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Using the User Cost

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Most studies employing the user cost of capital are inconsistent because their user cost assumes capital has an infinite life and depreciates geometrically, whereas they employ capital stock estimates that assume finite life and nongeometric depreciation. This study examines the significance of the inconsistency and derives a user cost formula that eliminates it.

Introduction

Since Dale Jorgenson (1963) explained how to measure the user cost of capital, it has been used in hundreds of studies. These studies have typically employed the user cost in one of two types of analyses. First, because the user cost is linked to the optimal size of the fixed capital stock and changes in the optimal stock are related to investment incentives, the user cost may be employed in the estimation of an investment function. In fact, the user cost has become the symbol of the "neoclassical investment function." Examples of this type of analysis are Jorgenson's (1963) seminal article and Hendershott and Hu (1981). Second, because the user cost represents the price of capital services, a comparison of one asset's user cost and stock size with another asset's user cost and stock size can reveal insights about resource usage and allocation. For example, Berndt and Christensen (1973) and Field and Grebenstein (1980) employ the user cost to analyze elasticities of substitution among factors of production, and Hendershott and Hu (1980) and Fullerton and Gordon (1983) employ the user cost to analyze the ways in which government tax policy distorts the distribution of capital throughout the U.S. economy.

Most of these studies are similar in that they empirically measure the user cost following the methodology outlined by Hall and Jorgenson (1967) and then run an analysis that relates the user cost to some measure of the capital stock. This introduces an inconsistency because the Hall-Jorgenson formulation assumes that capital has an infinite life and depreciates geometrically, whereas virtually all capital stock estimates assume that capital has a finite life and depreciates along a nongeometric path. In other words, many studies have employed a user cost derived under assumptions inconsistent with the assumptions used to derive their capital stock estimates. This study examines the significance of this inconsistency and, more importantly, derives a user cost formula that eliminates it.

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1. Development of the User Cost: Historically and Structurally

Although the user cost's fame is attributable to the modern contributions of Jorgenson, it was introduced to the economic profession long before that. Bohm-Bawerk (1888) described an asset's value as being equal to the discounted value of its capital service stream. He originally introduced the user cost concept with an example wherein an asset is expected to produce a stream of services worth \$100 per period:

> In the first year of its use the owner realises the "current" service with its value of 100. Naturally this service, thus consumed or rendered, comes off the value of the machine (which we may call the "bearer of the use"), and the good suffers a loss of value (p. 343).

Roughly a half century later, Keynes (1936) restated and renamed the concept: "Let us call this quantity, ... which measures the sacrifice of value involved in the production of A the user cost of A" (p. 53). Keynes envisioned that the returns to capital "consists of a series of user costs and profits earned in successive periods" (p. 73).

Jorgenson (1967) restated the concept within the framework of contemporary mathematical economics. If $c(t)$, the user cost, is the value of capital services at time t, then the competitive equilibrium price of capital, $q(s)$, equals the series of discounted user costs[1] as in

$$q(s)=\int_{s}^{\infty} e^{-(r+\delta)(t-s)}c(t)\ dt. \tag{1}$$

This equality characterizes the marginal capital investment as a 0 net present value alternative. Solving for the user cost of capital through differentiation with respect to time s shows

$$c(s)=q(s)(r+\delta-\pi),$$

where r is the discount rate, δ is the depreciation rate, and π is the expected inflation rate.

The user cost in the presence of taxes is derived by Hall and Jorgenson (1967). Let v be the effective rate of the investment tax credit allowed on investment expenditure, τ the marginal income tax rate, and $z(t)$ the proportion of the supply price taken as a tax depreciation deduction t periods after acquisition. The equality between the price of capital goods and the discounted value of capital services is

$$q(s)=vq(s)+\int_{s}^{\infty} e^{-r(t-s)}[(1-\tau)c(s)e^{-(\delta-\pi)(t-s)}+\tau q(s)z(t-s)]\ dt. \tag{2}$$

The user cost may be isolated as

$$c(s)=\frac{q(s)(r+\delta-\pi)(1-v-\tau Z)}{(1-\tau)}, \tag{3}$$

where Z is the present value of allowable depreciation deductions per dollar of investment; i.e., $Z=\int_0^\infty e^{-rt}z(t)\,dt$. Equation (3) is the standard Hall-Jorgenson formulation and assumes that capital has an infinite life and depreciates along an exponential path.

In order to modify the user cost formula so that it accommodates finite life capital that depreciates along a non-exponential path, let $h(j)$ be the percentage of the asset's original

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[1] Keynes' reference to "profits" in the previous quotation can be interpreted as a reference to the accounting profits qr. Economic profits are 0. Jorgenson's formulation, as did Bohm-Bawerk's, has the "profit" embedded within the user cost.

productive capacity lost at the jth moment after acquisition.[2] The cash net operating income generated by the marginal investment t periods after acquisition is $c(s)e^{\pi t}[1 - \int_0^t h(j)\,dj]$. The supply price is equated to the discounted value of capital services as

$$q(s) = vq(s) + \int_s^\infty e^{-r(t-s)} \Big\{ (1-\tau)c(s)e^{\pi(t-s)} \cdot \left[1 - \int_0^{t-s} h(j)\,dj \right] + \tau q(s) z(t-s) \Big\}\, dt. \tag{4}$$

The user cost may be isolated as

$$c(s) = \frac{q(s)(r-\pi)(1-v-\tau Z)}{(1-H)(1-\tau)}, \tag{5}$$

where

$$H = \int_0^\infty e^{-(r-\pi)j} h(j)\,dj.$$

Equation (5) is a general expression for the user cost that does not restrict capital to any particular depreciation path. Notice, though, that if capital has infinite life and depreciates along an exponential path, $h(j) = \delta e^{-\delta j}$, in which case $H = \delta/(r + \delta - \pi)$, and

$$c(s) = \frac{q(s)(r-\pi)(1-v-\tau Z)}{[1-\delta/(r+\delta-\pi)](1-\tau)}$$

$$= \frac{q(s)(r+\delta-\pi)(1-v-\tau Z)}{(1-\tau)}$$

This is the standard Hall-Jorgenson result. Thus, Equation (5) may be reduced to the standard result by imposing restrictive depreciation assumptions on capital.

2. Empirical Significance of the Depreciation-Path Specification

Imposing a depreciation path on the user cost that is inconsistent with the depreciation path inherent in the underlying asset is equivalent to misspecifying the marginal investment's expected return stream. Such an inconsistency violates the equilibrium assumption of marginal zero net present value. In order to determine whether or not this violation is empirically significant, this section reproduces a recent study by Feldstein (1982) that employs the user cost of equipment as constructed by the Hall-Jorgenson methodology. The Feldstein study contains the inconsistency discussed above because its user cost estimates assume that depreciation follows an infinite geometric path, whereas the capital stock estimates employed assume that capital depreciates along a finite straight-line path. The

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[2] Hall and Jorgenson derive the user cost in the presence of income taxes, depreciation deductions, and an investment tax credit. They assume constant prices. Coen (1975) derives the user cost under conditions of nongeometric depreciation, still maintaining prices constant. Hendershott and Hu (1980) derive the user cost given inflation, but geometric depreciation. The derivation herein combines these effects.

results obtained in the Feldstein reproduction are compared to those obtained when the "capital-consistent" user cost series constructed from Equation (5) is employed.

The reproduced Feldstein user cost[3] is \$21.51 in 1976. This estimate implies that the marginal capital acquisition in 1976 was expected to generate net operating income before depreciation, denoted NOI, of \$0.2151 per dollar of investment during its first period of use. The net return equals the NOI [$= c$] less gross taxes of \$10.32 [$= \tau c$] plus the investment tax credit of \$8.23 [$= vq$] plus \$10.28 from the tax savings on depreciation deductions [$= \tau q z(1)$; $z(1)$ equals 0.211 under the tax depreciation schedule described in footnote 3] and is \$29.70. During the second period of use the NOI would be expected to have increased to \$22.63 [$= (1 + \pi)c$], except that capacity depreciation will have occurred. In the Hall-Jorgenson methodology, depreciation is geometric so that the NOI would be \$19.51 [$= (1 + \pi)(1 - \delta)c$]. Gross taxes are \$9.36 and the depreciation tax shield is worth \$8.54 [$= \tau q z(2)$; $z(2)$ equals 0.175]. Thus, the second period net return consistent with the Hall-Jorgenson user cost is \$18.69.

In general, the Hall-Jorgenson formulation assumes that the net return expected during the tth period of use is equal to the prediscounted right-hand side of Equation (2), or, rather, its discrete counterpart, evaluated at time $s + t$. Equation (2) may be used to generate an infinitely long net return stream. The discounted value of the complete net return stream equals capital's supply price. Constructing the user cost from Equation (3) is assurance that the equality will hold. Because equipment has a finite life, however, the length of the expected net return stream is also finite; the implicit Hall-Jorgenson stream would be truncated, and the present value would not sum to the supply price, thereby violating the equilibrium assumption of zero net present value.

Column 1 of Table 1 presents the net returns of the implicit Hall-Jorgenson stream for the first 14 periods of use given the data in 1976. Fourteen years is a reasonable estimate of the average service life for the equipment capital stock estimates employed by Feldstein.[4] The discounted value of the 14-period net return stream is \$95.04, or about 6 1/2% less than the supply price of \$101.56, implying that if the user cost were actually \$21.51, then the marginal investment would have had a negative net present value.

In order to compute a user cost completely consistent with the capital stock estimates, one must use Equation (5) and compute H given a straight-line path of capacity depreciation; i.e., $h_j = 1/14$ for $j = 1, \cdots, 14$. In this case, H equals 0.698 and substitution into Equation (5) shows the user cost is \$19.50, roughly 10% less than the \$21.51 user cost of infinite life capital. The net return stream expected from the marginal investment given a user cost of \$19.50 and straight-line depreciation over a 14-year life is listed in column 2 of Table 1. The

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[3] The data used to reconstruct Feldstein's results are, if possible, identical to his. The series he did not publish have been obtained from the sources he listed. The correlation coefficient between his user cost series and my reconstruction over the 1954–1977 sample period is 0.985. The slight differences are apparently due to revisions having occurred since his collection. For the reconstructed user cost in 1976, $q = 101.56$, $r = 0.104$, $\pi = 0.052$, $v = 0.081$, $\tau = 0.48$, and $\delta = 0.138$. The present value of tax depreciation deductions, Z, is computed given a tax life of nine years and depreciation accounting equally divided between sum-of-years digits and double-declining balance with a switch to straight line at the midlife. In 1976, the reconstructed Z equals 70.70.

[4] The BEA capital stock estimates (1976) used in the Feldstein study assume a service life equal to 85% of the *Bulletin F* lives. A weighted average of these lives, using the BLS (1979) asset weights, results in an average service life of 14 years.

Table 1. Net Return Streams Under Alternative Depreciation Schedules

Period After Acquisition	Infinite Life, Geometric Path	14-Year Life, Straight-Line Path
1	\$29.70	\$28.66
2	18.69	18.45
3	16.27	16.69
4	14.14	15.07
5	12.25	13.56
6	11.01	12.55
7	9.83	11.46
8	8.71	10.30
9	6.65	8.05
10	4.64	5.71
11	4.21	4.81
12	3.81	3.79
13	3.46	2.66
14	3.14	1.40
Present Value	\$95.04	\$101.56
Net Present Value	−6.52	0.00

present value of the stream is 100% of the supply price, and the equilibrium condition of zero net present value is satisfied.[5]

Whether or not the user cost estimate resulting from Equation (5) is significantly different from the Hall-Jorgenson estimate from Equation (3) depends, of course, upon how the analysis employs the user cost. When the user cost (and stock size) of one asset is compared to the user cost of other assets, the important point may be how the asset-specific user costs are affected by the change in formulation. For example, the user cost of equipment consistent with capital stock estimates is shown above to be 10% less than the Hall-Jorgenson formulation. For structures, the user cost consistent with capital stock estimates is 32% larger than the Hall-Jorgenson formulation.[6] Thus, the standard formula tends to understate the user cost for some assets while overstating it for others. Although this potentially important interasset bias does not affect the Feldstein study, an implication is that researchers employing

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[5] In fact, with a user cost of \$19.50, the present value is \$98.54, or 97% of the supply price. This discrepancy arises because Equation (5) is derived in continuous time, which assumes the investment tax credit and first tax depreciation deduction occur simultaneously with the asset acquisition. Hence, they are assumed undiscounted. In discrete time, the tax credit and first depreciation deduction occur one period after acquisition and are discounted. Computing the user cost with Equation (5) modified so that the numerator contains $1 - [(v + \tau Z)/(1 + r)]$ instead of $[1 - (v + \tau Z)]$ results in a user cost of \$20.77 and a discounted value of the net return stream that is 100% of the supply price. The empirical work presented herein employs Equation (5) as it appears in the text because of its close connection to the widely used Hall-Jorgenson formula.

[6] The Hall-Jorgenson structures user cost is \$14.17 and takes r, q, τ, and π from Feldstein; Z is 0.24 and is from the SSRC model (1983); and δ is 0.03 and is from Feldstein and Summers (1978). The structures user cost consistent with structures capital stock estimates is \$18.75 and computes H given straight-line depreciation over a 28-year service life.

estimates of user costs for more than one asset may be very sensitive to the methodology used to construct their user costs. A researcher estimating elasticities of substitution or analyzing the distortionary impact of government tax policy on resource allocation may have significantly different results if he or she were to switch from a Hall-Jorgenson type user cost to a user cost consistent with capital stock estimates.

The Feldstein study is not sensitive to this interasset bias, because it employs the user cost of equipment to estimate an equipment investment function. To see the change in results induced by a switch from the Hall-Jorgenson user cost (Equation (3)) to a capital-consistent user cost (Equation (5)) in the Feldstein study, generating both user cost series annually over the 1954–1977 sample period is necessary. Column 1 of Table 2 lists the reconstructed Hall-Jorgenson series and column 2 the capital-consistent series. That the standard user cost overstates the capital-consistent user cost by around 9% with little intertemporal variation is striking. Because the correlation coefficient between the two series is 0.999, the substitution of one series for the other in an investment regression has little affect on the summary statistics.

Table 2. User Cost of Equipment Under Alternative Depreciation Schedules

Year	Infinite Life, Geometric Path	14-Year Life, Straight-Line Path
1954	0.233	0.213
1955	0.230	0.211
1956	0.226	0.208
1957	0.242	0.221
1958	0.223	0.205
1959	0.223	0.205
1960	0.228	0.210
1961	0.211	0.195
1962	0.225	0.205
1963	0.217	0.198
1964	0.201	0.183
1965	0.202	0.184
1966	0.221	0.200
1967	0.215	0.195
1968	0.218	0.199
1969	0.248	0.224
1970	0.257	0.232
1971	0.211	0.193
1972	0.203	0.185
1973	0.219	0.199
1974	0.241	0.216
1975	0.233	0.210
1976	0.215	0.195
1977	0.250	0.224

The investment function estimated by Feldstein is[7]

$$I^g_t = \alpha + \beta \sum_{j=1}^{5} \omega_j[(p/c)_{t-j}Q_{t-j} - (p/c)_{t-j-1}Q_{t-j-1}] + \gamma K_{t-1} + \epsilon_t, \tag{6}$$

where α, γ, and $\beta\omega_j$, $j = 1, \cdots, 5$, are estimated coefficients and ϵ is a normally distributed residual error. Table 3 shows the results obtained by estimating Equation (6) with the two user cost series listed in Table 2. Two points deserve mention. One is that the sum of lag weights drops from 0.0458 with the Hall-Jorgenson user cost down to 0.0422 with the capital-consistent user cost. This 9% decline occurs because the 9% average decline in user cost increases the independent variable by roughly 9%. The decline in estimated parameter roughly offsets the increase in independent variable. The second point is that the t statistics and R bar^2 are virtually identical in the two regressions. Even though the sum of squared errors is less with the capital-consistent series than with the Hall-Jorgenson series, the decline is insignificant. The bias inherent in the Hall-Jorgenson user cost is stable over time. Therefore, the change in formulation affects the magnitude of the estimators but not their significance or the equation's goodness of fit.

Table 3. Regression Results for the Estimation of Equation (6)

	User Cost Series	
Parameter	Hall-Jorgenson Infinite Life, Geometric Path	Capital-Consistent, 14-Year Life, Straight-Line Path
α	−1.7	−1.5
	(1.1)	(1.0)
$\beta\omega_1$	0.0121	0.0114
(t Stat)	(2.6)	(2.7)
$\beta\omega_2$	0.0171	0.0161
(t Stat)	(5.3)	(5.4)
$\beta\omega_3$	0.0122	0.0112
(t Stat)	(3.0)	(3.2)
$\beta\omega_4$	0.0043	0.0035
(t Stat)	(0.9)	(0.8)
γ	0.1735	0.1720
(t Stat)	(15.6)	(15.1)
R^2	0.987	0.987
SSE	109.8	108.0

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[7] Q is measured by the gross domestic product of nonfinancial corporations, p is the implicit price deflator for that output, K is the stock of equipment in nonfinancial corporations, and I^g is the gross investment in that equipment. The corrected series discussed above is computed with the relative price of capital goods and therefore already rejects fluctuations in p. The lagged weights are estimated with a zero endpoint constraint; $\omega_5 = 0$.

3. Summary

Previous empirical studies employing the Hall-Jorgenson user cost are inconsistent because their user cost assumes infinite life and geometric depreciation, whereas capital stock estimates assume finite life and nongeometric depreciation. A formula was derived in this article that allows computation of the user cost regardless of service life or depreciation path, thereby eliminating the inconsistency.

The empirical evidence indicates that the Hall-Jorgenson user cost overstates the price of the capital services of equipment by 10% and understates the price of the capital services of structures by 32%. Studies comparing one asset's user cost and stock size with those of other asset's might have substantially different empirical results if they were to employ a capital-consistent user cost instead of the Hall-Jorgenson formulation. In a reproduction of a recent study presenting an investment function for equipment, I have shown that replacing the standard user cost series with a capital-consistent user cost series reduces the estimated coéfficient by 9%. This substitution, although not affecting the summary statistics, does affect forecasts and predictions based upon the estimated equation.

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