Tax Efficiency and Marginal Effective Tax Rates for Capital Income

ABSTRACT Standard specification of marginal effective tax rate (*METR*) as expected pretax minus after-tax rates of return divided by pretax rate of return contains a fundamental flaw rendering the measure useless except in a few special cases. The current study exposes the flaw and introduces an alternative specification robust for a wider class of scenarios. Analysis reveals that asset characteristics such as service life or time-paths of pretax cash flow and economic depreciation are irrelevant to *METR*. Tax neutrality requires immediately expensing capital investments. Second best policy uses one tax depreciation schedule for all assets irrespective of underlying economic depreciation patterns.

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Assessing economic effects of taxes is essential for developing sound public policy. Indeed, recent Nobel Laureate Edward Prescott recaps for a *Wall Street Journal* interview that "… when it comes to taxes we should worry about policies from the perspective of their efficiency and distributional consequences" [*Outlook* column by Hilsenrath (October 18, 2004, page 2)]. Tax efficiency, as Harberger (1962) shows, diminishes as variation in effective tax rates increases. Underlying notion is that in the absence of taxes then economic agents reveal optimizing supply and demand of private goods and services across the spectrum of economic activity. Neutral tax policies exert equivalent across-the-board effects leading to efficient substitution of public for private goods. Inefficient tax policies exert differential effective tax rates; they are relatively high on some economic activities and relatively low on others. Revealed behavior distorts from optimum when effective tax rates differ across industries, sectors, factors of production, form of business organization, financing sources, etc., and efficiency losses accrue.

A substantial literature explores effects of tax policies on income from real capital assets. Emergent consensus is that the *marginal effective tax rate* ("*METR*") plays a significant role in the distribution of capital assets.[1] Fullerton (p. 270, 1999) offers the

[1] Since refinement of *METR* measurement procedures by King and Fullerton (1984) the concept has become a well-accepted policy tool around the globe. Anecdotal evidence attesting to this broad popularity includes: (1) An internet search for "metr tax rate" finds thousands of documents, including estimates of *METR* by public and private organizations in all developed economies, most developing ones, and even many lesser developed sub-Saharan countries in Africa. (2) The C.D. Howe Institute, Canada's most respected independent economic and social policy research institution, distributes an article (Chen 2000) for businessmen, politicians, and bureaucrats entitled "The Marginal Effective Tax Rate: The Only Tax Rate that Matters in Capital Allocation". The standard *METR* definition is gospel.

standard definition for *METR* as "the expected pretax rate of return minus the expected after-tax rate of return on a new marginal investment, divided by the pretax rate of return." Specification for the expected pretax rate of return, denoted IRR^{pre}, obtains from the framework of Hall and Jorgenson (1967):

$$\begin{aligned} IRR^{pre} &= \frac{c}{q} - \delta \\ &= \frac{(r + \delta - \pi)(1 - \tau Z)}{(1 - \tau)} - \delta \ . \end{aligned} \tag{1}$$

In this expression's first line, c represents *user cost of capital*, q is capital goods supply price for a new asset, and δ is the asset's geometric rate of decline in productive efficiency. Expected pretax rate of return IRR^{pre} equals *user cost* per dollar of asset minus depreciation rate δ. The second line shows user cost components: r is the investment after-tax financing rate, π is the expected inflation rate, τ is the marginal income tax rate, and Z is present value per dollar of supply price of tax depreciation deductions expected throughout the service life.

Fullerton describes several conceptually important special cases of equation 1 and explains how the *METR* relates to statutory tax rates, investment tax credits, and alternative tax policies. He also explains how equation 1 applies to corporate versus noncorporate sectors, to equity versus debt financing sources, and more. Those special cases reveal important lessons about tax efficiency.[2]

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[2] Among several important conceptual results that Fullerton reports are these. First, the *METR* equals the statutory rate if the investment tax credit is zero and depreciation allowances are based on replacement cost. Second, the *METR* equals the statutory rate if the investor receives an immediate deduction equal to the first-year recovery proposal of Auerbach and Jorgenson (1980). Third, the *METR* equals zero with expensing of new investment.

Specification of economic depreciation in the standard *METR* framework assumes that pretax cash flow from marginal investment declines smoothly through time at rate δ perpetually. This specific simplifying assumption is unnecessary and overly restrictive. Actual capital budgeting projects return cash flow streams following almost any conceivable shape. Some capital expenditures yield incremental pretax cash flow following fairly level time-path such as the one-hoss shay pattern. Other capital projects provide incremental pretax cash flows following a hump-shaped pattern, adding little incremental cash flow in the near term but then growing and ultimately subsiding. Yet other marginal investments may require back-loaded costs for reclamation or clean-up. And virtually every real capital expenditure promises a return stream of finite length.

This study modifies specification of user cost to accommodate any shape time-path of pretax cash flow. That trivial innovation allows cash flow analyses that reveal substantively important implications for interpretation and measurement of marginal effective tax rates. The assumption of geometric depreciation apparently obfuscates relation between tax effects, rates of return, and discounted cash flows.

Section I generalizes specification of user cost to allow for any pattern of expected pretax cash flow. Section II examines *METR* for several scenarios and establishes spurious connection between tax effects and the standard definition. The standard definition is correct <u>only</u> for a few special cases. Section III introduces a robust specification for marginal effective tax rate and Section IV discusses implications. A brief conclusion closes the study.

I. User cost with a dynamic pretax cash flow stream

Investment equilibrium in the market for real capital assets requires that the time *s* capital goods price q_s equals expected pretax cash flows net of proportional taxes plus tax savings from depreciation deductions discounted by *r,* the investor after-tax financing rate:

$$q_s = \sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} \left\{ (1-\tau)c_{s,t} + \tau q_s z_{s,t} \right\} \quad . \tag{2}$$

π equals expected inflation rate, τ is statutory tax rate, and $z_{s,t}$ is percentage *real* tax depreciation allowance (e.g., deflated MACRS weights) applicable to time *s* real investments. The term $c_{s,t}$ equals real pretax cash flow expected at time $s+t$ from the time *s* investment. First periodic pretax cash flow $c_{s,1}$ equals *user cost of capital* (c_s is identical to $c_{s,1}$). Obtain equilibrium user cost by simplifying equation 2 for c_s.[3]

Simplification requires specification of time-paths for pretax cash flow and for tax depreciation deductions. Let Z_s equal present value of tax depreciation deductions per dollar of time *s* marginal investment as in

$$Z_s = \sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} z_{s,t} \quad . \tag{3}$$

Substitute equation 3 into 2. Standard specification for *Z* naturally accommodates any time-path or schedule of tax depreciation weights.

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[3] Auerbach and Hassett (2003) examine importance of whether the source of equity financing is retained earnings or new share issuances. Personal taxes are important considerations in that analysis. My specification does not delve into effects of personal taxes. Similarly, Auerbach (1989) and Hall (2001) examine importance of capital stock adjustment costs. My analysis also ignores those important innovations. My focus is effect on marginal effective tax rates from relaxing restrictions on the pretax cash flow stream.

Consider the pretax cash flow stream. Let d_j denote proportional decline in real pretax cash flow occurring after receipt of the j'th cash flow ($d_0 = 0$). Expected real pretax cash flow at time $s+t$ from marginal real investment at time s is

$$c_{s,t} = c_s\left(1 - \sum_{j=1}^{t} d_{j-1}\right). \tag{4}$$

Define Δ as the d_j schedule discounted with real after-tax financing rate $r - \pi$:

$$\Delta = \sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} d_t \ . \tag{5}$$

Substitute equation 4 into 2 and simplify with equation 5. Δ equals present value per dollar of marginal investment of economic loss *in perpetuitas* due to depreciation. This specification relaxes restriction that periodic depreciation occurs at periodic rate δ.[4]

The d_j schedule for $j = 0,...,\infty$ accommodates any pretax cash flow time-path.[5] For level perpetual real cash flows $d_j = 0$ for $j = 0,\ldots,\infty$ and Δ from equation 5 equals 0 indicating present value of economic loss is zero. For a one-hoss shay pattern with level real pretax cash flow for N periods and subsequent immediate expiration then $d_N = 1$, $d_j = 0$ otherwise, and $\Delta = (1 + r - \pi)^{-N}$. For declining cash flow patterns typical of many capital budgeting projects $d_j > 0$. For example, with straight-line decline over N years $d_j = 1/N$ for $j = 1,\ldots,N$ and 0 otherwise. For growing streams $d_j < 0$. Hence, for a hump-shaped pretax cash flow stream near-term d_j are negative as periodic cash flows get larger and then remote d_j are positive as periodic incremental cash flows diminish

[4] Real periodic economic depreciation equals periodic decline in discounted value of remaining real after-tax cash flows. The d_j series implies a vector of endogenous asset price-changes dependent upon interaction between vectors of pretax cash flows and tax depreciation deductions. Samuelson (1964) and Fane (1987) explore this endogeneity issue.

[5] The d_j series embodies many implicit assumptions about production and cost functions (see Thomas (1969, pp. 41-47) for relevant discussion). For a standard set of assumptions the d_j series is equivalent to an exogenous capacity depreciation schedule. Caballero (1999) argues importance of endogenous depreciation. For my purposes, however, that innovation unnecessarily complicates analyses.

toward zero. For a one-period model d_1 = 1 and $\Delta = (1 + r - \pi)^{-1}$. Finally, the standard assumption that real pretax cash flow declines along a specific perpetual geometric path at rate δ assumes that $d_j = \delta(1-\delta)^{j-1}$ in which case $\Delta = \delta(r + \delta - \pi)^{-1}$. Present value of economic loss *in perpetuitas* per dollar of marginal investment for the standard case is $\delta(r + \delta - \pi)^{-1}$.

Obtain equilibrium user cost by substituting, simplifying, and rearranging equation 2:

$$c_s = \frac{q_s\ (r_s - \pi_s)\ (1 - \tau_s Z_s)}{(1 - \tau_s)\ (1 - \Delta_s)} \quad . \tag{6}$$

This important equation generalizes within user cost dynamic information about tax policies, economic depreciation, and pretax cash flows. For the special case when real pretax cash flow declines along a perpetual geometric path at rate δ then user cost c equals $q(r + \delta - \pi)(1 - \tau Z)/(1 - \tau)$, the Hall-Jorgenson standard. Even beyond boundaries of that special case, however, user cost is a powerful workhorse for economic analysis.

Glean fundamental and novel insight about dynamic relation between depreciation and user cost. Set τ = 0 and rearrange equation 6 to obtain:

$$q_s = \left(\frac{c_s}{r_s - \pi_s}\right)\left(1 - \Delta_s\right) \ . \tag{7}$$

First right-hand-side term $c \div (r - \pi)$ equals present value of periodic perpetual cash flow. In the absence of depreciation Δ = 0 and the asset supply price equilibrates to present value of the perpetuity. For a nondepreciating asset providing perpetual pretax cash flow c of \$1 and with real rate $r - \pi$ of 5 percent, for example, present value of the perpetuity is \$20. For this example peg the asset supply price q at \$20.

Now introduce periodic depreciation. Say the first pretax cash flow $c_{s,1}$ equals \$1 and declines by straight-line throughout a 4-year service life in which case $d_j = ¼$ for $j = 1,\ldots,4$ and Δ from equation 5 easily computes as 0.8865. Δ measures economic loss *in perpetuitas* due to depreciation and indicates that the 4-year cash flow stream loses 88.65 percent of the \$20 perpetuity value. Compute that present value of the 4-year real cash flow stream ($c_{s,1}$ = \$1.00; $c_{s,2}$ = \$0.75; $c_{s,3}$ = \$0.50; $c_{s,4}$ = \$0.25) equals \$2.2702, some 11.35 percent of the \$20 perpetuity value (the loss indeed is 88.65 percent *in perpetuitas*). Equilibrium user cost c computes from equation 6 as \$8.8096 [= \$20 × 0.05 ÷ (1 – 0.8865)].

Equation 6 reveals that dynamic response of user cost to change in Δ or τ is identical (with $Z = 0$). Lose, say, *nine-tenths* of perpetuity value because either Δ or τ goes from 0 to 0.90. Result is that *one-tenth* of perpetuity value remains and user cost c rises *ten-fold* to offset the leakage and restore zero net present value equilibrium toward fixed asset supply price q.[6]

These implications affect tax neutrality of user cost. Equation 7 specifies equilibrium condition in absence of taxes between user cost c and asset supply price q. Two different criteria for a neutral tax system exist. One criterion is that when c and q are invariant to taxes then certainly tax policy is neutral. Comparison of equations 6

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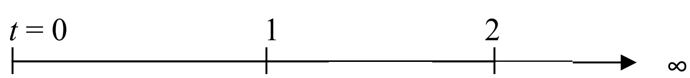
[6] An interesting special case of this multiplier effect occurs when real pretax cash flow is level at c for N periods (one-hoss shay). For that case $d_j = 0$ for $j = 1,\ldots,N-1$ and $d_N = 1$ and $\Delta = (1 + r - \pi)^{-1}$. Total differentiation of equation 7 with these settings (and differentials $dq = d(r - \pi) = 0$ and $dN = 1$) reveals that $dc \times FVIVA_{r-\pi,\, N} \approx -c$, where $FVIVA_{r-\pi,\, N}$ is the standard future value annuity factor. Explanation: increase real cash flow stream by one period and equilibrium user cost declines by dc such that the future value of N consecutive dc equals the appended c. User cost perfectly capitalizes the incremental c.

and 7 shows that immediate expensing of capital expenditures (Z = 1), and nothing else, satisfies this criterion. A second-best criterion for tax neutrality is equality of marginal effective tax rates across assets, sectors, financing sources, etc. With τ >0 and Z < 1 then either c or q or both vary with tax policy. Making further inferences requires investigation of *METR*.

II. Pedantries with *METR*

Throughout this illustration suppose that the expected real after-tax rate of return $r - \pi$ on a new marginal investment is 10 percent (= IRR^{post}). Consider a scenario in which a capital asset delivers real after-tax cash flow of \$90 per year perpetually. Asset supply price q equilibrates to total discounted after-tax cash flow of \$900. Obtain fundamental insights about the *METR* from a simplistic scenario devoid of tax shields, inflation, or depreciation (i.e, $\pi = \delta = Z = 0$). The graphic below illustrates the scenario.



After-tax cash flows:	\$90	\$90
Pretax cash flows:	\$120	\$120

present value$|_{10\%}$ *of after-tax cash flows = \$900;* $IRR|_{pv@\$900}$ *= 10.0 percent*
present value$|_{10\%}$ *of pretax cash flows = \$1,200;* $IRR|_{pv@\$900}$ *= 13.3 percent*

$$METR|_{IRR\ comparison} = (13.3\% - 10.0\%) \div 13.3\% = 25 \text{ percent}$$
$$METR|_{pv\ comparison} = (\$1,200 - \$900) \div \$1,200 = 25 \text{ percent}$$

User cost c equals pretax cash flow during first period of use such that, conditioned on all available information, the marginal real investment represents a zero net present value venture. Set statutory tax rate τ at 25 percent and, as the graphic shows, the perpetual pretax annual cash flow c equilibrates to \$120 [i.e., d_j = 0.0 and Δ = 0.0 and user cost from equation 6 is \$900 × 0.10 ÷ (1 – 0.25)]. The annual tax liability equals \$30. Expected pretax rate of return IRR^{pre} is the discount rate that equates the \$900 asset supply price to the stream of perpetual \$120 annual pretax cash flows and equals 13.3 percent; user cost per dollar of capital c/q equals 13.3 cents and depreciation δ equals zero. Compute with the standard definition that marginal effective tax rate, denoted $METR^{irr}$, is 25 percent. Likewise, comparison of total discounted pretax (\$1,200) and after-tax (\$900) cash flows reveals that taxes divert 25 percent of discounted cash flow. $METR^{irr}$ for this special case possesses intuitively pleasing properties.

A conceptually trivial adjustment to the simplistic illustration reveals a fundamental flaw with the standard, however. Suppose that this real capital asset delivers after-tax capital income of \$90 *for 1 year* (instead of perpetually) while still the relevant after-tax financing rate is 10 percent. Asset supply price q equilibrates to total discounted after-tax cash flow of \$82.

	$t = 0$	1	2
After-tax cash flows:		\$90	\$0
Pretax cash flows:		\$120	\$0

present value$|_{10\%}$ *of after-tax cash flows = \$82;* $IRR|_{pv@\$82} = 10.0$ *percent*
present value$|_{10\%}$ *of pretax cash flows = \$109;* $IRR|_{pv@\$82} = 46.7$ *percent*

$$METR|_{IRR\ comparison} = (46.7\% - 10.0\%) \div 46.7\% = 78.6 \text{ percent}$$
$$METR|_{pv\ comparison} = (\$109 - \$82) \div \$109 = 25 \text{ percent}$$

Except for the truncated horizon everything intuitively seems the same as before: statutory tax rate τ is 25 percent and $d_1 = 1.0$ and Δ from equation 5 equals 1.10^{-1} and pretax cash flow c equilibrates to \$120; that is, user cost from equation 6 equals \$82 × 0.10 ÷ ((1 – 0.25) × (1 – 1.10^{-1})). Because there are no tax shields the tax liability still equals \$30. Expected pretax rate of return IRR^{pre} equates \$82 asset supply price to pretax cash flow of \$120 and equals 46.7 percent; user cost per dollar of capital c/q equals 146.7 cents and depreciation δ is 100 percent. The standard definition comparing internal rates of return computes that $METR^{irr}$ is 78.6 percent. The asset, however, delivers total discounted pretax cash flow of \$109 and incurs total discounted taxes of \$27 – *discounted taxes still equal 25 percent of discounted pretax cash flow*.

The preceding one-period simplistic illustration strikes at the heart of the standard definition and reveals an insurmountable fatal flaw. Pay \$82 today and in one period receive pretax cash flow of \$120, pay taxes of \$30, and realize after-tax cash flow of \$90 for a 10 percent after-tax rate of return. Compare pretax and after-tax rates of return and compute that $METR^{irr}$ equals 78.6 percent. Compare cash flows and compute that $METR^{pv}$ equals 25 percent. The standard definition is intuitively implausible and yields an untrustworthy measure for assessing efficiency and distributional consequences of tax policy.

For this special case of the one period model devoid of tax shields and inflation (i.e, $\pi = Z = 0$ and $\delta = 1$) standard $METR^{irr}$ reduces to

$$METR^{irr} = \frac{\tau(1 + r)}{r + \tau} .$$

Vary after-tax discount rate r and obtain $METR^{irr}$ anywhere between τ and 100 percent! Suppose a capital asset delivers at $t = 1$ after-tax cash flow of \$90. With statutory tax rate τ of 25 percent pretax cash flow c equilibrates to \$120 irrespective of r. With r

equal to zero percent, for example, asset supply price q equals \$90, expected pretax rate of return IRR^{pre} equals 33.3 percent [= (\$120 – \$90) ÷ \$120], and $METR^{irr}$ is an implausible 100 percent [= (0.333 – 0) ÷ 0.333]. As r tends to infinity then q converges to zero and $METR^{irr}$ converges to the statutory tax rate τ of 25 percent. Irrespective of *r,* however, *discounted taxes for this one period model always equal 25 percent of discounted pretax cash flow*.

Consider now effect on $METR^{irr}$ as length of the \$90 after-tax cash flow stream increases (set r and τ at 10 and 25 percent, respectively, and $\pi = Z = 0$). $METR^{irr}$ declines monotonically from 78.6 percent down to 25 percent as service life increases from 1-year out to infinity. With a 14-year stream of \$90 after-tax cash flows and \$120 pretax cash flows, for example, the pretax rate of return IRR^{pre} equals 15.8 percent and $METR^{irr}$ equals 36.6 percent.[7] Discounted after-tax cash flows of \$663, however, are 25 percent less than discounted pretax cash flows of \$884. Even though $METR^{irr}$ attains between 25 and 78.6 percent as service life varies*, discounted taxes equal 25 percent of discounted pretax cash flow irrespective of service life*.

Now consider effects of tax shields. A fiscal policy introducing or enlarging a tax shield reduces discounted tax liabilities. The user cost framework implicitly assumes that the marginal capital goods investor capitalizes incremental tax savings while the asset supply price remains unchanged.[8] A fiscally stimulating tax policy presumably

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[7] Asset supply price q of \$663 equals discounted sum of \$90 for 14 years at 10%. With level pretax cash flow for 14 periods and subsequent expiration $d_{14} = 1.0$ *and* $d_j = 0$ otherwise and Δ from equation 5 equals 1.10^{-14}. User cost for this 14-year investment computes from equation 6 and equals \$120 {= \$663 × 0.10 ÷ [(1 – 0.25) × (1 – 1.10^{-14})] }.

[8] Goolsbee (1998) examines whether Congressionally-mandated tax savings flow to capital goods investor or supplier.

increases investment and capital accumulation because that specific asset which previously was zero or slightly negative net present value now becomes positive *NPV.* Investment and capital intensity increases until, due to diminishing marginal returns, discounted pretax cash flow declines by amount of incremental discounted tax savings and renders the marginal investment once again a zero net present value venture.

Revisit the simplistic illustration and say now that the capital asset provides annual tax depreciation deductions along a 12-year straight-line capital recovery period. With perpetual $90 after-tax annual cash flows and 10 percent after-tax rate of return r the capital asset provides annual tax deductions of $75 [= $900 ÷ 12] for twelve years, zero thereafter. Discounted tax depreciation deductions equal 56.8 cents per dollar of capital. Every $1 tax deduction directly saves 25 cents in taxes; pretax cash flow consequently equilibrates downward by 25 cents which saves even more in taxes (a *multiplier* effect). Every $1 of discounted tax deduction reduces total discounted taxes by 33.33 cents [= $1 × $\tau \div (1 - \tau)$] and likewise equilibrium discounted pretax cash flow declines by 33.33 cents.

The 12-year straight-line depreciation tax shield for the $900 capital asset provides total discounted incremental tax savings of $170 [=$900 × 0.5678 × 0.25 ÷ (1 – 0.25)]. Discounted pretax cash flow declines by $170 from $1,200 without the tax shield to $1,030 with it. Amortizing $170 at 10 percent along a perpetual horizon means equilibrium pretax cash flow declines $17 per year. Pretax cash flow c equilibrates to $103 per annum perpetually [i.e., for a perpetual and level pretax cash flow stream Δ = 0 and user cost from equation 6 is $900 × 0.10 × (1 – 0.25 × $0.568) ÷ (1 – 0.25)].

	$t = 0$	1	12	13	∞
After-tax cash flows:		\$96	\$96	\$77	
Tax deductions:		\$75	\$75	\$0	
Pretax cash flows:		\$103	\$103	\$103	

present value$|_{10\%}$ *of after-tax cash flows = \$900;* *IRR*$|_{pv@\$900}$ *= 10.0 percent*
present value$|_{10\%}$ *of pretax cash flows = \$1,030;* *IRR*$|_{pv@\$900}$ *= 11.4 percent*

$$METR|_{IRR\ comparison} = (11.4\% - 10.0\%) \div 11.4\% = 12.6 \text{ percent}$$

$$METR|_{pv\ comparison} = (\$1,030 - \$900) \div \$1,030 = 12.6 \text{ percent}$$

Expected pretax rate of return IRR^{pre} equals 11.4 percent [= \$103 ÷ \$900] and $METR^{irr}$ is 12.6 percent [= (0.114 – 0.10) ÷ 0.114]. The tax shield reduces the marginal effective tax rate (0.126) to about half the statutory tax rate (0.25). For this scenario discounted pretax cash flow equals \$1,030 and discounted after-tax cash flow equals \$900.[9] Discounted taxes also equal 12.6 percent of total discounted pretax cash flow. $METR^{irr}$ for this special case possesses intuitively pleasing properties.

Once again, however, intuitive plausibility of the standard definition shatters easily. Consider the scenario in which the asset provides a 14-year stream of level pretax cash flow and the capital goods supply price equilibrates to \$663 and the tax depreciation deduction is \$55 per year for twelve years [= \$663 ÷ 12] . Equilibrium pretax cash flow c equals \$103 per annum for 14 years [i.e., $d_j = 0.0$ for $j = 1,\ldots,13$ and $d_{14} = 1.0$ and user cost from equation 6 is \$685 × 0.10 × (1 – 0.25 × 0.568) ÷ {(1 – 0.25) × (1 – 1.10 $^{-14}$)}].[10]

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[9] With 12-year straight-line tax depreciation the after-tax cash flow equals \$95.97 [= \$102.97 × (1 – 0.25) + 0.25 × \$75] for the first twelve years and \$77.22 [= \$102.97 × (1 – 0.25)] thereafter perpetually.

[10] Discounted incremental tax savings equal \$125.49 [=\$663 × 0.5678 × 0.25 ÷ (1 – 0.25)]. Discounted pretax cash flow for 14-years therefore declines by \$125.49 to \$758.52 with the tax shield from \$884.00 without. Amortizing \$125.49 at 10 percent along a 14-year horizon means pretax cash flow declines \$17.03 per year. User cost falls by \$17.03 to \$102.97 with the tax shield from \$120.00 without.

	$t = 0$	1	12	13	14	15
After-tax cash flows:		\$91	\$91	\$77	\$77	\$0
Tax deductions:		\$55	\$55	\$0	\$0	\$0
Pretax cash flows:		\$103	\$103	\$103	\$103	\$0

present value$|_{10\%}$ *of after-tax cash flows = \$663;* $IRR|_{pv@\$663}$ *= 10.0 percent*
present value$|_{10\%}$ *of pretax cash flows = \$759;* $IRR|_{pv@\$663}$ *= 12.5 percent*

$$METR|_{IRR\ comparison} = (12.5\% - 10.0\%) \div 12.5\% = 20.4 \text{ percent}$$

$$METR|_{pv\ comparison} = (\$759 - \$663) \div \$759 = 12.6 \text{ percent}$$

Expected pretax rate of return IRR^{pre} equates the \$663 asset supply price to the 14-year discounted stream of \$103 per annum and equals 12.5 percent (user cost per dollar of capital c/q equals \$0.1553). $METR^{irr}$ equals 20.4 percent. Total discounted taxes, however, equal 12.6 percent of total discounted pretax cash flow. The emergent trend continues: a definition for marginal effective tax rate comparing present values has intuitively pleasing properties whereas, even with tax shields, comparison of internal rates of return is spurious.

Difficulties with standard *METR* mount even more with introduction of periodic depreciation. Revisit the scenario in which the \$900 capital asset promising perpetual pretax cash flows receives a tax shield from annual depreciation deductions of \$75 for twelve years except assume now that pretax cash flow declines 4 percent per year perpetually ($\delta = 0.04$). First period pretax cash flow c equilibrates to \$144 [i.e., user cost from equation 6 equals \$900 × (0.10 + 0.04) × (1 – 0.25 × 0.568) ÷ (1 – 0.25)].

	$t = 0$	1	2	12	13 $\rightarrow \infty$
After-tax cash flows:		\$127	\$123	\$88	$\$92(1 - 0.25)(1 - 0.04)^{t-12}$
Tax deductions:		\$75	\$75	\$75	\$0
Pretax cash flows:		\$144	\$138	\$92	$\$92(1 - 0.04)^{t-12}$

present value$|_{10\%}$ *of after-tax cash flows = \$900;* $IRR|_{pv@\$900}$ *= 10.0 percent*
present value$|_{10\%}$ *of pretax cash flows = \$1,030;* $IRR|_{pv@\$900}$ *= 12.0 percent*

$$METR|_{IRR\ comparison} = (12.0\% - 10.0\%) \div 12.0\% = 16.8 \text{ percent}$$
$$METR|_{pv\ comparison} = (\$1,030 - \$900) \div \$1,030 = 12.6 \text{ percent}$$

For the scenario in the graphic with δ of 4 percent the user cost per dollar of capital c/q equals \$0.160 and $METR^{irr}$ is 16.8 percent. Double δ to 8 percent and c/q increases to \$0.206 and $METR^{irr}$ becomes 20.6 percent. Increase δ to 88 percent and c/q increases to \$1.121 and $METR^{irr}$ becomes 58.5 percent. A rising δ certainly increases user cost c because near-term cash flows increase to offset relatively rapid decline in remote cash flows – *but they exactly offset leaving discounted pretax cash flows constant and net present value at zero irrespective of δ.*

Even though increasing δ increases $METR^{irr}$ there is no effect on discounted sum of pretax or after-tax cash flows or taxes. User cost adjusts to reestablish zero net present value marginal investment equilibrium and the asset supply price is constant. Relation between δ and $METR^{irr}$ is spurious. Capital goods supply price q capitalizes effects of economic depreciation, Z capitalizes discounted tax depreciation deductions, and δ is irrelevant to measurement of tax effects.

III. Resolution

Preceding difficulties with standard measurements for marginal effective tax rate $METR^{irr}$ arise for two reasons. One reason explains why the standard is valid in some special cases. The other reason explains why the standard is useless except for those special cases.

Derivation of Hall-Jorgenson user cost in equation 1 assumes a perpetual time-path of pretax cash flows. All illustrations herein with valid $METR^{irr}$ involve an infinite service life. Especially impressive is ability of user cost to amortize effects of a finite tax shield throughout an infinite pretax return stream. Hall-Jorgenson user cost is valid for perpetual pretax return streams and properly accounts for finite tax shields – thus far $METR^{irr}$ is valid too – yet things fall apart either of two ways and both pertain to introduction of economic depreciation.

(1) Hall-Jorgenson user cost properly accommodates $\delta \neq 0$ (subject to $r > -\delta$) but $METR^{irr}$ does not. For any perpetual pretax return stream subject to geometric depreciation the standard definition for marginal effective tax rate fails to properly measure tax effects.

(2) Hall-Jorgenson user cost, and consequently $METR^{irr}$ too, fail to account for economic depreciation that truncates the pretax return stream. That is, for every finite pretax return stream equation 1 incorrectly and equation 6 correctly specifies *user cost of capital*.[11]

[11] For a one period model equation 6 reduces to equation 1 with δ =100 percent.

The general difficulty rendering $METR^{irr}$ useless except for a few special cases is reliance on additive operation with internal rates of return $c/q - \delta$. The entire pretax return stream collapses into the summary statistic c/q while the entire stream of economic depreciation collapses into δ. The standard definition explicitly assumes that these internal rates of return, c/q and δ, are transitive and additive. In fact, they are not. Intertemporal cash flow distributions possess many well-known dynamic properties (see Hicks (1939, especially pp. 186-188) for discussion of an alternative summary statistic for intertemporal cash flow streams). But as Alchian (1955) explains, decision rules relying on internal rates of return are generally wrong.

Rates of return lack dynamic robustness necessary for collapsing an entire stream of cash flows into one number additive with other such collapsed numbers. Information content of the entire time-path is very relevant for determination of most outcomes. Present values properly collapse dynamic economic information into additive summary statistics.

A robust definition for the marginal effective tax rate is expected discounted taxes divided by expected discounted pretax cash flows. Equation 2 specifies present value of pretax cash flows (and also subtracts proportional taxes and adds discounted depreciation tax savings). The present value of after-tax cash flows equals asset supply price q. Find $METR^{pv}$ as:

$$METR_s^{pv} = \frac{\overbrace{\sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} c_{s,t}}^{\text{discounted pretax cash flow}} - q_s}{\sum_{t=1}^{\infty} (1 + r_s - \pi_s)^{-t} c_{s,t}} .$$

Simplify the preceding and obtain

$$METR_s^{pv} = \frac{\tau\,(1 - Z_s)}{1 - \tau\, Z_s} \quad . \tag{8}$$

Marginal effective tax rate $METR^{pv}$ depends exclusively on the statutory tax rate and discounted tax depreciation deductions – *asset characteristics such as service life or time-paths of pretax cash flow and economic depreciation are irrelevant*. Pretax cash flow equilibrates through user cost to underlying asset characteristics and maintains zero net present value of after-tax cash flows. This renders independence between real asset characteristics and tax policy.

Scenarios when standard $METR^{irr}$ are valid are special cases of $METR^{pv}$. For those special cases either definition yields the same number. More generally, however, standard $METR^{irr}$ correlates spuriously with tax effects whereas $METR^{pv}$ measures discounted taxes as a proportion of discounted pretax cash flow.

IV. Implications

Nearly a century of tax design builds on the paradigm that tax fairness and efficiency requires allocating capital expenditures along tax depreciation schedules reflecting economic depreciation of underlying assets.[12] From first issuance of *Bulletin F* in 1920 through today's *Modified Accelerated Cost Recovery System* (MACRS) intention intricately links tax with economic depreciation. According to equation 8, however, that paradigm is wrong. Instead, an analogue to Fisher (1930) separation for independence of financing and real investment cash flows exists for fiscal tax policy: *tax efficiency and marginal effective tax rates are independent of real asset characteristics*.

[12] Kern (2000) found that The Tariff Act of 1909 preceding ratification of the 16th amendment declares that business may deduct "a reasonable allowance for depreciation of property".

Table 1 provides evidence on possible empirical significance for this finding. Column 1 illustrates an asset with 7-year service life that depreciates for tax purposes along the 5-year MACRS schedule. Discounted sum of tax depreciation deductions slightly exceeds seventy-three cents (real financing rate $r - \pi$ is set at 8 percent). The column assumes that pretax cash flow declines along a geometric time-path at a double-declining rate of 2/7, that is 28.6 percent.[13] Statutory tax rate τ is set to 30 percent. User cost computes in row 3 from Hall-Jorgenson specification and equals \$0.4076. Internal rate of return IRR^{pre} for the pretax cash flow stream, $c/q - \delta$, equals 12.2 percent and $METR^{irr}$ equals 34.4 percent. Standard interpretation for $METR^{irr} > \tau$ is that rate of economic depreciation exceeds "rate" of tax depreciation.

With δ of 28.6 percent and pretax cash flow stream that truncates at 7 years the economic loss *in perpetuitas* Δ computes from equation 5 as 0.7934 (row 7).[14] Row 6 of column 1 computes user cost c from equation 6. Present value of pretax cash flow equals \$1.1145 per dollar of asset. Discounted taxes equal \$0.1145. And $METR^{pv}$ equals 10.3 percent, about one-third statutory rate τ. $METR^{irr}$ exceeds $METR^{pv}$ more than three-fold![15]

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[13] Fraumeni (1997, p. 7) summarizes "For most assets, empirical studies on specific assets conclude a geometric pattern of depreciation is appropriate." Settings in Table 1 are reasonable representations from her Table 3. For example, BEA asset type "Office, computing, and accounting machinery after 1978" has 7 year service life and 2.18 declining balance rate (31.2 percent) and is depreciated for tax purposes in the 5-year MACRS class. My column 1 fairly represents that asset type.

[14] A technicality arises when truncating a perpetual cash flow stream (recall Hall-Jorgenson is strictly correct only for perpetual streams or for a one-period model). Table 1 computes c in row 6 by setting $d_N = 1 - \sum d_j$ for $j = 1,\ldots, N - 1$. For this scheme pretax cash flow declines smoothly at rate δ for 7 periods and then immediately drops to zero. For a \$1 asset and a perpetual pretax cash flow stream IRR^{pre} equals as row 4 shows 12.2 percent (= $c_{equation\ 1} - \delta$). But for a 7-year pretax cash flow stream actual IRR is higher, 12.8 percent, and actual $METR^{IRR}$ is 37.6 percent. An internal inconsistency arises when applying equation 1 within a finite setting.

[15] Slemrod *et al* (2005) write "The empirical literature that seeks to measure the effective tax rate on new investment offers a striking paradox. On the one hand, summary measures of the effective tax rate on new investment are normally quite high. On the other hand, the amount of revenue actually collected is

Column 2 demonstrates that $METR^{irr}$ depends on real asset characteristics whereas $METR^{pv}$ does not. For a 12-year asset service life the economic depreciation rate δ (row 2) naturally is lower than in column 1 and user cost is less too (rows 3 and 6). The same 5-year MACRS tax depreciation schedule (row 1) applies to columns 1 and 2, however. For column 2 $METR^{irr}$ of 26.1 percent is less than statutory rate τ indicating that economic depreciation proceeds slower than tax depreciation.

Row 8 for columns 1 and 2 shows that discounted pretax cash flow per dollar of asset remains constant at $1.1145 irrespective of asset service life. This occurs because depreciation tax savings τZ are identical and asset supply price q is constant at one dollar. User cost equilibrates to changes in asset characteristics rendering discounted cash flows, pretax as well as after-tax, invariant to real asset characteristics. With 5-year MACRS tax depreciation, statutory tax rate τ of 30 percent, and 8 percent real rate $r - \pi$, discounted taxes always equal 10.3 percent of discounted pretax cash flow.[16]

Columns 3 and 4 show other representative assets. With 12-year asset service life and tax depreciation along the 10-year MACRS class $METR^{irr}$ equals 34.5 percent, about the same (34.1 percent) for the asset with 32-year service life depreciating in the 21-year MACRS class. Scanning along row 5 suggests little variation in marginal

apparently very low." Subsequent analyses lead them to "conclude that the effective tax rate does seem to be much lower than existing measures suggest."

[16] Actual time-paths of economic depreciation and pretax cash flows are irrelevant to measurement of marginal effective tax rates. With, for example, 7-year service life and 5-year MACRS tax schedule user cost c adjusts to $0.3480 for straight-line depreciation (d_j = 1/7 for j = 1,…7; Δ = 0.7438) or to $0.2141 for one-hoss shay (d_j = 0 for j = 1,…6 and d_7 = 1; Δ = 0.5845). For these and all cases, however, discounted pretax cash flow equals $1.1145 and $METR^{pv}$ equals 10.3 percent.

effective tax rates. Scanning along row 10 belies that suggestion. $METR^{pv}$ range between 10.3 and 24.6 percent.

Glean further insight about relation between $METR^{irr}$ and $METR^{pv}$ for the neoclassical case when pretax cash flow declines throughout a perpetual horizon at rate δ. For this special case $\Delta = \delta/(r + \pi - \delta)$ and user cost equation 6 simplifies to Hall-Jorgenson equation 1. Solve for $METR^{irr}$:

$$METR^{IRR} \;=\; \frac{(r + \delta - \pi)\,\tau\,(1-Z)}{(r + \delta - \pi)(1-\tau Z) - (1-\tau)\delta} \quad . \tag{9}$$

Notice that when δ equals zero then irrespective of Z equation 9 reduces to 8 indicating that $METR^{irr}$ equals $METR^{pv}$, a scenario that section II illustrates numerically. A gap between $METR^{irr}$ and $METR^{pv}$ opens when $\delta \neq 0$, however, and *ceteris paribus* the gap gets bigger as δ increases.

Further impose on equation 9 that real tax depreciation proceeds throughout a perpetual horizon at rate δ^{tax}. Solve equation 3 to find that $Z = \delta^{tax}/(r + \pi - \delta^{tax})$. Table 2 provides numerical estimates for this special case. For all entries statutory tax rate τ equals 0.30 and real rate $r - \pi$ equals 4 percent and asset supply price q equals \$1. Comparison of $METR^{irr}$ in row 1 across the three columns reveals well-known (see footnote 2) relation: *When tax depreciation rate δ^{tax} exceeds economic depreciation rate δ then $METR^{irr}$ is less than statutory tax rate τ and vice versa.*

Standard definition for $METR^{irr}$ implies that equivalence of tax with economic depreciation eliminates variation in marginal effective tax rates because for all assets $METR^{irr}$ equal τ and perfect tax neutrality occurs. This relation *appears* to justify linking tax policy with real asset characteristics. Rows 3-to-7 conduct a cash flow analysis on this special case, however, and find that the apparition is implausible.

For column 2, $\delta^{tax} = \delta$ and $METR^{irr} = \tau$. User cost c is first period's pretax cash flow and equals \$0.2971. With perpetual geometric depreciation the present value of pretax cash flow equals $c(r + \delta - \pi)^{-1}$ which, for column 2, is \$1.0612. Discounted tax depreciation deductions Z equal \$0.8571. Present value of after-tax cash flow equals discounted pretax cash flow net of proportional taxes, \$1.0612 × (1 – 0.30), plus discounted depreciation tax savings, \$0.8571 × 0.30. Discounted after-tax cash flow equals asset supply price of \$1 and satisfies zero net present value investment equilibrium. Discounted taxes of 6.12 cents equal 5.77 percent of discounted pretax cash flows. $METR^{irr}$ overstates $METR^{pv}$ more than five-fold!

Tax depreciation deductions for all scenarios in table 2 offset more than 90 percent of all proportional taxes. Even though $METR^{irr}$ range from 24 to 40 percent, discounted taxes as a proportion of discounted pretax cash flow extend over a relatively narrower range from 4.3 to 8.7 percent. Standard definition for marginal effective tax rate places false importance on measuring rate of economic depreciation. In equation 9 subtraction from the numerator of $(1 - \tau)\delta$ distorts measurement of marginal effective tax rate.

Table 3 delves further into the special case wherein $\delta^{tax} = \delta$ and $METR^{irr} = \tau$. Columns vary real after-tax financing rate $r - \pi$ and rows vary depreciation rates while maintaining $\delta^{tax} = \delta$ and consequently $METR^{irr}$ always equal statutory tax rate τ of 0.30. Table entries equal marginal effective tax rate $METR^{pv}$ from equation 8. All entries are substantially less than 30 percent.

The cell with δ = 24 percent and $r - \pi$ of 4 percent (row 4, column 2), for example, is fair representation of equipment in a typical economic environment. Present value of tax depreciation deductions Z and economic loss *in perpetuitas* Δ are equal at 0.8571; discounted depreciation tax savings equal 25.71 cents; user cost c equals \$0.2971; and discounted taxes of 6.12 cents equal 5.77 percent of discounted pretax cash flows. $METR^{irr}$ overstates $METR^{pv}$ five-fold!

Scan down column 2 and notice huge variation in $METR^{pv}$. These column entries conceivably represent cross-sectional variation in marginal effective tax rates for different asset types. Structures, for example, may have δ around 3 to 6 percent. Pursuing policies that link tax with economic depreciation causes tax inefficiencies.

User cost equation 6 shows that with immediate expensing (Z = 1) tax policy has no effect on user cost c or asset price q. Except for that policy nothing else works quite as well. A second-best solution applies the same tax depreciation schedule to all assets; e.g., depreciate 75 percent of capital expenditures at time 1 and 25 percent at time 2 irrespective of actual asset service life. This second-best solution equalizes marginal effective tax rates. Second-best solution, however, exerts pressure that causes divergence between user costs (or supply prices q) with and without taxes.

With immediate expensing Z = 1 and tax rate τ according to equation 6 exerts no influence on user cost c or asset supply price q. For a nondepreciating asset providing perpetual pretax cash flow c of \$1 and with real rate $r - \pi$ of 4 percent, for example, present value of the perpetuity is \$25. Asset supply price q for this example equals \$25. With immediate expensing at statutory tax rate τ of 30 percent immediate tax savings equal \$7.50. The perpetual stream of \$1 pretax cash flows incurs periodic taxes of 30 cents. With real rate $r - \pi$ of 4 percent present value of taxes equals \$7.50 (=\$0.30 ÷ 0.04). Present value of pretax cash flow net of proportional taxes is \$17.50 (=\$0.70 ÷ 0.04) which, added to immediate tax savings of \$7.50, satisfies zero net present value investment equilibrium. *First-best criterion for tax efficiency requires that present value of tax savings from capital income equal present value of taxes due*.

V. Conclusion

Analysis of efficiency and distributional consequences of tax policies relies on measurement of marginal effective tax rates ("*METR*"). Two decades of economic research build upon a standard that *METR* equals expected pretax minus after-tax rates of return divided by pretax rate of return. Measurement of economic depreciation is an important component for the standard approach.

My study generalizes specification of pretax cash flow streams thereby enabling detailed cash flow analysis of economic depreciation and tax effects. The analysis reveals a fundamental flaw rendering useless the standard measure except in a few special cases. Primarily the flaw occurs because the standard assumes that internal rates of return are additive and transitive when in fact they are not. The standard fails for capital assets that depreciate or have finite service life.

My study specifies *METR* as present value of expected taxes divided by present value of expected pretax cash flows. *METR* measurements with this approach are substantially less, on the order of one-fifth, than the standard based on internal rates of return.

My study suggests a separation theorem for tax policy on capital income: *tax efficiency and marginal effective tax rates are independent of real asset characteristics*.

Cause of the separation is that user cost capitalizes real asset characteristics and renders present values of cash flows invariant to time-paths for economic depreciation and pretax cash flow. Information requirements for the new framework are less than for the standard because actual asset service life, efficiency patterns, and economic depreciation are irrelevant to *METR* measurement.

My study also suggests that a first-best criterion for tax neutrality on capital income allows discounted tax depreciation deductions equal to expected discounted taxes. Immediate expensing satisfies this criterion. This policy imposes on capital income a zero net present value condition on discounted tax liabilities across industries, sectors, factors of production, form of business organization, financing sources, etc. This notion of tax efficiency fits easily within the Harberger model because prices equilibrate to present values of expected service streams. Conceivably a duality exists between sources and uses of tax revenues. That is, perhaps fiscal policy minimizes economic efficiency losses by equivalencing present values of taxes on capital income with tax savings and similarly by enabling expenditures that satisfy analogous albeit nebulous zero net present value criteria. Regardless, my study also shows that a second-best policy for tax neutrality is one that applies a uniform tax depreciation schedule to all assets irrespective of actual service life; e.g., depreciate 75 percent of capital expenditures during first year of use, 25 percent during second. Marginal effective tax rates across assets are equal with this second-best policy but pressure exerts on user costs and asset supply prices.

Kern (*op cit*) reports that during the 1920s businesses were loathe to claim depreciation deductions because the noncash charges depress reported earnings.

Policy-maker motivation, however, was to properly measure and tax periodic economic income. Immediate expensing depresses periodic earnings even more. My study suggests that properly measuring periodic economic income or economic depreciation is irrelevant for assessing efficiency and distributional consequences of tax policy. Sound public policy, just like many other decisions across the spectrum of economic activity, depends on assessing net present values for alternative courses of action.

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Table 1 – $METR^{irr}$ and $METR^{pv}$ for different real asset characteristics

	Service life MACRS class	*7 years* *5-year* - 1 -	*12 years* *5-year* - 2 -	*12 years* *10-year* - 3 -	*32 years* *21-year* - 4 -
1.	Z	\$0.7327	\$0.7327	\$0.6016	\$0.2385
2.	δ	28.6 percent	16.7 percent	16.7 percent	4.7 percent
3.	c *equation 1*	\$0.4076	\$0.2749	\$0.2888	\$0.1683
4.	IRR^{pre}	12.2 percent	10.8 percent	12.2 percent	12.1 percent
5.	$METR^{irr}$	34.4 percent	26.1 percent	34.5 percent	34.1 percent
6.	c *equation 6*	\$0.4829	\$0.2877	\$0.3022	\$0.1714
7.	Δ	0.7934	0.6901	0.6901	0.3810
8.	*PV pretax cash flows*	\$1.1145	\$1.1145	\$1.1707	\$1.3264
9.	*PV taxes*	\$0.1145	\$0.1145	\$0.1707	\$0.3264
10.	$METR^{pv}$	10.3 percent	10.3 percent	14.6 percent	24.6 percent

NOTES Exogenous settings include after-tax financing rate r = 0.12; inflation rate π = 0.04; tax rate τ = 0.30; and asset supply price q = \$1.00. Z computes from equation 3 and equals present value of tax depreciation deductions per dollar of asset. δ is rate of decline in pretax cash flow and for columns 1-to-3 is set to $2/N$, where N equals service life; $\delta = 1.5/N$ for column 4. Weights $d_j = \delta\,(1 - \delta\,)^{j-1}$ for $j = 1,\ldots,N$-1 and d_N equals one minus the others so that the sum of all is unity. User cost of capital c computes in row 3 from equation 1 and in row 6 from equation 6. Row 4 computes IRR^{pre} from equation 1. Δ computes from equation 5 and is present value of economic loss *in perpetuitas* due to depreciation. $METR^{irr}$ computes from the standard definition and $METR^{pv}$ computes from equation 8. $METR^{irr}$ is relatively uniform and suggests relatively high tax efficiency. Conversely, $METR^{pv}$ show large variation and imply tax inefficiency and resultant efficiency losses for the economy due to malallocation of capital.

Table 2 – Comparison with perpetual geometric depreciation of *METR*^{irr} and *METR*^{pv}

For all cells statutory tax rate τ = 0.30 and real rate $r - \pi$ = 0.04.		*Tax depreciation occurs at rate δ^{tax} Economic depreciation occurs at rate δ* $\delta = 0.24$, $\delta^{tax} = 0.14$ - 1 -	$\delta = 0.24$, $\delta^{tax} = 0.24$ - 2 -	$\delta = 0.24$, $\delta^{tax} = 0.34$ - 3 -
1.	*METR*^{IRR}	40.0 percent	30.0 percent	24.0 percent
2.	*user cost* *c*	0.3067	0.2971	0.2926
3.	*present value of pretax cash flows*	1.0952	1.0612	1.0451
4.	*discounted tax depreciation deductions Z*	0.7778	0.8571	0.8947
5.	*discounted depreciation tax savings τZ*	0.2333	0.2571	0.2684
6.	*present value of taxes*	0.0952	0.0612	0.0451
7.	*METR*^{PV}	8.70 percent	5.77 percent	4.32 percent

NOTES Exogenous settings include statutory tax rate τ = 0.30, real rate $r - \pi$ = 0.04, and asset supply price q = \$1.00. All table entries assume that perpetual pretax cash flow declines at geometric rate δ and tax depreciation allowances occur at rate δ^{tax}. Hence, $\Delta = \delta/(r + \delta - \pi)$ and $Z = \delta^{tax}/(r + \delta^{tax} - \pi)$. User cost of capital c computes from equation 1 or 6; for this special case they are identical. Present value of pretax cash flows equals $c/(r + \delta - \pi)$. For column 2 *METR*^{irr} equals statutory tax rate τ of 0.30 because $\delta^{tax} = \delta$. For columns 1 and 3 *METR*^{irr} is greater than and less than τ, respectively, because δ^{tax} is less than and greater than δ, respectively. In all cases *METR*^{irr} overstate *METR*^{pv} by about five-fold.

Table 3 – $METR^{pv}$ for different δ and real rates $r - \pi$ when $METR^{irr}$ equals statutory tax rate τ

For all cells $METR^{irr} = \tau = 0.30$		Real after-tax financing rate $r - \pi$ 2 percent - 1 -	4 percent - 2 -	6 percent - 3 -	8 percent - 4 -
1.	δ = 3 percent	14.6 percent	19.7 percent	22.2 percent	23.8 percent
2.	δ = 6 percent	9.7	14.6	17.6	19.7
3.	δ = 12 percent	5.8	9.7	12.5	14.6
4.	δ = 24 percent	3.2	5.8	7.9	9.7

NOTES Each table entry equals $METR^{pv}$ for the respective real rate $r - \pi$ and depreciation rate δ. Exogenous settings include statutory tax rate $\tau = 0.30$ and asset supply price $q = \$1.00$. All table entries assume that perpetual pretax cash flow declines at geometric rate δ and tax depreciation equals economic depreciation at replacement cost. Hence, $\Delta = Z = \delta/(r + \delta - \pi)$. User cost of capital computes from equation 1 or 6; for this special case they are identical. $METR^{irr}$ equals statutory tax rate τ of 0.30 for all cells and suggests identical marginal effective tax rates and tax neutrality. Conversely, $METR^{pv}$ show large variation and imply tax inefficiency and resultant efficiency losses for the economy due to malallocation of capital.