Is There a Lottery Premium in the Stock Market?

Persistent and significant.

Thomas W. Downs and Quan Wen

Perhaps risk and risk alone determines the level of equilibrium returns—but not all risk is bad. Risk relates to the propensity of a share to produce an extreme outcome, and stories of beneficial extreme returns dot the financial landscape.

An excerpt from the *Wall Street Journal* suggests that many investors actively pursue extreme outcomes:

> Federal Reserve Chairman Alan Greenspan says he has figured out what is driving the mania for Internet-related stocks: the same not quite-rational impulse that drives millions of people to pay more for lottery tickets than they are worth.
>
> "What lottery managers have known for centuries," Mr. Greenspan told the Senate Budget Committee yesterday in the Fed chief's most expansive public remarks on the current Internet mania, "is that you could get somebody to pay for a one-in-a-million shot more than the [pure economic] value of that chance."
>
> That's why lottery operations make money: The bigger the payoff, the more of a premium people are willing to pay for a chance at winning. Investors buying Internet stocks figure they either will be worthless or worth "some huge number," Mr. Greenspan said. They are paying handsomely to buy a chance it might be the latter, no matter how remote the prospect. Call it a "lottery premium," he said (Wessel [1999]).

Thomas W. Downs is the Harrison Professor of Teaching Excellence at the University of Alabama in Tuscaloosa (AL 35487).

Quan Wen is with at the Insightful Corporation in New York, NY.

Even though the latest craze for Internet stocks has waned, the claim that stock returns include a lottery premium is timeless. We define the lottery premium as the sacrifice in average return that investors pay for a chance to earn a huge although remote return. In this study, we reveal some pretty interesting patterns about return relations with size, beta, diversifiable risk, and the stock market lottery premium.

BACKGROUND

Malkiel and Xu [1997] report finding a relation between average return and diversifiable risk. Their procedure extends a remarkable study by Fama and French [1992]. Our execution of the same procedure yields data plotted in Exhibit 1.[1]

Exhibit 1 echoes Malkiel and Xu's Exhibit 4 [1997, p. 12]. It shows a clear tendency that higher levels of diversifiable risk are associated with higher average returns. Firm size, however, also covaries with diversifiable risk. Malkiel and Xu [1997] suggest that the predictive performance of size for explaining average returns might be an artifact of the underlying relationship between size and diversifiable risk. We formally test this suggestion by estimating Fama-MacBeth regressions.

For each month in the sample period, we estimate a cross-sectional regression of firm monthly return on that firm's equity market capitalization (ME), the corresponding portfolio's post-ranking beta (β), and the firm's contemporaneous monthly diversifiable risk (standard error of equation, SEE). The time series average coefficients from 432 cross-sectional regressions are:

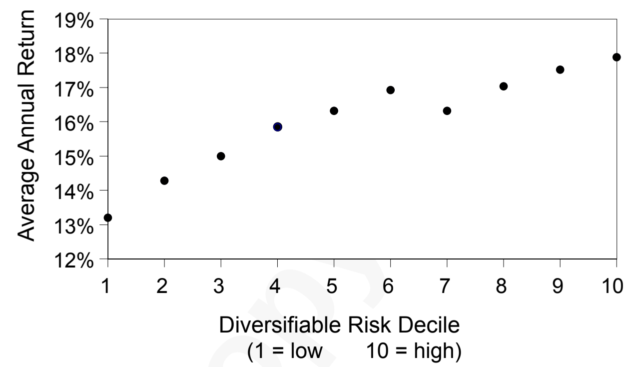
$$\begin{array}{l} \text{Return} = \underset{(2.85)}{0.90} - \underset{(-5.30)}{1.11\,\beta} + \underset{(0.73)}{0.03\,\text{ME}} + \underset{(6.51)}{14.66\,\text{SEE}} \end{array} \tag{1}$$

Time series t-statistics are in parentheses.[2]

The negative coefficient on β is statistically significant and violates the capital asset pricing model prediction of a positive premium for systematic risk. Malkiel and Xu [2000] explain that a negative coefficient on beta may result from negative correlation between alpha and beta in cross-sectional regressions.

The positive and statistically significant coefficient on SEE affirms the trend line from Exhibit 1. High diversifiable risk earns a positive risk premium. The coefficient on ME is insignificant when SEE is included. This supports the Malkiel and Xu [1997] suggestion that size is an imperfect proxy for diversifiable risk.

EXHIBIT 1
RELATIONSHIP OF RETURN AND DIVERSIFIABLE RISK: 1963-1999



Additional analysis below shows, however, that returns relate to size and diversifiable risk in a framework more complex than Equation (1) suggests.

EXTREME RETURNS ASSOCIATE WITH LOW-PRICED STOCKS

Malkiel [1973] persuasively argues that financial market equilibrium depends on investor perceptions of risk-return trade-offs. The capital asset pricing model establishes that in certain situations risk separates into systematic and diversifiable components. Equilibrium stock returns in these special situations include a risk premium for only systematic risk. Diversifiable risk receives no premium because some stocks win while other stocks lose, and idiosyncratic effects exactly offset.

Several studies investigate diversifiable risk. Levy [1978] extends the capital asset pricing model to account for diversification constraints. His theoretical model establishes that stock returns in this constrained equilibrium include a risk premium for diversifiable risk because investors are unable to perfectly diversify. Malkiel and Xu [1997] provide empirical evidence that high diversifiable risk correlates with high returns, and Campbell et al. [2001] find that firm diversifiable risk is higher today than decades ago.

Lottery risk, just like systematic risk and diversifiable risk, stems from the relation between investor utility and risk-return trade-offs. The utility-based model of Golec and Tamarkin [1998] establishes that risk-averse investors rationally sacrifice average return for the chance to win an extreme return. Unlike the systematic and diversifiable risk premiums, which are positive, the lottery risk premium is negative. Lottery risk represents the propensity of a share to provide an extreme return. Some shares represent gambles by investors for making money quickly, and, like lottery tickets, a price must be paid:

> Human nature desires quick results; there is a peculiar zest in making money quickly.... The game of professional investment is intolerably boring and over-exacting to anyone who is entirely exempt from the gambling instinct; whilst he who has it must pay to this propensity the appropriate toll (Keynes [1936, p. 157]).

The distribution of extreme returns incorporates information relevant to the lottery premium. Exhibit 2 shows that the 1.3 million monthly firm returns are not uniformly distributed throughout all size and SEE deciles. Approximately 50% of all observations occur for the two smallest size deciles. Likewise, approximately 50% of all observations occur for the three largest SEE deciles. Even though the 10 × 10 sort by ME and SEE yields 100 non-overlapping portfolios, about one-fifth of all stock returns fall within one portfolio: the smallest market capitalization decile with the largest SEE. The mass of returns flows into the diagonal toward the upper right corner of Exhibit 2.

The small ME, high SEE, portfolio not only has more observations, but it also has a greater representation of extreme observations. About one-quarter (24.0%) of the 287,676 monthly firm-returns in the small ME, high SEE, portfolio deviate (plus or minus) from market by more than 20%. Only 13.6% of returns in the big ME, high SEE, portfolio are extreme, and about 0.05% of returns in the big ME, small SEE, portfolio are extreme.

We attempt to identify a common characteristic of extreme returns. Our initial expectation was that most extreme returns must surely be caused by information events. A search for information relevant to a sampling of the most extreme returns, such as those deviating from market by more than 40% per month, shows a surprising absence of information events. For nine of the ten most extreme company stock returns, there are no news articles. This finding echoes the report by Cutler, Poterba, and Summers [1989] that extreme movements in aggregate stock market indexes are surprisingly unrelated to information events.

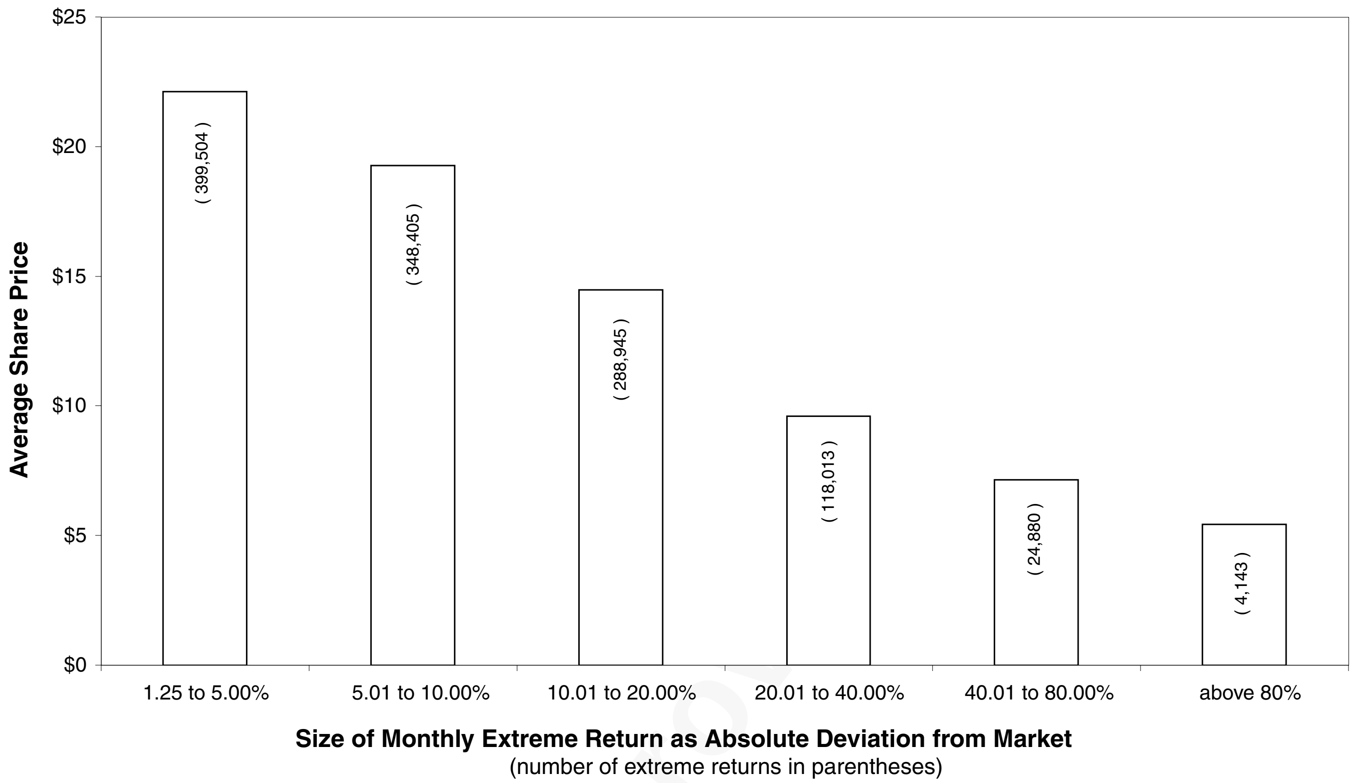
Further data analysis reveals that extreme returns associate with low share prices. Exhibit 3 illustrates the average share price for different levels of extreme returns. Throughout the 432 months in the sample period there are 4,143 monthly firm returns that deviate from market by more than 80%. The average share price for these stocks is $5.07. There are 399,504 monthly stock returns that deviate from market by between 1.25% and 5.00%, and the average share price for these stocks is $22.11. Exhibit 2 shows a clear trend: Extreme returns are associated with low-priced stocks.

We also investigate the average share price for portfolios sorted by ME and SEE. Within every SEE decile, small-cap stocks tend to be cheap, and large-cap stocks tend to be dear. The average share price in the smallest ME decile, $12, is one-fifth the size of the $60 average share price in the largest ME decile. For firms of similar size, the relation between SEE and average share price is mixed. For small-cap stocks, the average share price declines as SEE rises, while for mid- and large-cap stocks the inverse relation is less pronounced or absent.

EXHIBIT 2
NUMBER OF OBSERVATIONS (FIRM-MONTH RETURNS) FOR PORTFOLIOS SORTED BY SIZE (ME) AND DIVERSIFIABLE RISK (SEE)

	Low SEE	SEE-2	SEE-3	SEE-4	SEE-5	SEE-6	SEE-7	SEE-8	SEE-9	High SEE
Small ME	6239	5819	7199	10349	15599	23821	32160	52256	93408	287676
ME-2	3639	3783	4552	6516	9495	14458	16587	22001	29713	48334
ME-3	3288	3627	5356	6504	8601	11985	13574	15818	18590	24905
ME-4	3540	3296	5386	7264	9779	11225	12174	13213	14702	16398
ME-5	4400	4422	5859	7620	9428	9841	10756	10792	11104	10436
ME-6	5776	6047	7092	8357	9485	8996	8730	8133	7739	6898
ME-7	7476	7055	8271	9464	9537	8021	6683	5940	5829	3480
ME-8	8213	9123	10309	10303	8070	5777	5652	4840	3598	2227
ME-9	11187	12824	10858	8989	6308	5080	3988	2584	2097	984
Large ME	18181	16541	11014	7039	3792	3370	1856	1804	769	383

EXHIBIT 3
AVERAGE SHARE PRICE AT DIFFERENT EXTREME RETURNS



THE LOTTERY PREMIUM IS PERSISTENT AND SIGNIFICANT

We hypothesize that the share price embodies unique information for investors about the lottery premium. Perhaps because cheap stocks are different is why *Investor's Business Daily*, a newspaper popular among active traders, puts stock quotations under $7 into a separate table. The founder of *IBD*, William J. O'Neil, writes:

> What are some of the worst habits investors have? One is an overwhelming attraction to low-priced stocks. The idea of buying a large block of a $2, $5, or $10 stock and watching it double sounds wonderful. The only problem: your odds of winning the lottery may be better [2000, p. 15].

We glean insight about the lottery premium by modifying Fama-MacBeth regressions from Equation (1). For each month in the sample period, we estimate a cross-sectional regression of firm monthly return on four independent variables: β, ME, and SEE are the same as in Equation (1). We also include a dummy variable (PDUM), which is switched on when the share price is below a critical value. Our initial specification sets PDUM to unity when the share price for computing ME is under $7.

The time series average coefficients from the 432 cross-sectional regressions are:

$$\begin{aligned} \text{Return} = {} & \underset{(11.76)}{3.31} - \underset{(-6.21)}{1.27}\,\beta - \underset{(-12.68)}{0.41}\,\text{ME} + \\ & \underset{(10.22)}{22.82}\,\text{SEE} - \underset{(-24.48)}{4.06}\,\text{PDUM\$7} \end{aligned} \tag{2}$$

The size effect, absent in Equation (1), reappears when we introduce the price dummy. The negative and statistically significant coefficient on ME in Equation (2) suggests a fairly strong inverse relation between average return and size. The negative coefficient on beta repeats the anomalous finding from Equation (1). The positive and significant coefficient on SEE confirms that diversifiable risk is associated with high returns. The estimated coefficient on the price dummy, negative and statistically significant, implies that investment in stocks with prices of under $7 is associated with a significant decline in average return. The sacrifice in average return equals the lottery premium.

We check the robustness of our findings by switching on the price dummy at different critical values and reestimating. Repeated independent analysis of the 1.3 million monthly firm returns for the 432-month sample period yields results in Exhibit 4.

The average coefficient on the price dummy is negative and statistically significant for all critical values of PDUM. Moving down the PDUM column, we see that the sacrifice in average return is larger on a $1 stock than on a $7 stock. Moving down the SEE column, we see that inclusion of PDUM increases the statistical significance of the SEE coefficient. The evidence suggests that PDUM embodies the negative lottery risk premium.

EXHIBIT 4
FAMA-MACBETH CROSS-SECTIONAL REGRESSIONS OF STOCK RETURNS

PDUM = 1 for price less than	Average Estimated Coefficients (time series t-statistics over 432 monthly cross-sectional regressions)				
	Constant	β	ME	SEE	PDUM
a. no price dummy	0.90 (2.85)	-1.11 (-5.30)	0.03 (0.73)	14.66 (6.51)	....
b. $1	1.73 (5.87)	-1.56 (-7.45)	-0.10 (-2.56)	19.37 (8.43)	-6.14 (-21.86)
c. $3	2.59 (9.07)	-1.65 (-8.01)	-0.26 (-7.64)	22.68 (9.87)	-4.86 (-22.97)
d. $5	3.08 (10.95)	-1.50 (-7.32)	-0.36 (-11.02)	23.18 (10.27)	-4.32 (-22.91)
e. $7	3.31 (11.76)	-1.27 (-6.21)	-0.41 (-12.68)	22.82 (10.22)	-4.06 (-24.48)
f. $14	3.74 (12.73)	-0.76 (-3.73)	-0.45 (-12.86)	19.78 (8.91)	-3.63 (-31.99)

Post-Ranking Beta (β), Market Capitalization (ME), Diversifiable Risk (SEE), and Price Dummy Variable (PDUM).

RISE AND FALL OF RISK PREMIUMS WITH UP AND DOWN MARKETS

Additional analyses investigate the stability of the lottery premium in different market settings. We assign each of the 432 months in the 36-year sample to a quintile according to performance of the market index. Exhibit 5 presents average coefficients for the quintiles.

Coefficients in row (1) are averages from 432 independent monthly cross-sectional regressions, or the results of Equations (1) and (2). Entries in rows (2a) through (2e) use the same estimators, but average them in subsets based upon market performance during the particular month.

Trends in columns (1), (2), and (3) offer insights. Row (2a) shows average coefficients during months when the market moves up most dramatically. The average coefficient on β of 2.65 is significantly positive, and confirms a prediction from the capital asset pricing model: In up markets, high- beta stocks are associated with high returns. The coefficient on β in row (2e) is a statistically significant –5.04. This, too, is consistent with the capital asset pricing model: High-beta stocks in down markets are associated with high negative returns.

Looking down column (1), all in all, the ex post relation between beta and returns conforms with the capital asset pricing model. Empirical controversy such as that in Fama-French [1992] arises because for all market outcomes taken together, as in row (1), the relation between β and average returns is negative. This observation contradicts the capital asset pricing model prediction that, on average and in the long run, high-beta stocks earn high returns.

Looking down column (2), we see that the average coefficient on size also depends on the market outcome. Small-caps outperform large-caps in flat to falling markets. But large-caps outperform small-caps in up markets. Row (1) shows that the average relation between firm size and stocks is flat. ME apparently embodies a risk factor dependent on general market direction.

The trend in column (3) shows a similar story for SEE. High SEE is associated with high average returns, except in extreme down markets. In the lowest market quintile, there is a statistically significant negative relation between SEE and stock returns.

In up markets, average returns are related positively to three risk factors: β, SEE, and ME. In down markets, average returns are related negatively to these three risk factors. Whatever investor risk factors these three variables embody, the data suggest that their risk associates with high returns only in up markets—risk in down markets is costly!

Columns (4) through (7) present average coefficients for the regression that includes the $7 price dummy. Inferences with respect to beta, size, and diversifiable risk are the same as those found above. Average coefficients on β, ME, and SEE are positive in extreme up markets and negative in extreme down markets. The lottery premium coefficient is consistently negative. Investors in low-priced stocks persistently sacrifice average return as the toll for a chance, however remote, of quickly making a huge amount of money. Lottery tickets are never free, are they?

The magnitude of the lottery premium depends on market direction. In down markets, the average coefficient on PDUM$7, at –2.56, is half that in the highest market quintile (–5.62). Investors in down markets apparently are less willing to sacrifice average return, so the lottery premium is relatively small. In up markets, the lottery premium is relatively high, confirming Alan Greenspan's claim that "the bigger the payoff, the more of a premium people are willing to pay for a chance at winning."

EXHIBIT 5
FAMA-MACBETH CROSS-SECTIONAL REGRESSIONS OF STOCK RETURNS IN DIFFERENT MARKET PERFORMANCE QUINTILES

	Return = f(b, ME, SEE)			Return = f(b, ME, SEE, PDUM$7)			
	b (1)	ME (2)	SEE (3)	b (4)	ME (5)	SEE (6)	PDUM$7 (7)
	Average Coefficients for 432-Month Sample						
1. All Markets	-1.11	0.03	14.66	-1.27	-0.41	22.82	-4.06
(t-statistic)	(-5.30)	(0.73)	(6.51)	(-6.21)	(-12.68)	(10.22)	(-24.48)
	Average Coefficients for Months Within Market Quintile						
2. Market Quintiles							
2a. Highest Quintile:	2.65	0.79	34.80	2.39	0.18	46.09	-5.62
4.47% to 16.56%	(5.51)	(8.15)	(6.02)	(5.12)	(2.50)	(8.20)	(-14.17)
2b. Quintile 4:	0.80	0.19	25.29	0.59	-0.28	33.53	-4.46
2.09% to 4.46%	(2.38)	(2.60)	(5.27)	(1.80)	(-4.78)	(7.14)	(-15.91)
2c. Quintile 3:	-1.16	-0.03	15.25	-1.36	-0.43	22.71	-4.05
0.28% to 2.08%	(-3.51)	(-0.46)	(3.46)	(-4.17)	(-8.37)	(5.40)	(-14.61)
2d. Quintile 2:	-2.83	-0.24	6.75	-3.00	-0.64	14.53	-3.57
-2.25% to 0.27%	(4.41)	(-3.68)	(1.36)	(-10.11)	(-11.41)	(2.94)	(-13.73)
2e. Lowest quintile:	-5.04	-0.58	-9.05	-5.01	-0.91	-3.01	-2.56
-22.49% to -2.26%	(-13.76)	(-7.91)	(-2.53)	(-13.83)	(-13.38)	(-0.84)	(-5.15)

Post-Ranking Beta (β), Market Capitalization (ME), Diversifiable Risk (SEE), and $7 Price Dummy Variable (PDUM).

EXHIBIT 6
AVERAGE COEFFICIENT ON $7 PRICE DUMMY IN DIFFERENT MARKET PERFORMANCE QUINTILES AND CALENDAR PERIODS

	All 432 Months (1)	Return = f(b, ME, SEE, PDUM$7) 7/63 - 6/75 (2a)	7/75 - 6/87 (2b)	7/87-6/99 (2c)
	Average Coefficients for All Months in Period			
1. All Markets	-4.06	-2.67	-4.18	-5.33
(t-statistic)	(-24.48)	(-7.31)	(-23.87)	(-21.99)
	Average Coefficients for Months Within Each Market Quintile and Period			
2. Market Quintiles				
2a. Highest Quintile:	-5.62	-4.67	-5.05	-7.23
4.47% to 16.56%	(-14.17)	(-6.25)	(-11.27)	(-8.80)
2b. Quintile 4:	-4.46	-2.78	-4.90	-5.32
2.09% to 4.46%	(-15.91)	(-4.74)	(-13.81)	(-14.20)
2c. Quintile 3:	-4.05	-3.45	-4.17	-4.59
0.28% to 2.08%	(-14.61)	(-7.21)	(-9.02)	(-10.19)
2d. Quintile 2:	-3.57	-2.13	-3.82	-4.75
-2.25% to 0.27%	(-13.73)	(-3.94)	(-15.44)	(-9.87)
2e. Lowest Quintile:	-2.56	-0.82	-2.87	-4.76
-22.49% to -2.26%	(-5.15)	(-0.73)	(-11.79)	(-14.69)

A HIGHER LOTTERY PREMIUM

Exhibit 6 presents evidence that the lottery premium has grown more recently. The table partitions the 36-year sample into 3 subperiods of 12 years each. Row (1) shows a clear upward trend over time in the average coefficient on PDUM$7.

The lottery premium in 1987-1999 is twice the size as in 1963-1975. The lottery premium differs between up and down markets, however.

To control for differences among the three 12-year sample periods in the distribution of up and down markets, we therefore partition in rows (2a) through (2e) the monthly returns during each subperiod according to the market quintile breakpoints for the full sample. The lottery premium in all five quintiles is higher in the recent past than in the remote past.[3]

Exhibit 6 presents other evidence of an increasing propensity to gamble. During the remote past (column 2a), the spread between coefficients in rows (2a) and (2e) is relatively wide (more than fivefold). During the recent past (column 2c), however, the coefficients in rows (2a) and (2e) come closer together (less than a twofold difference). The spread between lottery premiums for up and down markets narrows over time.

The lottery premium is higher in the recent past than in the remote past, and the propensity to bet on low-priced stocks, even in down markets, has grown stronger over time.

SUMMARY

Risk and risk alone drives expected returns in an efficiently functioning financial market. Extreme outcomes are perhaps riskiest of all. We find that extreme returns tend to be associated with low-priced stocks. We also find that investment in low-priced stocks diminishes average returns, ceteris paribus. We infer that investors in low-priced stocks hope to be the beneficiaries of an unanticipated extreme return.

We define the lottery premium as the sacrifice in average return that investors pay for a chance to earn a huge although remote return. Analysis of more than 1.3 million stock returns spanning a 36-year sample period shows that the lottery premium is persistent and significant. It is higher on $1 stocks than on $7 stocks. It is greater in up markets than in down markets, and it is higher in the recent past than in the remote past.

ENDNOTES

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[1]The methodology follows Fama and French [1992] and Malkiel and Xu [1997]. The proxy for each firm's diversifiable risk is the standard error of equation (SEE) from a modified market model regression. Each June we compute SEE decile breakpoints for all NYSE stocks. Subsequently, firm monthly stock returns for the 12 months following the regression are sorted into decile portfolios. We annually rebalance the 10 portfolios using updated regressions and decile breakpoints. We collect for Exhibit 1 monthly returns for 10 portfolios throughout a 432-month sample period (July 1963–June 1999, 1.3 million observations). Each exhibit entry is the average annual percentage rate of return for each decile portfolio. See Downs and Ingram [2000] for additional information.

[2]Fama and French [1992] regress monthly returns on β and ME over 1963-1990. Our replication of that specific regression compares favorably with their benchmark results. Fama-French report slope coefficients (and t-statistics) of –0.37 (–1.21) on β and –0.17 (–3.41) on ME. Our replication (1963-1990) yields coefficients of –0.29 (–0.99) on β and –0.18 (–3.49) on ME. For the longer sample period, 1963-1999, coefficients are about the same: –0.25 (–0.97) on β and –0.18 (–4.10) on ME.

[3]Here are average coefficients in Exhibit 6 format for the other variables. For β, the average coefficients in the highest (row 2a) and lowest (row 2e) market deciles equal 3.76 and –4.54 in the remote past (column 2a), and 1.69 and –5.44 in the recent past (column 2c). For ME, the average coefficients in the highest and lowest market deciles equal 0.09 and –0.69 in the remote past, and 0.27 and –1.05 in the recent past. For SEE, the average coefficients in the highest and lowest market deciles equal 54.08 and –12.08 in the remote past, 38.83 and –5.27 in the middle subperiod, and 48.12 and 12.78 in the recent past. These estimates make the inferences in the text robust

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