The Neoclassical Theory of Term Structure

ABSTRACT
This study presents a specification for the neoclassical user cost of capital that reflects dynamic processes for debt maturity structure and for pretax cash flows. Equivalencing levered and unlevered user costs reveals tradeoffs between equilibrium financing rates and underlying processes that satisfy a dynamic no-arbitrage equilibrium condition between debt, equity, and costless reversible real investment. The primary finding is that an increase in debt ratio *or loan term* associates with an increase in equilibrium financing rate – invariance of the neoclassical user cost to leverage implies an endogenous upward sloped yield curve.

JEL codes: A10, D21, D92, E32, E43, G10

Keyword: user cost of capital

Corresponding Author:
Thomas W. Downs
Department of Economics & Finance
University of Alabama
Tuscaloosa, AL 35487-0224

email: tdowns@cba.ua.edu

The Neoclassical Theory of Term Structure

By* Thomas W. Downs

The neoclassical user cost of capital that Dale W. Jorgenson pioneers (1963, 1967) plays a central role linking asset prices, tax policies, financing rates, and other structural parameters, to determinants of capital market equilibrium. Robert E. Hall and Jorgenson (1967) obtain a user cost specification that reflects dynamic effects of tax policies. The Hall-Jorgenson specification remains the standard for analysis of real investment behavior, for capital allocation and efficiency studies, and for tax policy analysis. Since those pioneering studies, extensions to the user cost specification incorporate effects of other economic dynamics. Alan J. Auerbach (1989), for example, shows the effect on the user cost when adjustment costs for real capital stocks exist. Andrew B. Abel and Janice C. Eberly (1996) show the effect on user cost of costly reversible real investment. Hall (2001) calibrates a user cost framework with adjustment costs and concludes that the quantity of intangible capital is an increasingly important component of producer capital, an event that may provide a real explanation for the late 1990's monetary bull-market.

My specification sets real adjustment and reversibility costs at zero because for my purposes those complexities unnecessarily obscure the analysis. Instead, I focus on integrating into the user cost specification the equilibrium effects resulting from dynamic modeling of pretax cash flow streams and debt maturity structure. The result is a specification for user cost that includes two new dynamic variables. One variable measures the net present value to equity of loans that finance the marginal investment.

The second variable measures discounted pretax cash flows that the asset promises as a proportion of discounted cash flows promised by a level perpetuity.

When I restrict the general processes so that the pretax cash flow declines along a perpetual geometric path and the loan payments perfectly hold constant the debt-to-asset ratio then I find that my dynamic specification reduces to the Hall-Jorgenson standard *user cost of capital*. When further I require equivalence of levered and unlevered user costs of capital then I find that the equilibrium equity financing rate relates to the unlevered *equity cost of capital* just like Franco Modigliani and Merton H. Miller (1958) hypothesize. For this standard case the levered equity financing rate is an increasing linear function of the debt-to-equity ratio. For this special case dynamic processes reflecting debt maturity structure and pretax cash flow streams become irrelevant. Generally, however, these economic dynamics are very relevant to determination of equilibrium financing rates.

A significant finding from my study is that an increase in marginal debt ratio or loan-term exerts the same qualitative effect: the debt financing rate is less than the equity financing rate so therefore a transitory gain to equity from leverage accrues; sources of financing capitalize the gain and bid equilibrium financing rates higher until zero net present value recurs; the financing method (i.e., debt ratio or loan-term) becomes irrelevant. I further examine specification of a term-varying *debt-to-equity relative risk premium* and find deterministic yield curves that generally slope upward, but which under recessionary conditions tend to flatten or invert.

My study proceeds in Section I by embedding within the neoclassical user cost of capital dynamic processes for debt maturity and for the shape of the pretax cash flow stream. Section II introduces the debt-to-equity relative risk premium and explains effects of reward-sharing between debt and equity on an endogenous equilibrium term structure. A brief conclusion closes the study.

I. Integrating dynamic processes into the user cost specification

The *user cost of capital* derives from a dynamic specification of the zero net present value investment equilibrium in the market for real capital assets. The equilibrium condition specifies that the time s capital goods price q_s equals expected pretax cash flows net of proportional taxes plus tax savings from depreciation deductions discounted by the *unlevered equity cost of capital* ρ^u:

$$(1) \qquad q_s = \sum_{t=1}^{\infty} (1 + \overbrace{\rho_s^u - \pi_s}^{\text{real unlevered equity cost of capital}})^{-t} \underbrace{\{(1-\tau)c_{s,t} + \tau q_s z_{s,t}\}}_{\text{unlevered residual cash flow}} .$$

π is the expected inflation rate, τ is the corporate tax rate, and $z_{s,t}$ is percentage *real* tax depreciation allowance (e.g., MACRS weights) applicable to time s real investments. The term $c_{s,t}$ equals real pretax cash flow expected at time $s+t$ from the time s investment. The expression in curly brackets equals unlevered residual cash flow.[1] Each specific pretax cash flow $c_{s,t}$ and each specific unlevered residual cash flow is invariant to leverage. The first period's pretax cash flow $c_{s,1}$ equals *unlevered user cost of capital*.

Primal assumptions are that the asset supply price q and the unlevered equity cost of capital ρ^u are invariant to leverage. The marginal supply price q reflects information about the unlevered cash flow stream and about the marginal financing rate ρ^u associating with this specific capital asset. Almost surely q and ρ^u depend on supply and demand functions for capital as well as on investor risk preferences. The determination of q and ρ^u is beyond scope of this study, however.[2] For my partial equilibrium purposes, q and ρ^u are asset specific and in the short-run they are fixed and exogenous.

With leverage the zero net present value investment equilibrium supposes that the producer *supplies* equity financing of $(1-\alpha_s)q_s$ and *obtains* debt financing of $\alpha_s q_s$ from creditors.[3] The variable α_s represents the *marginal* debt-to-asset ratio. Real investment equilibrium with leverage satisfies:

$$(2) \qquad (1-\alpha_s)q_s = \sum_{t=1}^{\infty} \big(1 + \overbrace{\rho_s^{\ell} - \pi_s}^{\substack{\text{real levered equity} \\ \text{cost of capital}}} \big)^{-t} \underbrace{\left\{(1-\tau)c_{s,t} + \tau q_s z_{s,t} - B_{s,t}\right\}}_{\text{levered residual cash flow}},$$

where ρ^{ℓ} is the levered equity financing rate and $B_{s,t}$ equals the real after-corporate-tax loan payment made at time $s+t$ for the time s real investment. $B_{s,t}$ may include interest, principal repayment or issuance and any other debt related fees. There are no restrictions on the dynamic time path of loan payments – any loan contract fits. The expression in curly brackets equals levered residual cash flow.

Given costless and reversible real investment then a financial market no-arbitrage equilibrium condition requires that zero net present value investment equilibrium for real capital goods is invariant to financing method. Therefore, equate equations 1 and 2 as follows:

$$(3) \qquad \begin{aligned} &\sum_{t=1}^{\infty} (1+\rho_s^{\ell}-\pi_s)^{-t}\left\{(1-\tau)c_{s,t}+\tau q_s z_{s,t}-B_{s,t}\right\} \\ &\quad = (1-\alpha_s)\sum_{t=1}^{\infty} (1+\rho_s^{u}-\pi_s)^{-t}\left\{(1-\tau)c_{s,t}+\tau q_s z_{s,t}\right\} \end{aligned} .$$

Simplify equilibrium condition 3 by specifying dynamic time paths for pretax cash flows, tax depreciation deductions, and loan payments.

First consider pretax cash flows. Let d_j denote the proportional decline in real pretax cash flow occurring after the j'th cash flow is received (d_0 = 0). The series d_j for j = 0,...,∞ is a dynamic and exogenous capacity depreciation schedule that reflects the shape of the pretax cash flow stream.[4] Expected real pretax cash flow at time s+t from real investment at time s is

(4) $$c_{s,t} = c_s\left(1-\sum_{j=1}^{t} d_{j-1}\right).$$

c_s is the time *s user cost of capital* and equals pretax cash flow produced by one unit of new real assets during first period of use (c_s is identical to $c_{s,1}$). Let Δ_s^u and Δ_s^ℓ equal the capacity depreciation schedule discounted with real equity costs of capital $\rho^u - \pi$ and $\rho^\ell - \pi$, respectively, as in

(5) $$\Delta_s^\ell = \sum_{t=1}^{\infty}(1+\rho_s^\ell - \pi_s)^{-t} d_t \; .$$

Substitute equation 4 into 3 and simplify with equation 5.

Second consider specification of the discounted depreciation tax shield. Let Z_s^u and Z_s^ℓ equal present value of tax depreciation deductions per dollar of marginal investment when discounted with real equity costs of capital $\rho^u - \pi$ and $\rho^\ell - \pi$, respectively, as in

(6) $$Z_s^\ell = \sum_{t=1}^{\infty}(1+\rho_s^\ell - \pi)^{-t} z_t \; .$$

The series $z_{s,t}$ for t= 0,...,∞ is a dynamic and exogenous tax depreciation schedule. Substitute equation 6 into 3.

Third consider specification of loan payments. Let γ denote the real after-tax loan payment (interest, principal, and fees) at end of the asset's first period of use as a proportion of the asset's supply price:

$$\gamma_s = B_{s,1}/q_s \,, \tag{7}$$

and let

$$b_j = (B_{s,j} - B_{s,j+1})/B_{s,1}. \tag{8}$$

The variable b_j denotes the change in periodic real loan payment relative to $B_{s,1}$ occurring after j'th payment is made (b_0 = 0). The payment at time s+t attributable to the loan financing the time s marginal real investment is

$$B_{s,t} = \gamma_s q_s \Big\lfloor 1 - \sum_{j=1}^{t} b_{j-1} \Big\rfloor. \tag{9}$$

The series b_j for j = 0,...,∞ is a dynamic and exogenous schedule of debt maturity structure that reflects the shape of the marginal loan payment stream. Let Λ_s relate to the discounted loan payment stream as follows:

$$\Lambda_s = \alpha_s - \gamma_s(\rho_s^{\ell} - \pi_s)^{-1}(1 - \sum_{t=1}^{\infty}(1 + \rho_s^{\ell} - \pi_s)^{-t} b_t) \ . \tag{10}$$

Substitute equations 7-to-9 into 3 and simplify with equation 10.

Simplify, substitute, and rearrange the equilibrium condition in equation 3:

$$\underbrace{\frac{(\overbrace{\rho_s^{\ell} - \pi_s}^{\text{real levered equity cost of capital}})(1 \overbrace{- \tau_s Z_s^{\ell}}^{\text{discounted depreciation tax savings}} \overbrace{- \Lambda_s}^{\text{net present value to equity of loans}})}{(1-\tau_s)(1-\Delta_s^{\ell})}}_{\text{real levered user cost of capital}} = \underbrace{\frac{(\overbrace{\rho_s^{u} - \pi_s}^{\text{real unlevered equity cost of capital}})(1-\tau_s Z_s^{u})}{(1-\tau_s)(1-\Delta_s^{u})}}_{\text{real unlevered user cost of capital is invarint to financing method}} \ . \tag{11}$$

The equilibrium makes the net present value of the marginal real investment invariant to financing method. The left and right-hand-sides measure the real levered and unlevered user costs of capital.

Equation 11 contains two dynamic variables, Δ and Λ, that do not appear in the Hall-Jorgenson standard specification for user cost (conversely, dynamic variable Z appears in the Hall-Jorgenson standard). These two variables generalize economic dynamics associated with shape of the pretax cash flow stream and with debt maturity structure. Impose restrictions on the dynamic processes that Δ and Λ reflect and obtain the Hall-Jorgenson standard. By examining those restrictions glean insight about the benefits afforded by generalizing the processes.

Restrict the pretax cash flow stream in equation 4 so that pretax cash flows decline along a perpetual geometric path at rate δ. For this special case $d_t = \delta(1-\delta)^{t-1}$ and Δ^u equals $\delta/(\rho^u + \delta - \pi)$. Substitute Δ^u into the equilibrium condition and obtain the Hall-Jorgenson specification for the unlevered user cost of capital:

$$c_s = \frac{q_s\left(\rho_s^u + \delta - \pi_s\right)\left(1 - \tau_s Z_s^u\right)}{1 - \tau_s} .$$

The economic environment almost surely includes capital investments that promise perpetual geometric pretax cash flow streams in which case the standard user cost specification is valid. Certainly, too, many other investments promise cash flow streams that follow other patterns; for example, finite cash flow streams. The variable Δ reflects dynamic effects on user cost of expected pretax cash flows irrespective of the pattern.

Economic interpretation of Δ is straightforward. Suppose the pattern of pretax cash flow is level and *perpetual* at \$1 per period ($d_j = 0$ for $j = 0,\ldots,\infty$ and $\Delta = 0$). Present value of the pretax cash flow stream discounted with ρ is \$1/ρ. Now suppose the marginal capital investment offers \$1 of pretax cash flow for *one-period* and no more. Present value of this stream is \$1/(1 + ρ). The percentage of the level perpetuity value that the one-period stream retains equals [\$1/(1 + ρ) – \$1/ρ] ÷ \$1/ρ, which is $-(1 + \rho)^{-1}$. For a one-period model $d_1 = 1$ and $\Delta = (1 + \rho)^{-1}$. Hence, Δ represents present value of economic loss *in perpetuitas* caused by deviation of actual pretax cash flows from a level perpetual stream.[5] The term $(1 - \Delta)$ that appears in the denominator of the user cost specification in equation 11 measures the proportion of level perpetuity value that the discounted pretax cash flow stream retains.

Now restrict the loan payment stream. Suppose that pretax cash flow declines along a perpetual geometric path at rate δ and that for the marginal capital investment the periodic loan payment equals an amount that holds the debt-to-asset ratio constant at α_s. Let i_s denote the after-tax interest rate at time s. The first loan payment, $B_{s,1}$, comprises after-tax interest and principal equal to $\alpha q i$ and $\alpha q(\delta - \pi)$, respectively. Thus, γ equals $\alpha(i + \delta - \pi)$. Loan payments evolve along the time-path $b_t = (\delta - \pi)(1 - \delta + \pi)^{t-1}$ for $t = 1,\ldots,\infty$. Solution of equation 10 shows that Λ equals $\alpha(\rho^{\ell} - i)(\rho^{\ell} + \delta - \pi)^{-1}$. Substitute Δ^{ℓ} and Λ into the equilibrium condition obtain the Hall-Jorgenson specification for levered user cost:

$$c_s = \frac{q_s\left((1 - \alpha_s)\rho_s^{\ell} + \alpha_s i_s + \delta - \pi_s\right)\left(1 - \tau_s Z_s^{\ell}\right)}{1 - \tau_s}.$$

For this special case the Hall-Jorgenson standard contains the ubiquitous weighted average cost of capital.

Economic interpretation of Λ is straightforward. Suppose that for the dollar of marginal investment with net present value of zero the producer *obtains* debt financing of α from creditors with term of 1-year ($b_1 = 1$ and $b_t = 0$ otherwise). Interest accrues at the after-tax rate i. The first (and only) real loan payment $B_{s,1}$ equals $\alpha(1+ i - \pi)$. Present value to equity of the loan payment is $\alpha(1+ i - \pi)/(1+ \rho^{\ell}-\pi)$. Net present value to equity of the loan is $\alpha - \alpha(1+ i - \pi)/(1+ \rho^{\ell}-\pi)$. Substitute loan parameters into equation 10 and find that $\Lambda = \alpha - \alpha(1+ i - \pi)/(1+ \rho^{\ell}-\pi)$. Λ measures the net present value to equity of loans that finance the marginal dollar of real investment irrespective of the debt maturity structure.

Finally, substitute the standard Hall-Jorgenson levered and unlevered user costs into equilibrium condition 11, simplify, rearrange, and obtain:

$$(12) \qquad \rho_s^{\ell} = \rho_s^{u} + (\rho_s^{u} - i_s)\left(\frac{\alpha_s}{1-\alpha_s}\right).$$

Equation 12 shows that for this special case the equilibrium levered equity financing rate ρ^{ℓ} equals unlevered equity financing rate ρ^{u} plus the debt-to-equity ratio $\alpha \div (1 - \alpha)$ times the risk premium $(\rho^{u} - i)$. Equation 12 is Modigliani-Miller *Proposition 2* (1958).[6] The same restrictions that lead to Hall-Jorgenson also lead to Modigliani-Miller. The two blossom from the same root.[7]

Equation 11 is a dynamic financial market equilibrium condition that equates the levered and unlevered user costs of capital. Right-hand-side variables are invariant to leverage: Real unlevered user cost settles on a value dependent on production and opportunity costs. Joseph E. Stiglitz (1974) establishes that these factors are independent of company debt policy; customers possess preferences about the price and quality of the product, not about the producer's leverage ratio. Left-hand-side variables depend on debt decisions for the marginal real investment. In the absence of marginal debt financing Λ equals zero and as leverage increases then Λ increases. Λ measures net present value to equity of loans and represents a transitory gain for equity. The transitory gain creates arbitrage opportunities between debt, equity, and costless reversible real investment. To reestablish equilibrium the marginal levered equity financing rate ρ^{ℓ} rises, causing decline in present value to equity of unlevered residual cash flows. This transitory cost for equity from rising ρ^{ℓ} offsets transitory gain Λ thereby satisfying a dynamic no-arbitrage equilibrium condition.

II. Extracting the endogenous term structure within dynamic user cost

The extent by which unlevered equity financing rate ρ^{u} surpasses debt interest rate i determines Λ, the net present value to equity of marginal loan payments. Competitive financing sources capitalize transitory gain Λ and bid the levered equity financing rate ρ^{ℓ} higher. As ρ^{ℓ} rises then the decline in present value of unlevered residual cash flows imposes a transitory loss. ρ^{ℓ} continues to rise until transitory gain and loss perfectly offset at which point no-arbitrage equilibrium eventuates.

In the special case validating *Proposition 2* the levered equity financing rate ρ^{ℓ} rises above ρ^{u} by exactly $(\rho^{u} - i) \times \alpha \div (1 - \alpha)$. With i of 5 percent, ρ^{u} of 11 percent, and debt-to-asset ratio α of 25 percent, for example, an equilibrium risk-premium of 2.0 percent makes sources of equity financing indifferent between unlevered investment returning 11 percent and levered investment returning 13 percent because net present value of each is zero. Even though net present values are equal, however, wealth accumulations are not. One dollar certainly accumulates more wealth earning 13 percent than 11 percent.

Contemplate the source of the incremental wealth that accumulates at the higher equilibrium financing rates that associate with leverage. Because pretax cash flows are invariant to leverage the extra wealth does not come from capital. Labor and stakeholders certainly do not sacrifice wealth in order to compensate equity investors for leverage. And competitive capitalist creditors don't give it up either. Where does it come from?

Glean fundamental insight on the relation between Λ and user cost by considering the analogous yet simpler story pertinent to tax benefits. Examine equation 11 and see that right-hand-side unlevered user cost declines as depreciation tax savings τZ^{u} increase. Well-known interpretation is that equilibrium pretax cash flow is less with fiscally stimulating tax benefits than without because tax savings, like manna from heaven, descend from an unspecified government sector. Analogously, *as left-hand-side Λ increases from leverage then levered user cost would decline, too, <u>except</u> that financial market equilibrium requires irrelevance of financing method – pretax cash flow resists decline caused by rising Λ. Competitive financing sources capitalize the transitory gain and bid <u>financial</u> costs of capital higher, thereby restoring equilibrium between levered and unlevered <u>user</u> costs of capital*. Incremental wealth accumulates in the future even though the net present value remains at zero.

The valuable benefits provided by diversification are the source of future incremental wealth that results from the increase in equilibrium returns that associate with leverage. The potential for diversification benefits exists because real capital embodies equity with relatively high idiosyncratic risk. Real capital necessarily embodies equity irrespective of market valuation. Market value accrues only when financial markets capitalize embodied equity. Producers employ many types of real capital for producing portfolios of products and a single equity security often represents a claim on many different company divisions, product lines, and billions of dollars of real capital assets. To some extent, therefore, sources of equity possess as many diversification possibilities as creditors – but not to the full extent. Just as equity is claimant on residual cash flows, so too equity ultimately bears the idiosyncratic risk that real capital embodies. For clarity of thought, yet without losing generality, one may think of equity as residual claimant on a single underlying unit of real capital – the terms "firm" and "real capital asset" can therefore be used interchangeably.

Michael A. Klein (1973) establishes that equilibrium financing rates within a mean-variance framework decrease with divisibility. Imperfect divisibility, according to Klein, is a necessary condition for emergence of intermediaries. Real capital embodies relatively high idiosyncratic risk due to putty-clay technology and indivisibility of assets.[8] When creditors supply debt financing for indivisible real capital with relatively high idiosyncratic risk they create portfolios of claims on many real assets. Creditors operate like a clutch mechanism that intermediate relatively high equity financing rates with relatively low credit interest rates.[9] Creditors capitalize diversification benefits. Douglas W. Diamond (1984) establishes that diversification is a key to the net advantage of intermediation. *The capitalization of real diversification and divisibility opportunities is the source of the incremental wealth that accumulates at the higher equilibrium financing rates that associate with leverage.*

Sustainable increases in leverage signal higher equilibrium rates of return and creation of future incremental wealth. Capitalists bid required returns higher and perfectly offset the higher expected returns thereby restoring zero net present value investment equilibrium. In the Modigliani-Miller special case ρ^{ℓ} rises to reestablish equilibrium while the interest rate *i* remains constant thereby implying that for this special case incremental debt receives zero incremental compensation. For this special case all incremental wealth that leverage creates goes exclusively to equity. Because of agency relationships, however, seldom does equity have full use of incremental wealth. Creditors share with equity the risk of incremental leverage, surely they compete for incremental reward.

Explicitly specify debt and equity reward-sharing. Let $\lambda_{s,M}$ denote the debt-to-equity relative risk premium at time *s* for loans of term *M*:

(13) $$\lambda_{s,M} = \frac{i_{s,M} - i_s^u}{\rho_{s,M}^{\ell} - \rho_s^u} \ ,$$

where i^u denotes the short-term after-tax interest rate as both α and *M* approach zero (i.e., i^u is the instantaneous unlevered risk-free interest rate). λ specifies whether incremental risk premium and distribution of created-wealth is larger for debt ($\lambda>1$) or equity ($\lambda<1$). λ does <u>not</u> measure distribution of leverage-induced incremental <u>risk</u>, but rather λ measures <u>reward</u> for risk-sharing by specifying relative size of ex ante debt and equity risk premia. *Proposition 2* implicitly assumes that $\lambda = 0$ and that all incremental wealth flows to equity. In reality, competition between perfectly elastic sources of debt and equity may cause λ to vibrate around unity in response to changing market conditions.

Glean fundamental insight on forces underlying the equilibrium condition in equation 11 by setting parameters for this stark one-period model: $\pi = \tau = Z = 0$; $q = \$1$; $d_1 = b_1 = 1.0$; $\rho^u = 0.11$; $\alpha = 0.25$; and $i^u = 0.05$. Compute with equation 5 that $\Delta^u = 0.9009$. Substitute into equation 11 these numerical settings and substitute i from equation 13 to find that:

$$\frac{\rho^{\ell}\left(1-\overbrace{0.25\left\{1-\frac{1+\overbrace{\lambda\,(\rho^{\ell}-0.11)+0.05}^{i}}{\left(1+\rho^{\ell}\right)}\right\}}^{\Lambda}\right)}{1-\underbrace{\left(1+\rho^{\ell}\right)^{-1}}_{\Delta^{\ell}}}=\overbrace{\frac{0.11}{\left(1-\underbrace{0.9009}_{\Delta^u}\right)}}^{\text{unlevered user cost}}\ . \tag{14}$$

Equilibrium interest rate i equals λ times equilibrium equity leverage risk premium ($\rho^{\ell} - \rho^u$) plus short-term risk-free rate i^u.

Table I varies the setting for λ and iteratively solves for the value of ρ^{ℓ} that satisfies the no-arbitrage equilibrium condition in equation 14. Each row in the table depicts a unique scenario for equilibrium because net present values for debt and equity financing sources always equal zero, and levered user cost always equals unlevered user cost. Even though row 1 is consistent with *Proposition 2,* any one row is as consistent as any other with market equilibrium. *Proposition 2*, just like the Hall-Jorgenson standard specification of user cost, is valid only in special cases. Perhaps it is important to note that column 6 shows the sum for creditors and equity of leverage-induced incremental wealth is robust, ranging rather narrowly between 1.33 and 1.37 percent of the $1 real asset value even though reward-sharing ranges widely.

While each row of Table I illustrates a theoretically consistent scenario, consistency with empirical observation of one row rather than others is another story. And another good story is theoretical effect on λ of market segmentation or differential relative risk-aversion in debt and equity markets.[10] These intriguing stories, however, are beyond scope of this study. Certainly the one period model is too restrictive and requires relaxing. Equation 11 allows generalization of dynamic processes that reflect upon the user cost of capital.

Λ reflects dynamic effects on user cost of debt maturity structure irrespective of the pattern of loan payments. Loan payment schedules typically relate only loosely with the pattern of pretax cash flows. The most common loan-type in the U.S. credit markets is the debenture. The debenture loan payment stream includes constant interest payments throughout loan-life and repayment of principal *in toto* with the last payment. For a debenture with a face value of $\alpha_s q_s$, a term of M, and an after-corporate-tax coupon rate of i_s (annual coupon, no sinking fund), periodic real after-tax interest equals $\alpha_s q_s(i_s - \pi)$ and $B_{s,1}$, equals $\alpha_s q_s(i_s - \pi)$ and γ_s equals $\alpha_s(i_s - \pi)$. Subsequent payments $B_{s,2}$ through $B_{s,M-1}$ are the same as the first so $b_j = 0$ for $j = 0,...,M-2$. During period M the payment includes the coupon plus principal repayment so $b_{M-1} = -(i_s - \pi)^{-1}$. After period M the payment drops to zero so $b_M = (i_s - \pi)^{-1}(1+i_s - \pi)$. Substitution into equation 11 shows the dynamic no-arbitrage equilibrium condition for debentures:

$$\text{(15)}\qquad \underbrace{\frac{(\rho_s^{\ell}-\pi_s)}{(1-\Delta_s^{\ell})}\left\{1-\tau Z_s^{\ell}-\overbrace{\frac{\alpha_s\left(\rho_s^{\ell}-i_s\right)}{\rho_s^{\ell}-\pi_s}\left(1-\frac{1}{(1+\rho_s^{\ell}-\pi_s)^M}\right)}^{\substack{\Lambda\text{ is net present value}\\ \text{to equity of debenture loans}\\ \text{at rate } i \text{ for term } M}}\right\}}_{\text{real levered user cost of capital}} = \underbrace{\frac{(\rho_s^{u}-\pi_s)(1-\tau Z_s^{u})}{(1-\Delta_s^{u})}}_{\text{real unlevered user cost of capital}} \quad .$$

Maintenance of equation 15 is assurance that debenture financing for the marginal real investment has the same net present value irrespective of the debt ratio or loan term.[11]

Numerical illustrations below exemplify importance of reward-sharing on determination of equilibrium financing rates. All illustrations adopt these unlevered baseline conditions unless otherwise noted: 10-year asset service life declining by straight-line; 11 percent unlevered equity financing rate; 3 percent inflation rate; 7-year MACRS tax depreciation; 35 percent marginal tax rate. With these settings unlevered user cost equals \$0.2737 per dollar of capital.

To specify levered user cost all computations impose debenture financing with marginal debt-to-asset ratio α at 25 percent and after-tax short-term risk-free interest rate i^u at 5.0 percent (unless otherwise noted). Table II lists equilibrium interest rates given values for λ and M with subsequent iterative solution for i and ρ^{ℓ} from equation 15 subject to the constraint in equation 13. For columns 2-to-4 the debt-to-equity relative risk premium λ is 50 percent. With a 5-year loan the equilibrium interest rate is 6.10 percent implying a 110 basis point risk premium to creditors, much larger than the 45 basis point term premium on 2-year loans. This column reveals a yield curve with a normal upward slope.

Figure 1 fixes debt-to-equity relative risk premium λ at unity and illustrates effect on equilibrium term structure of varying unlevered equity risk premium $\rho^u - i^u$. The yield curve for which $\rho^u - i^u$ equals (0.11 – 0.05) is a plot of column 5, Table II. Increasing $\rho^u - i^u$ to (0.14 – 0.05) pushes the yield curve steeper and to a higher plateau. Leverage-induced incremental wealth is higher with unlevered equity financing rate of 14 percent than 11 percent because present value to equity of loan payments relates inversely with ρ^u. Higher incremental benefits from leverage support higher equilibrium interest rates. Conversely, yield curve flattens as $\rho^u - i^u$ diminishes.

Figure 2 fixes $\rho^u - i^u$ at (0.11 – 0.05) and illustrates that yield curve steepness increases and reaches a higher plateau as the debt-to-equity relative risk premium λ increases. Recall that λ does not affect distribution of risk resulting from increased α or lengthened M, rather λ affects distribution of reward. With $\lambda = 0$ creditors obtain none of the incremental wealth that leverage sustains (this is an assumption that underlies *Proposition* 2). Thus, the yield curve is perfectly flat at i^u of 5 percent. With $\lambda = 1$ the yield curve is a plot of column 5, Table II. With higher λ a larger proportion of leverage-induced incremental wealth flows to creditors, less to equity. Yield curve steepness is significantly greater with $\lambda = 6$. For a loan-term of 5 years (and $\lambda = 6$) the equilibrium interest and levered equity financing rates equal 9.37 percent and 11.73 percent, respectively. Levered equity risk premium $\rho^\ell - \rho^u$ of 73 basis points is one-sixth the interest term premium $i - i^u$ of 437 basis points.

Figures 1 and 2 reveal that the dynamic neoclassical user cost of capital enables rich insights about term structure.[12] Discussion of the model's transmission mechanism driving equilibrium forces is useful. Exogenous and deterministic characteristics of underlying real assets such as tax and capacity depreciation schedules, expected inflation, and capital goods price, jointly exert dynamic effects on user cost and predetermine the entire unlevered residual cash flow stream. Because ρ^u exceeds i^u then incremental increases in debt-to-asset ratio α or loan-term M disturb the zero net present value investment equilibrium. Notice that as $\alpha \rightarrow 0$ or $M \rightarrow 0$ then $\rho^\ell \rightarrow \rho^u$ and $i \rightarrow i^u$. Increasing α or M has the same qualitative effect: Financial portfolios spanning real capital assets create diversification benefits; financial markets capitalize diversification benefits and create incremental wealth; financing sources bid rates of return higher until zero net present value equilibrium recurs; marginal financing method (i.e., α or M) becomes irrelevant.

With ρ^u, i^u and λ fixed and exogenous the transmission mechanism driving equilibrium levered rates of return is independent of risk preferences. The role of risk preferences for determining the effect of leverage on equilibrium financing rates occurs exclusively because risk drives determination of financial variables ρ_s^u, i_s^u and $\lambda_{s,M}$. Specifying the general equilibrium determination of these variables is beyond scope of the current paper. Figures 1 and 2 nonetheless illustrate that through the user cost unspecified risk-return relations exert complex effects on equilibrium financing rates. For figure 1 decrease unlevered equity risk premium $\rho^u - i^u$ and equilibrium yield curve flattens. For figure 2 decrease levered equity risk premium $\rho^\ell - \rho^u$ and equilibrium yield curve steepens. Dynamic processes within the neoclassical user cost of capital transmit different dimensions of reward-sharing.

Diversification benefits relate directly to unlevered risk premium $\rho^u - i^u$. Variation in risk premium $\rho_s^u - i_s^u$ over time s causes change, as figure 1 illustrates, in yield curve slope. When diversification benefits are large and credit is widely available then a relatively large instantaneous unlevered risk premium causes a steep yield curve. Conversely, credit crunches and drying-up of diversification benefits causes flattening of this endogenous yield curve.[13]

The debt-to-equity relative risk premium $\lambda_{s,M}$ at time s for loans with term M depends on competitive position of financing sources for extracting incremental wealth created by increasing debt ratio or lengthening loan-term. Figure 3 shows an interesting case where λ declines with loan-term. These settings characterize a scenario in which equity in the short-term is willing to accept relatively little incremental wealth from leverage (they still demand some reward and therefore ρ^{ℓ} in the upper curve increases with loan-term). That is, perhaps equity "treads-water" expecting near-term bad times yet is unwilling to liquidate for the long-term. The endogenous yield curve for equilibrium interest rate i inverts – a tendency empirically consistent with onset of business cycle contractions.[14]

III. Conclusion

Dynamic processes within the user cost of capital imply equilibrium tradeoffs between levered equity financing rate ρ^{ℓ}, interest rate i, debt-to-asset ratio α, and loan-term M. Impose restrictions on the shape of the loan payment stream, on pretax cash flows, and on debt-to-equity reward-sharing and obtain the standard Hall-Jorgenson *user cost of capital* and the standard Modigliani-Miller *equity cost of capital*. For this special case the term of the loan payment stream becomes irrelevant. More generally, as figures 1-3 illustrate, dynamic tradeoffs within neoclassical user cost imply a deterministic yield curve for equilibrium financing rates.

Higher debt ratios and longer loan terms associate with an increase in equilibrium financing rates. The higher equilibrium rates of return imply the accumulation of future incremental wealth even though marginal net present values remain at zero. The capitalization of real diversification and divisibility opportunities is the source of the incremental wealth. The potential for incremental wealth exists because real capital necessarily embodies equity with relatively high idiosyncratic risk. The debt-to-equity relative risk premium λ measures the extent to which sources of marginal debt and equity financing share the incremental wealth that associates with leverage.

Financing rates that figures 1-3 illustrate are consistent with real and financial market equilibrium conditions. For each point on each yield curve the present value of (1) unlevered residual cash flows discounted with the unlevered equity cost of capital equals the capital goods price q; (2) levered residual cash flows discounted with the levered equity cost of capital equal the equity-provided financing $(1 - \alpha)q$; and (3) loan payments discounted with the interest rate equal the initial loan principal αq. For each point the decline in discounted unlevered residual cash flows as discount rate rises from ρ^u to ρ^ℓ equals the decline in discounted loan payments as discount rate rises from i to ρ^ℓ. Levered and unlevered user costs are always equal. Each point on each yield curve satisfies the dynamic no-arbitrage equilibrium condition in equation 11.

I offer a new yet fundamental explanation for term structure. Given i^u, ρ^u, λ, q and π, along with information about the time paths for tax policies, pretax cash flows, and loan payment schedules, my analysis of the dynamic processes reflecting within the neoclassical user cost of capital lead me to this conclusion:

The yield curve normally slopes upward because sustainable increases in loan-term or debt ratio signal creation of future incremental wealth and to reestablish the irrelevance of financing method to zero net present value equilibrium the financing sources capitalize incremental gains and bid rates of return higher – equilibrium debt and equity financing rates naturally rise with the duration of the real investment.[15]

References

Abel, Andrew B. and Eberly, Janice C. "Optimal Investment with Costly Reversibility." *Review of Economic Studies,* 1996, 63, pp. 581-593.

Albuquerque, Rui and Hopenhayn, Hugo A. "Optimal Lending Contracts and Firm Dynamics." *Review of Economic Studies,* April 2004, 71(2), pp. 285-315.

Ang, Andrew and Liu, Jun. "How to Discount Cashflows with Time-Varying Expected Returns." *Journal of Finance,* 2004, 59(6), pp. 2745-2783.

Ang, Andrew; Piazzesi, Monika, and Wei, Min. "What Does the Yield Curve Tell Us About GDP Growth?" *Journal of Econometrics,* 2005, forthcoming.

Auerbach, Alan J. "Tax Reform and Adjustment Costs: The Impact on Investment and Market Value." *International Economic Review*, 1989, 30(4), pp. 939-63.

Auerbach Alan J. and Hassett, Kevin A. "On the Marginal Source of Investment Funds." *Journal Of Public Economics,* January 2003, 87(1), pp. 205-32.

Bajeux-Besnainou, Isabelle; Jordan, James V.; and Portait, Roland. "Dynamic Asset Allocation for Stocks, Bonds, and Cash." *Journal of Business,* April 2003, 76(2), pp. 263-87.

Bansal, Ravi and Zhou, Hao. "Term Structure of Interest Rates with Regime Shifts." *Journal of Finance,* 2002, 57, pp.1997-2043.

Bansal, Ravi; Tauchen, George, and Zhou, Hao. "Regime-Shifts, Risk Premiums in the Term Structure, and the Business Cycle." *Journal of Business and Economic Statistics,* 2004, 22, pp. 396 - 409

Böhm-Bawerk, Eugen von. *The Positive Theory of Capital*. New York: G.E. Stechert and Co., 1888, translation by William Smart, 1891.

Brav, Alon; Constantinides, George C. and Geczy, Christopher C. "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence." *Journal of Political Economy,* 2002, 110, pp. 793-824.

Brock, William A. and Turnovsky, Stephen J. "The Analysis of Macroeconomic Policies in Perfect Foresight Equilibrium." *International Economic Review*, 1981, 22, 179-209.

Caballero, Ricardo J. “Aggregate Investment,” in John B. Taylor and Michael Woodford, eds. *Handbook of Macroeconomics Volume 1B*. Amsterdam: North-Holland, 1999, pp. 813-862.

Campbell, John Y. “Asset Pricing at the Millennium.” *Journal of Finance*, 2000, 55(4), pp. 1515-67.

Cooley, Thomas F. and Smith, Bruce D. “Indivisible Assets, Equilibrium, and the Value of Intermediation.” *Journal of Financial Intermediation*, 1995, 4, pp. 48-76.

Dai, Qiang and Singleton, Kenneth J. “Term Structure Dynamics in Theory and Reality.” *Review of Financial Studies*, Fall 2003, 16(3), pp. 631-78.

Diamond, Douglas W. “Financial Intermediation and Delegated Monitoring.” *Review of Economic Studies,* 1984,51*,* 393-414.

Gilchrist, Simon and Williams, John C. “Putty-Clay and Investment: A Business-Cycle Analysis.” *Journal of Political Economy*, 2000, 108(5), pp. 928-60.

Goolsbee, Austan. “Investment Tax Incentives, Prices, and the Supply of Capital Goods.” *Quarterly Journal of Economics,* 1998, 113(1), pp. 121–48.

Goolsbee, Austan. Taxes and the Quality of Capital.” *Journal of Public Economics.* March 2004, 88(3), pp. 519-43.

Graham, John R. “Taxes and Corporate Finance: A Review.” *Review of Financial Studies*, Winter 2003, 16(4), pp. 1075-1129.

Hall, Robert E. "The Stock Market and Capital Accumulation," *American Economic Review*, 2001, 91(5), pp. 1185-1202.

Hall, Robert E. and Jorgenson, Dale W. “Tax Policy and Investment Behavior.” *American Economic Review*, 1967, 57, pp. 391-414.

Hellwig, Martin F. “Financial Intermediation with Risk Aversion.” *Review of Economic Studies*, 2000, 67, pp. 719-742.

Hicks, John R. *Value and Capital*. Oxford University: Clarendon Press, 1939.

Jorgenson, Dale W. “Capital Theory and Investment Behavior.” *American Economic Review*, 1963, 53, pp. 247-259.

Jorgenson, Dale W. “The Theory of Investment Behavior,” in R. Ferber, ed., *The Determinants of Investment Behavior*. New York: Columbia University Press for the National Bureau of Economic Research, 1967, pp. 129-156.

Klein, Micheal A. "The Economics of Security Divisibility and Financial Intermediation", *Journal of Finance*, 1973, 28, pp. 923-931.

Modigliani, Franco and Miller, Merton H. "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review*, 1958, 48, pp. 261-297.

Modigliani, Franco and Miller, Merton H. "Taxes and the Cost of Capital: A Correction", *American Economic Review,* 1963, 53, pp. 443-453.

Stiglitz, Joseph E. "On the Irrelevance of Corporate Financial Policy," *American Economic Review,* 1974, 64, pp. 851-866.

Endnotes

* Thomas W. Downs, Department of Economics and Finance, University of Alabama, Tuscaloosa, AL 35487-0224, tdowns@cba.ua.edu

1 Equation 1 assumes that the unlevered equity financing rate at time s, ρ_s^u, is expected to remain constant throughout the real asset service life. As subsequent discussions show, this does not impose an implicit assumption that the yield curve of interest rates is flat. Equation 1 also assumes that there is no stochastic uncertainty about expected cash flows. Andrew Ang and Jun Liu (2004) present an analytical methodology for discounting stochastic cash flows that are correlated with risk premiums, risk-free rates and time-varying betas. My study proceeds without those interesting innovations.

2 Austan Goolsbee (1998, 2004) examines whether investment tax incentives affect the capital goods supply price q or whether the tax benefits flow to the capital goods investor. William A. Brock and Stephen J. Turnovsky (1981) present a general equilibrium model that links corporate debt and equity financing rates with household behavior in order to determine the equity cost of capital.

3 Auerbach and Kevin A. Hassett (2003) examine the importance of whether the source of equity financing is retained earnings or new share issuances. Personal taxes are important considerations in that analysis. My specification ignores effects of personal taxes.

4 Ricardo J. Caballero (1999) argues for the importance of endogenous depreciation. For my purposes, however, that assumption unnecessarily complicates analyses.

5 For the Hall-Jorgenson standard case when pretax cash flow declines along a perpetual geometric path at rate δ then the proportion of level perpetuity value that the discounted pretax cash flow stream retains equals $(\rho - \pi) \div (\rho + \delta - \pi)$. Yet Δ accommodates any stream, even complex non-monotonic ones. As one example, assume straight-line decline over N years ($d_j = 1/N$ for $j = 1,\ldots,N$ and $d_j = 0$ otherwise). Compute from equation 5 that Δ is $[1 - (1+\rho)^{-N}] \div (\rho N)$. With ρ of 10 percent, for example, the level perpetual \$1 stream has present value of \$10. The stream declining by straight-line over four years has

present value of \$2.075 (= \$1/1.1 + \$0.75/1.1^2 + \$0.50/1.1^3 + \$0.25/1.1^4). The straight-line stream loses 79.25 percent of the level perpetual value. Δ equals 0.7925 [= $(1 - 1.1^{-4}) \div (0.1 \times 4)$].

[6] Modigliani and Miller (1963) subsequently modify *Proposition 2* to account for the effects of the interest tax shield on company market value. I account for the interest tax shield by defining i as the after-corporate-tax interest rate. John R. Graham (2003) reviews empirical and theoretical arguments for how taxes affect corporate decision making.

[7] The standard specification for ρ^{ℓ} in *Proposition 2* is valid for a wider class of cases than is the Hall-Jorgenson standard user cost. Both are valid for the case that the text describes for perpetual and geometric capacity depreciation with a constant debt ratio. Both are valid for a one-period model. But *Proposition 2* is valid, and Hall-Jorgenson is not, for any pattern of pretax cash flow stream as long as the debt-to-asset ratio remains constant throughout the marginal asset service life.

[8] Simon Gilchrist and John C. Williams (2000) find that putty-clay characteristics of capital play a significant role explaining business cycle and medium-run dynamics.

[9] Thomas F. Cooley and Bruce D. Smith (1995) confirm the intuition of Klein that primary assets bear higher equilibrium rates of return than intermediary liabilities.

[10] The debt-to-equity relative risk premium relates to relative risk aversion and portfolio choice. Hellwig (2000) examines the role of risk aversion in intermediary behavior. Alon Brav, George C. Constantinides, and Christopher C. Geczy (2002) relate household relative risk aversion to the equity premium. Isabelle Bajeux-Besnainou, *et al*. (2003), investigate the role of different market parameters and relative risk aversion in portfolio strategies.

[11] My specification ignores creditor lending behavior in the presence of company borrowing constraints. Rui Albuquerque and Hugo A. Hopenhayn (2004) discuss this issue.

[12] Pure monetary theories of equilibrium financing rates culminate today with research on (1) financial asset pricing models (“ICAPMs”) and (2) dynamic term structure models (“DTSMs”). John Y. Campbell (2000) reviews ICAPMs and Qiang Dai and Kenneth J. Singleton (2003) survey DTSMs. Many of these

models integrate information from the real sector. Ravi Bansal and Hao Zhou (2002), for example, find that the yield curve slope relates statistically with business cycle. None of the preceding studies explain capital market rates of return by exploiting the information that reflects through the user cost of capital.

[13] The risk-premium $\rho_s^u - i_s^u$ depends on the regulatory environment, statutory seniority of debt, credit rating, and structural phenomena such as cash flows being better specified for debt than for equity. These phenomena seem short-term stable and likely change only during regime shifts. Bansal, George Tauchen, and Zhou (2004) find that the goodness-of-fit of dynamic term structure models relates to regime shifts.

[14] Andrew Ang, Monika Piazzesi, and Min Wei (2005) write that “every recession after the mid-1960s was predicted by a negative slope – an inverted yield curve – within 6 quarters of the impending recession.”

[15] The first to conclude that the equilibrium financing rate is generally an increasing function of the duration of real capital is Eugen von Böhm-Bawerk (1888) in *Positive Theory of Capital*. His modeling, methodology and assumptions obviously differ from mine. Still, Böhm-Bawerk introduces the user cost concept wherein he refers to it [p.343] as “bearer-of-the-use.” John R. Hicks reverently writes in *Value and Capital* (1939, p. 192): “Even to-day, the great name in this department of economics [that is, the study of economic dynamics] is the name of Böhm-Bawerk. This is so, not because his doctrine is generally accepted (it was not generally accepted even in his own time, and it has still fewer supporters in ours), but because it is a challenge that has somehow to be met. Nearly every one who comes to the study of capital falls a victim to Böhm-Bawerk’s theory at some stage or other. … Clearly Böhm-Bawerk was wrong; but there must have been something in what he said; you cannot construct such an elaborate theory as that out of nothing.” Hicks concludes [p. 222] that even though the Böhm-Bawerk theory does not generalize, it is correct for the special case where “all the input is utilized at one given date, and all the output comes to fruition at another given date.” As we see from Hall-Jorgenson and Modigliani-Miller, special cases may not generalize but they nonetheless may bear very long-lasting and durable fruit.

TABLE I – ALTERNATIVE EQUILIBRIUM SCENARIOS FOR A ONE-PERIOD MODEL

	λ debt-to-equity relative risk premium - 1 -	ρ^{ℓ} levered equity financing rate (percent) - 2 -	i interest rate (percent) - 3 -	*present value of incremental wealth* to equity (cents) - 4 -	to creditors (cents) - 5 -	sum (cents) - 6 -
row 1	0.0	13.00	5.00	1.33	0.00	1.33
row 2	0.5	12.71`	5.86	1.14	0.20	1.34
row 3	1.0	12.50	6.50	1.00	0.35	1.35
row 4	1.5	12.33	7.00	0.89	0.47	1.36
row 5	2.0	12.20	7.40	0.80	0.56	1.36
row 6	4.0	11.86	8.43	0.57	0.79	1.37
row 7	8.0	11.55	9.36	0.37	1.00	1.37
row 8	16.0	11.32	10.05	0.21	1.15	1.36

Exogenous settings include unlevered equity financing rate $\rho^u = 0.11$; inflation rate $\pi = 0.03$; tax rate $\tau = 0.35$; capacity depreciation is 1-year one-hoss shay ($d_1 = 1.0$); tax-life is 1-year ($z_1 = 1.0$); after-corporate-tax instantaneous unlevered interest rate $i^u = 0.05$; debt-to-asset ratio $\alpha = 0.25$, loan-term is 1-year. Column 1 is debt-to-equity relative risk premium λ and represents the reward-sharing ratio of leverage-induced incremental wealth flowing to creditors and shareholders. Row 1 is the special case pertinent to *Proposition 2* in which all rewards from leverage flow to equity. Row 3 partitions rewards for risk-sharing equally between debt and equity financing sources. Levered equity financing rate ρ^{ℓ} in column 2 and levered interest rate i in column 3 satisfy the dynamic no-arbitrage equilibrium condition in equation 14. Future incremental wealth to equity and creditors accrues because equilibrium financing rates ρ^{ℓ} and i in columns 2 and 3 exceed ρ^u and i^u, respectively. Columns 4 and 5 measure the present value of future incremental wealth. Without further specification of risk preferences any one row is as consistent with real and financial equilibrium as any other row.

TABLE II – VARYING LOAN-TERM AND RESULTANT EQUILIBRIUM INTEREST AND EQUITY RATES

Loan Term (M) - 1 -	$\lambda = 0.5$ i - 2 -	ρ^{ℓ} - 3 -	Λ - 4 -	$\lambda = 1.0$ i - 5 -	ρ^{ℓ} - 6 -	Λ - 7 -
0	5.00 percent	11.00 percent	\$0.00	5.00 percent	11.00 percent	\$0.00
1	5.22	11.45	0.0143	5.43	11.43	0.0138
2	5.45	11.90	0.1126	5.84	11.84	0.0264
3	5.67	12.34	0.0284	6.21	12.21	0.0378
4	5.89	12.78	0.0420	6.55	12.55	0.0480
5	6.10	13.19	0.0549	6.85	12.85	0.0571
6	6.29	13.58	0.0669	7.13	13.13	0.0651
7	6.47	13.95	0.0781	7.37	13.37	0.0721
8	6.64	14.28	0.0882	7.59	13.59	0.0783
9	6.79	14.58	0.0973	7.79	13.79	0.0837
10	6.93	14.85	0.1055	7.96	13.96	0.0885
15	7.40	15.80	0.1371	8.55	14.55	0.1047
20	7.65	16.29	0.1492	8.87	14.87	0.1130
24	7.75	16.50	0.1543	9.01	15.01	0.1167
25	7.77	16.53	0.1552	9.03	15.03	0.1174

This table sets exogenous unlevered equity rate $\rho^u = 0.11$; inflation rate $\pi = 0.03$; tax rate $\tau = 0.35$; capacity depreciation is 10-year straight-line; after-corporate-tax interest rate $i^u = 0.05$; debt-to-asset ratio $\alpha = 0.25$; and tax depreciation follows 7-year MACRS. Furthermore, debenture loan-term M is set in column 1. Columns 2-4 and 5-7, respectively, set debt-to-equity relative risk premium λ, that is the reward-sharing ratio of leverage-induced incremental wealth flowing to creditors and shareholders, at 50 percent and 100 percent. Levered equity financing rates ρ^{ℓ} and levered interest rates i satisfy the no-arbitrage equilibrium in equation 15 subject to the constraint in equation 13. Λ in columns 4 and 7 is the net present value to equity of marginal financing by debentures of term M. For a given debt-to-equity ratio there exists a term structure of equilibrium levered debt and equity financing rates. The yield curve slopes upward because sustainable increases in loan-term signal creation of incremental wealth and, to reestablish irrelevance of term to zero net present value equilibrium, financing sources capitalize incremental benefits and bid rates of return higher. Incremental wealth to equity and creditors accrues, however, because equilibrium ρ^{ℓ} and i exceed ρ^u and i^u, respectively. Without further specification of risk preferences the term structure in columns 2 and 3 is as consistent with real and financial equilibrium as is the term structure in columns 5 and 6.

FIGURE 1. EQUILIBRIUM TERM STRUCTURE AND VARYING UNLEVERED EQUITY RISK PREMIUM, $\rho^{u} - i$

This figure adopts exogenous settings from Table II and also sets the debt-to-equity relative risk premium λ at 100 percent, implying debt and equity financing sources share rewards for risk-sharing equally. Debenture loan-term M varies along the horizontal axis and unlevered equity financing rate ρ^u differs for each yield curve. Levered equity financing rates ρ^{ℓ} (not shown) and levered interest rates i satisfy equations 13 and 15. The yield curve slopes upward because sustainable increases in loan-term signal creation of incremental wealth and, to reestablish irrelevance of term to zero net present value equilibrium, creditors capitalize incremental benefits and bid rates of return higher. An increase in the unlevered equity risk premium $\rho^u - i^u$ signals an increase in future incremental wealth from leverage, equilibrium rates of return rise, and steepness of the yield curve increases.

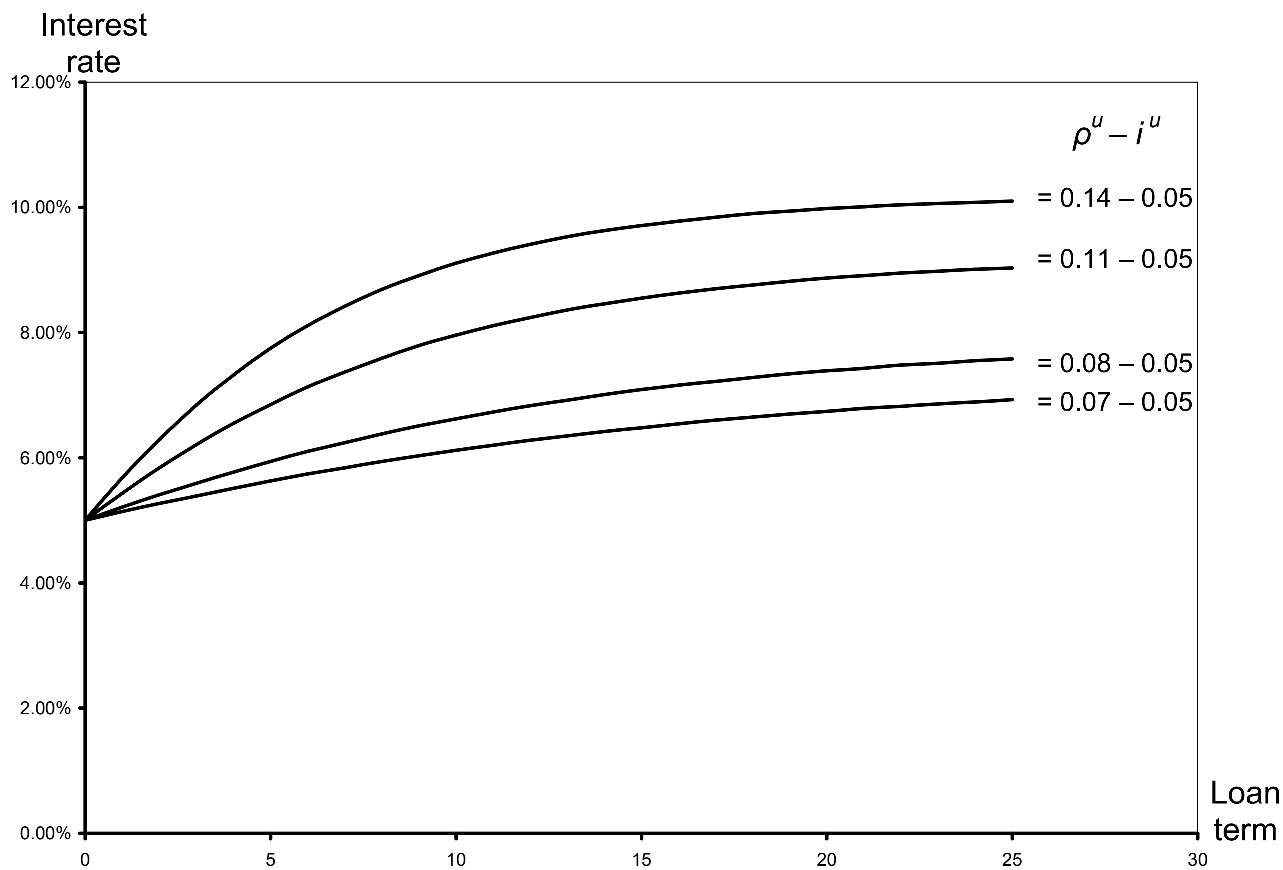


FIGURE 2. EQUILIBRIUM TERM STRUCTURE AND VARYING DEBT-TO-EQUITY RELATIVE RISK PREMIUM, λ

This figure adopts exogenous settings from Table II (including ρ^{u} = 0.11). Debenture loan-term *M* varies along the horizontal axis and the debt-to-equity relative risk premium *λ,* that is the reward-sharing ratio of leverage-induced incremental wealth flowing to creditors and shareholders, differs for each yield curve. Levered equity financing rates ρ^{ℓ} (not shown) and levered interest rates *i* satisfy equations 13 and 15. The yield curve slopes upward because sustainable increases in loan-term signal creation of future incremental wealth and creditors capitalize incremental benefits and bid equilibrium rates of return higher. Steepness increases with debt-to-equity relative risk premium because incremental benefits from leverage that flow to creditors are greater.

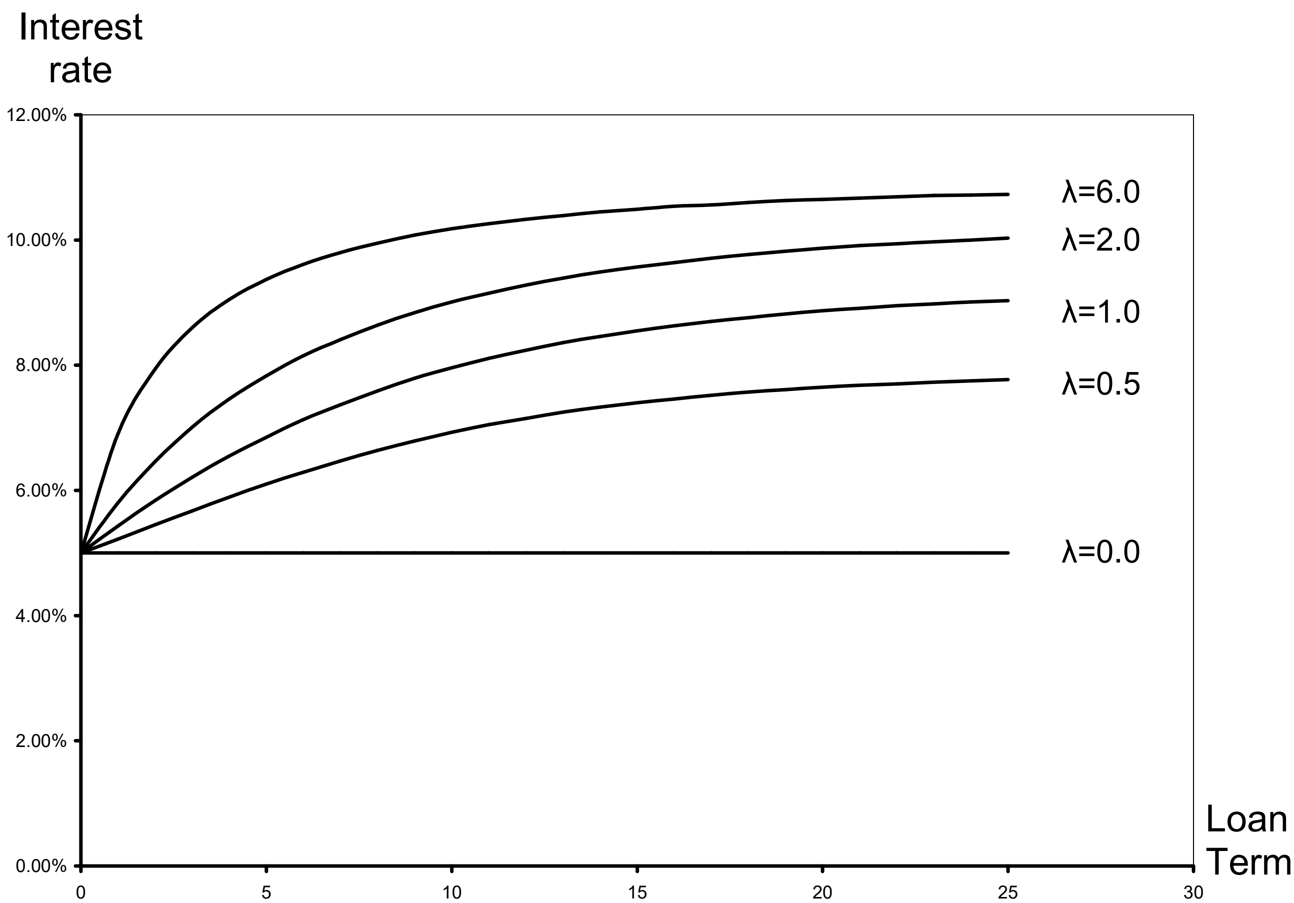
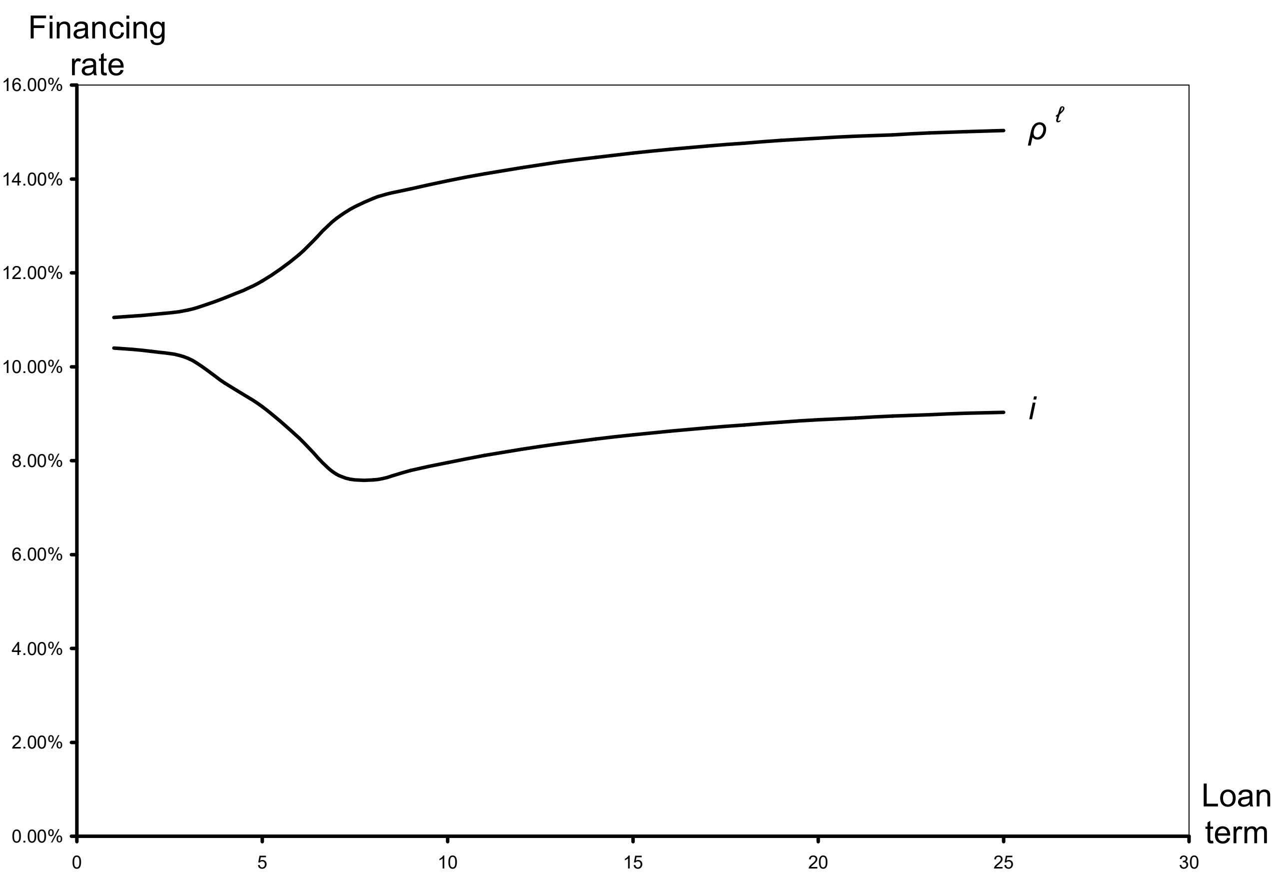


FIGURE 3. EQUILIBRIUM TERM STRUCTURE AND TERM-VARYING DEBT-TO-EQUITY RELATIVE RISK PREMIUM

This figure adopts exogenous settings from Table II (including $\rho^u = 0.11$). Debenture loan-term M varies along the horizontal axis. The debt-to-equity relative risk premium λ, that is the reward-sharing ratio of leverage-induced incremental wealth flowing to creditors and shareholders, varies by term. λ is large for short-term loans indicating equity receives little incremental reward. As loan-term increases λ declines toward unity according to this schedule: $\lambda_{s,M}$ = 100, 50, 25, 10, 5, 2.5, 1.25 for loan-term M = 1,…,7 years, and $\lambda_{s,M} = 1.0$ for $M \geq 8$ years. Levered equity financing rates ρ^{ℓ} in the upper curve and levered interest rates i in the lower curve satisfy equations 13 and 15. The term structure of interest rates inverts. This phenomenon typifies impending recession – equity in the short-term treads water and accepts less incremental wealth (and does not liquidate) but in the long-run equity insists more on reward-sharing.



ENDNOTES

---

* Thomas W. Downs, Department of Economics and Finance, University of Alabama, Tuscaloosa, AL 35487-0224, tdowns@cba.ua.edu

1 Equation 1 assumes that the unlevered equity financing rate at time s, ρ_s^u, is expected to remain constant throughout the real asset service life. As subsequent discussions show, this does not impose an implicit assumption that the yield curve of interest rates is flat. Equation 1 also assumes that there is no stochastic uncertainty about expected cash flows. Andrew Ang and Jun Liu (2004) present an analytical methodology for discounting stochastic cash flows that are correlated with risk premiums, risk-free rates and time-varying betas. My study proceeds without those interesting innovations.

2 Austan Goolsbee (1998, 2004) examines whether investment tax incentives affect the capital goods supply price q or whether the tax benefits flow to the capital goods investor. William A. Brock and Stephen J. Turnovsky (1981) present a general equilibrium model that links corporate debt and equity financing rates with household behavior in order to determine the equity cost of capital.

3 Auerbach and Kevin A. Hassett (2003) examine the importance of whether the source of equity financing is retained earnings or new share issuances. Personal taxes are important considerations in that analysis. My specification ignores effects of personal taxes.

4 Ricardo J. Caballero (1999) argues for the importance of endogenous depreciation. For my purposes, however, that assumption unnecessarily complicates analyses.

5 For the Hall-Jorgenson standard case when pretax cash flow declines along a perpetual geometric path at rate δ then the proportion of level perpetuity value that the discounted pretax cash flow stream retains equals $(\rho - \pi) \div (\rho + \delta - \pi)$. Yet Δ accommodates any stream, even complex non-monotonic ones. As one example, assume straight-line decline over N years ($d_j = 1/N$ for $j = 1,\ldots,N$ and $d_j = 0$ otherwise). Compute from equation 5 that Δ is $[1 - (1+\rho)^{-N}] \div (\rho N)$. With ρ of 10 percent, for example, the level perpetual \$1 stream has present value of \$10. The stream declining by straight-line over four years has

present value of $2.075 (= $1/1.1 + $0.75/1.1^2 + $0.50/1.1^3 + $0.25/1.1^4). The straight-line stream loses 79.25 percent of the level perpetual value. Δ equals 0.7925 [= $(1 - 1.1^{-4}) \div (0.1 \times 4)$].

[6] Modigliani and Miller (1963) subsequently modify *Proposition 2* to account for the effects of the interest tax shield on company market value. I account for the interest tax shield by defining *i* as the after-corporate-tax interest rate. John R. Graham (2003) reviews empirical and theoretical arguments for how taxes affect corporate decision making.

[7] The standard specification for ρ^{ℓ} in *Proposition 2* is valid for a wider class of cases than is the Hall-Jorgenson standard user cost. Both are valid for the case that the text describes for perpetual and geometric capacity depreciation with a constant debt ratio. Both are valid for a one-period model. But *Proposition 2* is valid, and Hall-Jorgenson is not, for any pattern of pretax cash flow stream as long as the debt-to-asset ratio remains constant throughout the marginal asset service life.

[8] Simon Gilchrist and John C. Williams (2000) find that putty-clay characteristics of capital play a significant role explaining business cycle and medium-run dynamics.

[9] Thomas F. Cooley and Bruce D. Smith (1995) confirm the intuition of Klein that primary assets bear higher equilibrium rates of return than intermediary liabilities.

[10] The debt-to-equity relative risk premium relates to relative risk aversion and portfolio choice. Hellwig (2000) examines the role of risk aversion in intermediary behavior. Alon Brav, George C. Constantinides, and Christopher C. Geczy (2002) relate household relative risk aversion to the equity premium. Isabelle Bajeux-Besnainou, *et al*. (2003), investigate the role of different market parameters and relative risk aversion in portfolio strategies.

[11] My specification ignores creditor lending behavior in the presence of company borrowing constraints. Rui Albuquerque and Hugo A. Hopenhayn (2004) discuss this issue.

[12] Pure monetary theories of equilibrium financing rates culminate today with research on (1) financial asset pricing models (“ICAPMs”) and (2) dynamic term structure models (“DTSMs”). John Y. Campbell (2000) reviews ICAPMs and Qiang Dai and Kenneth J. Singleton (2003) survey DTSMs. Many of these

models integrate information from the real sector. Ravi Bansal and Hao Zhou (2002), for example, find that the yield curve slope relates statistically with business cycle. None of the preceding studies explain capital market rates of return by exploiting the information that reflects through the user cost of capital.

[13] The risk-premium $\rho_s^u - i_s^u$ depends on the regulatory environment, statutory seniority of debt, credit rating, and structural phenomena such as cash flows being better specified for debt than for equity. These phenomena seem short-term stable and likely change only during regime shifts. Bansal, George Tauchen, and Zhou (2004) find that the goodness-of-fit of dynamic term structure models relates to regime shifts.

[14] Andrew Ang, Monika Piazzesi, and Min Wei (2005) write that "every recession after the mid-1960s was predicted by a negative slope – an inverted yield curve – within 6 quarters of the impending recession."

[15] The first to conclude that the equilibrium financing rate is generally an increasing function of the duration of real capital is Eugen von Böhm-Bawerk (1888) in *Positive Theory of Capital*. His modeling, methodology and assumptions obviously differ from mine. Still, Böhm-Bawerk introduces the user cost concept (and duration) wherein he refers to it [p.343] as "bearer-of-the-use." John R. Hicks reverently writes in *Value and Capital* (1939, p. 192): "Even to-day, the great name in this department of economics [that is, the study of economic dynamics] is the name of Böhm-Bawerk. This is so, not because his doctrine is generally accepted (it was not generally accepted even in his own time, and it has still fewer supporters in ours), but because it is a challenge that has somehow to be met. Nearly every one who comes to the study of capital falls a victim to Böhm-Bawerk's theory at some stage or other. … Clearly Böhm-Bawerk was wrong; but there must have been something in what he said; you cannot construct such an elaborate theory as that out of nothing." Hicks concludes [p. 222] that even though the Böhm-Bawerk theory does not generalize, it is correct for the special case where "all the input is utilized at one given date, and all the output comes to fruition at another given date." As we see from Hall-Jorgenson and Modigliani-Miller, special cases may not generalize but they nonetheless may bear very long-lasting and durable fruit.