THE USER COST AND CAPITAL BUDGETING

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ABSTRACT

This study describes the relationship between the user cost of capital and capital budgeting processes and presents a formula for computing the user cost of an investment, accommodating the existence of taxes, inflation, and a nonconstant level of production. A project acceptability criterion assigning positive net present value when cash operating income exceeds user cost is discussed and applied to several examples. The test is shown to be useful and easy to apply.

INTRODUCTION

A widely used tool in macroeconomic investment theory is the user cost of capital. The user cost concept was originally introduced by Bohm-Bawerk [2] and was subsequently discussed by Keynes [11]. It was not until Jorgenson [10] that an empirically useful construct of the user cost was developed. Subsequent contributions incorporating the impact of taxes have been Coen [3, 4] and Hall and Jorgenson [8]. The effect of inflation on the user cost has been discussed by Hendershott and Hu [9], and the importance of the asset depreciation path has been discussed by Coen [5]. The user cost, also known in the literature as the rental or shadow price of capital, is the dollar return capital assets must earn such that after the subtraction of taxes and economic depreciation, investors receive their required return. An equivalent definition of the user cost is that it is the cash operating income capital must earn in order to have a zero net present value.

A substantial body of research has firmly established that the user cost is a determinant of aggregate business fixed investment. Timely and exemplary articles are Feldstein [6] and Auerbach's [1] presentation to the House Congressional Committee on Banking, Finance, and Urban Affairs. The computation of the user cost depends upon variables such as the cost of funds, profit tax rates, and tax depreciation schedules. Because

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these variables are related to capital budgeting analyses, it is possible to relate the user cost to the firm's investment decisions. The purpose of this study is to describe the relationship between the user cost and capital budgeting processes.

SPECIFYING CASH FLOWS IN ORDER TO MEASURE THE USER COST

The decision to invest in an asset depends on the size and intertemporal distribution of the after-tax incremental cash flows the asset is expected to produce. Let the incremental production from an investment be F', the marginal physical product of capital, and let p be the cash operating income per unit of product. The investment's total cash operating income of pF' will be used by the firm to pay taxes and fair compensation to all sources of funds. In the absence of taxes, pF' is the cash operating income as well as the after-tax incremental cash flow. In the presence of taxes, discussed later, pF' is cash operating income, but after-tax incremental cash flow is pF' minus gross taxes plus the tax savings on depreciation deductions.

Say the asset acquisition cost is q and the asset is expected to generate a stream of cash flows that is as long as its useful service life. A firm would be indifferent to making marginal investment (i.e., acquiring new capital goods) if the present value of the expected cash flow stream equals q, at which point the investment has a zero net present value. In the absence of taxes and depreciation, this condition may be expressed as

$$q(s) = \int_{s}^{\infty} e^{-r(t-s)} p(t)F'(t)\, dt, \tag{1}$$

where s is the time of acquisition and r is the after-tax weighted average cost of funds.

The right-hand side of equation (1) is the incremental benefit associated with pursuing the investment, and the left-hand side is the incremental cost. A decision rule based upon this equation would be that when the right-hand side exceeds the left-hand side, the investment is financially feasible and should be pursued. To test the decision rule in a particular situation requires measurement of both sides of (1), meaning the cash flows must be fully specified by substituting all variables and terms with their expected values. For simplicity, say pF' is expected to be a constant, perpetual return, such as the one provided by consol bonds, and say that q is the observable supply price of capital.[1] One test for project acceptance consistent with the above decision rule is (NPV Test 1): When the right-hand side, pF'/r, exceeds the left-hand side, q, the investment has a positive net present value and should be pursued. An alternative test of project acceptability is (NPV Test 2): When the cash operating income, pF', exceeds the user cost, q times r, the investment has a positive net present value and should be pursued.

The NPV Tests are justified as follows. Because pF' is expected to remain constant, equation (1) may be rewritten as

$$q(s) = \int_s^\infty e^{-r(t-s)} p(s)F'(s)\, dt.$$

NPV Test 1 is based upon

$$q = pF'/r$$

and compares the supply price to the present value of the cash flow stream. NPV Test 2 is based on

$$qr = pF'$$

and compares the user cost of capital to the cash operating income. Regardless of which test is selected, the investment has a positive net present value whenever the right-hand side exceeds the left-hand side. Both NPV Tests are fundamentally the same and rely on identically the same amount of information. They always result in the same decision outcome.

The shape and other attributes of the cash flow stream may be made more complex, as shown below, but the fundamental NPV Tests remain the same. Suppose incremental production is expected to decline at the exponential rate δ over the asset's infinite life, and that operating income is expected to inflate at the rate π. The marginal condition equating supply price to the discounted cash flow stream is

$$q(s) = \int_s^\infty e^{-r(t-s)} p(s)e^{\pi(t-s)} F'(s)e^{-\delta(t-s)}\, dt. \qquad (2)$$

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[1]The supply price is observable as long as the payment for the asset is made in one lump sum. This term, the left-hand side of (1), is an expected value if a stream of acquisition costs is assumed.

Solving for an equality useful in NPV Test 2 yields

$$q(r + \delta - \pi) = pF'.$$

In this situation, the user cost is $q(r + \delta - \pi)$ and NPV Test 2 is that the net present value is positive whenever the expected cash operating income, pF', exceeds the user cost.[2]

Equation (2) may be modified so that it reflects virtually any production path. Let $h(t)$ be the proportion of original productive capacity lost in the t'th year after acquisition, so that $\int_0^t h(j)\,dj$ equals the cumulative proportion of original productive capacity lost over time. Two points about $\int h(j)\,dj$ merit comment. First, for finite life capital, the sum of $h(j)$ over the asset's service life is unity, and beyond the service life $h(j)$ is zero. In this case, the upper limit on equation (3) might as well be the service life rather than infinity. For example, in the case of an asset with a ten-year service life and a production path depreciating by straight line $h(j) = 1/10$ for $j = 1, \ldots, 10$ and $h(j) = 0$ for $j > 10$. An asset such as this might produce 100 units during the first year, 90 during the second, . . ., and 10 units during the tenth year. Second, when production increases during the early years of a project's life $h(j)$ begins as a negative number. For example, if production increases 10 percent during the first year of use $h(1)$ equals -0.10, but the sum of the entire sequence $\int h(j)\,dj$ is still unity. Regardless of production path, the marginal equilibrium is

$$q(s) = \int_s^{\infty} e^{-r(t-s)}\, p(t)F'(s)\,[1 - \int_0^{t-s} h(j)\,dj]\,dt.$$

The impact of the U.S. tax code may be introduced. Let τ be the profit tax rate, v the effective rate of the investment tax credit, and $z(j)$ the proportion of the supply price taken as a tax depreciation deduction j periods after acquisition. The marginal equilibrium is

[2]NPV Test 1 in this situation implies the project has a positive net present value whenever $pF'/(r + \delta - \pi)$ exceeds q. The well-known Gordon [7] constant dividend growth model is based upon this relationship. pF' is the expected dividend, δ is zero because equities do not depreciate, and π is the growth rate in dividends. The intrinsic value of a security is $pF'/(r - \pi)$, and if its price, q, is less than this, the investment has a positive net present value.

$$q(s) = vq(s) + \int_s^\infty e^{-r(t-s)} \Big\{ (1 - \tau)p(s)F'(s)e^{\pi(t-s)} \left[1 - \int_0^{t-s} h(j)dj \right] + \tau q(s)z(t - s) \Big\} dt. \quad (3)$$

The user cost, c, may be isolated as

$$\frac{q[r - \pi][1 - v - \tau Z]}{(1 - H)(1 - \tau)} = pF' = c, \quad (4)$$

where $Z = \int_0^\infty e^{-rj} z(j)dj$ and $H = \int_0^\infty e^{-(r-\pi)j} h(j)dj$. Z is the present value of tax depreciation deductions per dollar of investment, and H is the real product depreciated at the real rate. The user cost in (4) represents the cash operating income which must be earned during the first year of use for the investment to have a zero net present value. If the project is expected to be more profitable than this, the expected after-tax incremental cash flow stream has a positive net present value, and the investment should be pursued.

EXAMPLES

This section describes numerical applications of NPV Test 2. It is shown that the sign of an investment's net present value is easily inferred given a specification of the cash flow stream and subsequent computation of the user cost of capital. Consider an example in which a firm's marginal income tax rate is 46 percent, its cost of funds is 12 percent, and the operating income is expected to inflate 6 percent per year. Suppose the firm is exploring an investment qualifying for an 8 percent investment tax credit, that can be depreciated over a five-year cost recovery period, and that is expected to have a productive life of ten years. Say production is expected to decline along a straight line path.[3]

To apply NPV Test 2, it is necessary to compute the user cost of capital for this investment from equation (4). Table 1 lists the present value of tax depreciation deductions per dollar

[3]Capital stock estimates produced by the Bureau of Economic Analysis [12] for the corporate sector assume that productive depreciation follows a straight line path.

of investment, Z, for four recovery periods available with ACRS under various cost of funds. In the situation outlined above, the nominal cost of funds is 12 percent, and Z, the present value of depreciation deductions, is \$0.711 per dollar of investment.

Table 2 presents the value of H for different service lives and various real cost of funds. (The upper panel of Table 2 shows the value of H for the case of straight line depreciation described above. The lower panel shows H for the case of decelerated depreciation described later.) In this application, the real rate is 6 percent and $H = .736$. The user cost may now be computed. Notice it is not necessary to actually specify the incremental product nor the cash operating income per unit. Rather, it is sufficient to specify, as we have done, the path that the income is expected to follow (inflate at 6 percent) and the path real production follows (declines by straight line to zero over ten years).[4] Substitution of all variables into equation (4) shows

$$c(s) = \frac{q[.12 - .06][1 - .08 - .46(.711)]}{(1 - .736)(1 - .46)} \qquad (5)$$

$$= .2496q.$$

An investment must promise cash operating income of \$0.2496 per dollar of investment in order to have a zero net present value.

Table 3 allows insights into the relationship between an asset's user cost and its cash flows. Suppose the asset costs \$100 [= q] and has a zero net present value. It is expected to provide cash operating income of \$24.96 [= pF'] during the first year, and the tax, inflation, and depreciation assumptions imply the after-tax incremental cash flows described below. (The adjective after-tax is hereinafter dropped.)

The gross income tax equals \$11.48 [= $\tau pF'$], the tax savings on depreciation deductions equals \$6.90 [= $\tau qz(1)$; $z(1)$ equals .15 because ACRS five-year recovery allows 15 percent recovery in the first period], and the investment tax credit re-

[4]The user cost formula (4) separates the nominal changes in cash operating income brought about because of inflation from real changes brought about because of changes in the level of production. Nominal changes occur because of movement in the product price and are specified through π. Real changes are brought about because of changes in the asset's physical productivity *or* capacity utilization and are jointly specified in the $h(j)$ series.

TABLE 1

Z, THE PRESENT VALUE OF DEPRECIATION DEDUCTIONS ON A $1 ASSET UNDER ALTERNATIVE COST OF FUNDS AND RECOVERY PERIODS

Nominal Cost of Funds (%)	ACRS Recovery Period (years)			
	3	5	10	18
8	.851	.791	.682	.540
9	.835	.770	.653	.506
10	.819	.750	.626	.476
11	.804	.730	.601	.449
12	.790	.711	.578	.424
13	.775	.693	.556	.401
14	.761	.676	.535	.380
15	.748	.659	.516	.361
16	.735	.643	.497	.344

turns $8 [= vq]. Hence, the investment provides an incremental cash flow at the end of the first year equal to $28.38.

At the end of the second year, cash operating income would be expected to grow to $26.45 [= $pF'(1 + \pi)$] except that the asset's productive capacity will have declined by 10 percent [= $h(1)$], so that the asset is only expected to generate cash operating income of $23.81 [= $pF'(1 + \pi)(1 - h(1))$]. Subtracting gross taxes of $10.95 and adding back in the tax savings of $10.12 on depreciation deductions [= $\tau qz(2)$; $z(2) = .22$] brings the incremental cash flow up to $22.98.

The incremental cash flows are equal to the prediscounted right-hand side of equation (3) as t is incremented from 1 to 10. For years 3 through 10, those cash flows are $21.77, $20.90, $19.87, $9.02, $7.65, $6.08, $4.30, and $2.28. The large drop between years 5 and 6 reflects the expiration of the cost recovery period and subsequent loss of tax savings arising from tax depreciation deductions. The present value of the incremental cash flow stream is $100 when discounted by 12 percent.[5]

[5]In fact, the stream's capitalized value is $96.47. This sum can be brought to 100 percent of the supply price by computing the user cost with formula (4) modified so that the numerator subtracts $(v + \tau Z)/(1 + r)$ instead of $v + \tau Z$. The modification is needed because in discrete time the investment tax credit and first depreciation deduction are

TABLE 2

VALUES OF *H* UNDER ALTERNATIVE REAL COST OF FUNDS AND PRODUCTIVE SERVICE LIVES

Service Life (Years)	Real Cost of Funds								
	4	5	6	7	8	9	10	11	12
	Straight Line Capacity Depreciation								
4	.907	.886	.866	.847	.828	.810	.792	.776	.759
5	.890	.866	.842	.820	.799	.778	.758	.739	.721
6	.874	.846	.820	.794	.770	.748	.726	.705	.685
8	.842	.808	.776	.746	.718	.692	.667	.643	.621
10	.811	.772	.736	.702	.671	.642	.614	.589	.565
13	.768	.723	.681	.643	.608	.576	.546	.519	.494
16	.728	.677	.632	.590	.553	.520	.489	.461	.436
20	.680	.623	.573	.530	.491	.456	.426	.398	.373
30	.576	.512	.459	.414	.375	.342	.314	.290	.269
40	.494	.429	.376	.333	.298	.269	.244	.224	.206
	Decelerated Capacity Depreciation								
4	.883	.856	.831	.807	.784	.762	.740	.720	.700
5	.859	.828	.799	.771	.744	.718	.694	.671	.649
6	.836	.801	.768	.736	.707	.678	.652	.627	.603
8	.793	.750	.710	.673	.639	.606	.576	.548	.522
10	.752	.703	.658	.617	.578	.543	.511	.481	.454
13	.696	.639	.588	.542	.500	.463	.429	.399	.371
16	.645	.582	.527	.478	.435	.397	.363	.333	.307
20	.583	.515	.457	.407	.364	.326	.294	.266	.242
30	.458	.385	.326	.279	.241	.210	.184	.162	.145
40	.364	.294	.241	.200	.168	.144	.125	.109	.096

Hence, NPV Test 2: If the operating income expected during the first year of use exceeds the user cost, the net present value is greater than zero, and the investment is a feasible alternative.

The sensitivity of the user cost to alternative production paths may be determined by specifying alternative values for H. The lower panel of Table 2 presents values of H given production is characterized by a decelerated path. In this situation, production declines only slightly during the early years of the asset's life and falls off sharply during the latter years. For example, an asset producing 100 units in the first year character-

received one period after acquisition and hence are discounted one period. The examples presented herein employ (4) as it appears in the text because of its close connection to the widely used Hall-Jorgenson formula.

TABLE 3

DESCRIPTION OF AN INVESTMENT'S RETURN STREAM

Year of Use (J)	Cash Operating Income from a Nondepreciated Asset	Cash Operating Income from a Depreciated Asset	Gross Taxes	Tax Savings on Depreciation Deductions	Incremental Cash Flow
		Straight Line Capacity Depreciation			
1	$24.96	$24.96	$11.48	$ 6.90	$28.38*
2	26.45	23.81	10.95	10.12	22.98
3	28.04	22.43	10.32	9.66	21.77
4	29.72	20.80	9.57	9.66	20.90
5	31.50	18.90	8.70	9.66	19.87
6	33.40	16.70	7.68	0	9.02
7	35.40	14.16	6.51	0	7.65
8	37.52	11.26	5.18	0	6.08
9	39.78	7.96	3.66	0	4.30
10	42.16	4.22	1.94	0	2.28
		Decelerated Capacity Depreciation			
1	$19.26	$19.26	$ 8.86	$ 6.90	$25.30*
2	20.42	19.76	9.09	10.12	20.79
3	21.64	20.13	9.26	9.66	20.53
4	22.94	20.33	9.35	9.66	20.64
5	24.32	20.26	9.32	9.66	20.60
6	25.78	19.83	9.12	0	10.71
7	27.32	18.84	8.67	0	10.18
8	28.96	17.04	7.84	0	9.20
9	30.70	13.96	6.42	0	7.54
10	32.54	8.80	4.05	0	4.75

*This cash flow includes the investment tax credit of $8.

ized with the ten-year service life and decelerated depreciation produces 98 units in the second year, 72 units in the sixth year, and 27 units in its last (tenth) year.[6] Given the inflation and tax rates described above, $H = .658$, and the new "hurdle profit" may be computed from (4) as

$$c(s) = \frac{100[.12 - .06][1 - .08 - .46(.711)]}{(1 - .658)(1 - .46)}$$

$$= 19.26.$$

[6]The decelerated path discussed here is the one used by the Bureau of Labor Statistics [13] in their equipment capital stock estimates.

If the investment is expected to return cash operating income of $19.26 during its first year, it is a zero net present value alternative. If it is expected to return more than that, it has a positive net present value and should be considered a feasible investment. The incremental cash flows this investment would generate are shown in the lower panel of Table 3. Notice that cash operating income increases during the early years because the increasing per unit income more than offsets the slight production declines. The incremental cash flows follow a complex path; first falling, then rising, and finally falling monotonically towards zero.

NPV Test 2 is as easy to apply to long project lives as to short ones. For example, suppose an $80,000 asset has a forty-year service life, has productive capacity depreciation characterized by a straight line path, qualifies for a 6 percent investment tax credit, and is to be depreciated for tax purposes in the eighteen-year ACRS class. Say the cost of funds is 16 percent, the marginal tax rate is 30 percent, and the inflation rate is 4 percent per year. Applying NPV Test 1 requires the construction of a forty-year incremental cash flow stream and then its discounting. However, the hurdle profit for NPV Test 2 can be quickly computed as

$$c(s) = \frac{80{,}000[.16 - .04][1 - .06 - .30(.344)]}{(1 - .206)(1 - .30)}$$

$$= 14{,}453.50,$$

with the resulting rule that if the asset is expected to contribute more than $14,453.50 of cash operating income during the first year, it has a positive net present value and is a feasible investment alternative. The sensitivity of the decision outcome to any of the assumptions may be tested by recomputing the hurdle profit with alternative parameter values. Such an analysis is practicable, even if by hand.

SUMMARY

This study elaborates on the role of the neoclassical user cost in capital budgeting decisions. The study presents a formula for computing the user cost of an investment, accommodating the existence of taxes, inflation, and a nonconstant level of production. A project acceptance test is that if an investment promises a cash operating income exceeding its user cost of capital, the net present value of the project's after-tax incremental cash flows is positive, and the investment is a feasible alternative.

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