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BETA, SIZE, RISK, AND RETURN

Thomas W. Downs and Robert W. Ingram
The University of Alabama

Abstract

We relate the cross-section of stock returns to firm size, beta, and total risk. We find that as extreme monthly security returns are censored from the data, the significance level decreases rapidly for the size variable and increases for beta and total risk. An analysis of up and down markets reaffirms our findings. Consequently, average returns relate positively with beta, negatively with total risk, and not at all with firm size. We infer that investors willingly accept a lower average return on high-total-risk investments as the trade-off for buying a chance at an extreme positive return.

JEL classification: G1.

I. Introduction

Debate rages once again about the relation among beta, firm size, and stock returns. The capital asset pricing model of Sharpe (1964), Lintner (1965), and Black (1972) (the SLB model) implies that the security risk relevant for share market pricing is perfectly and completely reflected through the single-index beta coefficient. A study by Fama and French (1992), however, offers evidence inconsistent with the SLB model. First, Fama and French find that the relation between average returns and firm size is negative and statistically significant. Second, Fama and French do not find any relation between average returns and beta. Although both findings contradict the SLB model, the second finding is especially controversial because the absence of a reliable relation between average returns and beta strikes at the heart of the SLB model.

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Knez and Ready (1997), one of many studies inspired by Fama and French (1992),[1] use a least trimmed squares methodology and demonstrate that the negative relation between average returns and firm size is driven by a few extreme sample observations. After eliminating 1 percent of the most extreme returns, they find a positive and significant relation between average returns and firm size. The significance of the coefficient on firm size increases, too, as larger percentages of extreme observations are eliminated from the sample. Knez and Ready establish that the estimated relation between average returns and firm size is not robust across all data.

Knez and Ready offer no insight into the beta-return relation that lies at the heart of the SLB model. Their findings, however, hint at the possibility that perhaps the nonlinearity of the size-return relation biases estimates of the beta-return relation. We investigate the effect of extreme returns on the relation among beta, firm size, and stock returns.

We employ virtually the same methodology and data as Fama and French (1992), and we replicate their results. Our findings from the replication are virtually identical to theirs. Our extension of their analysis compares easily because we rebalance portfolios after eliminating extreme returns, we re-estimate post-ranking betas, and we re-estimate cross-sectional equations.

Our criterion for eliminating extreme returns differs slightly from Knez and Ready (1997). The Knez and Ready procedure eliminates observations that cause the largest absolute regression residuals. Our criterion instead eliminates monthly stock returns that deviate by a specific percentage from the respective monthly market return. Our first analysis, for example, eliminates monthly stock returns that deviate by more (+/−) than 50 percent from market; this filter eliminates about .5 percent of all observations. Although the two elimination criteria are similar, Knez and Ready report that the largest residuals associate with the most extreme stock returns, and our approach relies on the intuitive argument that monthly stock returns deviating by more than 50 percent from market almost surely are induced by unanticipated information events. These significant information events change underlying intrinsic values, and ex-post excess returns accrue. Estimating the risk-return relation without explicitly accounting for information-induced excess returns biases estimated coefficients.

We eliminate extreme returns and regress the cross-section of stock returns on three explanatory variables: firm size, beta, and total risk. We find that beta and total risk are statistically significant determinants for the cross-section of stock returns. Firm size is not statistically significant. Coefficients attain these significance levels when only 2.5 percent of the extreme observations are removed. As larger percentages of observations are eliminated, beta and total risk become increasingly significant and the size coefficient moves toward zero.

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[1]Other studies inspired by Fama and French (1992) include, among many others, Fama and French (1993, 1995, 1998).

We find that stock returns relate positively with beta, as predicted by the SLB model, and negatively with total risk; firm size is irrelevant. We conclude that firm size is a proxy for the possibility that a stock might garner an extreme positive return, but that total risk is a better proxy. Equilibrium returns decline as total risk increases because investors accept a lower average return as the price for the possibility of earning an extreme positive return.

II. Methodology and Replication of Fama and French

We present in Table 1 our replication of Fama and French (1992). The 100 cells within each panel represent unique portfolios. Each security is sorted into a portfolio according to its size decile (market equity, ME) and pre-ranking beta decile (PB). Securities are resorted annually, 1963 through 1990, and the out-of-sample monthly returns during the subsequent twelve months contribute to equally weighted portfolios. For the full twenty-eight-year sample period, we collect 336 monthly rates of return for each portfolio. The Appendix describes portfolio construction. Our procedure and findings for this replication are virtually identical to Fama and French.

Panel A lists the time-series average monthly return for each of the 100 portfolios throughout the 336 out-of-sample months in the study. Scanning down any column shows, as Fama and French report, the smallest firms (ME decile 1) garner the highest average monthly returns (1.63 percent), whereas the largest firms (ME decile 10) earn the smallest returns (0.89 percent). Scanning along any row shows that for similar size firms there is little association between average return and PB.

Panel B lists the post-ranking βs for each portfolio estimated from the 336 out-of-sample monthly returns. We once again find the same tendencies as reported by Fama and French. Scanning down any column shows that low ME stocks have large β and vice versa. Scanning along any row shows that sorting by PB yields a wide dispersion in β. Comparison of a given row from Panels A and B reveals the provocative conclusion advanced by Fama and French that the relation between β and average return is flat.

We also replicate the Fama-MacBeth (1973) analysis provided by Fama and French and obtain similar results. The analysis estimates a cross-sectional regression for each of the 336 months in the sample. The dependent variable is the cross-section of monthly stock returns. The independent variable(s) is (are) the respective ME as of the preceding sort or (and) the β of the portfolio into which the respective security is placed according to the preceding sort. The number of cross-sectional observations in any month equals the number of firms on the Center for Research in Security Prices (CRSP) tape for that month (and that have data for the preceding sort). Throughout the 336 months in the sample period there are 593,172 monthly rates of return. We average the slope coefficients from the 336 independent cross-

TABLE 1. Average Returns, Size, and β for Portfolios Sorted on Size (ME) and Pre-Ranking Beta (PB): All Stocks, July 1963–June 1991.

	All PB	Low PB	PB 2	PB 3	PB 4	PB 5	PB 6	PB 7	PB 8	PB 9	High PB
Panel A. Average Monthly Returns (in percent)											
All ME	1.34	1.26	1.33	1.34	1.35	1.38	1.36	1.35	1.34	1.39	1.30
Small ME	1.63	1.62	1.73	1.66	1.67	1.67	1.73	1.63	1.58	1.60	1.60
ME = 2	1.40	1.25	1.21	1.54	1.36	1.63	1.45	1.50	1.46	1.53	1.24
ME = 3	1.32	1.33	1.50	1.29	1.22	1.57	1.53	1.51	1.47	1.31	0.96
ME = 4	1.28	1.34	1.45	1.41	1.29	1.09	1.25	1.42	1.41	1.14	1.10
ME = 5	1.32	1.17	1.42	1.55	1.26	1.59	1.39	1.29	1.40	1.26	0.93
ME = 6	1.25	1.13	1.57	1.17	1.43	1.23	1.43	1.35	1.09	1.12	1.05
ME = 7	1.12	0.92	1.47	1.27	1.29	1.04	1.25	1.09	0.79	1.30	0.80
ME = 8	1.12	1.05	1.19	1.21	1.22	1.09	1.14	0.93	1.15	1.15	1.20
ME = 9	1.03	1.01	0.97	1.26	1.15	1.34	0.94	0.78	0.86	0.91	0.16
Big ME	0.89	1.01	0.97	1.03	1.00	0.77	0.77	0.82	0.39	0.86	0.43
Panel B. Post-Ranking β											
All ME	1.325	0.768	0.928	1.099	1.196	1.266	1.329	1.386	1.493	1.579	1.810
Small ME	1.547	1.088	1.124	1.236	1.378	1.448	1.489	1.467	1.587	1.662	1.861
ME =2	1.417	0.901	1.012	1.193	1.228	1.259	1.386	1.366	1.493	1.624	1.805
ME =3	1.366	0.853	0.884	1.159	1.191	1.253	1.335	1.481	1.456	1.530	1.747
ME =4	1.371	0.912	0.933	1.174	1.192	1.308	1.295	1.406	1.437	1.574	1.808
ME =5	1.277	0.608	0.912	1.145	1.209	1.177	1.275	1.356	1.543	1.504	1.727
ME =6	1.199	0.603	0.776	1.084	1.147	1.192	1.275	1.357	1.394	1.514	1.798
ME =7	1.195	0.579	0.910	1.051	1.116	1.248	1.329	1.341	1.409	1.435	1.714
ME =8	1.115	0.544	0.884	0.986	1.184	1.231	1.245	1.230	1.443	1.486	1.775
ME =9	1.026	0.595	0.817	0.944	1.076	1.177	1.159	1.231	1.384	1.315	1.504
Big ME	0.953	0.643	0.801	0.940	0.955	1.102	1.180	1.249	1.238	1.155	1.077
Panel C. Average Size (ln(ME))											
All ME	4.45	5.10	5.14	5.02	4.86	4.71	4.60	4.38	4.18	3.94	3.47
Small ME	2.28	2.25	2.29	2.32	2.33	2.33	2.29	2.32	2.32	2.26	2.23
ME = 2	3.57	3.58	3.58	3.57	3.58	3.57	3.58	3.57	3.58	3.58	3.56
ME = 3	4.07	4.07	4.08	4.07	4.07	4.06	4.08	4.07	4.07	4.07	4.06
ME = 4	4.51	4.52	4.51	4.51	4.50	4.51	4.51	4.51	4.51	4.51	4.51
ME = 5	4.93	4.95	4.94	4.93	4.94	4.95	4.93	4.94	4.93	4.93	4.92
ME = 6	5.35	5.36	5.36	5.36	5.36	5.35	5.35	5.36	5.36	5.34	5.35
ME = 7	5.79	5.79	5.79	5.79	5.80	5.80	5.78	5.79	5.79	5.78	5.78
ME = 8	6.30	6.34	6.31	6.30	6.30	6.30	6.31	6.32	6.27	6.30	6.28
ME = 9	6.89	6.89	6.91	6.91	6.89	6.89	6.87	6.86	6.87	6.65	6.33
Big ME	8.01	8.03	8.13	8.10	8.06	7.91	7.92	7.84	7.74	7.14	5.28

Note: This table replicates Fama and French (1992). Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) and pre-ranking beta (PB) deciles are determined in June of year t (t = 1963–90) using all NYSE stocks on the monthly CRSP tape. The PB for individual stocks are estimated with two to five years of monthly returns (as available) ending in June of year t. Every stock on the tape is allocated to one of ten ME deciles and one of ten PB deciles using the NYSE breakpoints. The equally weighted monthly returns on the resulting 100 portfolios are then calculated for July of year t to June of year t+1. The ME and PB criteria and breakpoints are subsequently recomputed in June of t+1 and portfolios are rebalanced. The procedure is repeated for each of the twenty-eight years in the sample, yielding 336 monthly rates of return for each of 100 portfolios. The average monthly return in Panel A is the time-series average of the 336 monthly equally weighted portfolio returns, in percent. The post-ranking βs in Panel B use the full sample of post-ranking monthly portfolio returns. The PB and βs are the sum of the slopes from a regression of monthly returns on the current and prior month's returns of the CRSP value-weighted market index. The average size in Panel C is the time-series average of monthly averages of ln(ME) for stocks in the portfolio at the end of June of each year, with ME denominated in millions of dollars. Portfolios are sorted by size (down) and pre-ranking beta (across). The All PB column shows statistics for equally weighted portfolios of the stocks in every size decile (ME) group. The All ME row shows statistics for equally weighted portfolios of the stocks in every PB group.

sectional regressions and compute the time-series sample *t*-statistic. A univariate regression of monthly security return on ME yields an average estimated slope coefficient of −0.15 (*t*-statistic = −2.39); Fama and French report −0.15 (*t*-statistic = −2.58). Regressing monthly return on β yields an average slope coefficient of 0.26 (*t*-statistic = 0.78); Fama and French report 0.15 (0.46). This result statistically confirms that the relation between average returns and beta is flat. Including both ME and β in a multivariate regression shows the coefficient on size retains its statistical significance, whereas the coefficient on beta still is indistinguishable from zero.

The similarity of our findings with Fama and French indicates that our procedures are sound. If our analysis were to end here the evidence would lead us, like Fama and French, to reject the SLB model. In the following section, however, we account for the effect of extreme returns on estimates of the risk-return relation.

III. Revealing the Risk-Return Relation

Accounting for Information-Induced Changes in Intrinsic Value

Noise causes the shareholder ex-post rate of return to deviate from its ex-ante expected value. There are two primary sources for noise. First, white noise occurs, which is random and cannot be attributed to anything. Second, some noise occurs because of unanticipated changes in the information set. As the share market assimilates new information, the stock price presumably follows intrinsic value to a new basis reflective of revised expectations. This fundamental yet unexpected change in stock price represents a windfall gain or loss to existing shareholders. Unexpected changes in intrinsic value constitute an important component of stock returns. Unexpected changes are noise that obscure empirical estimation of the risk-return relation.

Dybvig and Ross (1985) discuss problems associated with estimation of the beta-return relation when changes in information shape ex-post returns. Information-induced changes in intrinsic value bias the estimation except when the information changes are distributed as white noise. The bias increases in significance as (a) the unanticipated changes in information approach once-in-a-horizon frequency, and (b) the absolute magnitude of the fundamental revaluation increases relative to the systematic return. Although difficult, in principle it is desirable to separate white noise from unanticipated, information-induced changes in intrinsic value.

The simplest procedure to account for unanticipated changes in intrinsic value is to censor the returns data. A monthly stock return that deviates from the market index by more than 50 percent, for example, almost surely was unanticipated and was caused by new information. We account for the change in intrinsic value as follows: when collecting equally weighted portfolio returns for the twelve out-of-sample months following each annual sort, we eliminate any monthly return that deviates by more than 50 percent from that month's market return. For example, if a particular month's market return is 2 percent, we eliminate monthly security returns greater than 52 percent or less than −48 percent. Except for this extension, the procedure is the same as the Fama and French procedure from section II. We collect monthly returns for each of the 100 portfolios throughout the 336 out-of-sample months in the study, estimate out-of-sample post-ranking βs with the censored data, estimate 336 independent monthly Fama-MacBeth (1973) equations, average the estimated coefficients, and compute the time-series *t*-statistics.

Table 2 illustrates the effect of censoring on the number of observations and on average returns. Panels A and B list the distribution of observations throughout the ME and PB portfolios, respectively. The upper left cell of Panel A shows that with the uncensored data, we collect 593,172 monthly rates of return for the Fama-MacBeth analysis. Panel B shows that after eliminating monthly security returns deviating from market by more than 50 percent, 590,090 monthly rates of return remain. Removal of these extreme observations eliminates about .5 percent of all monthly rates of return. Close inspection of the table reveals that most censored observations are in the small size portfolios; censoring eliminates 2,279 returns from small-ME portfolios and 6 returns from big-ME portfolios. Likewise, censoring eliminates more returns from high-PB portfolios (927 returns) than from low-PB portfolios (217 returns). Panel C shows for the ME and PB portfolios the change in average monthly return caused by the censoring. Even though the filter censors abnormally low returns as well as abnormally high returns, the average return declines for virtually every portfolio. This indicates that extreme returns tend to be positive rather than negative. Censoring extreme returns from a portfolio diminishes average returns.

Censoring introduces uncertain biases because data are eliminated. Nonetheless, when a security's monthly rate of return deviates from market by more than 50 percent, an unanticipated information event pertinent to the security likely exists. Leaving in the sample monthly returns that contain such large information-induced fundamental revaluations biases estimates from the Fama-MacBeth cross-sectional regressions.

Table 3 presents summary information about the portfolios after imposing the 50 percent censor filter on security returns. Panels A and B list the portfolio average monthly returns and post-ranking βs. Scanning down any column in Panel A shows that with the censored data, there is little apparent relation between average returns and firm size. Scanning along any row shows that, likewise, there is little relation between average returns and beta (either PB or β). The Fama-MacBeth equations estimated with the censored data are listed in rows 1, 2, and 3 of Panel C. The coefficient on the size variable (ME) is positive and statistically indistinguishable from zero in the univariate (row 2) and multivariate (row 3) equations. The coefficient on β also is statistically insignificant for both equations.

TABLE 2. Number of Observations for the Monthly Rates of Return in the Size (ME) and Pre-Ranking Beta (PB) Portfolios.

	All PB	Low PB	PB 2	PB 3	PB 4	PB 5	PB 6	PB 7	PB 8	PB 9	High PB
Panel A. Uncensored Data											
All ME	593172	54848	52425	53337	53575	54637	56327	59298	61277	65400	82048
Small ME	172125	11879	10157	10744	11553	12816	14186	17459	19665	24320	39346
ME = 2	63232	4462	4115	4694	5125	5664	5658	6771	7318	8630	10795
ME = 3	52043	3119	3580	4205	4318	4670	5067	5326	6007	6966	8785
ME = 4	47542	2806	3446	3858	4300	4007	5035	4799	6243	5563	7485
ME = 5	45034	4345	3746	3688	3853	4233	4679	4987	5167	5268	5068
ME = 6	43548	4718	4072	4293	4083	4456	4428	4976	4662	4453	3407
ME = 7	43038	4431	4197	4385	4867	4715	4780	4489	3886	3926	3362
ME = 8	42469	5547	5195	4905	5115	5188	4071	3702	3682	2963	2101
ME = 9	41800	6397	5827	5464	5405	4874	4291	3762	2650	1986	1144
Big ME	42341	7144	8090	7101	4956	4014	4132	3027	1997	1325	555
Panel B. Data Censored with the 50 Percent Filter											
All ME	590090	54631	52306	53207	53416	54459	56088	59053	60919	64890	81121
Small ME	169846	11696	10071	10653	11442	12690	14024	17288	19404	23952	38626
ME = 2	62929	4451	4103	4679	5114	5644	5633	6741	7282	8576	10706
ME = 3	51860	3110	3573	4198	4309	4656	5043	5311	5985	6940	8735
ME = 4	47422	2800	3443	3852	4290	3999	5028	4789	6232	5544	7445
ME = 5	44954	4342	3744	3686	3849	4230	4669	4977	5156	5246	5055
ME = 6	43508	4716	4071	4289	4081	4455	4423	4975	4652	4445	3401
ME = 7	43013	4430	4196	4383	4863	4714	4778	4486	3886	3921	3356
ME= 8	42443	5547	5192	4904	5113	5186	4068	3699	3677	2957	2100
ME= 9	41780	6395	5824	5463	5400	4872	4290	3762	2648	1984	1142
Big ME	42335	7144	8089	7100	4955	4013	4132	3025	1997	1325	555
Panel C. Change in Average Monthly Return That Occurs Because of Censoring (basis points)											
All ME	-31	-22	-14	-14	-16	-20	-23	-23	-34	-45	-69
Small ME	-79	-92	-46	-48	-60	-61	-72	-56	-77	-89	-117
ME = 2	-26	-16	-44	-7	0	-18	-24	-22	-28	-16	-49
ME = 3	-17	-11	-14	-9	-11	-17	-20	-14	-22	-15	-28
ME = 4	-13	-9	-8	-8	-3	-8	-7	-16	-9	-24	-23
ME = 5	-9	-2	-3	-3	-6	-3	-4	-14	-13	-17	-19
ME = 6	-4	0	-2	-7	-3	-1	-4	-1	-12	-4	-5
ME = 7	-3	-1	-3	-2	-7	-2	-4	-1	0	-8	-6
ME = 8	-3	—	-2	-1	-3	-1	-5	-5	-8	-10	-6
ME = 9	-3	-2	-1	-1	-5	-3	-2	—	-3	-9	-4
Big ME	0	—	0	0	0	0	—	0	—	—	—

Note: Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) and pre-ranking beta (PB) deciles are determined in June of year *t* (*t* = 1963–90) using all NYSE stocks on the monthly CRSP tape. The PB for individual stocks are estimated with two to five years of monthly returns (as available) ending in June of year *t*. Every stock on the tape is allocated to one of ten ME deciles and one of ten PB deciles using the NYSE breakpoints. The equally weighted monthly returns on the resulting 100 portfolios are then calculated for July of year *t* to June of year *t*+1. The ME and PB criteria and breakpoints are subsequently recomputed in June of *t*+1 and portfolios are rebalanced. The procedure is repeated for each of the twenty-eight years in the sample, yielding 336 monthly rates of return for each of 100 portfolios. The PB and βs are the sum of the slopes from a regression of monthly returns on the current and prior month's returns of the CRSP value-weighted market index. Panel A presents number of monthly returns collected for the full, uncensored sample in each of the 100 ME/PB portfolios. Panel B shows number of monthly returns for the equally weighted portfolios subject to the condition that monthly security returns are eliminated when they deviate (+/−) from the monthly market return by more than 50 percent.

TABLE 3. Results After Censoring with a 50 Percent Filter.

Panel A. Average Monthly Returns (in percent)

	All PB	Low PB	PB 2	PB 3	PB 4	PB 5	PB 6	PB 7	PB 8	PB 9	High PB
All ME	1.03	1.04	1.19	1.20	1.19	1.18	1.13	1.12	1.00	0.94	0.61
Small ME	0.84	0.70	1.27	1.18	1.17	1.06	1.01	1.07	0.81	0.71	0.43
ME = 2	1.14	1.09	1.06	1.36	1.22	1.39	1.29	1.29	1.19	1.15	0.75
ME = 3	1.15	1.22	1.36	1.20	1.11	1.43	1.33	1.37	1.25	1.16	0.68
ME = 4	1.15	1.25	1.37	1.33	1.26	1.01	1.18	1.26	1.32	0.90	0.87
ME = 5	1.23	1.15	1.39	1.52	1.20	1.56	1.35	1.15	1.27	1.09	0.74
ME = 6	1.21	1.13	1.55	1.10	1.40	1.22	1.39	1.34	0.97	1.08	1.00
ME = 7	1.09	0.91	1.44	1.25	1.22	1.02	1.21	1.08	0.79	1.22	0.74
ME = 8	1.09	1.05	1.17	1.20	1.19	1.08	1.09	0.88	1.07	1.05	1.14
ME = 9	1.00	0.99	0.96	1.25	1.10	1.31	0.92	0.78	0.83	0.82	0.12
Big ME	0.89	1.01	0.98	1.02	0.99	0.75	0.77	0.80	0.39	0.86	0.43

Panel B. Post-Ranking β

	All PB	Low PB	PB 2	PB 3	PB 4	PB 5	PB 6	PB 7	PB 8	PB 9	High PB
All ME	1.273	0.740	0.904	1.083	1.174	1.238	1.288	1.352	1.436	1.500	1.679
Small ME	1.406	0.997	1.035	1.172	1.300	1.355	1.365	1.384	1.437	1.498	1.636
ME =2	1.386	0.894	0.984	1.198	1.213	1.242	1.355	1.354	1.479	1.590	1.724
ME =3	1.338	0.839	0.884	1.158	1.196	1.232	1.290	1.428	1.426	1.508	1.701
ME =4	1.351	0.906	0.932	1.154	1.215	1.293	1.274	1.424	1.415	1.537	1.742
ME =5	1.264	0.599	0.908	1.151	1.195	1.176	1.259	1.326	1.538	1.487	1.706
ME =6	1.198	0.603	0.776	1.081	1.143	1.195	1.282	1.361	1.403	1.488	1.782
ME =7	1.189	0.572	0.907	1.047	1.114	1.249	1.333	1.329	1.409	1.413	1.697
ME =8	1.110	0.544	0.877	0.984	1.190	1.215	1.241	1.223	1.440	1.457	1.783
ME =9	1.025	0.594	0.820	0.959	1.059	1.176	1.166	1.231	1.380	1.307	1.500
Big ME	0.953	0.643	0.803	0.939	0.956	1.099	1.180	1.248	1.238	1.155	1.077

Panel C. Fama-MacBeth Regressions with Censored Data (50 percent filter)

	Average Slope Coefficients	
β	ME	SD
-0.39	—	—
(-1.22)		
—	0.03	—
	(0.56)	
-0.35	0.00	—
(-1.15)	(0.01)	
—	—	-0.05
		(-1.29)
0.59	-0.13	-0.18
(1.78)	(-3.14)	(-4.85)

Note: Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) and pre-ranking beta (PB) deciles are determined in June of year t (t = 1963–90) using all NYSE stocks on the monthly CRSP tape. The PB for individual stocks are estimated with two to five years of monthly returns (as available) ending in June of year t. Every stock on the tape is allocated to one of ten ME deciles and one of ten PB deciles using the NYSE breakpoints. The equally weighted monthly returns on the resulting 100 portfolios are then calculated for July of year t to June of year t+1. The ME and PB criteria and breakpoints are subsequently recomputed in June of t+1 and portfolios are rebalanced. The procedure is repeated for each of the twenty-eight years in the sample, yielding 336 monthly rates of return for each of 100 portfolios. The average monthly return in Panel A is the time-series average of the 336 monthly equally weighted portfolio returns, in percent. The post-ranking βs in Panel B use the full sample of post-ranking monthly portfolio returns. The PB and βs are the sum of the slopes from a regression of monthly returns on the current and prior month's returns of the CRSP value-weighted market index. The equally weighted monthly returns for the 100 ME and PB portfolios are collected subject to the condition that monthly security returns are eliminated when they deviate (+/−) from the monthly market return by more than 50 percent. That is, if the monthly market return is 2 percent, security returns are eliminated if they exceed 52 percent or are less than −48 percent. For Panel C, the dependent variable is the monthly stock return and the independent variables include that respective stock's β, ME, and SD. An independent cross-sectional regression is estimated for each of the 336 months in the sample. For the monthly cross-sectional regressions from July of year *t* until June of year *t*+1, a stock's β is the full-sample post-ranking beta for the ME/PB portfolio containing the stock in June of year *t*. The stock's size is its ME in June of year *t*. The stock's SD is the full-sample standard deviation of monthly security returns for the ME and PB portfolio containing the stock in June of year *t*. The average slope is the time-series average of the slopes from the 336 independent monthly cross-sectional regressions from July 1963 through June 1991. The *t*-statistic is the average slope divided by its time-series standard error.

Relating Stock Returns to Total Risk

Several studies conjecture whether total risk plays an important role in determining equilibrium returns. Roll and Ross (1980) employ a total risk variable in testing the arbitrage pricing theory (APT). Total risk, in fact, is the sole variable in addition to the APT factor loadings that Roll and Ross include in their empirical analysis. Levy (1978) analytically extends the SLB model and finds that a security's equilibrium return depends on the security's total risk when investors encounter restrictions about the number of securities they may own. Consequently, we include a total risk variable in our Fama-MacBeth (1973) regressions.

Table 4 summarizes information about total risk for securities in the 100 ME and PB portfolios. Each table entry equals the standard deviation (SD) of all monthly security returns composing the respective portfolio throughout the 336 months in the sample. Panels A and B list results with the censored and uncensored data, respectively. Looking along any row reveals that SD correlates inversely with size. Small firms experience large variation in returns and vice versa. Looking along any row shows that SD correlates positively with PB. The trends are the same for Panels A and B. Magnitudes of SD, however, are lower for censored data, especially for small ME.

The inclusion of SD is analogous to inclusion of β. For the monthly cross-sectional regressions from July of year *t* until June of year *t*+1, a stock's β is the full-sample post-ranking beta for the ME and PB portfolio containing the stock in June of year *t*. The stock's SD is the full-sample standard deviation of monthly returns for the ME and PB portfolio containing the stock in June of year *t*. The β and SD depend on the censor filter employed.

Inclusion of β, ME, and SD in the 336 multivariate Fama-MacBeth equations with the censored data yields the estimates in row 5 of Panel C in Table 3. The average coefficients on all three variables are statistically significant. In univariate equations regressing the cross-section of stock returns on any one variable, the average estimated coefficient is indistinguishable from zero. Inclusion of all three variables, however, reveals significant relations among beta, size, total risk, and the cross-section of stock returns.

TABLE 4. Standard Deviations (SD) of Monthly Returns.

	All PB	Low PB	PB 2	PB 3	PB 4	PB 5	PB 6	PB 7	PB 8	PB 9	High PB
Panel A. Standard Deviation of Monthly Security Returns for Uncensored Data (in percent)											
All ME	12.76	10.37	9.86	10.43	11.06	11.66	12.44	12.66	13.66	15.38	16.96
Small ME	16.98	16.65	14.19	14.21	14.87	15.76	16.62	15.69	16.75	18.92	19.36
ME = 2	13.35	10.90	10.37	11.45	11.38	12.26	12.86	13.31	13.71	14.61	16.20
ME = 3	12.37	9.59	10.34	10.79	11.07	11.09	12.36	12.58	13.14	13.12	14.55
ME = 4	11.76	9.40	9.16	9.91	10.27	11.20	10.83	11.64	12.17	13.26	14.54
ME = 5	11.04	7.63	8.45	9.64	10.57	9.82	10.83	11.44	12.11	12.66	14.05
ME = 6	10.24	7.01	7.91	9.28	9.72	9.71	10.27	10.45	11.52	12.19	13.55
ME = 7	9.80	6.83	8.44	8.85	9.27	9.54	10.14	10.06	10.27	11.65	12.95
ME = 8	9.27	6.51	8.11	8.54	9.20	9.30	9.63	9.74	10.43	11.34	12.64
ME = 9	8.49	6.48	7.68	7.96	8.65	8.87	8.93	9.06	9.77	10.30	12.02
Big ME	7.78	6.97	7.39	7.53	7.74	7.75	8.26	8.39	8.97	9.56	10.99
Panel B. Standard Deviation of Monthly Security Returns for Data Censored with the 50 Percent Filter											
All ME	11.30	8.86	9.15	9.75	10.30	10.66	11.14	11.66	12.30	13.07	14.61
Small ME	13.85	12.61	12.12	12.30	12.77	13.19	13.44	13.63	14.03	14.59	15.66
ME = 2	12.33	10.31	9.81	10.85	10.94	11.39	11.90	12.48	12.73	13.31	14.61
ME = 3	11.69	8.86	9.83	10.47	10.63	10.46	11.50	11.95	12.23	12.52	13.73
ME = 4	11.20	8.79	8.85	9.52	9.95	10.80	10.54	11.25	11.81	12.42	13.44
ME = 5	10.64	7.34	8.34	9.56	10.31	9.68	10.43	10.88	11.70	11.98	13.40
ME = 6	10.02	6.92	7.87	8.86	9.60	9.64	10.10	10.41	11.06	11.86	13.24
ME = 7	9.65	6.76	8.27	8.75	9.06	9.46	10.01	9.93	10.27	11.31	12.68
ME = 8	9.14	6.51	8.00	8.51	9.13	9.21	9.45	9.57	10.21	10.92	12.57
ME = 9	8.37	6.41	7.58	7.94	8.48	8.77	8.88	9.06	9.61	9.76	11.54
Big ME	7.75	6.97	7.38	7.48	7.70	7.68	8.26	8.28	8.97	9.56	10.99

Note: Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) and pre-ranking beta (PB) deciles are determined in June of year *t* (*t* = 1963–90) using all NYSE stocks on the monthly CRSP tape. The PB for individual stocks are estimated with two to five years of monthly returns (as available) ending in June of year *t*. Every stock on the tape is allocated to one of ten ME deciles and one of ten PB deciles using the NYSE breakpoints. The equally weighted monthly returns on the resulting 100 portfolios are then calculated for July of year *t* to June of year *t*+1. The ME and PB criteria and breakpoints are subsequently recomputed in June of *t*+1 and portfolios are rebalanced. The procedure is repeated for each of the twenty-eight years in the sample, yielding 336 monthly rates of return for each of 100 portfolios. Each entry equals the standard deviation of all the security returns composing the respective portfolio.

Table 5 summarizes the effect of changing the censor filter. Row 1 presents the multivariate regression with uncensored data. This estimation uses the same data as our Fama and French replication from section II, plus SD as an additional explanatory variable. Row 2 presents results after censoring monthly returns deviating from market by more than 200 percent; column 2 shows these extreme returns are rare because censoring eliminates only 34 observations (of 593,172). The filter tightens with every row. In row 8, censoring eliminates monthly returns deviating from market by more than 20 percent; still, more than 93 percent of the original observations remain in the analysis. Inspection of the slope coefficients in columns 4 through 6 highlights the trends. As extreme observations are removed from the data, the significance level on the average coefficient decreases for ME and increases for SD and β.

TABLE 5. Censored Data: Average Slopes (and t-statistics) from Monthly Regressions of Stock Returns on β, ME, and SD.

Censor Filter	Number of Monthly Returns in Fama-MacBeth Regressions	b_0	Return = $b_0 + b_1\beta + b_2\text{ME} + b_3\text{SD}$ b_1	b_2	b_3
None (Uncensored)	593,172	2.68 (8.31)	-0.06 (-0.16)	-0.20 (-4.79)	-0.03 (-1.29)
200%	593,138	2.70 (8.14)	0.01 (0.03)	-0.20 (-4.66)	-0.05 (-1.57)
100%	592,861	2.75 (7.94)	0.13 (0.39)	-0.18 (-4.40)	-0.07 (-2.38)
75%	592,322	2.71 (7.53)	0.24 (0.71)	-0.16 (-3.89)	-0.10 (-2.95)
50%	590,090	2.86 (7.47)	0.59 (1.78)	-0.13 (-3.14)	-0.18 (-4.85)
40%	587,111	2.98 (7.60)	0.83 (2.51)	-0.09 (-2.37)	-0.25 (-6.10)
30%	579,535	3.37 (8.34)	1.34 (4.19)	-0.06 (-1.54)	-0.40 (-8.43)
20%	553,987	3.85 (9.05)	2.14 (6.85)	0.01 (0.38)	-0.65 (-10.81)

Note: From July of year t through June of year t+1 the monthly return is collected for every security on the CRSP tape that satisfies data requirements. Monthly returns are eliminated from the sample when they deviate (+/-) from the monthly market return by more than the censor filter shown in column 1. In every month a cross-sectional regression is estimated. The dependent variable is the monthly stock return, and the independent variables include the respective stock's β, ME, and SD. For the monthly cross-sectional regressions from July of year t until June of year t+1, a stock's β is the full-sample post-ranking beta for the ME and PB portfolio containing the stock in June of year t. The stock's size is its ME in June of year t. The stock's SD is the full-sample standard deviation of monthly security returns for the ME and PB portfolio containing the stock in June of year t. Post-ranking β and SD are recomputed for every censor filter. The average slope presented in the table is the time-series average of the slopes from the 336 independent monthly cross-sectional regressions from July 1963 through June 1991. The t-statistic is the average slope divided by its time-series standard error.

Looking down columns 4 through 6 of Table 5 reveals the cross-section of stock returns relates positively to systematic risk (β) and negatively to total risk (SD). Firm size tends toward irrelevancy. When extreme observations remain in the data, as for the uncensored data, ME apparently explains more about extreme returns than SD. Extreme stock returns embed large amounts of noise that stem from information effects, and ME does a better job than SD at measuring this type of noise. Censoring the data reduces information-induced noise. After eliminating fewer than 2.5 percent of all observations (row 7), the relation between size and the cross-section of stock returns is insignificant. Equilibrium stock returns relate exclusively to risk: systematic and total.

TABLE 6. Average Slopes (and t-statistics) from Monthly Regressions of Returns on β, ME, and SD.

	Return = f(β,ME)		Return = f(β, ME, SD) Uncensored Data			Return = f(β, ME, SD) Censored Data, 30% Filter		
	β	ME	β	ME	SD	β	ME	SD
Panel A. Average Coefficients for the Full 336-Month Sample								
Reprinted from								
Fama and French	-0.37	-0.17	—	—	—	—	—	—
(t-statistic)	(-1.21)	(-3.41)						
Our analysis	-0.24	-0.17	-0.06	-0.20	-0.03	1.34	-0.06	-0.40
(t-statistic)	(-0.83)	(-3.21)	(-0.16)	(-4.79)	(-1.29)	(4.19)	(-1.54)	(-8.43)
[Observations]	[593,172]			[593,172]			[579,535]	
Panel B. Average Coefficients for the Months Within Each Market Decile								
Highest decile	5.90	0.32	6.86	0.14	-0.17	7.67	0.40	-0.41
5.86% to 16.53%	(7.46)	(1.44)	(7.75)	(0.92)	(-1.58)	(7.45)	(2.85)	(-2.31)
	[60,980]			[60,980]			[58,332]	
Decile 9	3.72	0.27	4.78	0.08	-0.18	4.89	0.31	-0.28
4.54% to 5.85%	(4.11)	(1.54)	(4.72)	(0.64)	(-1.99)	(4.97)	(2.81)	(-1.46)
	[61,061]			[61,061]			[59,336]	
Decile 8	2.90	-0.11	3.16	-0.15	-0.04	3.24	0.14	-0.07
3.14% to 4.53%	(4.13)	(-0.78)	(3.82)	(-1.67)	(-0.42)	(3.44)	(2.05)	(-0.41)
	[57,972]			[57,972]			[56,533]	
Decile 7	2.68	-0.23	3.03	-0.27	-0.06	3.26	-0.01	-0.15
1.97% to 3.13%	(4.38)	(-1.33)	(4.41)	(-2.32)	(-0.73)	(4.25)	(-0.13)	(-1.06)
	[56,823]			[56,823]			[55,226]	
Decile 6	-0.40	-0.09	0.26	-0.21	-0.11	1.52	-0.06	-0.42
0.89% to 1.96%	(-0.73)	(-0.70)	(0.41)	(-1.86)	(-1.89)	(2.72)	(-0.65)	(-4.11)
	[54,877]			[54,877]			[53,918]	
Decile 5	-0.67	-0.35	-0.84	-0.31	0.03	0.53	-0.08	-0.30
0.00% to 0.88%	(-1.18)	(-1.79)	(-1.27)	(-1.83)	(0.46)	(0.73)	(-0.72)	(-2.95)
	[61,070]			[61,070]			[59,602]	
Decile 4	-2.64	-0.33	-3.08	-0.25	0.08	-0.94	-0.20	-0.41
-0.99% to -0.01%	(-5.40)	(-3.19)	(-5.01)	(-2.93)	(1.14)	(-1.51)	(-2.72)	(-3.68)
	[59,372]			[59,372]			[58,577]	
Decile 3	-2.91	-0.22	-2.83	-0.22	-0.01	-0.88	-0.18	-0.52
-2.24% to -1.00%	(-4.76)	(-2.11)	(-4.20)	(-2.49)	(-0.24)	(-1.23)	(-2.31)	(-3.98)
	[59,575]			[59,575]			[58,690]	
Decile 2	-3.83	-0.60	-4.09	-0.55	0.04	-1.54	-0.49	-0.60
-4.25% to -2.25%	(-9.11)	(-3.41)	(-6.22)	(-3.92)	(0.56)	(-1.75)	(-4.30)	(-4.06)
	[61,844]			[61,844]			[60,703]	
Lowest decile	-7.39	-0.40	-8.01	-0.29	0.10	-4.48	-0.40	-0.84
-21.83% to -4.26%	(-10.9)	(-2.43)	(-9.01)	(-1.77)	(1.31)	(-4.72)	(-2.85)	(-5.74)
	[59,598]			[59,598]			[58,618]	

Note: From July of year t through June of year t+1 the monthly return is collected for every security on the CRSP tape that satisfies data requirements. Monthly returns are eliminated from the sample when they deviate (+/-) from the monthly market return by more than the censor filter shown in column 1. In every month a cross-sectional regression is estimated. The dependent variable is the monthly stock return, and the independent variables include the respective stock's β, ME, and SD. For the monthly cross-sectional regressions from July of year t until June of year t+1, a stock's β is the full-sample post-ranking beta for the ME and PB portfolio containing the stock in June of year t. The stock's size is its ME in June of year t. The stock's SD is the full-sample standard deviation of monthly security returns for the ME and PB portfolio containing the stock in June of year t. Post-ranking β and SD are recomputed for every censor filter. The average slope presented in the table is the time-series average of the slopes from the 336 independent monthly cross-sectional regressions from July 1963 through June 1991. The t-statistic is the average slope divided by its time-series standard error.

The SLB model hypothesizes a positive relation between stock returns and beta. Our analysis with censored data reveals this positive relation. Compensation for beta risk seems to increase in importance as the likelihood of extreme returns diminishes. That is, the slope coefficient on beta increases as the censor filter is tightened (column 1 of Table 5).

We find the empirical relation between average return and total risk is inconclusive when the data include extreme returns. As the censor filter tightens, however, the relation between average returns and total risk becomes neative and significant. The negative relation between total risk and censored average return does not imply a convoluted risk-return relation. To the contrary, the evidence suggests investors accept lower average returns on high-total-risk stocks as the price for a chance, albeit small, of garnering an extreme positive return.

IV. An Assessment of Robustness by Partitioning into Up Markets and Down Markets

The preceding results reflect averages of coefficients for 336 independent monthly regressions. The following analysis assesses the robustness of our findings by presenting disaggregate information about the coefficients. In our analysis we use coefficients identical to those from sections II and III, but we average them differently. We collect and compute decile breakpoints for the 336 monthly market returns in the full sample and present averages of the Fama-MacBeth coefficients for all months in respective market deciles. Table 6 presents average coefficients for three estimations: the regression of monthly security returns (1) on β and ME; (2) on β, ME, and SD with uncensored data; and (3) on β, ME, and SD with censored data (30 percent filter).

Table 6 reveals the beta-return relation two ways. First, censoring the data shows that full-sample average returns relate positively to β (row 2, column 6). Second, partitioning the data into up and down markets shows the beta-return relation in different market deciles is strong. Even for our Fama and French replication in columns 1 and 2, the significance level throughout the market deciles generally is greater for β than for ME. The beta-return relation is positive in up markets, negative in down markets, and nonexistent in flat markets. Aggregating all market outcomes, thereby including extreme observations caused by changes in intrinsic values, obscures the underlying relation between β and the cross-section of stock returns.

The average coefficient for ME shows the size effect is market specific. That is, ME relates positively to average returns in up markets and negatively in down markets. In extreme up markets, for example, large firms garner more than small firms from the size effect. The well-known inverse relation between firm size and average return seems to exist predominantly in flat-to-falling markets (this finding is true irrespective of whether extreme returns are eliminated).

As the data are censored and sorted by market outcome, the coefficients on ME generally attain higher significance levels and less consistency. In other words, the coefficients in column 7 exhibit a wider and more significant swing from positive to negative than the coefficients in column 4. This suggests ME is an instrument for extreme returns.

The average coefficients on SD are not specific to the market outcome. For the uncensored data (column 5), the coefficients generally are indistinguishable from zero. As we censor the data (column 8), the coefficients indicate a persistent negative relation between censored average returns and total risk. The negative coefficient on total risk suggests investors accept a lower average return as the trade-off for a chance at an extreme positive return.

V. Conclusions

The claim by Fama and French (1992) that beta from the capital asset pricing model is an irrelevant risk factor, whereas size is significant, has stimulated research on one of the fundamental issues in finance. Our analysis recognizes that two sources of noise cause actual returns to deviate from expected returns. First, excess returns accrue because of pure white noise. Second, excess returns accrue because of changes in intrinsic values caused by unanticipated information events. Excess returns induced by information events are not randomly distributed; rather, they tend to be positively skewed and larger for small firms than for large firms. Analysis of data without accounting for information-induced fundamental revaluations biases estimates of the risk-return relation. Our procedure censors the data to eliminate extreme observations that occur because of unanticipated changes in intrinsic value.

Analysis of the censored data reveals that the cross-section of stock returns relates exclusively to systematic risk and total risk. Firm size generally is irrelevant. Our findings support the central proposition of the capital asset pricing model that average returns relate positively with beta. The share market, on average, compensates investors for bearing systematic risk.

We also find that average returns relate negatively with total risk. Investors buy high-total-risk securities even though average returns diminish. The reduction in average return associated with increases in total risk presumably reflects the equilibrium price paid by investors for a chance to garner an extreme positive return.

Our analysis of stock returns during up and down markets provides additional support for our inferences. The beta-return relation is positive in up markets, negative in down markets, and nonexistent in flat markets. The well-known inverse size-return relation exists predominantly in flat-to-falling markets. Generally, however, we believe size simply reflects the propensity of a share to earn extreme stock returns. Elimination of extreme returns causes the significance level to decrease for size and increase for beta and total risk.

Appendix

Our procedure, like Fama and French (1992), constructs portfolios by sorting a sample of securities according to criteria known to affect ex-post stock returns. The sample consists of all stocks on the 1993 monthly returns tape from from CRSP, excluding financial corporations (Standard Industrial Classification codes 6000–6999). The procedure computes two criteria, size and beta, for every firm on the CRSP tape in June of year *t* (*t* = 1963, . . ., 1990). The CRSP tape we use has data for only securities listed on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX). The CRSP tape(s) used by Fama and French (1992) additionally includes, after 1972, stocks listed with Nasdaq. The replication establishes that this sample difference does not materially affect the quantitative or qualitative findings.

The subset of NYSE firms is isolated and decile breakpoints are computed for each criteria. Every firm is subsequently assigned to a category based on the location of its two criteria in the NYSE deciles. Consequently, a firm is placed into one of 100 portfolios (i.e., 10 times 10). Its location within this 10×10 array may change from year to year. Also, the total number of firms for June of any year changes because of entry into or exit from the CRSP tape. Data are collected only for firms that are on the tape during the preceding June's computation of the criteria. Hence, the number of firms in a given portfolio may decline from month to month within a year because of exits from CRSP. The number will not increase, however, until the subsequent June.

Firm size (market value of equity, ME) is the first criterion. The procedure employs this sorting variable because size correlates highly with beta. Sorting solely by beta makes it impossible to separate the effect of beta on average return from that of size. In June of each year, therefore, the size of each firm is computed and its location within the NYSE decile breakpoints is found.

The second criterion is the pre-ranking beta (PB). Sorting by PB allows for variation in average returns that is unrelated to size effects. In June of each year, therefore, the PB of each firm is estimated and its location relative to the NYSE decile breakpoints is found. The estimation uses between twenty-four and sixty months of observations for the period immediately preceding June, as available. All betas in our analysis, pre-ranking and post-ranking, are estimated by regressing the security or portfolio's monthly rate of return on the current and prior month's CRSP value-weighted index rate of return. Beta is set equal to the sum of the two slope coefficients. Fama and French also use this procedure.

Sorting each firm by the ten ME and ten PB decile breakpoints creates 100 portfolios for June of year *t*. The equally weighted monthly stock returns for each portfolio throughout the succeeding twelve months (July of year *t* through June of year *t*+1) are collected. In June of year *t*+1, the ME and PB criteria are recomputed for each available firm, the 100 portfolios are rebalanced according to the new NYSE breakpoints, and the monthly equally weighted portfolio returns throughout the succeeding twelve months are collected. It is at this stage that the censor filter (if any) is invoked. Repeating the process throughout the twenty-eight-year sample period (1963–90) results in 336 monthly observations for each of the 100 portfolios. A full-sample post-ranking beta (β) for each portfolio results from regressing the 336 monthly portfolio returns on the market index.

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