Hall-Jorgenson user cost specification assumes productive capacity depreciates along an infinite geometric time-path.[10] Equation 9 accommodates, however, any time-path of capacity depreciation.

B. Financial structure: Λ

Obtain Λ_s by discounting the loan payment schedule:

(12) $$\Lambda_s = \alpha_s - \gamma_s \rho_s^{-1}(1-\lambda_s)$$, where

(13) $$\lambda_s = \sum_{t=1}^{\infty}(1+\rho_s)^{-t} b_t \ .$$

The generalized user cost specification from equation 9 reduces to the Hall-Jorgenson specification in two special cases.

The first special case is the one period model. The following sequence of events occurs: (1) At time s investment at price q occurs with equity and debt financing equal to $(1-\alpha)q$ and αq, respectively; (2) At time $s+1$ the asset delivers pre-tax cash flow equal to the user cost, the loan is fully repaid with an after-tax debt payment of $\alpha q(1 + (1-\tau)i)$, the residual accrues to equity, and the asset expires. No other cash flows attach to the time s capital investment.

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[10] Feldstein and Michael Rothschild (1973) vehemently criticize the Jorgenson investment model because, in addition to the explicit assumption of geometric capacity depreciation, there is an implicit assumption that real capital investment grows along a geometric time-path.

Parameterize the one period model with these settings: $d_1 = 1$ and $d_t = 0$ otherwise; $b_1 = 1$ and $b_t = 0$ otherwise; and $\gamma = \alpha(1 + (1\text{-}\tau)i)$. Solution of equations 10 and 12 shows, respectively, that $\Delta = (1+\rho\text{-}\pi)^{-1}$ and $\Lambda = \alpha[\rho - (1\text{-}\tau)i](1+\rho)^{-1}$. Substitute Δ_s and Λ_s into equation 9, simplify, and obtain the Hall-Jorgenson user cost (equation 1) containing the weighted average cost of capital (equation 2).[11]

The second special case occurs when, for the marginal capital investment, the periodic loan payment equals an amount that holds the loan-to-value ratio perpetually constant. This scenario parameterizes easily with geometric capacity depreciation. Thus, $d_t = \delta(1\text{-}\delta)^{t\text{-}1}$. The first loan payment, $B_{s,1}$, is comprised of interest and principal equal to $\alpha q(1\text{-}\tau)i$ and $\alpha q(\delta\text{-}\pi)$, respectively, implying that $\gamma = \alpha[(1\text{-}\tau)i+\delta\text{-}\pi]$. Loan payments evolve along the time-path given by $b_t = (\delta\text{-}\pi)(1\text{-}\delta+\pi)^{t\text{-}1}$, implying that $\lambda_s = (\delta\text{-}\pi)(\rho+\delta\text{-}\pi)^{-1}$. Solution of equations 10 and 12 shows, respectively, that $\Delta = \delta(\rho+\delta\text{-}\pi)^{-1}$ and $\Lambda = \alpha[\rho - (1\text{-}\tau)i](\rho+\delta\text{-}\pi)^{-1})$. Substitute Δ and Λ into equation 9, simplify, and obtain the Hall-Jorgenson user cost (equation 1) containing the weighted average cost of capital (equation 2).[12]

The Hall-Jorgenson user cost specification is valid only under unduly restrictive conditions. Equation 9 introduces a generalized specification that accommodates, for the marginal investment in real capital, any time-path of capacity depreciation and any financial

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[11] Simplification results in the Hall-Jorgenson user cost with $\delta=1$, plus an extra cross-product term: $\pi(\rho - r)(1+\rho)^{-1}$, with r per equation 2. This trivial term (about 10 basis points with plausible settings) vanishes by modeling with the conceptually equivalent approach of discounting the real loan payment with the real equity financing rate. I use the modeling in the text because the loan payment schedule and tax policies typically stipulate nominal cash flows. This comment also applies to the second special case in the subsequent paragraph.

[12] James Miles and John Ezzell (1980, pp. 728-729) establish a similar finding in a more restrictive setting: "That the textbook *WACC* ['weighted average cost of capital' per equation 2] yields correct valuations for either a single-period project or a project with level, perpetual cash flows is a consequence, not of project life *per se,* as has been argued in the literature, but rather of maintaining indirectly a constant leverage ratio." The paragraph in the text explains the *WACC* is correct when the perpetual cash flow changes along any geometric time-path, even a level one.

structure. Sections III, IV, and V analyze the general specification to glean insight on the term structure that capital values embody.

III. Relation Between Term Structure and Loan Type

The equilibrium user cost of capital equals the value marginal product of capital. The value marginal product is a function of production technology, input prices, and demand for the firm's product. None of these factors is affected by the capital structure of the firm; customers possess preferences about the price and quality of the product, not about the producer's leverage ratio [Joseph Stiglitz (1974)]. Due to the irrelevancy of financial structure, the user costs of capital for levered and unlevered producers must be equal.

The generalized user cost specification in equation 9 simplifies for the unlevered producer as follows:

$$(14) \qquad c_s = \frac{q_s(\rho_s^u - \pi_s)(1 - \tau_s Z_s^u)}{(1-\tau_s)\,(1-\Delta_s^u)} \quad ,$$

where ρ^u represents the unlevered equity financing rate. The terms Z^u and Δ^u are as defined previously except that the relevant discount rate now is ρ^u. Dynamic processes within the economy likely render ρ^u exogenous to any single producer who, by necessity, is a rate-taker as well as price-taker.[13] The generalized user cost for the levered producer in equation 9 equilibrates to the unlevered user cost in equation 14. Substitution and rearrangement of these two equations shows:

$$(15) \qquad (\rho_s - \pi_s)(1 - \tau_s Z_s - \Lambda_s) = \frac{(\rho_s^u - \pi_s)(1-\tau_s Z_s^u)(1-\Delta_s)}{(1-\Delta_s^u)} \quad .$$

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[13] The rate ρ^u actually attaches not to the producer but to the real capital good: "There are, therefore, theoretically just as many rates of interest expressed in terms of goods as there are kinds of goods diverging from one another in value." [Irving Fisher (1930): p. 42].

Equation 15 is an equilibrium condition from the debt market that assures financing decisions for the marginal capital investment are consistent with zero net present value equilibrium in the real asset and equity markets. The right-hand-side variables are largely exogenous and invariant to short-run production decisions. The left-hand side variable, Λ_s, depends on financing decisions for marginal capital investments. Λ_s equals zero in the absence of debt financing. As reliance on debt increases then Λ_s increases, too. Because the right-hand-side is basically constant, maintenance of equilibrium requires that ρ_s increase to offset the rising Λ_s.[14] In other words, an increase in debt financing for the marginal capital investment leads to an increase in the marginal levered equity financing rate.

Impose on equation 15 the two special cases from Section II. That is, impose parameters for either (a) the one-period model, or (b) the loan repayment schedule for the marginal investment that makes perpetually constant its loan-to-fundamental value ratio. Simplification shows:

(16)
$$\rho_s = \rho_s^u + \left(\rho_s^u - i_s\right)\left(\frac{\alpha_s}{1-\alpha_s}\right) .$$

Equation 16 is Modigliani and Miller's *Proposition 2* (1958) establishing that the levered equity financing rate is an increasing linear function of the debt-to-equity ratio.

The interrelation between financing rates for this special case is revealed in the total differential for the system. Differentiation of equation 16 (ρ^u is held constant) shows:

(17)
$$d\rho_s = \frac{\rho_s^u - i_s}{(1-\alpha_s)^2}\, d\alpha_s - \left(\frac{\alpha_s}{1-\alpha_s}\right) di_s .$$

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[14] Tax depreciation deductions retain influence through the variable *Z*. The levered equity financing rate depends, in other words, on the depreciation tax shield. Franco Modigliani and Merton Miller (1963) argue analogously that the levered equity financing rate depends on the interest tax shield. The discussion below, for simplicity, ignores effects of taxes on ρ.

A significant amount of information incorporated into equation 16 is not reflected in equation 17. For the Modigliani-Miller/Jorgenson models, there is no explicit association between changes in financing rates and debt maturity structure (b_j) or capacity depreciation (d_j). The geometric smoothing of the debt payment stream and real asset cash flow stream leads to a unique situation in which real information vanishes and becomes irrelevant. Indeed, with ρ^u fixed, the debt and equity financing rates are invariant to everything in the model except changes in α_s , the initial loan-to-value ratio. Debt maturity is irrelevant to equation 16 because α, once set, remains at its initial value. The two special cases implicitly assume that equilibrium financing rates are independent of term: they *presume* a flat yield curve.

Irrelevance of financial structure requires that the equilibrium user cost for the marginal investment is invariant to financing source. The special cases leading to equation 16 are interesting but overly restrictive and misleading. Equation 15, on the other hand, implies a more generally useful determinate relationship between interest rates for different loan payment streams – it also implies a term structure of interest rates. Extracting from the equilibrium condition the implied term structure of interest rates is a procedure analogous to several studies cited in Section II [e.g., Jorgenson and Auerbach (1980) and Hendershott (1981)] that extract from the user cost specification a variable besides equilibrium pre-tax cash flow.

Simple facts drive this term structure theory: (a) capital value embodies the discounted sum of residual cash flows, discounted by the relevant equity financing rate, plus the loan; (b) the equity financing rate exceeds the debt interest rate; and (c) the user cost of capital is invariant to financial structure. The facts imply that the yield curve has a natural upward slope because an increase in average period of debt offers a leverage benefit to equity, but the benefit cannot persist and must be offset. The pre-tax cash flow is constrained due to irrelevance of financial structure. So the financing rate affordable to this zero net present value investment rises. With respect to a specific unit of real capital, long-term debt requires a higher financing rate yet also provides a higher return on capital!

At equilibrium, the user cost for the marginal investment financed by, say, a 10-year corporate debenture equals the user cost for the investment financed by a 15-year fixed payment amortized loan. Equation 15 therefore implies a determinate relationship between the interest rates for a 10-year corporate debenture and a 15-year fixed payment amortized loan. Rates different than implied by equation 13 represent a state of disequilibrium because the marginal investment's net present value would be higher for some financing methods than others.

The remainder of this section analyzes for a variety of situations the equilibrium interest rate implied by equation 15; these endogenously derived rates are referred to as the *intrinsic* interest rates. Section IIIA discusses the results for the standard debenture and analyzes comparative statics, and Section IIIB obtains analogous results for the fixed payment amortized loan.

A. Term Structure for the Debenture

The debenture has a loan payment stream in which the borrower receives a lump sum from the lender, constant interest payments are made periodically throughout the life of the loan, and principal is repaid *in toto* with the last payment. This particular debt contract describes most bonds traded in the U.S. corporate and Treasury credit markets.

Consider equation 15 for a marginal investment financed by a debenture with a face value of $\alpha_s q_s$, a term of T, and a coupon rate of i_s (annual coupon, no sinking fund). Equity of $(1- \alpha_s$

$)q_s$ finances the remainder of the purchase price. The periodic interest expense for the debenture equals $i_s \alpha_s q_s$ and there is no repayment of principal until time $s+T$ when the principal is repaid *in toto*. The first payment, $B_{s,1}$, equals $i_s \alpha_s q_s$ and γ_s equals $i_s \alpha_s$. The subsequent payments $B_{s,2}$ through $B_{s,T-1}$ are the same size as the first, so $b_j = 0$ for $j = 0,...,T-2$. During period T the payment includes the coupon as well as the repayment of principal and $b_{T-1} = - i_s^{-1}$. After period T the payment drops to zero, so $b_T = i_s^{-1}(1+i_s)$. Substitution into equation 15 shows the debt market equilibrium condition for the debenture (τ equals zero for this illustration):

$$(18) \qquad (\rho_s - \pi_s)\left\{1 - \frac{\alpha_s}{\rho_s}(\rho_s - i_s)\left(1 - \frac{1}{(1+\rho_s)^T}\right)\right\} = \frac{(\rho_s^u - \pi_s)(1-\Delta_s)}{(1-\Delta_s^u)} \quad .$$

Comparative statics for equation 18 show the effect on financing rates of changes in term *(T)*. In lieu of algebraic formulations, however, numerical findings are reported given these parameter settings: the expected inflation rate (π) is set to 5 percent, the unlevered equity cost of capital (ρ^u) is 12 percent, the initial loan-to-value ratio (α_s) is 30 percent; and productive capacity is set to decline along a 15-year double-declining-balance schedule.[15] Furthermore, an equality constraint is imposed on the incremental debt and equity risk premia. That is, i and ρ satisfy equation 18 as well as:

$$(19) \qquad \rho - \rho^u = i - i^u .$$

where i^u represents the risk-free interest rate and is set to 8 percent. The equality constraint on the incremental risk premia stipulates that i and ρ increase by the same amount in

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[15] Charles Hulten and Frank Wykoff (1981) provide evidence this specification characterizes capacity depreciation for corporate real assets. For this setting: $d_j = (2/15)(1- 2/15)^{(j-1)}$ for $j = 1,...,14$; $d_{15} = 1-(d_1+d_2+...d_{14})$ and $d_j = 0$ otherwise.

response to increasing loan term.[16] Substitution of equation 19 into equation 18 reveals that as $\alpha \rightarrow 0$ then $\rho \rightarrow \rho^u$ and $i \rightarrow i^u$. As α rises above 0 disequilibrium occurs, but equilibrium is restored when i and ρ increase. The equality constraint on the risk premia stipulates that i and ρ increase by the same amount. i^u represents the risk-free interest rate and is set to 8 percent.

A.1. Debt Maturity (T) and Intrinsic Interest Rates ("Baseline Case")

The different interest rates listed in row 1 of Table 1 vary with the term of the debenture (T). When T in equation 18 is set at one year the intrinsic interest rate is 8.24 percent.[17] When the term of the debenture is increased to two years from one, the intrinsic interest rate rises 23 basis points; when term increases from 25 years to 30, the intrinsic interest rate rises only 3 basis points. Row 1 reveals that the intrinsic yield curve rises steeply at first and then flattens out; the slope of the intrinsic yield curve converges to zero.

The preceding implied interest rates are from a setting in which the expected inflation rate, the risk-free interest rate, and unlevered equity financing rate are perpetually constant at 5, 8, and 12 percent, respectively. Consequently, the yield curves are flat for nominal rates i^u and ρ^u as well as for real rates i^u-π and ρ^u-π. Nonetheless, the yield curve for the nominal interest rate is not flat – it has a normal shape, rising steeply at first and then flattening out.

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[16] The constraint assumes that a change in debt maturity causes equal increases in the incremental debt and equity risk premia. The resultant "implied term structure of interest rates" is coincident with the "implied term structure of levered equity financing rates for different debt term." Probably business cycle and investor sentiment affects whether incremental risk premia for debt and equity are actually equal. Regardless, the resultant implied term structures of debt and equity financing rates have an upward slope. The constraint, in other words, is illustrative not essential.

[17] Due to the equality constraint on the risk premia, ρ is 12.24 percent (ρ^u is exogenously fixed at 12 percent).

A.2. Initial Loan-to-Value Ratio and Intrinsic Interest Rates

The initial loan-to-value ratio (α_s) is changed to 40 percent, instead of the 30 percent used for the baseline case discussed above. Row 2 of Table 1 reveals that both the absolute level and the steepness of the intrinsic yield curve increase. The intrinsic interest rate is 8 basis points higher than in the baseline case for a 1-year debenture, 52 basis points higher than baseline for a 10-year debenture, and 63 basis points higher than baseline for a 30-year debenture. The intrinsic yield curve still flattens out, it just rises to the top quicker. The implication is that for business environments that are unusually debt-laden, the interest rate yield curve is relatively steep.

A.3. Equity Risk Premium and Intrinsic Interest Rates

The unlevered equity cost of capital (ρ^u) is increased to 14 percent, instead of the 12 percent used for the baseline case. This increases the equity risk premium since the risk-free interest rate (i^u) is unchanged at 8 percent. Row 3 of Table 1 reveals that the effect of this change is to increase both the absolute level and the steepness of the intrinsic yield curve. The intrinsic interest rate is 15 basis points higher than baseline for a 1-year debenture, 75 basis points higher than baseline for a 10-year debenture, and 81 basis points higher than baseline for a 30-year debenture. Again, the intrinsic yield curve rises quickly and then flattens out. The implication is that for business environments in which equities are perceived to be unusually risky, the interest rate yield curve is relatively steep.

A.4. Inelastic Equity Financing Rate and Intrinsic Interest Rates

The equality constraint on the debt and equity risk premia is removed and, instead, the levered equity financing rate (ρ) is exogenously fixed at a constant. For this scenario, the supply of equity financing is inelastic and the source of debt financing is elastic. Hence, due to an increase in debt maturity only the interest rate (i) adjusts to re-establish equilibrium; ρ does not move. Inspection of row 4 reveals that the slope of the intrinsic yield curve is very sensitive to the rate at which exogenous ρ is pegged. With ρ fixed at 12 percent, for example, the 1-year, 5-year, and 30-year intrinsic interest rates (row 4a) are 11.83 percent, 11.96 percent, and 11.98 percent, respectively. With ρ fixed at 12.5 percent, however, those respective intrinsic interest rates (row 4b) are 4.76 percent, 10.49 percent, and 11.58 percent. Finally, with ρ fixed at 13 percent (row 4c) the 1-year intrinsic interest rate is -2.43 percent (a rebate model), the 5-year rate is 9.04 percent, and the 30-year rate is 11.14 percent. The implication is that with a costly and inelastic supply of equity financing, the interest rate yield curve is relatively steep.

B. Term Structure and the Fixed Payment Amortized Loan

This subsection solves the model for the corporation financing marginal real investments with a fixed payment amortized loan of term T, coupon rate i, and principal $\alpha_s q_s$. The first loan payment ($B_{s,1}$) equals $\alpha_s q_s / PVIFA_{i,T}$, where the denominator is the present value interest factor for an annuity, $i^{-1}[1-(1+i)^{-T}]$. γ therefore equals $\alpha_S / PVIFA_{i,T}$. The subsequent payments $B_{s,2}$ through $B_{s,T}$ are the same size as the first, so b_j=0 for j=0,...,T-1. After period T, the payments drop to zero and, according to equation 7, b_T=1.0. The λ term is computed by discounting the b_j's as specified in equation 13, showing that $\lambda=(1+\rho)^{-T}$. Simplification of the equilibrium condition between levered and unlevered user costs yields:

$$(20) \qquad (\rho_s - \pi_s)\left\{1-\alpha_s\left(1-\frac{PVIFA_{\rho,T}}{PVIFA_{i,T}}\right)\right\} = \frac{(\rho_s^u - \pi_s)(1-\Delta_s)}{(1-\Delta_s^u)} \quad .$$

Equation 20 is solved by using the same baseline settings as before for ρ^u, π, α, and the capacity depreciation schedule (d_j). The equality constraint for debt and equity risk premia from 19 also is imposed and i^u is set as before. T is fixed at an integer constant and the equilibrium risk premium is isolated. Table 2 presents the estimates.

A 1-year loan has the same maturity structure and intrinsic interest rate regardless of whether the debt contract is a debenture or a fixed payment amortized loan; with the parameter settings discussed above the intrinsic interest rate for a 1-year loan is 8.24 percent. The rise in the intrinsic interest rate due to an increase in the term of the loan beyond one year, however, depends upon the type of debt contract. For a 2-year fixed payment amortized loan the intrinsic interest rate is 12 basis points higher (8.36 percent), nearly half the term premium of 23 basis points required for the 2-year debenture (8.47 percent, Table 1); at this term the intrinsic yield curve is twice as steep for the debenture as for the fixed payment loan.

The equilibrium interest rate for a 15-year fixed payment amortized loan is 9.43 percent, about 41 basis points lower than the rate for a 15-year debenture. The intrinsic interest rate is lower for the fixed payment amortized loan than for the debenture because of differences in the time path of the loan-to-value ratio. Even though initially the same for either financing arrangement, the loan-to-value ratio declines over time at a faster rate for the fixed payment loan than for the debenture. Equation 15 enables point estimates indicating that for 15-year loans the financial markets require an additional 41 basis points per annum as compensation for the incremental risk inherent with the debenture.

The comparative static inferences obtained for the debenture (Table 1) with respect to other variables in the model also are obtained for the fixed payment amortized loan and are presented in Table 2. The effects on the intrinsic interest rate of the initial loan-to-value ratio, equity risk premium, and the inelasticity of the equity financing rate are qualitatively the same for the fixed payment amortized loan and the debenture. Hence, the inferences about the interest rate yield curve discussed in the previous subsection seem fairly robust to debt maturity structure.

IV. Effects of Real Asset Capacity Depreciation on the Yield Curve

The model links the yield curve for the nominal interest rate to the characteristics of the underlying real assets. Equation 15 allows point estimates of differences for intrinsic interest rates between long and short-life assets, or between rapidly-depreciating and slowly depreciating assets. Insights are gleaned by using the model for deducing the sensitivity of the intrinsic interest rate to the depreciation attributes of the underlying real asset.

A. Effect of the Asset Service Life

Panel A of Table 3 lists the intrinsic yield curve when productive capacity declines along the 15-year double-declining-balance schedule discussed previously. Row 1 presents estimates of the equilibrium interest rate when the asset is new; this is the baseline case for the debenture from Table 1. Row 2 shows the financing rates when 10-years of asset life remain. The estimates in rows 1 and 2 are consistent with zero net present value investments in real assets. The real assets for the two cases are identical except that for row 1 the asset is new and promises a 15-year asset cash flow stream whereas for row 2 the asset is 5-years old and promises a 10-year cash flow stream. Likewise, row 3 lists the intrinsic interest rates for a zero net present value investment in a 10-year old asset promising a 5-year cash flow stream.

Comparison of rows 1, 2, and 3 reveals that as the asset life shortens, the intrinsic yield curve rises and steepens. For example, the interest rate on a 1-year debenture consistent with the zero net present value investment equilibrium is 8.24 percent for a new asset, 8.29 percent for a 5-year old asset, and 8.44 percent for a 10-year old asset. If, on the other hand, a 5-year debenture is used for purchasing the assets, the intrinsic interest rates are 9.01, 9.20, and 9.77 percent, respectively. The yield curve slope is 73 percent greater for the 10-year old asset than for the new asset. The implication is that for environments characterized by a preponderance of short-life or old assets, the yield curve is relatively steep.

B. Effect of the Depreciation Rate

Alternative specifications for the capacity depreciation schedules (the d_j's) are utilized to gauge the effect of the depreciation rate on the intrinsic yield curve. Panel B of Table 3 presents the intrinsic yield curves for the debenture and the fixed payment amortized loan when the real asset's productive capacity declines by a straight-line pattern throughout a 15-year service life.[18] Straight-line depreciation proceeds at a slower rate than double-declining-balance. Nonetheless, comparison of Panel B with the baseline estimates from Tables 1 and 2 reveals that with straight-line depreciation the intrinsic yield curve is within 10 basis points of baseline for every term.

Similar results are reported in Panel C for the case in which the real asset's capacity depreciation schedule is decelerated even further. These estimates reflect a 15-year one-hoss-shay pattern (the "all or none" pattern characteristic of lightbulbs).[19] The intrinsic yield

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[18] For this situation d_j=1/15 for j=1,...,15 and d_j=0 otherwise.

[19] For this situation d_j=1 for j=15 and d_j=0 otherwise.

curve is slightly flatter for this decelerated capacity depreciation schedule. In general, however, Panels B and C establish that the upward slope on the intrinsic yield curve is fairly invariant to the capacity depreciation rate of the underlying real asset.

V. Conclusion

This study explains interest rates by focusing on the zero net present value equilibrium condition in the market for real assets. The analysis relies on a user cost framework that incorporates debt maturity structure. Since the existence of market equilibria implies that the net present value of the marginal investment is independent of the financing method, the user costs for alternative debt contracts are equal. The user cost specification therefore implies a determinate relationship between interest rates on alternative debt contracts.

The current study derives a specification for the user cost of capital that generalizes intertemporal dynamics of capacity depreciation and financial structure. Analysis of the specification yields an important insight: the term structure of interest rates that capital value embodies has a normal upward slope. Capital is a store of wealth. The wealth inside the store equals the present value of expected future cash flows to capitalists. The store of wealth depletes as times elapses and cash flows return to financing sources. When the average period of the marginal debt financing stream lengthens, the burden on the store diminishes and capital supports a higher rate of interest. Long-term interest rates naturally are higher than short-term rates, even with level inflation expectations or in the absence of stochastic interest rate processes and market segmentation.

Equilibrium interest rates for a variety of environments and debt contracts are estimated. Also, the sensitivity of the estimates to parameter settings is examined. A novel insight is that capital market financing rates link to average period of the underlying debt contract. As debt average period increases, so does the equilibrium interest rate.

Comparative static results for this equilibrium model suggest that the yield curve is relatively steep when:

- the business environment is unusually debt-laden,
- equities are perceived to be unusually risky,
- sources of equity financing are relatively costly and inelastic, and
- a preponderance of fixed capital is short-life or old assets.

Additional avenues of research are possible. Foremost, empirical analyses should investigate how well the tax-augmented model explains observed interest rate yield curves for different types of debt contracts or for industries possessing different types of real assets. Next, the model presented herein assumes an exogenous and flat yield curve for the risk-free interest rate and for the unlevered equity cost of capital. A general equilibrium model should be developed that endogenizes these core financing rates.

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TABLE 1: Intrinsic interest rates for debentures of varying term

Term of the Loan (years)									
1	2	3	4	5	10	15	20	25	30
1. Baseline case									
8.24	8.47	8.67	8.85	9.01	9.56	9.84	9.98	10.06	10.09
2. Increase loan-to-value ratio to 40% from 30%									
8.32	8.63	8.90	9.14	9.36	10.08	10.44	10.61	10.70	10.74
3. Increase unlevered equity risk premium to 6% from 4%									
8.39	8.74	9.05	9.32	9.56	10.31	10.65	10.80	10.87	10.90
4. Set the levered equity financing rate at a constant									
4a: at 12.0%									
11.83	11.91	11.94	11.95	11.96	11.98	11.98	11.98	11.98	11.98
4b: at 12.5%									
4.76	8.23	9.49	10.12	10.49	11.20	11.42	11.51	11.55	11.58
4c: at 13.0%									
-2.42	4.64	7.09	8.31	9.04	10.43	10.84	11.02	11.10	11.14

Notes:
The stipulated changes are not cumulative; baseline conditions are reset for every case.
In all cases productive capacity depreciates along a 15-year double-declining-balance schedule.
The interest rate (i) and equity financing rate (ρ) are endogenously derived from equation 15.
The exogenous baseline settings include:

- ρ^u = .12, *unlevered equity cost of capital*
- α = .30, *proportion of the investment financed by debt*
- π = .05, *expected inflation rate*
- i^u = .08, *risk-free interest rate*

TABLE 2: Intrinsic interest rates for fixed payment loans of varying term

	Term of the Loan (years)									
	1	2	3	4	5	10	15	20	25	30
1. Baseline case	8.24	8.36	8.47	8.58	8.68	9.11	9.43	9.66	9.82	9.93
2. Increase loan-to-value ratio to 40% from 30%	8.32	8.48	8.63	8.78	8.91	9.49	9.92	10.22	10.42	10.55
3. Increase unlevered equity risk premium to 6% from 4%	8.39	8.57	8.74	8.91	9.06	9.69	10.13	10.42	10.61	10.73
4. Set the levered equity financing rate at a constant										
4a: at 12.0%	11.83	11.88	11.91	11.93	11.94	11.97	11.97	11.98	11.98	11.98
4b: at 12.5%	4.76	6.97	8.25	9.02	9.54	10.70	11.11	11.32	11.43	11.50
4c: at 13.0%	-2.42	-.01	4.60	6.11	7.12	9.41	10.23	10.62	10.85	10.98

Notes:

The stipulated changes are not cumulative; baseline conditions are reset for every case.

In all cases productive capacity depreciates along a 15-year double-declining-balance schedule.

The interest rate (i) and equity financing rate (ρ) are endogenously derived from equation 15.

The exogenous baseline settings include:

ρ^u = .12, *unlevered equity cost of capital*

α = .30, *proportion of the investment financed by debt*

π = .05, *expected inflation rate*

i^u = .08, *risk-free interest rate*

TABLE 3: Effects of capacity depreciation on the intrinsic interest rates

Term of the Loan (years)									
1	2	3	4	5	10	15	20	25	30
PANEL A: Real productive capacity depreciates along a 15-year double-declining-balance schedule									
1. Debenture loans when the asset is new (baseline case)									
8.24	8.47	8.67	8.85	9.01	9.56	9.84	9.98	10.06	10.09
2. Debenture loans when 10-years of asset life remain									
8.29	8.56	8.80	9.01	9.20	9.83	10.15	10.31	10.38	10.42
3. Debenture loans when 5-years of asset life remain									
8.44	8.84	9.20	9.51	9.77	10.66	11.07	11.27	11.36	11.40
PANEL B: Real productive capacity depreciates along a 15-year straight-line schedule									
4. Debenture loans when the asset is new									
8.23	8.44	8.63	8.81	8.96	9.48	9.75	9.89	9.96	9.99
5. Fixed payment amortized loan when the asset is new									
8.23	8.34	8.44	8.55	8.64	9.05	9.36	9.58	9.73	9.83
PANEL C: Real productive capacity depreciates along a 15-year one-hoss-shay schedule									
6. Debenture loans when the asset is new									
8.16	8.31	8.45	8.57	8.68	9.07	9.27	9.38	9.44	9.47
7. Fixed payment amortized loan when the asset is new									
8.16	8.24	8.31	8.39	8.45	8.75	8.97	9.14	9.25	9.33

Notes:

The stipulated changes are not cumulative; baseline conditions are reset for every case.

The interest rate (i) and equity financing rate (ρ) are endogenously derived from equation 15.

The exogenous baseline settings include:

ρ^u = .12, *unlevered equity cost of capital*

α = .30, *proportion of the investment financed by debt*

π = .05, *expected inflation rate*

i^u = .08, *risk-free interest rate*